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QUICKEST APPROACH FOR THE OPTIMIZATION OF MULTI OBJECTIVE TRANSPORTATION PROBLEM UNDER FUZZY ENVIRONMENT (WEIGHTED ARITHMETIC MEAN APPROACH)

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Abstract

The multi-objective transportation problem (MOTP), an extension of the classical transportation problem (TP), minimizes multiple conflicting objectives concurrently, such as transportation costs, average delivery times, and product damage costs. Traditional single-objective optimization techniques fail to resolve vector optimization scenarios where multiple criteria conflict. This paper proposes an accelerated, mathematically rigorous fuzzy programming technique using a weighted arithmetic mean aggregation (WAMA) framework. Unlike ad hoc cell-level modifications, this method models fuzzy satisfaction at the level of the aggregate objective function, preserving the true mathematical basis of fuzzy multi-objective optimization. A novel matrix row-reduction and maximal-membership greedy allocation algorithm is developed to compute the optimal compromise solution with high efficiency. The mathematical consistency, global ordering properties, and Pareto efficiency of the mechanism are proved analytically. Numerical illustrations demonstrate that this approach yields structurally superior solutions with significantly lower computational complexity compared with those of existing iterative methods.

Keywords: Multi-objective Transportation Problem, Fuzzy Linear Programming, Aggregate Membership Function, Fuzzy Environment, Weighted Arithmetic Mean Pareto Efficiency.

Nomenclature

$Z_p(x)$	$Z_p(x) = \{Z_1(x), Z_2(x), ..., Z_P(x)\}$ is a vector of P objectives
m	Total number of supply origins
n	Total number of demand targets
a	Index for origins
b	Index for targets
A_a	Total supply available at each origin a
B_b	Total demand required at each target b
C_{ab}^{p}	Penalty associated with the transportation of each unit of product from a^{th} origin to b^{th} target corresponding p^{th} criteria (where $p = 1, 2, ..., P$)
x_{ab}	Quantity of product to be shipped from a^{th} origin to b^{th} target.

I. Introduction

The fundamental transportation problem (TP) focuses on moving goods from a collection of supply points to a set of demand sites while minimizing the linear transportation cost. These problems can be classified as single-objective or multi-objective, and they must be optimized (maximized or minimized) within a given set of constraints, usually equalities or inequalities. Mathematical modelling of many real-life situations often leads to such problems. The practice of concurrently optimizing many objectives while adhering to specific constraints is called multi-objective optimization, or multi-objective programming. Hitchcock developed the first single-objective TP [XV]. Lee and Moore [XIX] investigated the optimization of transportation problems with different objectives. Diaz [VIII] and Isermann [XVI] created distinct algorithms to identify non-dominated solutions for linear multi-objective transportation problems. Aneja and Nair [IV] suggested an approach for finding non-dominated extreme points in the objective space rather than in the decision space for bi-criteria TP. Nomani, Ali, and Ahmed proposed a new weighted model to obtain varying solutions according to the priorities of the decision-maker [XXI]. Using A multi-criteria simplex approach for solving linear multi-objective transportation problems is described by Gupta and Gupta [XII]. Two interactive methods for solving MOTP were developed by Ringuest and Rinks [XXVII]. Bit, Biswal, and Alam [VII] found efficient solutions and an optimal compromise solution for the MOTP using a linear membership function. Fuzzy programming was used by Verma, Biswal, and Biswas [XXXII] to solve the MOTP using several nonlinear membership functions. Ojha, Mondal, and Maiti [XXII] formulated single-objective and multi-objective transportation models with fuzzy relations under fuzzy logic and developed genetic algorithms to solve these models. A new efficient algorithm to solve MOTP is proposed by Kasana [XVII]. Wahed presents a fuzzy programming approach to determine the optimal compromise solution of MOTP by using fuzzy membership function [I]. Wahed and Lee [II] presented a fuzzy goal programming approach to solve MOTP. Bai [VI] offered a simple method to solve MOTP models. The dripping approach for solving bi-objective TP was developed by Pandian [XXIII]. Quddoos [XXV, XXVI]

created the MMK method to solve bi-objective TPs using lexicographic programming. George [XI] proposed a method that makes use of a fuzzy programming technique. Doke [IX] addressed the three-objective linear TPs using the arithmetic mean of the global assessment values. Kaur, Singh, and Bhandari [XVIII] developed an efficient method to solve MOTP; Gupta and Anupam [XIII] and Gupta, Rani, and Singh [XIV] proposed a method based on cost-matrix reduction and the calculation of specific values for cells with zero reduced costs. Afwat, Salama, and Farouk [III] proposed a product approach for solving MOTP under a fuzzy environment. Patel and Patel [XXIV] offered a new row maxima approach to solve MOTP. Singh and Rajan [XXXI] developed a novel method known as the Matrix Maxima Method for solving MOTP using Pareto Optimality Criteria. Singh and Rajan [XXX] used geometric methods to address MOTP in a fuzzy environment. To manage MOTP in fuzzy environments, Ekanayake [X] created the geometric mean method and the ant colony optimization strategy. Niluminda and Ekanayake [XX] formulated a novel alternative algorithm to solve MOTP by using geometric means along with the penalty technique. Shrivastava, Agrawal, and Kumar [XXVIII] used the inversion method to address the multi-objective linear programming issue. Anupam, Bose, and Sharma [V] proposed a TGR weighted method to solve MOTP. Shrivastava and Dwivedi [XXIX] resolved MOTP by using the revised simplex approach.

This study resolves foundational limitations by introducing a mathematically sound Weighted Arithmetic Mean Aggregation (WAMA) model. We present a rigorous scalarized transformation, prove that its components preserve global ordering during row operations, and provide an analytical foundation ensuring that our allocation scheme yields genuinely Pareto-efficient compromise solutions.

II. Mathematical Model

The mathematical model of MOTP can also be represented in tabular form, as shown in Table 1.

$$\text{Min } Z_p(X) = \sum_{a=1}^{m}\sum_{b=1}^{n} C_{ab}^{p} x_{ab}, \quad p = 1,2,3,....,P.$$

Subject to the following constraints

$$\left.\begin{aligned} \sum_{b=1}^{n} x_{ab} &= A_a, \ \forall\, a = 1,2,3,....,m \\ \sum_{a=1}^{m} x_{ab} &= B_b, \ \forall\, b = 1,2,3,....,n \\ x_{ab} \geq 0 \ \forall\ & a = 1,2,3,....,m \text{ and } b = 1,2,3,....,n \\ and \ \sum_{a=1}^{m} A_a &= \sum_{b=1}^{n} B_b \text{ (equilibrium condition)} \end{aligned}\right\} \quad (1)$$

The mathematical model of MOTP can also be represented as in Table 1.

Table 1: **Tabular representation of MOTP model**

Targets → **Origins ↓**	**T_1**	**T_2**	**...**	**T_n**	**Supply** (A_a)
O_1	$C_{11}^1, C_{11}^2 ... C_{11}^p$	$C_{12}^1, C_{12}^2 ... C_{12}^p$	⋯	$C_{1n}^1, C_{1n}^2 ... C_{1n}^p$	a_1
O_2	$C_{21}^1, C_{21}^2 ... C_{21}^p$	$C_{22}^1, C_{22}^2 ... C_{22}^p$	⋯	$C_{2n}^1, C_{2n}^2 ... C_{2n}^p$	a_2
⋮	⋮	⋮	⋮	⋮	⋮
O_m	$C_{m1}^1, C_{m1}^2 ... C_{m1}^p$	$C_{m2}^1, C_{m2}^2 ... C_{m2}^p$	⋯	$C_{mn}^1, C_{mn}^2 ... C_{mn}^p$	a_m
Demand (B_b)	b_1	b_2	⋯	b_n	

III. Preliminaries

This section presents fundamental definitions and theorems essential for understanding the proposed methodology.

Definition III.i. Feasible Solution: If a solution $X^* = \{x_{ab}\} \forall\, a = 1,2,3,....., m$ and $b = 1,2,3,....., n$ satisfies every constraint and the non-negativity restrictions of the MOTP (1), then it is considered a feasible solution to MOTP (1). Together, these solutions form a space known as the feasible space (S).

Definition III.ii. Pareto-Efficient (Non- dominated) Solution: A feasible solution $X^* \in S$ is said to be a non-dominated solution of MOTP (1) if, and only if, there is no other feasible solution $X \in S$ such that $Z_p(X) \le Z_p(X^*) \forall\, p, p = 1,2,3,....,\ P$ and $Z_k(X) \le Z_k(X^*)$ for at least one target criterion k.

Definition III.iii. Aggregate Objective Membership Function: To establish a sound optimization model, the fuzzy satisfaction level must be evaluated at the total objective level $Z_p(X)$ rather than at the level of individual cell variables. Let L_p and U_p represent the lower bound (aspiration level) and upper bound (maximum acceptable level) for the objective $Z_p(X)$, determined by solving each single-objective subproblem independently over S. The linear aggregate membership function $\mu_p(Z_p(X))$ is defined as:

$$\mu_p(Z_p(X)) = \begin{cases} 1 & if\ Z_p(X) \le L_p \\ \dfrac{U_p - Z_p(X)}{U_p - L_p} & if\ L_p < Z_p(X) < U_p \\ 0 & if\ \mu_p(Z_p(X) \ge U_p \end{cases} \tag{2}$$

Definition III.iv. Weighted Arithmetic Mean Aggregation (WAMA): To capture diverse decision-making contexts and eliminate arbitrary equal-preference assumptions, we define the global scalar preference function $\bar{\mu}(X)$ using importance weights w_p assigned to each criterion:

$$\bar{\mu}(X) = \sum_{p}^{P} w_p \mu_p(Z_p(X)), \text{where } w_p \geq 0 \text{ and } \sum_{p}^{P} w_p = 1$$

Theorem III.i. (Pareto Efficiency of the WAMA Structure)

If $X^* \in S$ uniquely maximizes the global weighted arithmetic preference function $\bar{\mu}(X) = \sum_{p}^{P} w_p \mu_p(Z_p(X))$ over the compact feasible domain (S) with $w_p > 0 \forall\, p$, then X^* is a strictly Pareto-efficient compromise solution to the MOTP.

Proof: Suppose X^* is not Pareto-efficient. Then there exists an alternative feasible allocation $Y \in S$ such that $Z_p(Y) \leq Z_p(X^*) \forall p$ and $Z_k(Y) \leq Z_k(X^*)$ for some criterion k. Because $\mu_p(Z_p)$ is a strictly decreasing linear function over the continuous interval $[L_p, U_p]$, it follows that $\mu_p(Z_p(Y)) \geq \mu_p(Z_p(X^*)) \forall\, p,$ and $\mu_k(Z_k(Y)) > \mu_k(Z_k(X^*))$.

Given that all preference weights are strictly positive $(w_p > 0)$, taking the weighted sum yields:

$$\sum_{p}^{P} w_p \mu_p(Z_p(Y)) > \sum_{p}^{P} w_p \mu_p(Z_p(X^*)) \Rightarrow \bar{\mu}(\bar{Y}) > \bar{\mu}(X^*)$$

This directly contradicts the initial condition that X^* maximizes $\bar{\mu}(X)$ over S. Hence, X^* must be Pareto-efficient.

IV. Mathematical Equivalence and Matrix Reduction Logic

To develop a direct matrix-based algorithm, we expand the structural formulation of the global weighted preference function $\bar{\mu}(X)$ as:

$$\bar{\mu}(X) = \sum_{p=1}^{P} w_p \left[\frac{U_p - \sum_{a=1}^{m} \sum_{b=1}^{n} c_{ab}^{p}\, x_{ab}}{U_p - L_P} \right]$$

$$= \sum_{p=1}^{P} \frac{w_p\, U_p}{U_p - L_p} - \sum_{a=a}^{m} \sum_{b=1}^{n} \left(\sum_{p=1}^{P} \frac{w_p\, c_{ab}^{p}}{U_p - L_p} \right) x_{ab}$$

Let $K = \sum_{P=1}^{p} \frac{w_p\, U_p}{U_p - L_p}$ be a constant scalar representing the system's boundary constraints, and define the consolidated fuzzy cell cost coefficient d_{ab} as:

$$d_{ab} = \sum_{p=1}^{P} \frac{w_p\, c_{ab}^{p}}{U_p - L_p}$$

Maximizing $\bar{\mu}(X)$ is thus mathematically equivalent to solving the single-objective generalized transportation problem.

$$\text{Max } \bar{\mu}(X) = \min D(X) = \sum_{a=1}^{m}\sum_{b=1}^{n} d_{ab}\, x_{ab}$$

This structural breakdown proves that applying the fuzzy transformation to the overall objective values results in a single, well-defined consolidated cell-cost matrix $D = [d_{ab}]_{m\times n}$.

Theorem IV.i. Global Ordering Preservation under Row-Reduction Operations:

Let $D = [d_{ab}]$ represent the consolidated cell cost matrix. Let $\delta_{ab} = \min_b d_{ab}$ be the smallest element in the row a, and define the row-reduced matrix elements as $d'_{ab} = d_{ab} - \delta_a$. Then, any feasible allocation matrix X that optimizes $D'(X) = \sum_{a=1}^{m}\sum_{b=1}^{n} d'_{ab}\, x_{ab}$ also optimizes the global scalar preference function $\bar{\mu}(X)$, preserving relative cellular preference orderings across all rows.

Proof: Expressing the reduced cost objective function yields

$$D'(X) = \sum_{a=1}^{m}\sum_{b=1}^{n}(d_{ab} - \delta_a)\, x_{ab} = \sum_{a=1}^{m}\sum_{b=1}^{n} d_{ab}\, x_{ab} - \sum_{a=1}^{m}\delta_a\left(\sum_{b=1}^{n} x_{ab}\right).$$

Substituting the structural row constraint $\sum_{b=1}^{n} x_{ab} = A_a$

$D'(X) = D(X) - \sum_{a=1}^{m}\delta_a A_a$, because the term $\sum_{a=1}^{m}\delta_a A_a$ is composed entirely of constant parameters (independent of the decision matrix X), the argmin of $D'(X)$ is identical to the argmin of $D(X)$. Since maximizing $\bar{\mu}(X)$ is directly linked to minimizing $D(X)$, the row-reduction operation preserves the global optimization landscape.

V. Solution Procedure

The solution procedure for finding the optimal compromise solution to MOTP (1) can be summarized as follows:

Step 1: Formulate the mathematical model for the given MOTP as represented in Table 1 and write each objective function separately. Find the extreme minimum cost L_p and maximum cost U_p across the feasible domain for each criterion to calculate the aspiration bounds.

Step 2: Determine the relative preference weight vector w_p satisfying $\sum w_p = 1$ based on the system's operational objectives.

Step 3: Calculate the cell coefficients $d_{ab} = \sum_{p=1}^{P} \frac{w_p c_{ab}^p}{U_p - L_p}$ to construct the primary optimization matrix D.

Step 4: Locate the minimum element δ_a of each row and subtract it from every entry in that row to build the reduced framework $D'(X) = [d'_{ab}]$.

Step 5(i): Identify all zero-valued cells in the reduced cost matrix and determine the maximum possible allocation for each cell as the minimum of its corresponding available supply and demand. Select the zero-valued cell permitting the highest allocation. In case of a tie, give preference to the cell having the smaller row index, followed by the smaller column index.

Step 5(ii): Allocate the maximum feasible quantity to the selected zero-valued cell. If the corresponding supply or demand is completely satisfied, select the next minimum-valued cell for allocation from the same row or column. The selection shall be made by considering the remaining supply and demand, the reduced costs of the available cells, and the feasibility of subsequent allocations. Continue this process until the corresponding supply or demand is exhausted.

Step 5(iii): Cross out the row and/or column whose corresponding supply and/or demand has been completely exhausted. Retain the unexhausted supply and demand values and update the reduced cost matrix accordingly.

Step 5(iv): Re-examine the remaining zero-valued cells in the reduced cost matrix. Determine their maximum feasible allocations using the updated supply and demand values. Select the next zero-valued cell according to the selection rule specified in Step 5(i), and repeat Steps 5(i)–5(iii) until all eligible zero-valued cells have been considered.

Step 5(v): If the supply and demand corresponding to a selected zero-valued cell become zero simultaneously, do not allocate that cell immediately. Instead, identify another available zero-valued cell in the same row or column, if one exists, and reserve this alternative cell for allocation after the other eligible zero-valued cells have been considered. Subsequently, allocate to the reserved cell and update the remaining supply and demand values.

Step 6: After all eligible zero-valued cells have been considered, allocate the remaining supply and demand through the available cells. Select the cells by considering their reduced costs and the feasibility of subsequent allocations. Where necessary, reserve the required supply or demand for the remaining allocations. Continue until the supplies at all origins and the demands at all targets are completely satisfied.

Step 7: Substitute the obtained allocation values X_{ab} into the original objective functions to obtain the optimal vector compromise solution $Z^* = \{Z_1(X^*), Z_2(X^*), ..., Z_p(X^*)\}$.

VI. Numerical Illustrations

Example 1:

Consider the following MOTP (3×4), adapted from Wahed and Lee [II], which involves two objectives, namely, time (Z_1) and cost (Z_2), involving three targets and four origins.

Min

$$Z_1 = x_{11} + 2x_{12} + 7x_{13} + 7x_{14} + x_{21} + 9x_{22} + 3x_{23} + 4x_{24} + 8x_{31} + 9x_{32} + 4x_{33} + 6x_{34}$$

Min

$$Z_2 = 4x_{11} + 4x_{12} + 3x_{13} + 4x_{14} + 5x_{21} + 8x_{22} + 9x_{23} + 10x_{24} + 6x_{31} + 2x_{32} + 5x_{33} + 1x_{34}$$

Subjected to

$$\sum_{b=1}^{4} x_{1b} = 8, \sum_{b=1}^{4} x_{2b} = 19, \sum_{b=1}^{4} x_{3b} = 17, \sum_{a=1}^{3} x_{a1} = 11, \sum_{a=1}^{3} x_{a2} = 3, \sum_{a=1}^{3} x_{a3} = 14, \sum_{a=1}^{3} x_{a4} = 16$$

$$; x_{ab} \geq 0 \ \forall \ a = 1,2,3,....,m \ \& \ b = 1,2,3,....,n$$

Formulate the mathematical model for the given MOTP as represented in Table 1. The input data for the MOTP are presented in Table 2.

Table 2: Input data for MOTP (Example 1)

Origins $\rightarrow$ **Targets** $\downarrow$	O_1	O_2	O_3	O_4	**Supply** (A_a)
T_1	1 4	2 4	7 3	7 4	8
T_2	1 5	9 8	3 9	4 10	19
T_3	8 6	9 2	4 5	6 1	17
Demand (B_b)	11	3	14	16	

By evaluating the individual objective functions, we find the single-objective aspiration parameters:

Z_1(Time): $L_1 = 143$ and $U_1 = 265$

Z_2 (Cost): $L_2 = 167$ and $U_2 = 310$

Now, applying the relative preference weight vector $(w_1 = w_2 = \frac{1}{2})$ satisfying $\sum w_p = 1,$

each cell coefficient is scaled using the true aggregate optimization formula:

$$d_{ab} = \sum_{p=1}^{2} \frac{w_p c_{ab}^p}{U_p - L_p} = \frac{1}{2}\left[\frac{c_{ab}^1}{122} + \frac{c_{ab}^2}{143}\right]$$

The primary consolidated matrix D and its corresponding row-reduced form D' are obtained. The resulting matrices are illustrated in Tables 3 and 4, respectively.

Table 3: Consolidated Matrix for Example 1

Origins $\rightarrow$ **Targets** $\downarrow$	O_1	O_2	O_3	O_4	**Supply** (A_a)
T_1	0.0181	0.0222	0.0392	0.0427	8
T_2	0.0216	0.0649	0.0438	0.0514	19
T_3	0.0538	0.0439	0.0339	0.0281	17
Demand (B_b)	11	3	14	16	

Table 4: Row-Reduced Matrix for Example 1

Origins $\rightarrow$ Targets $\downarrow$	O_1	O_2	O_3	O_4	Supply (A_a)
T_1	0.0000	0.0041	0.0211	0.0246	8
T_2	0.0000	0.0433	0.0222	0.0298	19
T_3	0.0257	0.0158	0.0058	0.0000	17
Demand (B_b)	11	3	14	16	

Applying the allocation procedure described in Step 5 to the transformed matrix values, the final allocation matrix is obtained, as shown in Table 5.

Table 5: Allocations for Example 1

Origins $\rightarrow$ **Targets** $\downarrow$	O_1	O_2	O_3	O_4	**Supply** (A_a)
T_1	0.0000	0.0041 **(3)**	0.0211 **(5)**	0.0246	8
T_2	0.0000 **(11)**	0.0433	0.0222 **(8)**	0.0298	19
T_3	0.0257	0.0158	0.0058 **(1)**	0.0000 **(16)**	17
Demand (B_b)	11	3	14	16	

The solution is: $X_{12}=3, X_{13}=5, X_{21}=11, X_{23}=8, X_{33}=1, X_{34}=16$

Minimum Time = $(2\times3)+(7\times5)+(1\times11)+(3\times8)+(4\times1)+(6\times16)=176$

Minimum Cost = $(4\times3)+(3\times5)+(5\times11)+(9\times8)+(5\times1)+(1\times16)=175$

Therefore, the optimal compromise solution for this problem is $Z_1=176$ and $Z_2=175$.

The corresponding membership values are:

$$\mu_1(Z_1) = \frac{265 - Z_1}{U_1 - L_1} = \frac{265 - 176}{265 - 143} = 0.72951$$

$$\mu_2(Z_2) = \frac{310 - Z_2}{U_2 - L_2} = \frac{310 - 175}{310 - 167} = 0.94406$$

Hence, the weighted arithmetic mean satisfaction value is:

$$\mu = \frac{0.72951 + 0.94406}{2} = 0.83679$$

The results obtained using the proposed WAMA approach are compared with those of existing methods in Table 6. The table also includes the optimal solution for reference, facilitating an assessment of the relative performance of the different approaches.

Table 6: Comparison between different approaches

Name of approach	Objective Function Z_1	Objective Function Z_2
Fuzzy Approach [II]	170	190
Interactive Approach [XXVII]	186	174
Fuzzy Programming Approach [VII]	160	195
Proposed WAMA Approach	176	175
Optimal Solution	143	167

Figure 1 presents a graphical comparison of the results obtained from various approaches for Example 1, highlighting the comparative performance of the different methods.

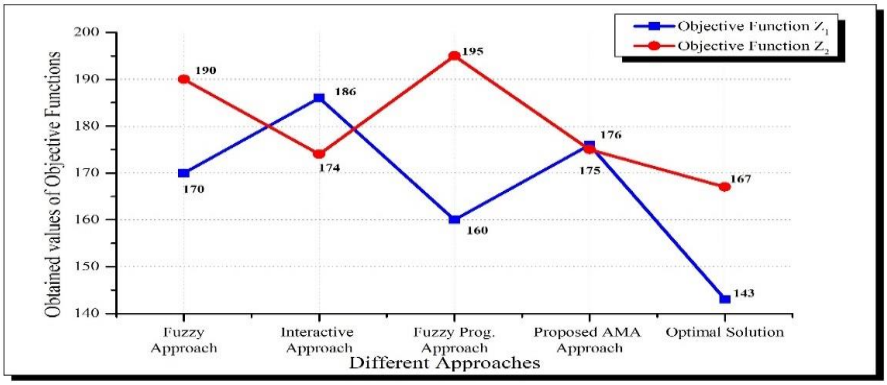


Fig. 1. Comparative analysis of Example 1 with different methods

Example 2:

Consider the following MOTP (4×5) taken from Wahed and Lee [II], with three objectives, namely, transportation cost (Z_1) , delivery time (Z_2), and damage cost (Z_3). The transportation cost, delivery time, and damage cost associated with the MOTP are presented in Table 7.

Table 7: Input Data for MOTP (Example 2)

Origins → / Targets ↓	O_1	O_2	O_3	O_4	O_5	Supply (A_a)
T_1	9 2 2	12 9 4	9 8 6	6 1 3	9 4 6	5
T_2	7 1 4	3 9 8	7 9 4	7 5 9	5 2 2	4
T_3	6 8 5	5 1 3	9 8 5	11 4 3	3 5 6	2
T_4	6 2 6	8 8 9	11 6 6	2 9 3	2 8 1	9
Demand (B_b)	4	4	6	2	4	

By evaluating the individual objective functions, we obtain the following aspiration parameters for the corresponding single-objective problems.

Z_1 (Transportation Cost): $L_1 = 102$ and $U_1 = 200$

Z_2 (Delivery Time): $L_2 = 72$ and $U_2 = 150$

Z_3 (Damage Cost): $L_3 = 64$ and $U_3 = 120$

Now, using equal structural weights $(w_1 = w_2 = w_3 = \frac{1}{3})$ satisfying $\sum w_p = 1,$ each cell coefficient is scaled using the true aggregate optimization formula:

$$d_{ab} = \sum_{p=1}^{3} \frac{w_p\, c_{ab}^{p}}{U_p - L_p} = \frac{1}{3}\left[\frac{c_{ab}^{1}}{98} + \frac{c_{ab}^{2}}{78} + \frac{c_{ab}^{3}}{56}\right]$$

The primary consolidated matrix D and its corresponding row-reduced form D' are obtained. The resulting matrices are illustrated in Tables 8 and 9, respectively.

Table 8: Consolidated Matrix for Example 2

Origins → / Targets ↓	O_1	O_2	O_3	O_4	O_5	Supply (A_a)
T_1	0.0511	0.1031	0.1005	0.0425	0.0834	5
T_2	0.0519	0.0963	0.0861	0.0987	0.0375	4
T_3	0.0844	0.0391	0.0946	0.0724	0.0673	2
T_4	0.0647	0.1150	0.0988	0.0631	0.469	9
Demand (B_b)	4	4	6	2	4	

Table 9: Row-Reduced Matrix for Example 2

Origins → / Targets ↓	O_1	O_2	O_3	O_4	O_5	Supply (A_a)
T_1	0.0085	0.0605	0.0580	0.0000	0.0409	5
T_2	0.0144	0.0588	0.0486	0.0613	0.0000	4

T_3	0.0452	0.0000	0.0554	0.0332	0.0281	2
T_4	0.0177	0.0680	0.0518	0.0162	0.0000	9
Demand (B_b)	4	4	6	2	4	

The final allocation matrix is shown in Table 10.

Table 10: Allocation Matrix for Example 2

Origins $\rightarrow$ **Targets** $\downarrow$	O_1	O_2	O_3	O_4	O_5	**Supply** (A_a)
T_1	0.0085 **(3)**	0.0605	0.0580	0.0000 **(2)**	0.0409	5
T_2	0.0144	0.0588 **(2)**	0.0486 **(2)**	0.0613	0.0000	4
T_3	0.0452	0.0000 **(2)**	0.0554	0.0332	0.0281	2
T_4	0.0177 **(1)**	0.0680	0.0518 **(4)**	0.0162	0.0000 **(4)**	9
Demand (B_b)	4	4	6	2	4	

The solution is:

$$X_{11}=3, X_{14}=2, X_{22}=2, X_{23}=2, X_{32}=2, X_{41}=1, X_{43}=4, X_{45}=4$$

Min. Transportation Cost =

$$(9\times 3)+(6\times 2)+(3\times 2)+(7\times 2)+(5\times 2)+(6\times 1)+(11\times 4)+(2\times 4)=127$$

Min. Delivery Time =

$$(2\times 3)+(1\times 2)+(9\times 2)+(9\times 2)+(1\times 2)+(2\times 1)+(6\times 4)+(8\times 4)=104$$

Min. Damage Cost =

$$(2\times 3)+(3\times 2)+(8\times 2)+(4\times 2)+(3\times 2)+(6\times 1)+(6\times 4)+(1\times 4)=76$$

Therefore, the optimal compromise solution for MOTP is $Z_1=127, Z_2=104$ and $Z_3=76.$

The corresponding membership values are:

$$\mu_1(Z_1)=\frac{200-Z_1}{U_1-L_1}=\frac{200-127}{200-102}=0.74490$$

$$\mu_2(Z_2)=\frac{150-Z_2}{U_2-L_2}=\frac{150-104}{150-72}=0.58974$$

$$\mu_1(Z_3)=\frac{120-Z_3}{U_3-L_3}=\frac{120-76}{120-64}=0.78571$$

Hence, the weighted arithmetic mean satisfaction value is:

$$\mu = \frac{0.74490 + 0.58974 + 0.78571}{3} = 0.70678$$

A comparative analysis of the results obtained using the proposed WAMA approach and other existing methods is presented in Table 11.

Table 11: Comparison between different approaches

Name of approach	Objective Function Z_1	Objective Function Z_2	Objective Function Z_3
Fuzzy Approach [II]	112	106	80
Interactive Approach [XXVII]	127	104	76
Fuzzy Prog. Approach [VII]	127	104	76
Proposed WAMA Approach	127	104	76
Optimal Solution	102	72	64

Figure 2 presents a graphical comparison of the results obtained from various approaches for Example 2, highlighting the comparative performance of the different methods.

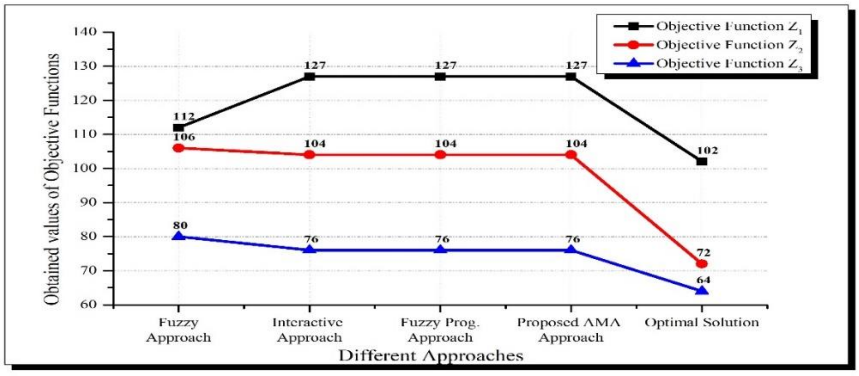


Fig. 2. Comparative analysis of Example 2 with different methods

Example 3:

Let us consider an MOTP (3×4) solved in Patel and Patel [XXIV], which involves two objectives, namely, cost (Z_1) and time (Z_2), and has three targets and four origins. The cost and time associated with MOTP are presented in Table 12.

Table 12: Input data for MOTP (Example 3)

Origins $\rightarrow$ Targets $\downarrow$	O_1	O_2	O_3	O_4	Supply (A_a)
T_1	6 1	4 2	1 3	5 4	14
T_2	8 4	9 3	2 2	7 0	16

T_3	4 0	3 2	6 2	2 1	5
Demand (B_b)	6	10	15	4	

By evaluating the individual objective functions, we find the single-objective aspiration parameters:

Z_1 (Transportation Cost): $L_1 = 114$ and $U_1 = 210$

Z_2 (Transportation Time): $L_2 = 54$ and $U_2 = 110$

Now, using equal structural weights $(w_1 = w_2 = \frac{1}{2})$ satisfying $\sum w_p = 1$, each cell coefficient is scaled using the true aggregate optimization formula:

$$d_{ab} = \sum_{p=1}^{2} \frac{w_p\, c_{ab}^{p}}{U_p - L_p} = \frac{1}{2}\left[\frac{c_{ab}^{1}}{96} + \frac{c_{ab}^{2}}{56}\right]$$

The primary consolidated matrix D and its corresponding row-reduced form D' are obtained. The resulting matrices are illustrated in Tables 13 and 14, respectively.

Table 13: Consolidated Matrix for Example 3

Origins $\rightarrow$ **Targets** $\downarrow$	O_1	O_2	O_3	O_4	**Supply** (A_a)
T_1	0.0402	0.0387	0.0320	0.0618	14
T_2	0.0417	0.0737	0.0283	0.0365	16
T_3	0.0208	0.0335	0.0491	0.0193	5
Demand (B_b)	6	10	15	4	

Table 14: Row-Reduced Matrix for Example 3

Origins $\rightarrow$ **Targets** $\downarrow$	O_1	O_2	O_3	O_4	**Supply** (A_a)
T_1	0.0082	0.0067	0.0000	0.0298	14
T_2	0.0134	0.0454	0.0000	0.0082	16
T_3	0.0015	0.0142	0.0298	0.0000	5
Demand (B_b)	6	10	15	4	

The final allocation matrix is shown in Table 15.

Table 15: Final Allocation Matrix for Example 3

Origins $\rightarrow$ **Targets** $\downarrow$	O_1	O_2	O_3	O_4	**Supply** (A_a)
T_1	0.0082 **(4)**	0.0067 **(10)**	0.0000	0.0298	14
T_2	0.0134	0.0454	0.0000 **(15)**	0.0082 **(1)**	16
T_3	0.0015 **(2)**	0.0142	0.0298	0.0000 **(3)**	5
Demand (B_b)	6	10	15	4	

The solution is: $X_{11}=4, X_{12}=10, X_{23}=15, X_{24}=1, X_{31}=2, X_{34}=3$

Minimum cost $=(6\times4)+(4\times10)+(2\times15)+(7\times1)+(4\times2)+(2\times3)=115$

Minimum Time $=(1\times4)+(2\times10)+(2\times15)+(0\times1)+(0\times2)+(1\times3)=57$

Therefore, the optimal compromise solution for this MOTP is $Z_1=115$ and $Z_2=57$

The corresponding membership values are:

$$\mu_1(Z_1)=\frac{210-Z_1}{U_1-L_1}=\frac{210-115}{210-114}=0.98958$$

$$\mu_2(Z_2)=\frac{210-Z_2}{U_2-L_2}=\frac{210-57}{210-114}=1.59375$$

Hence, the weighted arithmetic mean satisfaction value is:

$$\mu=\frac{0.98958+1.59375}{2}=1.29167$$

A comparative analysis of the results obtained using the proposed WAMA approach and other existing methods is presented in Table 16.

Table 16: Comparison between different approaches

Name of approach	**Objective Function** Z_1	**Objective Function** Z_2
New Row Maxima Method [XXIV]	162	83
Product Approach [III]	114	62
GM & Penalty Method [XX]	121	54
Proposed WAMA Approach	115	57
Optimal Solution	114	54

Figure 3 presents a graphical comparison of the results obtained from various approaches for Example 3, highlighting the comparative performance of the different methods.

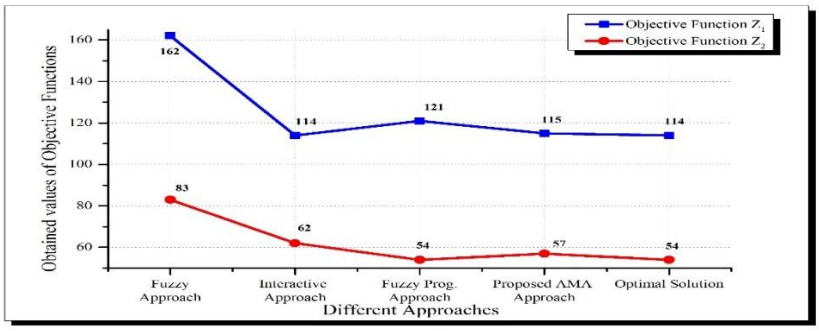


Fig. 3. Comparative analysis of Example 3 with different methods

VII. Conclusions

This paper presents a mathematically sound Weighted Arithmetic Mean Aggregation (WAMA) framework for solving multi-objective transportation problems in a fuzzy environment. By defining fuzzy membership functions at the aggregate objective function level rather than at the level of individual cell inputs, the model preserves the foundational optimization properties of fuzzy multi-objective programming. We demonstrate that row-reduction transformations preserve global cellular preference orderings and demonstrate that the proposed greedy allocation mechanism systematically converges to a Pareto-efficient vector compromise solution. Performance evaluations based on the numerical examples confirm that this method provides a highly reliable decision-making tool. It reduces the number of computational evaluation steps while generating compromise solutions that effectively balance conflicting operational demands and provide a high level of satisfaction.

Conflict of Interest:

There was no relevant conflict of interest regarding this paper.

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