



DOUBLY ISOLATED EDGE GEODETIC NUMBER OF A GRAPH

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Abstract

Here we study the new concept of a doubly isolated edge geodetic set for a connected graph $G = (V, E)$ of order $n \geq 3$. The edge geodetic set $S \subseteq V$ of a connected graph G is said to be a doubly isolated edge geodetic set if the induced subgraphs $\langle S \rangle$ and $\langle V(G) - S \rangle$ have at least one isolated vertex. The doubly isolated edge geodetic number is the minimum cardinality of a doubly isolated edge geodetic set of G and is denoted by $g_{1di}(G)$. The doubly isolated edge geodetic number of some standard graphs, Cartesian product, and corona graphs is determined. It is shown that for any three positive integers a, b and c with $a \leq b \leq c$ there exists a connected graph G of order n with $g(G) = a$, $g_1(G) = b$ and $g_{1di}(G) = c$.

Keywords: Cartesian product, Corona, Edge geodetic set, Extreme vertex, Full vertex, Isolate edge geodetic number, Support vertex.

I. Introduction

In this paper, we consider only finite, connected, and simple graphs $G = (V, E)$. In [VI], Harary et al. introduced a graph-theoretical parameter, the geodetic number of a graph, and further studied it in [IV, VIII, IX]. The distance between any two vertices u, v is the length of the shortest $u - v$ path in a connected graph G . An $u - v$ path of length $d(u, v)$ is called an $u - v$ geodesic. The closed interval $I[u, v]$

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consists of all vertices lying on some $u - v$ geodesic of G , and for a non-empty subset $S \subseteq V(G)$, $I[S] = \bigcup_{u,v \in S} I[u, v]$. The set S of vertices is a geodetic set in G if $I[S] = V(G)$, and the minimum cardinality of a geodetic set is the geodetic number and is denoted as $g(G)$. This concept was introduced in [VI]. The edge geodetic set $S \subseteq V(G)$, in which all edges of G is included in a geodesic connecting some pair of vertices in S . The edge geodetic number $g_1(G)$ is the number that represents the cardinality of the minimum edge geodetic set. The concept of edge geodetic number was introduced in [I].

The concept of a doubly geodetic set was introduced in [II]. The concept of isolate geodetic set was introduced in [XII]. The vertex v in the graph G is said to be an extreme if a subgraph induced by its neighbours is a complete graph. The vertex v is termed as an isolated vertex if $\deg(v) = 0$, and the vertex v is called as the pendant vertex if $\deg(v) = 1$. The vertex v is called a full vertex or dominating vertex of a graph G if $\deg(v) = n - 1$, and any vertex adjacent to the pendant vertex is called a support vertex. Antipodal vertices u and v whose distance $d(u, v)$ equals the diameter of the graph. A graph formed by taking a single copy of G with $|V(G)| = n$ copies of H . By joining i^{th} vertex of G with each vertex in the i^{th} copy of H , then it is said to be a Corona $G \circ H$ of two graphs G and H . The Cartesian product of any two graphs G, H is a graph with vertex set $V(G) \square V(H) = (u_i, v_j)$ where $u_i \in V(G), v_j \in V(H), 1 \leq i \leq m$ and $1 \leq j \leq n$. Any two distinct vertices $(u_i, v_j), (u_k, v_l)$ are adjacent in $G \square H$ if and only if either $u_i = u_k$ and $v_j v_l \in E(H)$ or $v_j = v_l$ and $u_i u_k \in E(G)$. Any undefined terms may be found in [V, X, XI].

Theorem 1.1 [I] Any graph G with at least two vertices of degree $p - 1$, $g_1(G) = p$.

Theorem 1.2 [I] Each extreme vertex of a graph G belongs to every edge geodetic cover of G . In particular, each end vertex of G belongs to every edge geodetic cover of G .

Theorem 1.3 [I] If G has exactly one vertex v of degree $p - 1$, then $g_1(G) = p - 1$.

In [X], the doubly connected geodetic number of graphs and further in [XI], the doubly connected geodetic number on operations in graphs. Motivated by the above definitions, we introduce the doubly isolated edge geodetic number of graphs.

II. Doubly Isolate Edge Geodetic Number

Definition 2.1 The edge geodetic set $S \subseteq V(G)$ in a graph G is said to be a doubly isolated edge geodetic set if the induced subgraphs $\langle S \rangle$ and $\langle V(G) - S \rangle$ have at least one isolated vertex. The cardinality of a minimum doubly isolated edge geodetic set is the doubly isolated edge geodetic number and is denoted by $g_{1di}(G)$.

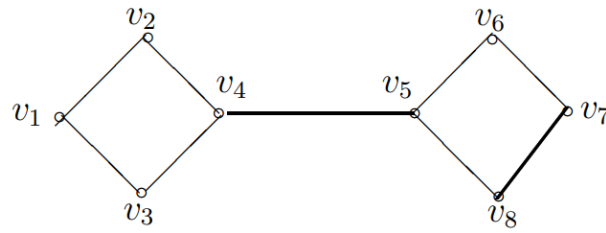


Fig. 1.

The graph G in Figure 2.1; consider the set $S = \{v_1, v_4, v_7\}$. Here, the $\langle S \rangle$ and $\langle V(G) - S \rangle$ have isolated vertices. Therefore, $g_{1di}(G) = 3$.

Observation 2.2 All graphs need not have a doubly isolated edge geodetic set. For example, complete graphs, wheel and fan graphs have no doubly isolated edge geodetic set.

Observation 2.3 In a connected graph G with $n \geq 3$, if $g_{1di}(G) = 2$ then $g(G) = g_1(G) = 2$.

Observation 2.4 Every full vertex v in a graph G does not belong to a doubly isolate edge geodetic set.

Observation 2.5 In a cycle C_n , $n \geq 4$, $g_{1di}(C_n) = \begin{cases} 2 & \text{for } n = 4, \\ 3 & \text{otherwise.} \end{cases}$

Observation 2.6 Every doubly isolated edge geodetic set of G contains all its extreme vertices. In particular, all end vertices in a graph G belong to a doubly isolated edge geodetic set.

Observation 2.7 For a graph G , $g(G) \leq g_{1di}(G)$.

Observation 2.8 For a graph G of order n , $2 \leq g_{1di}(G) \leq n - 1$.

Theorem 2.9 The connected graph G with order $n \geq 3$ has exactly one full vertex if and only if $g_{1di}(G) = n - 1$ if and only if G has at least one pendant vertex.

Proof. Consider the graph G of order $n \geq 3$ that has exactly one full vertex v_f and $g_{1di}(G) = n - 1$. Assume that the graph G is connected and has no pendant vertex. By the definition, $|S| = n - 1$, but $\langle S \rangle$ has no isolated vertex, which is a contradiction. Thus, G has at least one pendant vertex.

Conversely, suppose G has exactly one full vertex v_f and contains at least one pendant vertex, say, v_p . By Theorem 1.3, $g_1(G) = n - 1$ and g_1 -set S contains v_p . Clearly, $\langle S \rangle$ contains the isolate vertex v_p . Also $\langle V(G) - S \rangle$ contains the isolate vertex v_f . Hence $g_{1di}(G) = n - 1$.

Theorem 2.10 If the graph G contains two or more full vertices, then there is no doubly isolated edge geodetic set in G .

Proof. If G has at least two vertices of degree $n - 1$, then by Theorem 1.1, the edge geodetic set $g_1(G) = n = |S|$. It is clear that $\langle S \rangle$ is connected and $\langle V(G) - S \rangle$ is

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an empty graph. Thus, there is no doubly isolated edge geodetic set in a graph G containing two or more full vertices.

Theorem 2.11 In a tree T with at least two internal vertices and k pendant vertices, the doubly isolated edge geodetic number $g_{1di}(T) = k + 1$.

Proof. Consider $S = \{v_1, v_2, \dots, v_k\}$ is an edge-geodetic set in a tree T and let $\{u_1, u_2, \dots, u_x\}$ be the set of internal vertices of a tree T . By Observation 2.6, all the vertices of S belong to a doubly isolated edge geodetic set, but $\langle V(T) - S \rangle$ is connected. Consider $S' = S \cup \{u_i\}$, where u_i is an arbitrary internal vertex adjacent to a support vertex in T . Clearly, S' and $\langle V(T) - S' \rangle$ have isolated vertices. Hence, S' is a doubly isolated edge geodetic set. Thus, $g_{1di}(T) = k + 1$.

Theorem 2.12 In a graph G , $g_{1di}(G) = 2$ if and only if there exists a doubly isolated edge geodetic set containing two antipodal vertices u, v in G such that $d(u, v) = 2$.

Proof. Suppose that $g_{1di}(G) = 2$ and $S = \{u, v\}$ is a doubly isolated edge geodetic set in G . Assume that $d(u, v) > 2$; then the graph G has at least two internal vertices. Clearly, $\langle S \rangle$ has at least one isolate vertex and $\langle V(G) - S \rangle$ is connected, so $g_{1di}(G) > 2$, which is a contradiction. Hence, $d(u, v) = 2$.

Conversely, assume that the edge geodetic set $S = \{u, v\}$ of a graph G such that $d(u, v) = 2$. The set S containing two antipodal vertices forms a doubly isolated edge geodetic set in G . Hence, $g_{1di}(G) = 2$.

Theorem 2.13 For a complete bipartite graph $K_{l,m}$ with $l \geq 1$ and $m > 1$,

$$g_{1di}(K_{l,m}) = \begin{cases} m & \text{for } l = 1, \\ \min(l, m) & \text{for } l > 1. \end{cases}$$

Proof. If $l = 1$, by Observation 2.6, $g_{1di}(K_{l,m}) = m$.

Suppose $l > 1$. Let $A = \{a_i\}$ where $1 \leq i \leq l$ and $B = \{b_k\}$ where $1 \leq k \leq m$ be the partite sets of $K_{l,m}$. Consider $l \leq m$. By definition, $S = A$ is a $g_1(G)$ -set. Since every vertex b_k where $1 \leq k \leq m$ lies on $a_i - a_j$ geodesic for $1 \leq (i \neq j) \leq l$. Then, S is a g_{1di} -set of $K_{l,m}$. Suppose if the set X contains $m - 1$ vertices, $|X| < |S|$ and for any arbitrary vertex $a_m \notin X$ and hence $a_m \notin I[X]$. So, X is not $g_1(G)$ -set. Thus, S is g_{1di} -set. Therefore, $g_{1di}(K_{l,m}) = \min(l, m)$.

III. Doubly isolated edge geodetic number of operations on graphs

In this section, we determine the doubly isolated edge geodetic number for the corona and cartesian product of graphs.

Theorem 3.1 If T is a non-trivial tree of order n , then $g_{1di}(T \circ K_1) = n + 1$.

Proof. By Theorem 2.11, the result is obvious.

Theorem 3.2 For any cycle C_n , $g_{1di}(C_n \circ K_1) = n + 2$.

Proof. Let C_n be a cycle of order n . By the definition of corona graph, $|V(C_n \circ K_1)| = 2n$ and $C_n \circ K_1$ has n end vertices. By Observation 2.6, the set S of n end vertices belongs to a doubly isolated edge geodetic set, and it is clear that $\langle S \rangle$ has isolated

vertices but $\langle V(G) - S \rangle$ is connected, so $g_{1di}(G) > n$. Consider $S' = S \cup \{u_i, u_j\}$ and it forms a doubly isolated edge geodetic set $u_i, u_j \in V(C_n), d(u_i, u_j) = 2$. Thus, $g_{1di}(G) = n + 2$.

Theorem 3.3 If $K_{l,m}$ is a complete bipartite graph of order $l + m$ and $l \leq m$, then $g_{1di}(K_{l,m} \circ K_1) = 2l + m$.

Proof. Let $K_{l,m}$, $l \leq m$ be a complete bipartite graph with two bipartite sets $X = \{\cup_{i=1}^l u_i\}$, $Y = \{\cup_{j=1}^m v_j\}$ and hence $V(K_{l,m} \circ K_1) = V(K_{l,m}) \cup \{x_k\}$, $1 \leq k \leq l + m$ where $\{x_1, x_2, \dots, x_{l+m}\}$ are end vertices in $K_{l,m} \circ K_1$. By Observation 2.6, the set $S = \{x_1, x_2, \dots, x_{l+m}\}$ is a g_1 -set. It is clear that, $\langle S \rangle$ has isolated vertices but $\langle V(K_{l,m} \circ K_1) - S \rangle$ is a complete bipartite graph with no isolated vertex. Now, consider $S' = S \cup X$. Clearly, S' forms a g_{1di} -set. Hence, $g_{1di}(K_{l,m} \circ K_1) = 2l + m$.

Theorem 3.4 For any cycle C_{n_1} and any path P_{n_2} , $n_1 \geq 4$ and $n_1 \geq n_2$,

$$g_{1di}(C_{n_1} \square P_{n_2}) = \begin{cases} 4 & \text{if } n_1 \text{ is even,} \\ 5 & \text{if } n_1 \text{ is odd.} \end{cases}$$

Proof. Consider $C_{n_1} \square P_{n_2}$. Let $V(C_{n_1}) = \{v_1, v_2, \dots, v_{n_1}\}$, $W(P_{n_2}) = \{w_1, w_2, \dots, w_{n_2}\}$ and $V(C_{n_1} \square P_{n_2}) = V(C_{n_1}) \cup W(P_{n_2})$.

Case i) Suppose n_1 is even.

In $C_{n_1} \square P_{n_2}$, the set $X = \{(v_i, w_{n_2}), (v_{i+\frac{n_1}{2}}, w_1)\}$ where $1 \leq i \leq \frac{n_1}{2}$ is g_1 -set with $\text{diam}(C_n \square P_m) = d\{(v_i, w_m), (v_{i+\frac{n}{2}}, w_1)\}$. Clearly, $\langle X \rangle$ has isolated vertices but $\langle V(C_{n_1} \square P_{n_2}) - X \rangle$ is connected, so $g_{1di}(C_{n_1} \square P_{n_2}) > 2$. Consider $S = X \cup \{(v_{i+2}, w_{n_2}), (v_{i+1}, w_{n_2-1})\}$ forms g_{1di} -set. Thus, $g_{1di}(C_{n_1} \square P_{n_2}) = 4$.

Case ii) Suppose n_1 is odd.

In a graph $C_{n_1} \square P_{n_2}$, the set $X = \{(v_i, w_1), (v_{i+\lceil \frac{n_1}{2} \rceil}, w_1), (v_{i+1}, w_{n_2}), (v_{i-1}, w_{n_2})\}$ where $1 \leq i \leq \lceil \frac{n_1}{2} \rceil$ is g_1 -set and the induced subgraph $\langle X \rangle$ has isolated vertices but $\langle V(C_{n_1} \square P_{n_2}) - X \rangle$ has no isolated vertex and so, $g_{1di}(C_{n_1} \square P_{n_2}) \geq 5$. The set $S = X \cup \{(v_i, w_{n_2-1})\}$ forms g_{1di} -set. Therefore, $g_{1di}(C_{n_1} \square P_{n_2}) = 5$.

Theorem 3.5 For any two paths P_{m_1}, P_{m_2} where $m_1, m_2 \geq 2$,

$$g_{1di}(P_{m_1} \square P_{m_2}) = \begin{cases} 3 & \text{for } m_1 = 2, m_2 > 2, \\ 4 & \text{otherwise.} \end{cases}$$

Proof. Consider $P_{m_1} \square P_{m_2}$. Let $V(P_{m_1}) = \{\cup_{i=1}^{m_1} v_i\}$, $V(P_{m_2}) = \{\cup_{j=1}^{m_2} w_j\}$ and $V(P_{m_1} \square P_{m_2}) = \{(v_i, w_j) / 1 \leq i \leq m_1, 1 \leq j \leq m_2\}$. The graph $(P_{m_1} \square P_{m_2})$ is formed from m_1 -copies of P_{m_2} . We have the following cases:

Case i) Suppose that $m_1 = 2$ and $m_2 > 2$. Let $S = \{(v_1, w_1), (v_i, w_j)\}$, where $d\{(v_1, w_1), (v_i, w_j)\} = \text{diam}(P_{m_1} \square P_{m_2})$ be g_1 -set of $P_{m_1} \square P_{m_2}$. It is clear that $\langle S \rangle$ has isolated vertices, but there is no isolated vertex in $\langle V(P_{m_1} \square P_{m_2}) - S \rangle$, so

$g_{1di}(P_{m_1} \square P_{m_2}) \geq 3$. Consider $S' = S \cup \{(v_1, w_{m_2-1})\}$. Clearly, $\langle V(P_{m_1} \square P_{m_2}) - S' \rangle$ has isolate vertex. Thus, $g_{1di}(P_{m_1} \square P_{m_2}) = |S'| = 3$.

Case ii) Suppose that $m_1 > 2$. Let $S = \{(v_1, w_1), (v_i, w_j)\}$, where $d\{(v_1, w_1), (v_i, w_j)\} = \text{diam}(P_{m_1} \square P_{m_2})$ be a g_1 -set of $P_{m_1} \square P_{m_2}$. Consider $S' = S \cup \{(v_1, w_{m_2-1}), (v_2, w_{m_2})\}$. Clearly, $\langle V(P_{m_1} \square P_{m_2}) - S' \rangle$ has isolate vertex. Therefore, $g_{1di}(P_{m_1} \square P_{m_2}) = |S'| = 4$.

IV. Adding a Leaf Vertex to a Cycle

Theorem 4.1 Let G' be the graph formed by adding the leaf vertex $v \notin G$ to any one of the vertices in a cycle $G = C_n$ where $n \geq 4$. Then $g_{1di}(G') = 3$.

Proof. Let $G = C_n$, $n \geq 4$ where $V(G) = \{u_i\}_{i=1}^n$ and let G' be a graph formed from G by adding the leaf vertex $v \notin G$ to any one of the vertices $u_i \in G$. We discuss the following cases:

Case i) Suppose n is even. Let $S = \{v, u_i\}$ be g_1 -set, where $v \notin G$ is a leaf vertex of G' and $\text{diam}(G') = v - u_i$ path. Clearly, $\langle S \rangle$ has isolated vertices, but there is no isolated vertex in $\langle V(G') - S \rangle$. Let us consider $S' = \{v, u_i, u_{i+2}\}$. It is clear that both $\langle S' \rangle$ and $\langle V(G') - S' \rangle$ have isolated vertices. Thus, S' is g_{1di} -set. Therefore, $g_{1di}(G') = 3$.

Case ii) Suppose n is odd. Let $S = \{v, u_i, u_{i+2}\}$ be g_1 -set, where $v \notin G$ is a leaf vertex of G' and $u_i, u_{i+2} \in G$ such that $d(u_i, v) = \text{diam}(G')$ and $d(v, u_i) = d(v, u_{i+2}) + 1$. Clearly, both $\langle S \rangle$ and $\langle V(G') - S \rangle$ have isolated vertices. Thus, S is g_{1di} -set. Therefore, $g_{1di}(G') = 3$.

Theorem 4.2 Let G' be the graph formed by joining k -leaf vertices $\{v_j\} \notin G$ where $1 \leq j \leq k$ to any one of the vertices $u_i \in G = C_n$, where C_n is a cycle of order $n \geq 4$. Then $g_{1di}(G') = k + 2$.

Proof. Let $G = C_n$, $n \geq 4$ where $V(G) = \{u_i\}_{i=1}^n$ and let G' be a graph formed from G by joining k -leaf vertices $X = \{v_j\}_{j=1}^k \notin G$ to any one of the vertices $u_i \in G$. We discuss the following cases:

Case i) Let $G = C_{2r}$, $S = X \cup \{u_i\}$ be g_1 -set, where $X = \{v_j\}_{j=1}^k \notin G$ are leaf vertices of G' and $u_i \in G$ such that $\text{diam}(G') = d(X, u_i)$. Clearly, $\langle S \rangle$ has isolated vertices, but there is no isolated vertex in $\langle V(G') - S \rangle$. Let us consider $S' = S \cup \{u_{i+2}\}$. It is clear that both $\langle S' \rangle$ and $\langle V(G') - S' \rangle$ have isolated vertices. Thus, S' is g_{1di} -set. Therefore, $g_{1di}(G') = k + 2$.

Case ii) Let $G = C_{2r+1}$, $S = X \cup \{u_i, u_{i+2}\}$ be g_1 -set, where $X = \{v_j\}_{j=1}^k \notin G$ is a set of leaf vertices of G' and $u_i, u_{i+2} \in G$ such that $d(u_i, X) = \text{diam}(G')$ and $d(X, u_i) = d(X, u_{i+2}) + 1$. Clearly, both $\langle S \rangle$ and $\langle V(G') - S \rangle$ have isolated vertices. Thus, S is g_{1di} -set. Therefore, $g_{1di}(G') = k + 2$.

Theorem 4.3 If the graph G' is formed by joining k -leaf vertices $v_j \notin G$ where $1 \leq j \leq k$ to each vertex in $G = C_n$ of order $n \geq 4$. Then, $g_{1di}(G') = nk + 2$.

Proof. The graph G' formed by joining k -leaf vertices $X = \{v_j\}$ where $1 \leq j \leq k$ to each vertex in $G = C_n$. By Observation 2.6, the set X of nk pendant vertices belongs to a g_{1di} -set and it is clear that $\langle V(G') - X \rangle$ is connected and hence $g_{1di}(G') > nk$. Consider the set $S = X \cup \{u_i, u_{i+2}\}$ forms g_{1di} -set where $u_i, u_{i+2} \in G$ and $d(u_i, u_{i+2}) = 2$. Clearly, both $\langle S \rangle$ and $\langle V(G') - S \rangle$ have isolated vertices. Therefore, $g_{1di}(G') = nk + 2$.

V. Doubly Isolated Edge Geodetic Number and Diameter

Theorem 5.1 *In a graph G with diameter $d = 1$, there is no doubly isolated edge geodetic number.*

Proof. If $d = 1$ then $G = K_n$ with $g_1(G) = n$. Clearly, $\langle S \rangle$ has no isolated vertex and $\langle V(G) - S \rangle$ is a null graph. By Observation 2.2 and Theorem 2.10, the result is obvious.

Theorem 5.2 *In a nontrivial tree T with diameter $d \geq 3$ of order n , if T is a caterpillar, then $g_{1di}(T) = n - d + 2$.*

Proof. Suppose T is a caterpillar tree of order n with k pendant vertices and $d = d(u, v)$ is a diameter. By Theorem 2.11, $g_{1di}(T) = k + 1 = n - (d - 1) + 1$. Thus, T is a caterpillar if and only if $g_{1di}(T) = n - d + 2$.

Theorem 5.3 *For three positive integers $a, b, c \geq 2$ and $a \leq b \leq c$, then the graph G exists which is connected such that $g(G) = a$, $g_1(G) = b$ and $g_{1di}(G) = c$.*

Proof. Case i) Suppose that $a = b = c$. Consider the path $P_3: u_0, u_1, u_2$ and set of pendant vertices $\{\cup_{j=1}^{a-2} v_j\}, \{\cup_{k=1}^{n-a-1} w_k\}$. The graph G shown in Figure 5.1 is formed by adding $a - 2$ pendant vertices $\{\cup_{j=1}^{a-2} v_j\}$ to $u_1 \in P_3$ and set of $n - a - 1$ vertices $\{\cup_{k=1}^{n-a-1} w_k\}$ to the two end vertices $u_0, u_2 \in P_3, |V(G)| = n$. Let $S = \{u_0, u_2, v_1, v_2, \dots, v_{(a-2)}\}$ be a g -set with $|S| = a$. Clearly, all the edges of G are covered by the vertices of S , so that $g_1(G) = a$. The induced subgraphs $\langle S \rangle, \langle V(G) - S \rangle$ have isolated vertices. Thus, the set S itself forms g -set, g_1 -set and g_{1di} -set. Therefore, $g(G) = g_1(G) = g_{1di}(G) = a$.

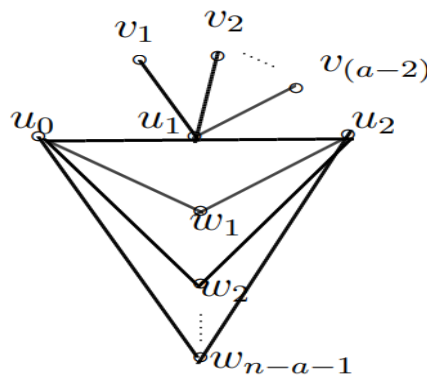


Fig. 2.

Case ii) Suppose that $a = b < c$ and $c = a + 1$. Consider $P_3: u_0, u_1, u_2$. The graph G shown in Figure 5.2 is formed by adding $a - 1$ pendant vertices $\{\cup_{j=1}^{a-1} v_j\}$ to $u_2 \in P_3$ and $\{\cup_{k=1}^{n-a-2} w_k\}$ to both $u_0, u_2 \in P_3$. Let $S = \{u_0, v_1, v_2, \dots, v_{(a-1)}\}$ be g_1 -set with $|S| = a$. Clearly, $\langle S \rangle$ has isolated vertices and $\langle V(G) - S \rangle$ is connected. Thus, the set S is not g_{1di} -set and so $g_{1di}(G) > a$. Consider $S' = S \cup \{u_2\}$ forms g_{1di} -set with $|S'| = a + 1$. Thus, $g(G) = g_1(G) = a$ and $g_{1di}(G) = c = a + 1$.

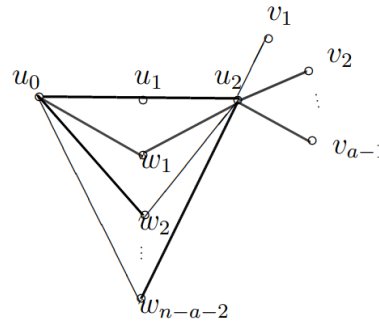


Fig. 3.

Case iii) Suppose that $a < b = c$. Consider $P_3: u_0, u_1, u_2$ and the set of pendant vertices $\{\cup_{j=1}^{a-2} v_j\}, \{\cup_{k=1}^{b-a} w_k\}$. The graph G shown in Figure 5.3 is formed by adding $\{\cup_{j=1}^{a-2} v_j\}$ to $u_1 \in P_3$ and $\{\cup_{k=1}^{b-a} w_k\}$ to all the three vertices in $P_3, V(G) = \{u_0, u_1, u_2\} \cup \{\cup_{j=1}^{a-2} v_j\} \cup \{\cup_{k=1}^{b-a} w_k\}$. Let $S = \{u_0, u_2\} \cup \{\cup_{j=1}^{a-2} v_j\}$ be a g -set of G but not g_1 -set so $g_1(G) > a$. Consider the set $S' = S \cup \{w_i, 1 \leq i \leq (b - a)\}$ is g_1 -set, and it is clear that the induced subgraphs $\langle S' \rangle, \langle V(G) - S' \rangle$ have isolated vertices. Thus, the set S' forms g_{1di} -set. Therefore, $g(G) = a$ and $g_1(G) = g_{1di}(G) = b - a + a - 2 + 2 = b > a$.

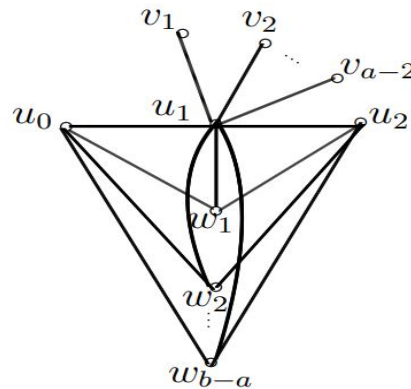


Fig. 4.

Case iv) Suppose that $a < b < c$ and $c = b + 1$. Consider the path $P_5: u_0, u_1, u_2, u_3, u_4$ and set of vertices $\{\cup_{j=1}^{a-1} v_j\} \cup \{\cup_{k=1}^{2a-b-2} l_k\} \cup \{\cup_{r=1}^{b-a} w_r\}$. The graph G is formed by joining $a - 1$ pendant vertices $\{\cup_{j=1}^{a-1} v_j\}$ to the end vertex $u_4 \in$

P_5 also join the set of vertices $\{\cup_{k=1}^{2a-b-2} l_k\} \cup \{\cup_{r=1}^{b-a} w_r\}$ to both $\{u_1, u_3\} \in P_5$. Each vertex in $\{\cup_{r=1}^{b-a} w_r\}$ induces the complete graph is shown in Figure 5.4. Let $S = \{u_0, v_1, v_2, \dots, v_{(a-1)}\}$ be a g -set of G but not g_1 -set, so $g_1(G) > a$. Consider $S' = S \cup \{w_r\}$ where $1 \leq r \leq (b-a)$, $|S'| = b$. Clearly all the edges of G are covered by the vertices of S' , so that $g_1(G) = a - 1 + 1 + b - a = b$ and the induced sub graph $\langle S' \rangle$ has isolated vertices but there is no isolate vertex in $\langle V(G) - S' \rangle$. Choose the internal vertex $u_3 \in P_5$ and $S'' = S' \cup \{u_3\}$ forms g_{1di} -set of G . Hence, $g_{1di}(G) = |S''| = b + 1 = c$.

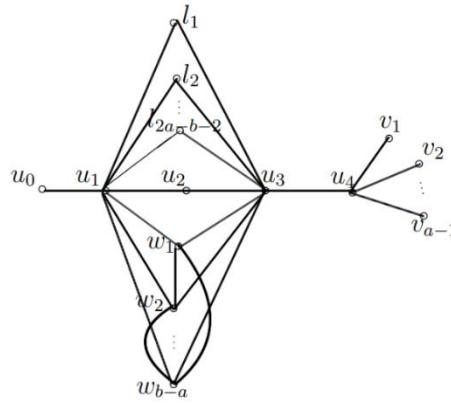


Fig. 5.

Theorem 5.4 For the two integers $r, d > 2$ and also $r \leq d \leq 2r$, then there exists the graph G which is connected of order n , $\text{rad}G = r$ and $\text{diam}G = d$ with $g_1(G) = a$ and $g_{1di}(G) = b = a + 1$.

Proof. Case i) Suppose that $r < d < 2r$. Consider a path $P_d: u_0, u_1, \dots, u_d$ and two sets of pendant vertices $X = \{\cup_{j=1}^{a-2} v_j\}$, $Y = \{\cup_{k=1}^{n-d-a+1} w_k\}$. The graph G shown in Figure 5.5 is formed by adding the vertices in X to $u_1 \in P_d$ and vertices in Y to both $\{u_0, u_2\} \in P_d$. Let $S = X \cup \{u_0, u_d\}$ be g_1 -set with $|S| = a$. Clearly, $\langle S \rangle$ has isolated vertices and $\langle V(G) - S \rangle$ has no isolated vertex. So, $g_{1di}(G) > a$. Consider $S' = S \cup \{u_2\}$ where $u_2 \in P_d$ forms g_{1di} -set. Hence, $g_{1di}(G) = |S'| = a + 1 = b$.

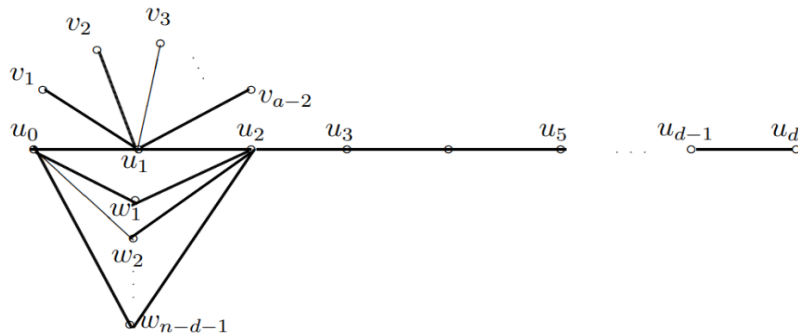


Fig. 6.

Case ii) Suppose that $r = d < 2r$. Consider a cycle $C_{2r}: v_1, v_2, \dots, v_{2r}, v_1$ of order $2r$ and set of vertices $X = \{\cup_{i=1}^{a-2} w_i\}$. The graph G of order n with diameter d , radius r is obtained by joining each $(a-2)$ vertices in X to both $\{v_2, v_{2r}\} \in C_{2r}$ is shown in Figure 5.6. Let $S = \{v_1, v_{r+1}\} \cup X$ covers all the edges, so S is g_1 -set with $|S| = a$. Clearly, $\langle S \rangle$ has isolated vertices, but $\langle V(G) - S \rangle$ has no isolated vertex. Now, choose the vertex $\{v_j\} \in C_{2r}$ such that $d(v_1, v_j) = d(v_{r+1}, v_j) = 2$ and $S' = S \cup \{v_j\}$ forms g_{1di} -set in G in which $\langle V(G) - S' \rangle$ has isolate vertex. Hence, $g_{1di}(G) = |S'| = a + 1 = b$.

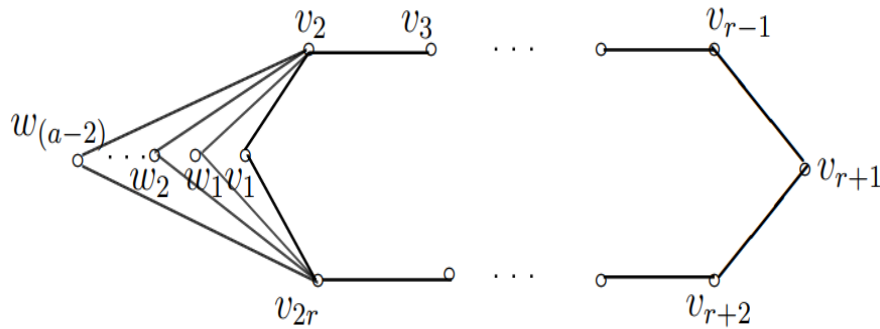


Fig. 7.

Case iii) Suppose that $r < d = 2r$. The graph D_4^a is a Dutch windmill graph containing a number of cycles C_4 meet at a common vertex x of radius r and diameter $d = 2r$, $V(D_4^a) = \{\cup_{i=1}^a z_i\} \cup \{\cup_{j=1}^a w_j\} \cup \{\cup_{k=1}^a y_k\} \cup \{x\}$ and $|V(D_4^a)| = 3a + 1$ is shown in Figure 5.7. Consider $S = \{\cup_{i=1}^a z_i\}$ is g_1 -set of D_4^a . Clearly, $\langle S \rangle$ has isolated vertices and $\langle V(D_4^a) - S \rangle$ is connected, so that S is not g_{1di} -set. Let us consider the set $S' = S \cup \{x\}$ containing $a + 1$ vertices forms g_{1di} -set in which $\langle V(D_4^a) - S' \rangle$ has isolated vertices. Therefore, $g_1(D_4^a) = a$ and $g_{1di}(D_4^a) = b$.

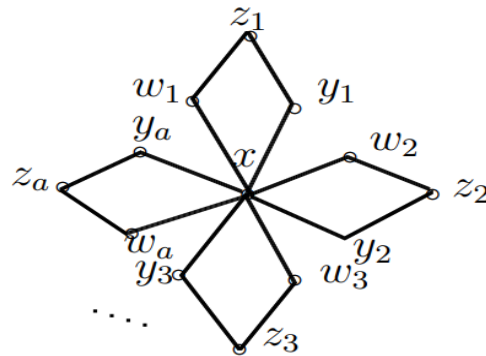


Fig. 8

VI. Conclusion

In this paper, we obtained the results on the doubly isolated edge geodetic number of several special graphs, as well as some general properties of corona and cartesian product of graphs. Further studied the results on integers.

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Conflict of Interest:

The authors declare that there was no relevant conflict of interest regarding this paper.

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