JOURNAL OF MECHANICS OF CONTINUA AND MATHEMATICAL SCIENCES
www.journalimcms.org

ISSN (Online) : 2454 -7190 Vol.-21, No.-11, November (2026) pp 208-230 ISSN (Print) 0973-8975

IMPACT OF PROBABILITY DISTRIBUTION SELECTION ON TRANSPORTATION COST AND RISK IN STOCHASTIC TRANSPORTATION PROBLEMS

Amarjit Singh[1], Amarinder Singh[2]

[1,2] Chitkara University Institute of Engineering & Technology, Chitkara University, Rajpura, Punjab, India.

Email : [1]amarjit.harsi@gmail.com, [2]amarinder77@gmail.com,

Corresponding Author: **Amarinder Singh**

https://doi.org/10.26782/jmcms.2026.11.00010

(Received: August 29, 2026; Revised: October 29, 2026; Accepted: November 12, 2026

Abstract

Transportation problems help in making smart choices in logistics, supply chains, and modern transport systems. At their core, they play a huge role in balancing economic factors, addressing social needs, and keeping our transit networks safe and secure. However, in the real world, several parameters become uncertain, making it harder to determine reliable transportation plans. This study presents an ambiguity-bounded framework for a chance-constrained Stochastic Classical Transportation Problem (SCTP) that handles demand uncertainty. The historical demand at each destination is examined by fitting four candidate distributions, namely, Normal, Lognormal, Gamma, and Weibull. The most appropriate distribution is selected through the information criteria, i.e., Akaike Information Criterion and Bayesian Information Criterion, and the goodness-of-fit test, i.e., the Kolmogorov-Smirnov test. Worst-case marginal quantiles from these ambiguity sets are then used to convert stochastic demands into deterministic equivalents across various confidence levels, and the ensuing transportation problems are optimally solved. The effects of distribution selection on demand estimates and transportation costs are investigated through extensive Monte Carlo simulations. The study also evaluates the Value-at-Risk and Conditional Value-at-Risk to compute the contingency and worst-case budget requirements under extreme demand scenarios. The findings show that while multiple distributions provide an adequate statistical fit and yield similar demand estimates in many cases, their tail- risk behaviours diverge significantly. Specifically, the ambiguity-bounded consensus envelope approach excels at limiting extreme cost outcomes and producing more reliable worst-case budget estimates. Summarily, the framework provides decision-makers with systematic tools to assess contingencies, manage risk, and execute dynamic transportation planning under uncertainty.

Keywords: Stochastic transportation problem, Chance-constrained programming, Distribution fitting, AIC, BIC, Demand uncertainty, Simulations.

I. Introduction

The transportation problem (TP) is an optimization problem associated with logistics and supply chain management. Its primary objective is to minimize the total cost or time required to transport goods from manufacturers or distributors (sources) to consumers or retailers (destinations). The TP originated in the early 20th century when it was first mathematically formulated by Hitchcock [X]. Consequently, it is also commonly known as the Hitchcock problem or the classical transportation problem (CTP).

Consider

- m sources: $S_1, S_2, \ldots\ldots\ldots\ldots S_m$
- n destinations: $D_1, D_2, \ldots\ldots\ldots\ldots D_n$
- a_i : no. of units of product available at source S_i
- b_j: no. of units of product demanded at destination D_j
- c_{ij} : cost of transporting one unit of product from S_i to D_j
- x_{ij} : no. of units of products to be transported from S_i to D_j

The problem is to determine x_{ij} so that it minimizes the total transportation cost while satisfying the availability and demand constraints.

Mathematically, the CTP is presented as:

$$\text{Minimize } Z = \sum_{j=1}^{n}\sum_{i=1}^{m} c_{ij}x_{ij} \tag{1}$$

Subject to the constraints:

$$\sum_{j=1}^{n} x_{ij} \quad \leq \quad a_i \quad ; \; 1 \leq i \leq m \tag{2}$$

$$\sum_{i=1}^{m} x_{ij} \quad \geq \quad b_j \quad ; \; 1 \leq j \leq n \tag{3}$$

Where

$$x_{ij} \geq 0 \quad ; \; 1 \leq i \leq m, 1 \leq j \leq n \tag{4}$$

Equation (1) represents the cost-minimization linear objective function, while constraints (2) and (3) are the upper and lower bounds on the availability and demand levels of products at the respective sources and destinations. Equation (4) is referred to as the feasibility condition. Here, the x_{ij}s are not only non-negative, but also integers since the no. of units cannot be in fractions.

Furthermore, the problem is called balanced if

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \tag{5}$$

In case equation (5) does not hold, then the problem is called unbalanced.

However, in real-life problems, the parameters of TP are not deterministic; rather, they are uncertain, and it poses a challenging situation for the decision makers to take optimal decisions. The uncertainty of parameters can be addressed with fuzzy and stochastic approaches. When parameters are uncertain and no previous relevant data is available, then the fuzzy set theory approach is used, where only expert judgement about the uncertain parameter is used, and the problem is named as a fuzzy transportation problem [VII]. On the other hand, when enough past data is available, which may follow the trend of some theoretical probability distribution, then the problem is addressed as the Stochastic Transportation Problem (SCTP) [XXIV].

In recent works, authors solved the models of SCTP by chance-constraint programming, in which constraints are taken as probabilistic. To define an SCTP: Consider the parameters as defined in the CTP where a_i and b_j are stochastic and assumed to follow some probability distribution. Agrawal et al. [II] defined SCTP with the same linear objective function (Equation (1)) and constraints as probabilistic ones as follows:

$$P\left(\sum_{j=1}^{n} x_{ij} \leq a_i\right) \geq \alpha_{a_i} \; ; \; 1 \leq i \leq m \tag{6}$$

$$P\left(\sum_{i=1}^{m} x_{ij} \geq b_j\right) \geq \alpha_{b_j} \; ; \; 1 \leq j \leq n \tag{7}$$

where α_{a_i}, α_{b_j} are the confidence levels of fulfilment of particular supply and demand constraints and represent satisfaction probabilities. Depending upon the fitted probability distribution, the stochastic constraints are changed to corresponding deterministic forms.

Let $F_{a_i}(a)$ denote the cumulative distribution function (CDF) of the random supply a_i.

By definition,

$$F_{a_i}(a) = P(a_i \leq a)$$

Let $S_i = \sum_{j=1}^{n} x_{ij}$ be the total quantity transported from source *i;* then, from equation (6), we have

$$P(S_i \leq a_i) \geq \alpha_{a_i} \;\; or \;\; P(a_i \geq S_i) \geq \alpha_{a_i} \;\; or \; 1 - P(a_i \leq S_i) \geq \alpha_{a_i}$$

Thus $1 - F_{a_i}(S_i) \geq \alpha_{a_i}$ *or* $F_{a_i}(S_i) \leq 1 - \alpha_{a_i}$

Or $S_i \leq F_{a_i}^{-1}(1 - \alpha_{a_i})$

On substituting the value of S_i, we get

$$\sum_{j=1}^{n} x_{ij} \leq F_{a_i}^{-1}(1 - \alpha_{a_i}), \quad 1 \leq i \leq m. \tag{8}$$

Again, let $F_{b_j}(b)$ denote the cumulative distribution function (CDF) of the random demand b_j.

Therefore $F_{b_j}(b) = P(b_j \leq b)$

Let $D_j = \sum_{i=1}^{m} x_{ij}$ be the total quantity delivered to destination *j*

Starting with equation (7), we get

$P(D_j \geq b_j) \geq \alpha_{b_j}$ *or* $P(b_j \leq D_j) \geq \alpha_{b_j}$. Implying $F_{b_j}(D_j) \geq \alpha_{b_j}$ *or* $D_j \geq F_{b_j}^{-1}(\alpha_{b_j})$

On substituting the value of D_j

$$\sum_{i=1}^{m} x_{ij} \geq F_{b_j}^{-1}(\alpha_{b_j}), \quad 1 \leq j \leq n. \tag{9}$$

Equations (8) and (9) represent the equivalent deterministic constraints obtained from equations (6) and (7), respectively. Where F^{-1} is the inverse cumulative distribution function of the concerned probability distribution. The reduced model was then solved by existing methods.

Mahapatra et al. [XIV] and Roy et al. [XVI] used the lognormal distribution and exponential distribution, respectively, to change SCTP to deterministic form. Mahapatra et al. [XIII] converted the SCTP using the probability density function (*p.d.f.*) of extreme value distribution and then used LINGO to find the solution. Roy [XVII] used a Weibull distribution wherein the multi-choice cost coefficients were converted to a single number using binary numbers. A multi-objective TP with fuzzy supplies and demands and stochastic constraints following a normal distribution was solved by Acharya et al. [I]. Roy [XVIII] defined the demand and supply as logistically distributed and used Lagrange's interpolating polynomial to convert multi-choice numbers to a single value. Singh et al. [XXI] solved the SCTP in which parameters follow a gamma distribution. Al Qahtani et al. [V] proposed an approach of goal programming to solve multi-choice and multi-objective SCTP in which demand and supply constraints are taken to follow extreme value distribution. Gupta et al. [IX] used a maximum likelihood estimator to determine the scale and location of uncertain parameters, which followed a gamma distribution. Agrawal et al. [III] solved the SCTP following a Weibull distribution by using the water cycle algorithm and compared its performance with a neural network algorithm. Bhavana and Priya [VIII] solved the SCTP in which parameters followed Lomax distribution. Joshi et al. [XII] used Newton's divided difference interpolations to convert multi-choice costs to a single value and solved the multi-choice solid SCTP in which parameters followed a normal distribution.

The review of these articles helped in identifying the patterns in the approaches followed by the authors. The authors fixated a single distribution to all the sources and destinations and to the respective supplies and demands. However, the real-life data may not correspond to a single distribution since all the parameters are independent of each other, raising doubts about the efficiency of estimations of the uncertain parameters used in these articles. This identification augments the need for fitting the suitable probability distribution to the sources/destinations/supplies/demands.

Ashour et al. [VI] solved a case study where the demands are uncertain and fit the normal distribution to all the destinations, estimating the minimum values of demands resulting in the minimum cost solution. For estimation of uncertain parameters, a range of values can be developed by using the inverse cumulative distribution function of the fitted distribution; for the normal distribution, the range of estimated parameters can

be formed by using $\mu + z_\alpha \sigma$, but Ashour et al. specifically estimated demand with μ−1.96σ that corresponds to the least demand requirement. Such a solution plan is not truly helpful to decision makers under uncertainty.

The SCTP requires a vigorous solution plan that proposes optimal approaches during uncertain situations. In this paper, an algorithm is proposed to solve the problem undertaken by Ashour et al. [VI] and carries out a comprehensive analysis following different scenarios. The main process of the algorithm is to fit a suitable probability distribution, from among the Normal, Lognormal, Gamma, and Weibull distributions, to the available past data. The best-fit distributions for the uncertain parameters are identified through the statistical tests Akaike Information Criterion (AIC) [IV], Bayesian Information Criterion (BIC) [XX], and Kolmogorov–Smirnov (KS) [XXII]. Secondly, the demands are estimated using the inverse cumulative distribution function of the respective best-fit distribution to transform the problem into a deterministic form that is amenable to solution by the traditional methods VAM and MODI [XXIII]. To validate the fitted probability distribution and to analyse the effect of the probability distribution in demand estimation, the Monte Carlo simulation technique [XIX] is employed, and a robust solution plan is designed to take a reliable decision during uncertainty.

The successive sections of this article include the statistical concepts useful for estimation of demands, methodology, results and comparisons, managerial implications and conclusion.

II. Statistical Concepts Useful for Estimation of Demands

In case of uncertain transportation models, if enough past data is available, then to predict uncertain parameters, some probability distribution can be fitted. Theoretical distributions [XI] which are frequently used for fitting are given in the following subsections.

A. Probability distributions useful for estimation of uncertain parameters based on the past available data

1) *Normal distribution:* If a random variable X follows a normal distribution with mean μ and standard deviation σ, then we write $X \sim N(\mu, \sigma^2)$.

The probability density function (p.d.f.) is:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad -\infty < x < \infty$$

and the cumulative distribution function (c.d.f.) is

$$F(x) = \Phi\left(\frac{x-\mu}{\sigma}\right) \tag{10}$$

where Φ(·) denotes the standard normal cumulative distribution function.

The inverse cumulative distribution function is obtained from

$$\alpha = \Phi\left(\frac{x-\mu}{\sigma}\right) \tag{11}$$

Which gives

$$x = \mu + \sigma\Phi^{-1}(\alpha) \tag{12}$$

Since $\Phi^{-1}(\alpha)$ has no closed-form expression, it is evaluated numerically using approximation or iterative methods such as the Newton–Raphson method.

2) *Gamma distribution:* The Gamma distribution is again a continuous probability distribution broadly used in Statistics and reliability analysis. If a random variable X follows a gamma distribution with shape parameter *(k)* and scale parameter (θ), *i.e., X* ∼ Gamma*(k,θ),* then its *p.d.f.* is given as

$$f(x) = \frac{x^{k-1}e^{-x/\theta}}{\Gamma(k)\,\theta^k}, \quad x > 0$$

where $k > 0$, $\theta > 0$ and $\Gamma(k)$ denotes the gamma function.

Shape and scale parameters are computed using the relations: $k = \frac{\mu^2}{\sigma^2}, \theta = \frac{\sigma^2}{\mu}$

The cumulative distribution function is

$$F(x) = \frac{\gamma(k, x/\theta)}{\Gamma(k)} \tag{13}$$

where $\gamma(k,x/\theta)$ is the lower incomplete gamma function. The inverse cumulative distribution function is defined implicitly by

$$\alpha = F(x) = \frac{\gamma(k, x/\theta)}{\Gamma(k)} \tag{14}$$

Since this equation cannot be solved analytically, the inverse *c.d.f.* is computed numerically.

3) *Lognormal Distribution:* A random variable X is assumed to follow a lognormal distribution if the natural logarithm of X follows a Normal Distribution.

i.e. If $ln(X) \sim N(\mu,\sigma^2)$ then $X \sim$ Lognormal(μ,σ^2)

where μ mean of *ln(X)* and σ^2 variance of *ln(X)*

The p.d.f. of the lognormal distribution is

$$f(x) = \frac{1}{x\sigma\sqrt{2\pi}}e^{-\frac{(\ln x-\mu)^2}{2\sigma^2}}, \quad x > 0$$

The cumulative distribution function is

$$F(x) = \Phi\left(\frac{\ln x-\mu}{\sigma}\right) \tag{15}$$

The inverse cumulative distribution function is derived by solving

$$\alpha = \Phi\left(\frac{\ln x - \mu}{\sigma}\right) \tag{16}$$

which gives

$$\ln x = \mu + \sigma\Phi^{-1}(\alpha) \tag{17}$$

Thus,

$$x = \exp\left(\mu + \sigma\Phi^{-1}(\alpha)\right) \tag{18}$$

Hence, the inverse *c.d.f.* of the lognormal distribution is evaluated using the inverse standard normal function.

4) *Weibull Distribution:* The Weibull distribution is a continuous probability distribution that has wide applications in reliability analysis, survival analysis, and failure time modelling. If a random variable X follows a Weibull distribution with shape parameter *(β)* and scale parameter *(η), i.e., X* ~ Weibull *(β, η)*

Then its probability density function is given by:

$$f(x) = \frac{\beta}{\eta}\left(\frac{x}{\eta}\right)^{\beta-1}\exp\left[-\left(\frac{x}{\eta}\right)^{\beta}\right], \qquad x \geq 0 \tag{19}$$

The cumulative distribution function is

$$F(x) = 1 - \exp\left[-\left(\frac{x}{\eta}\right)^{\beta}\right] \tag{20}$$

The inverse cumulative distribution function is obtained by solving

$$\alpha = 1 - \exp\left[-\left(\frac{x}{\eta}\right)^{\beta}\right] \tag{21}$$

which gives

$$x = \eta\left[-\ln(1-\alpha)\right]^{1/\beta} \tag{22}$$

Thus, the inverse *c.d.f.* of the Weibull distribution is also available in closed form.

B. Statistical tests to decide about best fit distribution:

To identify the most appropriate probability distribution for the given data, model selection criteria such as Akaike Information Criterion (AIC) [IV] and Bayesian Information Criterion (BIC) [XX] were employed. These criteria provide a trade-off between goodness of fit and model complexity.

The AIC is defined as:

$$AIC = -2ln(L) + 2k$$

while the BIC is given by

$$BIC = -2ln(L) + kln(n)$$

where *L* represents the likelihood function, k is the number of estimated parameters, and n is the sample size. The model with the minimum AIC and BIC values is considered the best fit to the data.

Kolmogorov–Smirnov (KS) Test:

To assess the goodness of fit of the potential probability distribution, the Kolmogorov–Smirnov (KS) test [XXII] is used, which is a goodness-of-fit test and compares the empirical cumulative distribution function of the sample data with the theoretical cumulative distribution function as obtained using the fitted distribution. Two hypotheses are formulated,

Null hypothesis H_0: The sample data follows the specified theoretical distribution.

Alternative hypothesis H_1: The sample data does not follow the specified theoretical distribution. The KS statistic (D) is computed as the maximum absolute difference between the observed and theoretical cumulative distribution functions:

$$D = \max |F_n(x) - F(x)|$$

where $F_n(x)$ represents the empirical/observed cumulative distribution function of the sample of size *n,* and *F(x)* represents the theoretical cumulative distribution corresponding to a specific distribution.

For statistical decision-making, a significance level of 0.05 is considered, and if the p-value obtained from the KS test is greater than 0.05, then the null hypothesis is accepted, indicating that the specified distribution provides an adequate fit to the data. Otherwise, the null hypothesis is rejected, i.e., the specified distribution does not adequately represent the observed data.

C. *Computation of Demand Using Inverse Cumulative Distribution Function*

In the proposed model, deterministic demand corresponding to a confidence level α is computed as

$$b_j^{\alpha} = F^{-1}(\alpha_{b_j}) \tag{23}$$

where $F^{-1}(\alpha)$ represents the inverse cumulative distribution function of the selected best-fit distribution for destination *j*.

III. Methodology

An integrated framework is proposed to estimate uncertain demands from historical data using candidate probability distribution fitting, distributional ambiguity set construction, and Monte Carlo risk propagation analysis. The procedure involves the following stages:

a) Candidate Probability Distributions and Parameter Estimation

Let b_j denote the random demand at destination *j*. Historical demand records across *T* days are evaluated against four candidate continuous probability distributions:

$$\mathcal{F} = \{F_{\mathrm{N}}, F_{\mathrm{LN}}, F_{\mathrm{G}}, F_{\mathrm{W}}\}$$

representing the Normal, Lognormal, Gamma, and Weibull distributions, respectively. For each destination *j*, distribution parameters are estimated independently via Maximum Likelihood Estimation (MLE), yielding candidate Cumulative Distribution Functions (CDFs) $\hat{F}_{jk}$.

b) Model Selection and Distributional Ambiguity Set Construction

Selecting a single distribution introduces model-selection uncertainty when multiple candidates provide comparable statistical fit. To address this, a distributional ambiguity framework is established.

For destination *j*, candidate model fit is evaluated using AIC and the KS goodness-of-fit test. The relative AIC difference is computed as:

$$\Delta\mathrm{AIC}_{jk} = \mathrm{AIC}_{jk} - \min_{r} \mathrm{AIC}_{jr}$$

A candidate distribution F_{jk} is classified as statistically admissible for destination *j* if it satisfies three strict conditions:

1. Relative AIC Criterion: $\Delta AIC_{jk} \leq 2$, retaining candidates with substantial empirical support.
2. Goodness-of-Fit Criterion: $p_{jk}^{KS} \geq 0.05$, ensuring the candidate is not rejected at the 5% significance level.
3. Parameter Validity: Estimated parameters are finite, consistent with distribution support, and derived from a convergent MLE procedure.

The marginal distributional ambiguity set for destination *j*, denoted A_j, is defined as:

$$A_j = \left\{ F_{jk} \in \mathcal{F} \;\middle|\; \Delta\mathrm{AIC}_{jk} \leq 2,\ p_{jk}^{\mathrm{KS}} \geq 0.05,\ \text{and parameter validity holds} \right\}$$

For all the n destinations, the joint marginal ambiguity space is given by the Cartesian product $A = A_1 \times A_2 \times \ldots \times A_n$.

c) Estimation of Worst-Case Marginal Quantile Demands

To guard against model-selection uncertainty without making overreaching parametric claims, demand estimation employs a conservative marginal bounding strategy over the ambiguity set. The worst-case marginal demand quantile for destination j at confidence level $\alpha \in \{0.90, 0.95, 0.99\}$ is formulated as:

$$q_j^{\mathrm{WC}}(\alpha) = \max \left\{ F_{jk}^{-1}(\alpha) \;\middle|\; F_{jk} \in A_j \right\}$$

where $F_{jk}^{-1}(\alpha)$represents the inverse CDF (quantile function). For integer-valued transportation demand allocations, the conservative envelope demand estimate is $D_j^{envelope}(\alpha) = \lceil q_j^{WC}(\alpha) \rceil$.

Methodological Qualification: This marginal worst-case envelope provides destination-wise risk protection. It acts as a practical decision-support heuristic and does not analytically guarantee an identical simultaneous coverage probability across all the destinations.

d) Deterministic Transportation Problem Optimization

Using the envelope demand vector $\hat{D}(\alpha) = [D_1^{envelope}(\alpha), \ldots, D_n^{envelope}(\alpha)]$ the stochastic model transforms into a deterministic linear programming problem:

$$\text{Minimize } Z = \sum_{j=1}^{n} \sum_{i=1}^{m} c_{ij} x_{ij} \tag{24}$$

Subject to :

$$\sum_{j=1}^{n} x_{ij} \leq S_i, \quad i = 1, \ldots, m \tag{25}$$

$$\sum_{i=1}^{m} x_{ij} \geq D_j^{\text{envelope}}(\alpha), \quad j = 1, \ldots, n \tag{26}$$

$$x_{ij} \geq 0, \quad \forall i, j \tag{27}$$

The deterministic problem represented by equations (24) – (27) is solved on MATLAB using the Vogel's Approximation Method (VAM) and Modified Distribution (MODI) method to establish baseline allocations x_{ij}^*.

e) Empirical Cross-Destination Correlation Analysis

To evaluate whether historical observations across the destinations exhibit multivariate dependence, Pearson correlation coefficients between destination D_r and D_s are computed across the T observation periods as follows:

$$\rho_{D_r D_s} = \frac{\sum_{t=1}^{T} (D_{rt} - \overline{D}_r)(D_{st} - \overline{D}_s)}{\sqrt{\sum_{t=1}^{T} (D_{rt} - \overline{D}_r)^2 \sum_{t=1}^{T} (D_{st} - \overline{D}_s)^2}}$$

where:

- D_{rt}= Demand at destination D_r on day t.
- D_{st}= Demand at destination D_s on day t.
- $\overline{D_r}$= Mean demand at destination D_r.
- $\overline{D_s}$ = Mean demand at destination D_s.
- $t = 1,2, \ldots, T$.
- $r, s = 1,2, \ldots, n$ and $r \neq s$.

If the pairwise correlations are empirically weak ($\rho_{D_r D_s} \approx 0$), independent marginal sampling serves as a robust baseline. If strong dependencies or tail co-movements are present, copula-based multivariate joint distributions $C\ (F_1(b_1), \ldots, F_n(b_n))$ should be constructed, where $F_j(b_j)$ is the marginal cumulative distribution function (CDF) of the random demand b_j at j^{th} destination.

f) Monte Carlo Simulation and Tail Risk Evaluation

To examine uncertainty propagation and cost variance, Monte Carlo simulations are executed under an independent joint demand CDF formulation:

$$F_B(b_1, \ldots, b_n) = \prod_{j=1}^{n} F_j(b_j)$$

A total of N = 1,000 simulation runs are performed individually for each candidate distribution model (Normal, Lognormal, Gamma, Weibull) as well as the consensus fit model, yielding 5,000 stochastic simulation experiments.

For each simulation run $s = 1 \ldots N$:

1. Random demand vectors $D^s$are generated via independent marginal sampling.
2. The transportation allocation problem is solved, and the minimum cost C_s is recorded.

Statistical Performance and Stability Metrics:
Comparative evaluation across models utilizes standard variance metrics and financial tail-risk indicators:
1) *Mean Transportation Cost (μC):*

$$\mu_C = \frac{1}{N}\sum_{s=1}^{N} C_s,$$

where C_s denotes transportation cost in simulation s and $N = 1000$.
2) *Standard Deviation of Cost (σ_C):*

$$\sigma_C = \sqrt{\frac{1}{N-1}\sum_{s=1}^{N}(C_s - \mu_C)^2}$$

3) Coefficient of Variation (CV):

$$CV = \frac{\sigma_C}{\mu_C} \times 100$$

CV is the relative variability of transportation cost, and a model with low CV is considered more stable.
4) Value-at-Risk (VaR): The Value-at-Risk α-level is

$$VaR_\alpha = \inf\{c : P(C \le c) \ge \alpha\}$$

VaR helps to observe the tail risk behaviour of the model, VaR_α is the threshold value of cost at confidence level α. The percentage deviation of VaR from the mean is defined as

$$VaR_{Diff\%} = \frac{VaR - \mu_C}{\mu_C} \times 100$$

As the problem is of cost optimization so smaller value of VaR_α and $VaR_{Diff\%}$ is preferred.

5) Conditional Value-at-Risk (CVaR): Compute the Conditional Value-at-Risk as

$$CVaR_\alpha = E[C \vee C \ge VaR_\alpha]$$

Conditional value at risk is also useful to study the tail risk behaviour of the model and helpful for worst-case preparedness, $CVaR_\alpha$ is the average transportation cost in the worst-case tail region with confidence level α and distribution having minimum value of $CVaR_\alpha$is chosen.

6) Failure Rate: Estimate the failure rate as

$$\text{Failure Rate} = P\left(\sum_{j=1}^{n} D_j > \sum_{i=1}^{m} S_i\right)$$

Failure rate determines the probability when total demand is greater than the supply, and a zero value of Failure rate means total demand is less than the supply.

7) Robustness Index: The stability of the model is decided through CV; a model with lower CV is termed as more stable than others. To quantify the stability of a model on the scale from 0 to 1, the Robustness Index (RI) is defined as

$$RI = \frac{1}{1 + CV}$$

Higher robustness index values are obtained when the value of CV is close to zero; this indicates greater stability of the model under uncertainty.

Thus, the proposed algorithm ensures a systematic integration of statistical modelling and optimization techniques for handling demand uncertainty in transportation problems and validating it using the Monte Carlo Simulation. Detailed steps to execute the algorithm in MATLAB are given in the following pseudocode:

Algorithm Distribution Fitting, Demand Estimation and Monte Carlo-Based Transportation Cost Analysis

Phase-1 : Distribution fitting and Finding the Solution

Require: Historical demand data $D_j = \{d_{1j}, d_{2j}, \ldots, d_{pj}\}$ for each destination $j = 1, 2, \ldots, n$ and p =size of historical data; supply vector S_i; transportation cost matrix C_{ij}; candidate distributions $\mathcal{F} = \{F_{\mathrm{N}}, F_{\mathrm{LN}}, F_{\mathrm{G}}, F_{\mathrm{W}}\}$; confidence levels $\alpha = \{0.90, 0.95, 0.99\}$; number of simulations $N = 1000$

Ensure: Estimated stochastic demands, optimal transportation plans and comparative robustness results

Initialize result tables for fitted parameters, AIC, BIC, estimated demands and transportation costs
for each destination $j = 1, 2, \ldots, n$ **do**
  for each candidate distribution $F_r \in \mathcal{F}$ **do**
    Estimate distribution parameters using maximum likelihood estimation (MLE)
    Compute log-likelihood value LL_r
    Compute AIC and BIC values:

$$AIC_r = 2k_r - 2LL_r, \qquad BIC_r = k_r \ln(n) - 2LL_r$$

    Apply goodness-of-fit test to check statistical acceptability of F_r
  end for
  Identify statistically acceptable distributions with close AIC/BIC values
  Construct the marginal distributional ambiguity set A_j for destination j.
  for each confidence level $\alpha \in \{0.90, 0.95, 0.99\}$ **do**
    Estimate worst-case marginal quantile demands using inverse cumulative distribution function:

$$q_j^{\mathrm{WC}}(\alpha) = \max\left\{ F_{jk}^{-1}(\alpha) \;\middle|\; F_{jk} \in A_j \right\}$$

    Obtain integer-valued transportation demand allocations as the conservative envelope demand estimate.

$$D_j^{\text{envelope}}(\alpha) = \lceil q_j^{\mathrm{WC}}(\alpha) \rceil$$

  end for
end for
for each confidence level $\alpha \in \{0.90, 0.95, 0.99\}$ **do**
  Form deterministic transportation problem using estimated demand vector

$$\hat{D}^{(\alpha)} = \left(D_1^{\text{envelope}}(\alpha), \ldots, D_n^{\text{envelope}}(\alpha) \right)$$

  Solve the transportation problem:

$$\min Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

subject to

$$\sum_{j=1}^{n} x_{ij} \le S_i, \quad i = 1, 2, \ldots, m$$

$$\sum_{i=1}^{m} x_{ij} \ge D_j^{\text{envelope}}(\alpha), \quad j = 1, \ldots, n$$

$$x_{ij} \ge 0$$

Record optimal allocation x_{ij}^* and minimum transportation cost Z_α^*
end for
Phase-2 : Monte Carlo Simulation
for each distribution model $F_r \in \mathcal{F}$ and best-fit model **do**
  for simulation $s = 1, 2, \ldots, 1000$ **do**
    Generate random demand scenario $D^{(s)}$ from the fitted distribution model
    Solve the transportation problem for simulated demand $D^{(s)}$
    Store simulated transportation cost Z_s
  end for
  Compute performance statistics:

$$\overline{Z} = \frac{1}{1000}\sum_{s=1}^{1000} Z_s$$

$$SD(Z) = \sqrt{\frac{1}{999}\sum_{s=1}^{1000}(Z_s - \overline{Z})^2}$$

  Compute minimum cost, maximum cost, coefficient of variation, VaR and CVaR
end for
Compare fitted distributions and best-fit approach using mean cost, standard deviation, cost variability and robustness
Identify the most reliable distributional approach for stochastic transportation planning

IV. Results and Comparisons

The proposed algorithm is applied to the case study undertaken by Ashour et al. [VI], which deals with the transportation of gasoline product from three warehouses(sources) to six main fuel stations(destinations) for the Oil Products Distribution Company of the Iraqi Ministry. The supply available at warehouses is fixed, but demand at all six stations is uncertain, and data on the past 30 demands required at destinations is available. The transportation cost matrix is given in Table I, whereas the historical demand values over a period of 30 days are presented in Table II. Each row of Table II represents one day, and the six observations in a particular row correspond to the same day, thereby synchronising the demand across all the destinations. This data is used to estimate the marginal demand at each destination and the dependence structure among the destination demand.

TABLE I: Per unit cost (in dinars) of transporting gasolene from three warehouses to six stations

Stations(destinations) / Warehouses(sources)	D1 (Al-Mathnaa)	D2 (Al-Mansour)	D3 (Al-Mustansiriya)	D4 (Al-Idrisi)	D5 (Al-Kilani)	D6 (Al-Bunk)	Supply
S1 (Al-Dawruh)	836	1097	1202	1254	1045	1620	440
S2 (Al-Karkh)	560	2717	2926	3396	2665	3762	504
S3 (Al-Rusafa)	508	2874	1855	1515	1672	1881	560
Demand	D1	D2	D3	D4	D5	D6	

In order to validate the deterministic equivalent of the supply constraint given by equation (8), an illustration is constructed using the warehouse capacities given in Table I. Here, using the mean supply $\mu = 501.33$and an assumed standard deviation $\sigma = 0.1\mu = 50.13$, the lower quantile bounds for Normal, Gamma, and Weibull distributions for 90%, 95%, and 99% confidence levels are computed and presented in Table III. The empirical probabilities obtained by 5000 simulations are very close to their specified confidence levels. Small differences arise from Monte Carlo sampling variation. As shown in the table, an increase in the confidence level strictly decreases the maximum allowable threshold ($F_{a_i}^{-1}(1-\alpha_{a_i})$), thereby ensuring that the empirical satisfaction probability matches the specified confidence level.

TABLE II: Day wise demand of destination D1 to D6

Destination / Day	D1	D2	D3	D4	D5	D6
1	140	250	160	140	260	120
2	155	260	153	143	255	110
3	146	255	165	145	259	115
4	162	259	169	156	261	118
5	154	262	159	156	265	125
6	159	260	154	152	254	129
7	149	259	152	154	268	124
8	160	258	156	146	259	119
9	164	266	158	148	253	116
10	153	265	159	149	257	125
11	150	261	162	146	269	128
12	164	258	167	142	261	116
13	163	263	168	141	263	113
14	169	257	163	147	267	124
15	158	256	165	159	264	128
16	151	268	164	153	261	123
17	149	269	167	156	256	117
18	146	262	160	154	258	113
19	142	265	150	158	257	112
20	147	258	162	157	261	114
21	146	251	158	158	264	120
22	158	269	157	152	268	127
23	162	266	169	142	255	110
24	157	255	163	141	255	128
25	146	254	162	151	266	126
26	149	262	151	156	257	119
27	153	267	150	153	251	126
28	159	256	158	149	261	128
29	157	251	162	148	264	124
30	152	259	161	147	259	126
Mean	154	260.03	160.13	149.97	260.27	120.77
S.D.	7.24	5.33	5.55	5.83	4.81	6.03

TABLE III: Supply Thresholds and Empirical Satisfaction Probabilities

Distribution	Specified α	Supply threshold	Empirical satisfaction probability
Normal	0.90	437.08	0.900089
Normal	0.95	418.87	0.950005
Normal	0.99	384.71	0.990039
Gamma	0.90	438.25	0.899849
Gamma	0.95	421.82	0.950283
Gamma	0.99	392.12	0.989988
Weibull	0.90	434.52	0.899756
Weibull	0.95	409.53	0.949711
Weibull	0.99	358.13	0.989860

The multivariate nature of 30-day demand data across the six destinations provided in Table II is addressed by evaluating the empirical dependence structure before distributional fitting. This is achieved by computing the Pearson correlation coefficient across the historic data and is presented in Table IV.

TABLE IV: Pairwise Correlation Matrix of Destination Demands

	D1	D2	D3	D4	D5	D6
D1	1.000	0.141	0.316	-0.323	-0.008	0.085
D2	0.141	1.000	-0.110	0.203	-0.313	-0.178
D3	0.316	-0.110	1.000	-0.289	0.214	-0.150
D4	-0.323	0.203	-0.289	1.000	0.127	0.138
D5	-0.008	-0.313	0.214	0.127	1.000	0.342
D6	0.085	-0.178	-0.150	0.138	0.342	1.000

It is pertinent to note that all pairwise coefficients lie between -0.323 and $+0.342$, indicating weak linear dependency across destinations during the 30-day period. This implies that independent sampling serves as an accurate approach for the subsequent simulation experiments. However, Ashour et al. [VI] concluded that all the destinations followed a normal distribution and calculated all the demands using $\mu - 1.96\sigma$ that estimate the demand values with a lower range of data and misses the major portion of demand. The authors, on the other hand, propose estimating the demand using $\mu + 1.96\sigma$, a true prediction that will secure 97.5 % of the demand.

The results obtained using the proposed framework are presented in the following subsections.

A. Distribution Fitting Analysis

The histograms of demands at different destinations are plotted; however, is insufficient to identify a suitable probability distribution. This identification is carried out through the goodness-of-fit statistical tests, viz. AIC and BIC. Among the candidate distributions, i.e., Normal, Lognormal, Gamma, and Weibull, the distribution having the smallest values of AIC and BIC is selected as the best-fit distribution. The results of AIC and BIC values at all destinations are given in Table V. The graphical representation of distribution fittings at all six destinations is given in Figure 1, which shows almost the same behaviour near the mean point, but tail risk values vary among the distributions.

B. Marginal Worst-Case Demand Estimation

The close AIC and BIC values suggest considering multiple distribution fittings to estimate demands within the ambiguity set A_j instead of relying on a single distribution. The worst-case marginal demand for a destination is determined by taking the maximum quantile across all candidate distributions within A_j. The selected distributions and the resulting ambiguity-bounded marginal demands across different scenarios are presented in Table VI, which demonstrate the direct role of confidence level α on escalation of demands, i.e., an increase in α-value effects an increase in the demand.

C. Transportation Cost Analysis

To address the level of uncertainty, the following three different scenarios are taken:

- Scenario-1: confidence level is taken as 90%, which is good for routine planning.

• Scenario-2: confidence level is taken as 95%, which corresponds to conservative robust planning.

• Scenario-3: confidence level is taken as 99%, which is good for high-risk-averse planning and essential for emergency preparedness.

Upon estimating the demands, the problem is transformed to deterministic form and an optimal cost is found for each of the three scenarios as given in Table VII. An important observation from the optimal cost of different scenarios, with an increase in the total demands, the optimal cost increases steadily, with an upper bound of 1586899 dinars corresponding to Scenario-3. The optimal solution is directly dependent on the demand at destinations. The optimal solution proposed here is more than that obtained by Ashour et al. due to the low demands estimated by them. The upper bound of optimal cost as found in Scenario-3 is still 11% less than the budget approved by the company. It is also important to observe how the allocation routes are selected during uncertainty. Allocation matrices for different scenarios are given in Table VIII that indicate that source-destination allocation routes are almost the same for all three scenarios.

D. Monte Carlo Simulation Analysis

The availability of multiple plausible distributions fitting the data poses the following questions:

• Is it necessary to fit a probability distribution to historical data?

• How reliable is the ambiguity-bounded consensus fit compared to individual candidate distributions?

To investigate uncertainty propagation and cost variability, a total of 5000 Monte Carlo replications were performed across five alternative distributional specifications: the ambiguity-bounded consensus fit (Best-fit), Normal, Lognormal, Gamma, and Weibull distributions. Each of these was evaluated using 1000 independent runs, initialized using MATLAB's random-number-generator seed `rng(1)` to support reproducibility.

The historical dataset comprised 30 daily demand observations for each of six destinations. Four candidate probability distributions were fitted independently to each destination's data. Under the consensus envelope approach, the distribution minimizing the Akaike Information Criterion (AIC) was selected separately for each destination. Under the four comparative approaches, all six destinations were simulated using the corresponding fitted distribution family, with destination-specific estimated parameters.

In each Monte Carlo replication, a six-dimensional demand vector was generated independently from the fitted marginal distributions. The generated values were rounded upward to the nearest integer, and a minimum demand of one unit was imposed. This vector was supplied to a linear programming transportation model to determine the minimum transportation cost. Supply and demand imbalances were handled through the addition of a zero-cost dummy destination or dummy source, as applicable.

The joint random-generation procedure was based on independent marginal sampling. Consequently, the simulation preserved the fitted marginal distributions but did not

explicitly model cross-destination correlation structures, copulas, or joint tail co-movements. The resulting transportation costs from the five sets of 1000 simulations were subsequently used to compute the mean, standard deviation, coefficient of variation, Value-at-Risk (VaR), Conditional Value-at-Risk (CvaR), failure rate, and robustness index.

The findings should therefore be interpreted as conditional on the assumption of independent destination demands. A dependence-preserving simulation, based on a suitably estimated copula or paired historical resampling, could be used in future work to assess the sensitivity of transportation costs and reliability measures to cross-destination demand dependence. The graphical representation of transportation costs of the simulation for the best-fit distribution is shown in Figure 3, showing that costs vary in the range between 1357000 and 1514000 dinars.

E. Comparative Distribution Study and Robustness Analysis

The values of various statistical measures for the considered distributions are given in Table IX. The observations about these measures are:

• Mean transportation costs obtained in the case of different distributions are close but are minimum for the Gamma distribution.

• Standard deviation of cost in the case of the Weibull distribution is slightly higher relative to other distributions.

• CV of cost is almost similar for all distributions except Weibull, in which case it is slightly higher.

• Minimum and maximum costs are similar for all distributions.

• Failure rate is zero for all distributions, indicating sufficient availability of supplies to tackle demands under uncertainty.

• Robustness index values are similar for all distributions.

Although the mean cost is similar for all destinations, tail risk behaviour may be different. It is important to record how cost varies at α values 90%, 95% and 99%. Table X represents the value at risk and conditional value at risk with different distributions and α levels. The percentage difference in VaR from the mean cost is also computed, and its minimum value is considered as a more stable fit. The comparative cost box-plot and Mean and SD comparison graphs shown in Figures 2 and 4, respectively, indicate the closeness among the different distribution models; the uncertain cost range is nearly the same among the distributions, but the best-fit distribution result is better in comparison to the other candidate distributions.

V. Managerial Implications

Table XI compares the probability distributions in terms of their impact on transportation budgets. The Gamma distribution gives the lowest expected transportation cost and needs the smallest contingency at the 90% confidence level. The lowest coefficient of variation corresponding to the Normal, Lognormal, and Gamma distributions implies they produce more stable and predictable budget estimates. The increase in confidence level from 90% to 99% results in an increase of

both the contingency and reserve budgets. It is significant to note that across all confidence levels, the ambiguity-bounded consensus fit yields the minimum worst-case budget (CVaR), exhibiting its strength at limiting extreme cost scenarios. Thus, it can be safely implied that if the objective is to minimize average costs, the Gamma distribution works best; however, for safeguarding the budget from cost spikes, the ambiguity-bounded consensus fit is the safer choice.

The decision-makers should not rely on a single fitted distribution to represent the uncertain data. It is better to consider several statistically plausible distributions and see how each affects expected cost and risk metrics before taking the final call. Tools like Monte Carlo simulation and tail-risk measures (VaR and CVaR) help logistics teams estimate contingency budgets, test worst-case scenarios, and make transportation plans that are more robust.

TABLE V: All distribution fitting result

Destination	Distribution	Log Likeli-hood	AIC	BIC	KS-h	KS p-Value	KS Statistic	Estimated demand at $\alpha = 0.90$	Estimated demand at $\alpha = 0.95$	Estimated demand at $\alpha = 0.99$
1	Normal	-102.48	208.95	211.76	0	0.93	0.09	163.29	165.92	170.86
1	Lognormal	-101.47	206.94	209.75	0	0.89	0.1	163.41	166.23	171.65
1	Gamma	-101.46	206.91	209.72	0	0.89	0.1	163.2	165.91	171.06
1	Weibull	-102.7	209.39	212.19	0	0.91	0.1	163.16	165.01	168.09
2	Normal	-92.25	188.5	191.31	0	0.95	0.09	266.87	268.8	272.43
2	Lognormal	-92.27	188.53	191.34	0	0.94	0.09	266.91	268.9	272.68
2	Gamma	-92.25	188.5	191.31	0	0.92	0.1	266.78	268.72	272.38
2	Weibull	-93.3	190.6	193.4	0	0.7	0.12	266.67	267.97	270.11
3	Normal	-93.49	190.97	193.77	0	0.91	0.1	167.25	169.27	173.05
3	Lognormal	-93.63	191.25	194.05	0	0.87	0.1	167.35	169.49	173.56
3	Gamma	-93.57	191.13	193.94	0	0.87	0.1	167.19	169.25	173.15
3	Weibull	-93.48	190.97	193.77	0	0.96	0.09	166.79	168.1	170.26
4	Normal	-94.95	193.9	196.7	0	0.77	0.12	157.44	159.56	163.53
4	Lognormal	-95.05	194.09	196.9	0	0.78	0.12	157.55	159.79	164.1
4	Gamma	-95	194	196.8	0	0.74	0.12	157.38	159.54	163.66
4	Weibull	-94.88	193.77	196.57	0	0.72	0.12	156.93	158.29	160.54
5	Normal	-90.16	184.33	187.12	0	0.85	0.11	265.43	267.18	270.45
5	Lognormal	-89.14	182.28	185.09	0	0.88	0.1	266.46	268.25	271.64
5	Gamma	-89.14	182.28	185.08	0	0.88	0.1	266.34	268.09	271.39
5	Weibull	-90.74	185.49	188.29	0	0.35	0.16	266.38	267.57	269.55
6	Normal	-96.46	196.93	199.72	0	0.31	0.17	127.52	129.68	131.79
6	Lognormal	-96.19	196.38	199.18	0	0.28	0.18	128.67	131.04	135.62
6	Gamma	-96.1	196.19	199	0	0.27	0.18	128.47	130.74	135.07
6	Weibull	-95.04	194.08	196.88	0	0.6	0.13	128.73	130.18	134.79

TABLE VI: Best-fit distributions and obtained ambiguity-bounded marginal demands

Destination	Best Fit Distributions	Scenario - 1, Demand at $\alpha = 0.90$	Scenario - 2, Demand at $\alpha = 0.95$	Scenario - 3, Demand at $\alpha = 0.99$	Demand estimated by Ashour et al. [VI]
D1	Lognormal, Gamma	164	167	172	140
D2	Normal, Lognormal, Gamma	267	269	273	250
D3	Normal, Lognormal, Gamma, Weibull	168	170	174	149
D4	Normal, Lognormal, Gamma, Weibull	158	160	165	139
D5	Lognormal, Gamma	267	269	272	251
D6	Weibull	129	131	135	109

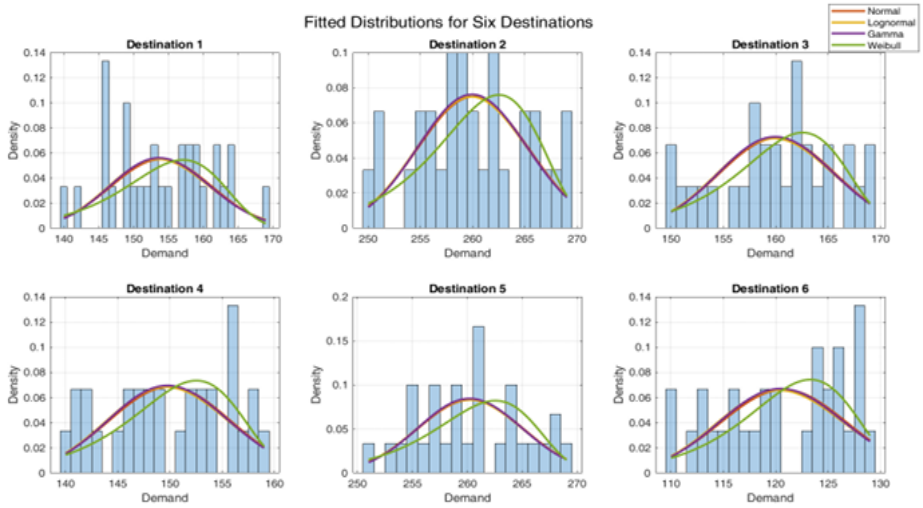

Fig. 1: Distribution fitting at all destinations

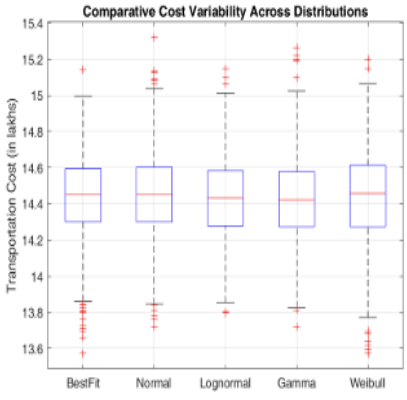

Fig. 2: Comparative Cost Box-plot

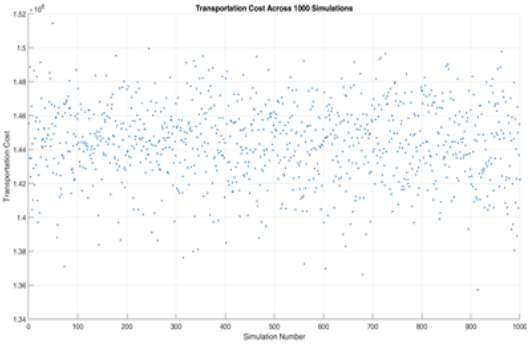

Fig. 3: Best fit distribution simulation plot

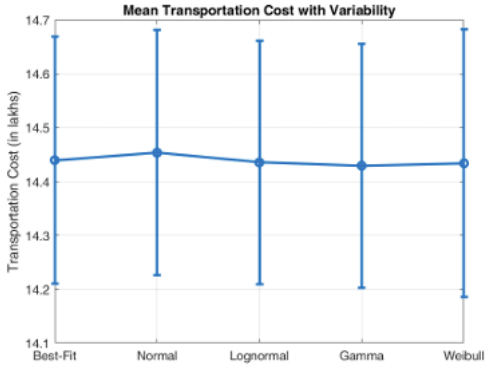

Fig. 4: Mean and SD comparison from simulations

TABLE VII: Comparison of solution obtained vs demands

Demand estimation method	D1	D2	D3	D4	D5	D6	Total Demand	Optimal Solution (in dinars)
Ashour et al.	140	250	149	139	251	109	1038	1336016
Scenario-1 ($\alpha = 0.90$)	164	267	168	158	267	129	1153	1511411
Scenario-2 ($\alpha = 0.95$)	167	269	170	160	269	131	1166	1530853
Scenario-3 ($\alpha = 0.99$)	172	273	174	165	272	135	1191	1586899

TABLE VIII: Allocation matrix in all scenarios

Scenario	Source	Destinations					
		D1	D2	D3	D4	D5	D6
Scenario-1 ($\alpha = 0.90$)	S1	0	267	168	0	5	0
	S2	153	0	0	0	0	0
	S3	11	0	0	158	262	129
Scenario-2 ($\alpha = 0.95$)	S1	0	269	170	0	1	0
	S2	166	0	0	0	0	0
	S3	1	0	0	160	268	131
Scenario-3 ($\alpha = 0.99$)	S1	0	266	174	0	0	0
	S2	172	7	0	0	12	0
	S3	0	0	0	165	260	135

TABLE IX: Robustness Index computed by simulations

Distribution	Mean Cost	Std Cost	CV Cost	Min Cost	Max Cost	Failure Rate	Robustness Index
Best fit	1443972	22945	1.59	1357346	1514432	0	0.9844
Normal	1445360	22745	**1.57**	1372153	1531976	0	0.9845
Lognormal	1443546	22633	**1.57**	1379523	1514657	0	0.9846
Gamma	**1442947**	22652	**1.57**	1372090	1526696	0	0.9845
Weibull	1443393	24848	1.72	1357239	1520117	0	0.9831

TABLE X: $VaR, CVaR, VaR_{Diff\%}$ for different distributions at various α levels

Distribution	$\alpha = 90\%$			$\alpha = 95\%$			$\alpha = 99\%$		
	VaR	CVaR	$VaR_{Diff\%}$	VaR	CVaR	$VaR_{Diff\%}$	VaR	CVaR	$VaR_{Diff\%}$
Best fit	1472442	**1481579**	1.9716	**1479791**	**1487566**	2.4806	1492244	1497294	3.343
Normal	1474336	1486722	2.0048	1484787	1494339	2.7278	1500837	1509347	3.8383
Lognormal	1473840	1483985	2.0986	1481720	1490594	2.6445	1498009	1503459	3.7729
Gamma	**1471001**	1483435	**1.9442**	1479885	1491001	2.5599	1498189	1509782	3.8284
Weibull	1473858	1482583	2.1107	1480013	1488654	2.5371	**1490403**	1500735	3.2569

TABLE XI: Best fit distribution relative to minimum cost of various measures

Source of comparison: Min value of	Distribution(s)	Remark
Mean cost	Gamma	Normal budget
CV	Normal/Lognormal/Gamma	Most predictable cost
VaR ($\alpha = 90\%$)	Gamma	Lowest budget risk
VaR ($\alpha = 95\%$)	Best Fit	Moderate budget risk
VaR ($\alpha = 99\%$)	Weibull	High budget risk
CVaR ($\alpha = 90\%$)	Best fit	Lowest worst case exposure
CVaR ($\alpha = 95\%$)	Best fit	Moderate worst case exposure
CVaR ($\alpha = 99\%$)	Best fit	High worst case exposure
$VaR_{Diff\%}$ ($\alpha = 90\%$)	Gamma	Lowest budget risk
$VaR_{Diff\%}$ ($\alpha = 95\%$)	Best Fit	Moderate budget risk
$VaR_{Diff\%}$ ($\alpha = 99\%$)	Weibull	High budget risk

VI. Conclusion

Unlike the classical transportation problem, where all parameters are deterministic, the stochastic classical transportation problem deals with the optimization under uncertain input parameters. Thus, choosing a suitable probability distribution for the uncertain parameters is a key step. This paper demonstrated the effects of probability distributions on demand estimates for the SCTP. It was observed that several distributions fit the historical data and raise comparable demand estimates but diverge under the risk-focused measures. In particular, the ambiguity-based consensus fit performed best at limiting extreme cost outcomes and required smaller worst-case budgets when assessed with tail-risk measures like CVaR. The authors accentuate that the optimal transportation cost under uncertainty is not a single fixed number, but rather a range of possible outcomes governed by confidence levels and risk metrics. The evaluation of VaR and CVaR helps the decision-makers to compute contingencies and prepare for unfavourable demands also. The objectives of the study were attained by developing a MATLAB framework that included distribution fitting, demand estimation, optimization of the transportation problem, and Monte Carlo simulation for validation. This framework offers a methodical way to evaluate expected costs and budget risks, helping managers make more robust decisions under uncertainty. The approach can be adapted to other stochastic combinatorial optimization problems bearing uncertain parameters.

Conflict of Interest:

There was no relevant conflict of interest regarding this paper.

References

I. Acharya, Srikumar, et al. “Computation of a Multi-Objective Fuzzy Stochastic Transportation Problem.” *International Journal of Fuzzy Computation and Modelling,* vol. 1, no. 2, 2014, pp. 212–233. 10.1504/IJFCM.2014.067129.

II. Agrawal, P., and T. Ganesh. “Multi-Choice Stochastic Transportation Problem Involving Logistic Distribution.” *Advances and Applications in Mathematical Sciences,* vol. 18, no. 1, 2018, pp. 45–58. https://www.mililink.com/upload/article/1203372211aams_vol_181_nov_2018_a4_p45-58_prachi_agrawal_and_talari_ganesh.pdf.

III. Agrawal, P., et al. "An Artificial Intelligence Approach for Solving Stochastic Transportation Problems." *Computers, Materials & Continua,* vol. 70, no. 1, 2022, pp. 817–829. https://doi.org/10.32604/cmc.2022.019685.

IV. Akaike, Hirotugu. "A New Look at the Statistical Model Identification." *IEEE Transactions on Automatic Control,* vol. 19, no. 6, 1974, pp. 716–723. https://doi.org/10.1109/TAC.1974.1100705.

V. Al Qahtani, Hadeel, et al. "A Goal Programming Approach to Multichoice Multiobjective Stochastic Transportation Problems with Extreme Value Distribution." *Advances in Operations Research,* vol. 2019, 2019, article 9714137. 10.1155/2019/9714137.

VI. Ashour, Marwan Abdul Hameed, Alyaa Abdulameer Ahmed, and Ammar Sh. Ahmed. "Optimization Algorithms for Transportation Problems with Stochastic Demand." *Periodicals of Engineering and Natural Sciences,* vol. 10, no. 3, 2022, pp. 172–179. https://doi.org/10.21533/pen.v10.i3.654.

VII. Bellman, Richard E., and Lotfi A. Zadeh. "Decision-Making in a Fuzzy Environment." *Management Science,* vol. 17, no. 4, 1970, pp. B-141–B-164. https://doi.org/10.1287/mnsc.17.4.B141.

VIII. Bhavana, Pulloru, and D. Kalpana Priya. "The Stochastic Transportation Problem with Imprecise Data Using Lomax Distribution." *European Journal of Pure and Applied Mathematics,* vol. 17, no. 2, 2024, pp. 1228–1243. 10.29020/nybg.ejpam.v17i2.5162.

IX. Gupta, Srikant, Harish Garg, and Sachin Chaudhary. "Parameter Estimation and Optimization of Multi-Objective Capacitated Stochastic Transportation Problem for Gamma Distribution." *Complex & Intelligent Systems,* vol. 6, 2020, pp. 651–667. 10.1007/s40747-020-00156-1.

X. Hitchcock, Frank L. "The Distribution of a Product from Several Sources to Numerous Localities." *Journal of Mathematics and Physics,* vol. 20, nos. 1–4, 1941, pp. 224–230. 10.1002/sapm1941201224.

XI. Johnson, Norman L., Samuel Kotz, and Narayanaswamy Balakrishnan. "Continuous Univariate Distributions." 2nd ed., vol. 1, Wiley, 1994. https://www.wiley-vch.de/en/areas-interest/mathematics-statistics/continuous-univariate-distributions-978-0-471-58495-7.

XII. Joshi, Vishwas Deep, Medha Sharma, and Jagdev Singh. "Solving Multi-Choice Solid Stochastic Multi-Objective Transportation Problem with Supply, Demand and Conveyance Capacity Involving Newton Divided Difference Interpolations." *Journal of Computational Analysis and Applications,* vol. 33, no. 1A, 2024, pp. 372–395. https://eudoxuspress.com/index.php/pub/article/view/30.

XIII. Mahapatra, Deshabrata Roy, Sankar Kumar Roy, and M. P. Biswal. "Multi-Choice Stochastic Transportation Problem Involving Extreme Value Distribution." *Applied Mathematical Modelling,* vol. 37, no. 4, 2013, pp. 2230–2240. 10.1016/j.apm.2012.04.024.

XIV. Mahapatra, Deshabrata Roy, Sankar Kumar Roy, and M. P. Biswal. "Multi-Objective Stochastic Transportation Problem Involving Log-Normal." *Journal of Physical Sciences,* vol. 14, 2010, pp. 63–76. https://d1wqtxts1xzle7.cloudfront.net/110417413/Art7-libre.pdf.

XV. Rockafellar, R. Tyrrell, and Stanislav Uryasev. "Optimization of Conditional Value-at-Risk." *The Journal of Risk,* vol. 2, no. 3, 2000, pp. 21–41. 10.21314/JOR.2000.038.

XVI. Roy, Sankar Kumar, Deshabrata Roy Mahapatra, and M. P. Biswal. "Multi-Choice Stochastic Transportation Problem with Exponential Distribution." *Journal of Uncertain Systems,* vol. 6, no. 3, 2012, pp. 200–213. https://jus.worldscientific.com/pdf/jusVol06No3paper03.pdf.

XVII. Roy, Sankar Kumar. "Multi-Choice Stochastic Transportation Problem Involving Weibull Distribution." *International Journal of Operational Research,* vol. 21, no. 1, 2014, pp. 38–58. https://doi.org/10.1504/IJOR.2014.064021.

XVIII. Roy, Sankar Kumar. "Transportation Problem with Multi-Choice Cost and Demand and Stochastic Supply." *Journal of the Operations Research Society of China,* vol. 4, no. 2, 2016, pp. 193–204. 10.1007/s40305-016-0125-3.

XIX. Rubinstein, Reuven Y., and Dirk P. Kroese. "Simulation and the Monte Carlo Method." 3rd ed., Wiley, 2016. 10.1002/9781118631980.

XX. Schwarz, Gideon. "Estimating the Dimension of a Model." *The Annals of Statistics,* vol. 6, no. 2, 1978, pp. 461–464. 10.1214/aos/1176344136.

XXI. Singh, S., A. Pradhan, and M. P. Biswal. "Multi-Objective Solid Transportation Problem under Stochastic Environment." *Sādhanā,* vol. 44, no. 5, 2019, article 105. 10.1007/s12046-019-1094-0.

XXII. Stephens, M. A. "EDF Statistics for Goodness of Fit and Some Comparisons." *Journal of the American Statistical Association,* vol. 69, no. 347, 1974, pp. 730–737. 10.1080/01621459.1974.10480196.

XXIII. Taha, Hamdy A. "Operations Research: An Introduction." *Pearson Education,* 2013. https://books.google.com/books?vid=ISBN9789332518223.

XXIV. Williams, A. C. "A Stochastic Transportation Problem." *Operations Research,* vol. 11, no. 5, 1963, pp. 759–770. 10.1287/opre.11.5.759.