



SKIP LOT SAMPLING PLAN OF TYPE SKSP-R BASED ON PERCENTILES OF TYPE II GENERALIZED HALF-LOGISTIC DISTRIBUTION

M. Rami Reddy^{1*}, Sd. Jilani², B. Srinivasa Rao³, P. Chiranjeevi⁴,
K. Vijaya Bhaskara Reddy⁵

¹Department of Mathematics, Vasireddy Venkatadri International
Technological University, Andhra Pradesh, India. 522508.

²Department of Science and Humanities, QIS College of Engineering and
Technology, Andhra Pradesh, India. 523272.

^{3,4} Department of Mathematics and Humanities, RVR & JC College of
Engineering, Guntur, Andhra Pradesh, India. 522019.

⁵Department of Humanities and Sciences, St. Ann's College of Engineering
and Technology, Andhra Pradesh, India. 523187.

Email: ¹ramireddy.munnangi@gmail.com, ²jilanisyed1992@gmail.com,
³boyapatisrinu@yahoo.com, ⁴chirupunugupati@gmail.com,
⁵kvbryashwanth@gmail.com

Corresponding Author: **M. Rami Reddy**

<https://doi.org/10.26782/jmcms.2026.10.00004>

(Received: July 02, 2026; Revised: September 21, 2026; Accepted: October 07, 2026)

Abstract

This article develops a time-truncated Skip Lot Sampling Plan of model R (SkSP-R) assuming that the product lifetime follows a Type II Generalized Half-Logistic Distribution (Type II GHLD). The distributional properties of Type II GHLD were briefly discussed. The percentiles of the Type II GHLD are used in the preparation of skip lot sampling plan of SkSP-R model for a life test. A reference single sampling plan is used to determine the minimum sample size required to ensure the specific percentile life of the product. The parameters of the sampling plan are estimated using a two-point approach, specifically the Limiting Quality Level (LQL) and Acceptable Quality Level (AQL). The Operating Characteristic (OC) values and Average Sample Number (ASN) values are calculated for the estimated plan parameters using OC function and ASN of the SkSP-R plan, the applicability of the proposed sampling plan is illustrated with a real-data example.

Keywords: Type II Generalized Half-Logistic Distribution, Percentiles, SkSP-R, Life Test, Skip Lot Sampling.

M. Rami Reddy et al

I. Introduction

The Skip-Lot Sampling Plan is a quality-control method used in manufacturing and production processes to efficiently assess the quality of lots produced. This sampling plan is designed to provide a balance between quality assurance and inspection costs by allowing a part of the lots to be skipped under certain conditions, thus reducing the overall inspection effort while maintaining a high level of quality control.

To maintain the desired quality level of the product in any industry it is crucial to inspect the quality of the product that they manufacture. At the inspection level it is not possible to check the quality of each and every item. Testing a sample of product is required to sustain the expected quality of the product. The acceptance sampling plans have been extensively used in industries for maintaining the high-quality level of the product at the minimum inspection cost. An acceptance sampling plan supports the manufacturers to produce high-quality product with assurance.

Various types of acceptance sampling plans like single sampling plan, double sampling plan, multiple sampling plan, sequential sampling plan, resampling schemes, repetitive group sampling plan, group acceptance sampling plan, and skip-lot sampling plans have been proposed for the attributes and variables quality characteristics. Acceptance sampling plans provide the decision (accept or reject) about the submitted lot on the basis of sample information selected from the lot. Through these plans, there is a chance of rejecting a good lot (producer's risk) and accepting a bad lot (consumer's risk). Therefore, new acceptance sampling plans are designed to minimize these two risks.

Several methodologies have been studied by various authors. Some of these studies are, Acceptance sampling plans based on the percentiles of Exponentiated Inverse Kumaraswamy Distribution [XX], resubmitted lot-based group acceptance sampling plans [XIX], multiple dependent state and repetitive sampling plan for Type II generalized Half-Logistic distribution [XI, XVIII].

In view of minimizing the inspection cost of the product quality the skip-lot Sampling plan (SkSP) is considered as more effective than the Single Sampling Plan. The plans comprise two phases of inspection, normal inspection phase and skipping inspection phase, which makes it preferable for real time use. The model SkSP-1 is the primary plan of SKSPs, instigated by Dodge [IX]. The enhanced variety of SkSP-1 plan designated as SkSP-2 was suggested by Perry [XVII] by incorporating the concept of reference plan. A modified skip-lot sampling plan was proposed in [XVI] and these plans were suggested with some adjustments for inspection error by Carr [VIII]. Optimal design of skip-lot group acceptance sampling plan and two-point method were designed by Aslam et. al. [II, III]. A new approach of skip-lot sampling plans having a capability of reducing normal inspection was discussed in [VI]. Further the skip-lot sampling plans were designed with some modifications by reducing the time and cost of inspection. Some of these models were studied and verified by several authors, Aslam et. al., [I] proposed the model SkSP-2 with two-stage group acceptance sampling plan as reference plan. The skip-lot sampling plan of model

SkSP-V designed for exponentiated Weibull distribution [V] and SkSP-V for life tests based on percentiles of Exponentiated Rayleigh distribution [XIV].

The present model is a new method of skip-lot sampling plan named as SkSP-R plan by incorporating the concept of resampling introduced by Balamurali et al. [VII]. A few more studies on SkSP-R model are [IV] and SkSP-R plan using popular statistical distributions [X], SkSP-R plan under the assumption of Gompertz distribution [XXI], percentile-based sampling plan of type SkSP-R based on exponentiated Rayleigh distribution was established in [XV]. The SkSP-R is a sampling procedure of continuous sampling along with the resampling structure for inspection.

The sampling plans are designed to confirm the lifetime of the item by estimating the probability of survival up to a predefined testing time (Failure probability) with the help of cumulative probability function of the corresponding distribution. To assess the mean life of the product, we use the mean or percentile value of the distribution. In this article, we established the SkSP-R sampling plan for the inspection of the variables by assuming that the quality characteristic follows Type II Generalized Half-Logistic Distribution (Type II GHLD). This article emphasizes the development of a SkSP-R life-test plan based on the percentiles of the chosen distribution. The structure of this paper is organized as follows: Section II presents the chief properties of Type II GHLD. Section III describes the design of the proposed SkSP-R plan. Section IV presents the optimization results and tables. Section V compares the proposed plan with the reference single sampling plan. Section VI illustrates the application using a real-data set. Finally, Section VII summarizes the main findings and concludes the paper.

II. Properties of Type II Generalized Half-Logistic Distribution

Consider a series system of θ components with individually and identically distributed (i.i.d) individual lifetimes with distribution function $G(t)$. The reliability function of such a system is given by $[1 - G(t)]^\theta$; hence, the cumulative distribution function (cdf) of the lifetime random variable of that series system is $1 - [1 - G(t)]^\theta$. Considering $G(t)$ as the distribution function of standard Half-Logistic model, Kantam *et al.* [XII] defined a new model called the standard Type II GHLD, whose cdf is given by

$$\begin{aligned} F(t; \theta) &= 1 - [1 - G(t)]^\theta \\ &= 1 - \left[1 - \left(1 - \frac{2}{1+e^t}\right)\right]^\theta = 1 - \frac{2^\theta}{(1+e^t)^\theta}; \quad 0 < t < \infty, \theta > 0 \end{aligned} \quad (1)$$

where θ is the shape parameter and the corresponding probability density function (pdf) is obtained as

$$f(t; \theta) = \frac{2^\theta \theta e^t}{(1+e^t)^{\theta+1}}; \quad 0 < t < \infty, \theta > 0 \quad (2)$$

Some properties of standard Type II GHLD are

$$\text{Median} = \log \left[2^{\frac{(\theta+1)}{\theta}} - 1 \right]$$

M. Rami Reddy et al

$$Mode = 2^\theta \theta \left[\frac{(1+e^t)e^t - (\theta+1)e^{2t}}{(1+e^t)^{\theta+2}} \right]$$

Survival function:

$$S(t) = 1 - F(t) = [2/(1 + e^t)]^\theta$$

and the reliability function is

$$R(x) = 1 - \left[1 - \frac{2^\theta}{(1+e^t)^\theta} \right] \quad (3)$$

The hazard function is a conditional failure rate which gives the instantaneous potential for failing at time t per unit time for an individual surviving to time t. The hazard function is given by

$$h(t) = f(t)/S(t) = \theta/(1 + e^{-t}) \quad (4)$$

The graphs for pdf, survival function and hazard function of standard Type II GHLD for different

values of shape parameters are shown in the following figures.

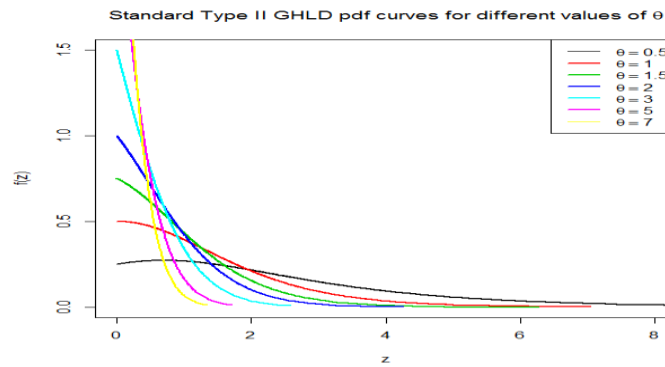


Fig.1. pdfs of standard Type II GHLD

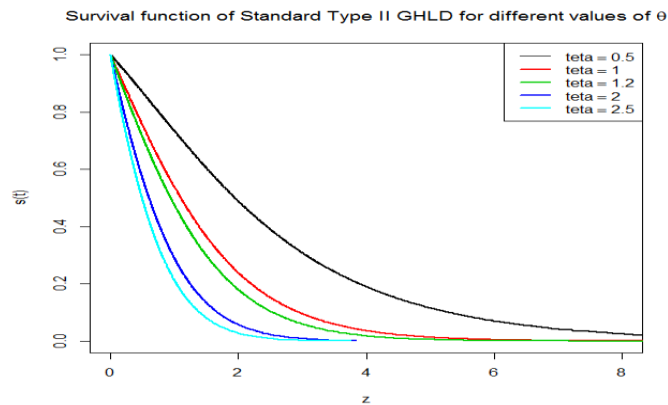


Fig. 2. Survival functions of standard Type II GHLD

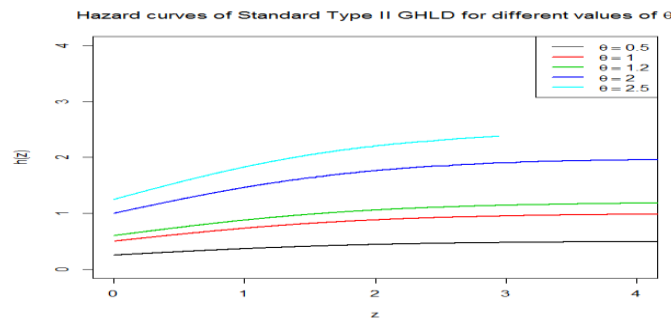


Fig. 3. Hazard functions of standard Type II GHLD

III. Design of SkSP-R Plan based on Time Truncated Life Test

As per the working process of SkSP-R plan suggested by Balamurali et al., (VII), the following are the steps of the SkSP-R plan under truncated life tests.

- Step-1:** Conduct a normal inspection for the failure items by considering a sample of n items randomly from the proposed lot and conduct the life test for the sample items for a specified period t_0 . Identify and count the failed items within the examination time, say, d . Accept the lot if $d \leq c$, reject the lot if $d > c$.
- Step-2:** When 'i' successive lots are accepted under normal inspection based on time truncated life test, terminate the normal check-up, and shift to the skipping inspection.
- Step-3:** During the skipping of inspection, select a fraction 'f' of lots in random for the inspection and inspect the lots using the procedure as given in Step-1. Continue the skipping inspection until a rejected lot is found.
- Step-4:** If a lot is rejected after 'k' consecutive lots have been accepted and then execute the resampling process for the next lot as described in Step-5, otherwise switch to normal inspection as in Step-1.
- Step-5:** During resampling process, conduct the inspection based on the reference plan and continue the skipping inspection if the lot is accepted. If the lot is not accepted, resampling is performed 'm' times and accept the lot if it has been accepted in $(m-1)^{\text{th}}$ resubmission or else reject the lot.
- Step-6:** Shift back to normal inspection as in Step-1 if a lot is rejected under resampling technique.
- Step-7:** Replace or remove the unsuccessful units with good one which are identified in the rejected lot.

The intended SkSP-R plan is totally specified by 6 parameters as mentioned below.

- I. 'n', the size of sample of the reference plan.
- II. 'c', Acceptance number of the reference plan.
- III. 'f', fraction of lots that are inspected in skipping sampling mode, ($0 < f < 1$)
- IV. 'i', the clearance number of the normal inspection mode.
- V. 'k', the clearance number of skipping inspection mode.
- VI. 'm', the number of times, resubmission of lots for inspection is allowed.

M. Rami Reddy et al

The intended plan is specified as “SkSP-R(n, c, f, i, k, m)”. According to Balamurali et al.(2014), the Operating Characteristic (OC) function of the SkSP-R plan is defined as

$$P_a = \frac{(1-f)P^i + fP^k(P^i - P)(1-Q^m) + fP}{P^i(1+fQP^k) + f(1-P^i)[1-P^k(1-Q^m)]} \quad (5)$$

And the Average Sample Number (ASN) function of the SkSP-R plan is

$$ASN = \frac{n f Q P^{i+k} - n f P^k (1-P^i)(1-Q^m) + n f}{P^i(1+f Q P^k) + f(1-P^i)[1-P^k(1-Q^m)]} \quad (6)$$

As per the reference plan SSP the acceptance probability of a lot is defined as P and Q=1-P, which are computed as

$$P = \sum_{j=0}^c \binom{n}{j} p^j (1-p)^{n-j} \quad (7)$$

Where $p = F(t, \delta_0)$ represents the failure probability at time t given a determined 100q-th percentile of the lifetime t_q^0 and p depends only on $\delta_0 = \frac{t}{t_q^0}$.

We generally consider $t_0 = \delta \mu_0$ for a given constant (termination ratio), since it is appropriate to set the termination time as a multiple of specified median value μ_0 .

Percentile and Failure probability function of the distribution

Let η_p denote the p -th percentile. Then

$$p = 1 - \left(\frac{2}{1 + e^{\eta_p}} \right)^\theta$$

Thus, the percentile function of Type II GHLD is given by

$$\eta_p = \ln \left[\frac{2}{(1-p)^{1/\theta}} - 1 \right]$$

For $p = 0.5$, we get the median of the distribution as

$$\eta_{0.5} = \ln(2^{(\theta+1)/\theta} - 1)$$

Since $\mu = \sigma \eta_p$, $\mu_0 = \sigma_0 \eta_p$, and

$$r = \frac{\mu}{\mu_0}, \quad \delta = \frac{t_0}{\mu_0}$$

we have

$$\frac{t_0}{\sigma} = \frac{\delta \mu_0}{\sigma} = \frac{\delta \eta_p}{r}.$$

Therefore,

$$F(t_0) = 1 - \left[\frac{2}{1 + \exp(t_0/\sigma)} \right]^\theta$$

and hence

$$p = 1 - \left[\frac{2}{1 + \exp(\delta/r)} \right]^\theta$$

Where $r = \frac{\mu}{\mu_0}$

Thus, the failure probability at the termination time t_0 , expressed in terms of the shape parameter θ , termination ratio δ , and quality ratio μ/μ_0 , is given by

$$p = 1 - \left[\frac{2}{1 + \exp\left(\frac{\delta}{\frac{\mu}{\mu_0}}\right)} \right]^\theta \quad (8)$$

This failure probability is used to obtain the acceptance probability of the reference single sampling plan and, subsequently, the OC and ASN of the SkSP-R plan. Thus, for a reference single sampling plan with parameters (n, c) the acceptance probability is given by

$$P = \sum_{j=0}^c \binom{n}{j} p^j (1-p)^{n-j}$$

where p is given by Equation (8).

The major objective of designing sampling plans is minimizing the producer's risk (α) and consumer's risk (β). It is essential that the probability of acceptance of a good lot be greater than $(1 - \alpha)$ at Acceptable Quality Level (AQL) and the probability of acceptance of poor-quality lot should be less than β at Limiting Quality Level (LQL). The AQL and LQL are represented by p_0 and p_1 respectively. The sampling plan is designed in accordance with the OC curve passes through the points $(p_0, 1 - \alpha)$ and (p_1, β) . The following nonlinear optimization problem will be used to determine the plan parameters of the SkSP-R model, where the objective function is to minimize ASN function at LQL and the corresponding constraints are the lot acceptance probabilities at AQL and LQL respectively.

Minimize

$$ASN(p_1) = \frac{nfQ_1P_1^{i+k} - nfP_1^k(1 - P_1^i)(1 - Q_1^m) + nf}{P_1^i(1 + fQ_1P_1^k) + f(1 - P_1^i)[1 - P_1^k(1 - Q_1^m)]} \quad (9)$$

Subject to

$$P_a(p_0) = \frac{(1 - f)P_0^i + fP_0^k(P_0^i - P_0)(1 - Q_0^m) + fP_0}{P_0^i(1 + fQ_0P_0^k) + f(1 - P_0^i)[1 - P_0^k(1 - Q_0^m)]} \geq 1 - \alpha \quad (10)$$

$$P_a(p_1) = \frac{(1 - f)P_1^i + fP_1^k(P_1^i - P_1)(1 - Q_1^m) + fP_1}{P_1^i(1 + fQ_1P_1^k) + f(1 - P_1^i)[1 - P_1^k(1 - Q_1^m)]} \leq \beta \quad (11)$$

For the numerical calculations, $\alpha=0.05$ and $\beta \in \{0.25, 0.10, 0.05, 0.01\}$. The termination ratios are $\delta=0.5$ and 0.75 , and $\mu/\mu_0 \in \{2, 4, 6, 8, 10, 12\}$. The resubmission parameter is fixed at $m=2$. In the numerical search the values $i=2$, $f=0.5$, $k=1$ is fixed, while integer values n and $c=0$ to $n-1$, are examined. For each

M. Rami Reddy et al

candidate plan, both AQL and LQL constraints are evaluated. Only feasible plans satisfying both constraints are retained, and the plan with minimum ASN at LQL is selected. The residuals are defined as $R_AQL = L(AQL) - 0.95$ and $R_LQL = \beta - L(LQL)$. The smaller residual is reported as the limiting constraint. Thus, the discrete search establishes the minimum ASN among the enumerated feasible integer plans without requiring simulation.

IV. Description of Tables and Optimization Results

To exhibit the implementation of the designed SkSP-R plan, Tables 1–4 present the feasible plan parameters for the selected values of θ , δ , β and μ/μ_0 . For every row, $L(AQL)$ and $L(LQL)$ are reported separately, together with their constraint residuals and the ASN at LQL. The parameters $i=2$, $f=0.5$, $k=1$ and $m=2$ are fixed throughout. The tables show that, in general, larger quality ratios and larger termination ratios permit smaller feasible sample sizes and/or lower ASN, while stricter consumer's-risk requirements increase the required sample size.

For each combination of the distributional and quality parameters, an integer search is performed over n and c , while $i=2$, $f=0.5$, $k=1$ and $m=2$ are kept fixed. For each combination of (n, c) , the OC is evaluated separately at AQL and LQL. Only plans satisfying both constraints are retained. Among the feasible plans, the plan with minimum ASN at LQL is selected.

Table 1: Feasible plan parameters of SkSP-R with $\theta=1.5$ and $\delta=0.5$.

β	μ/μ_0	n	c	$L(AQL)$	$L(LQL)$	R_AQL	R_LQL	Limiting	ASN(LQL)
0.25	2	31	8	0.950877	0.239178	0.000877	0.010822	AQL	28.802405
	4	11	2	0.961506	0.242909	0.011506	0.007091	LQL	10.200072
	6	8	1	0.956056	0.201318	0.006056	0.048682	AQL	7.574442
	8	8	1	0.974159	0.201318	0.024159	0.048682	AQL	7.574442
	10	8	1	0.982956	0.201318	0.032956	0.048682	AQL	7.574442
	12	8	1	0.987907	0.201318	0.037907	0.048682	AQL	7.574442
0.10	2	53	13	0.955327	0.089296	0.005327	0.010704	AQL	52.322148
	4	18	3	0.959346	0.092628	0.009346	0.007372	LQL	17.753981
	6	14	2	0.973494	0.098364	0.023494	0.001636	LQL	13.786725
	8	10	1	0.960235	0.099431	0.010235	0.000569	LQL	9.844672
	10	10	1	0.973629	0.099431	0.023629	0.000569	LQL	9.844672
	12	10	1	0.981203	0.099431	0.031203	0.000569	LQL	9.844672
0.05	2	67	16	0.955108	0.045097	0.005108	0.004903	LQL	66.758656
	4	25	4	0.961034	0.038006	0.011034	0.011994	AQL	24.934911
	6	17	2	0.955795	0.037837	0.005795	0.012163	AQL	16.956113
	8	17	2	0.978537	0.037837	0.028537	0.012163	LQL	16.956113
	10	12	1	0.962649	0.048371	0.012649	0.001629	LQL	11.950663
	12	12	1	0.973281	0.048371	0.023281	0.001629	LQL	11.950663
0.01	2	100	23	0.956785	0.009419	0.006785	0.000581	LQL	99.982741
	4	34	5	0.953527	0.009396	0.003527	0.000604	LQL	33.994160
	6	26	3	0.961534	0.008412	0.011534	0.001588	LQL	25.996410
	8	22	2	0.957905	0.007203	0.007905	0.002797	LQL	21.997765
	10	22	2	0.975890	0.007203	0.025890	0.002797	LQL	21.997765
	12	22	2	0.984886	0.007203	0.034886	0.002797	LQL	21.997765

Table 2: Feasible plan parameters of SkSP-R with $\theta=1.5$ and $\delta=0.75$.

β	μ/μ_0	n	c	L(AQL)	L(LQL)	R_AQL	R_LQL	Limiting	ASN(LQL)
0.25	2	24	9	0.959713	0.213918	0.009713	0.036082	AQL	22.586441
	4	8	2	0.955539	0.183450	0.005539	0.066550	AQL	7.636393
	6	5	1	0.963297	0.234549	0.013297	0.015451	AQL	4.656802
	8	5	1	0.978570	0.234549	0.028570	0.015451	LQL	4.656802
	10	5	1	0.985937	0.234549	0.035937	0.015451	LQL	4.656802
	12	5	1	0.990059	0.234549	0.040059	0.015451	LQL	4.656802
0.10	2	34	12	0.952364	0.088802	0.002364	0.011198	AQL	33.569505
	4	12	3	0.963473	0.093948	0.013473	0.006052	LQL	11.831738
	6	10	2	0.970347	0.068968	0.020347	0.031032	AQL	9.920301
	8	7	1	0.957887	0.076729	0.007887	0.023271	AQL	6.932114
	10	7	1	0.972127	0.076729	0.022127	0.023271	AQL	6.932114
	12	7	1	0.980157	0.076729	0.030157	0.023271	LQL	6.932114
0.05	2	46	16	0.959385	0.042756	0.009385	0.007244	LQL	45.850203
	4	17	4	0.962422	0.032852	0.012422	0.017148	AQL	16.966494
	6	11	2	0.961506	0.041542	0.011506	0.008458	LQL	10.966083
	8	11	2	0.981524	0.041542	0.031524	0.008458	LQL	10.966083
	10	8	1	0.963799	0.043314	0.013799	0.006686	LQL	7.973300
	12	8	1	0.974159	0.043314	0.024159	0.006686	LQL	7.973300
0.01	2	66	22	0.956359	0.008770	0.006359	0.001230	LQL	65.990106
	4	23	5	0.955608	0.007759	0.005608	0.002241	LQL	22.997293
	6	17	3	0.966300	0.008774	0.016300	0.001226	LQL	16.997449
	8	14	2	0.964331	0.008656	0.014331	0.001344	LQL	13.997955
	10	14	2	0.979749	0.008656	0.029749	0.001344	LQL	13.997955
	12	11	1	0.952355	0.007595	0.002355	0.002405	AQL	10.998759

Table 3: Feasible plan parameters of SkSP-R with $\theta=2$ and $\delta=0.5$.

β	μ/μ_0	n	c	L(AQL)	L(LQL)	R_AQL	R_LQL	Limiting	ASN(LQL)
0.25	2	27	9	0.959500	0.237254	0.009500	0.012746	AQL	25.111301
	4	9	2	0.955238	0.203109	0.005238	0.046891	AQL	8.514060
	6	6	1	0.957968	0.215210	0.007968	0.034790	AQL	5.643036
	8	6	1	0.975296	0.215210	0.025296	0.034790	AQL	5.643036
	10	6	1	0.983714	0.215210	0.033714	0.034790	AQL	5.643036
	12	6	1	0.988450	0.215210	0.038450	0.034790	LQL	5.643036
0.10	2	42	13	0.951411	0.081376	0.001411	0.018624	AQL	41.546394
	4	14	3	0.958400	0.090122	0.008400	0.009878	AQL	13.817927
	6	11	2	0.971890	0.089928	0.021890	0.010072	LQL	10.857502
	8	8	1	0.956458	0.083815	0.006458	0.016185	AQL	7.908816
	10	8	1	0.971092	0.083815	0.021092	0.016185	LQL	7.908816
	12	8	1	0.979375	0.083815	0.029375	0.016185	LQL	7.908816
0.05	2	53	16	0.950933	0.040363	0.000933	0.009637	AQL	52.845278
	4	19	4	0.963224	0.042695	0.013224	0.007305	LQL	18.938294
	6	13	2	0.955932	0.038371	0.005932	0.011629	AQL	12.965531
	8	13	2	0.978603	0.038371	0.028603	0.011629	LQL	12.965531
	10	10	1	0.955576	0.031899	0.005576	0.018101	AQL	9.981372
	12	10	1	0.968170	0.031899	0.018170	0.018101	LQL	9.981372

0.01	2	79	23	0.951695	0.007918	0.001695	0.002082	AQL	78.990322
	4	30	6	0.967425	0.007448	0.017425	0.002552	LQL	29.996744
	6	20	3	0.960872	0.008135	0.010872	0.001865	LQL	19.997415
	8	17	2	0.956339	0.006519	0.006339	0.003481	LQL	16.998583
	10	17	2	0.974963	0.006519	0.024963	0.003481	LQL	16.998583
	12	17	2	0.984290	0.006519	0.034290	0.003481	LQL	16.998583

Table 4: Feasible plan parameters of SkSP-R with $\theta=2$ and $\delta=0.75$.

β	μ/μ_0	n	c	L(AQL)	L(LQL)	R_AQL	R_LQL	Limiting	ASN(LQL)
0.25	2	19	9	0.964304	0.247467	0.014304	0.002533	LQL	17.575453
	4	6	2	0.961296	0.222623	0.011296	0.027377	AQL	5.622303
	6	4	1	0.961137	0.219150	0.011137	0.030850	AQL	3.754709
	8	4	1	0.977281	0.219150	0.027281	0.030850	AQL	3.754709
	10	4	1	0.985077	0.219150	0.035077	0.030850	LQL	3.754709
	12	4	1	0.989444	0.219150	0.039444	0.030850	LQL	3.754709
0.10	2	29	13	0.961867	0.097362	0.011867	0.002638	LQL	28.566293
	4	10	3	0.954759	0.067597	0.004759	0.032403	AQL	9.923202
	6	8	2	0.967831	0.060406	0.017831	0.039594	AQL	7.950134
	8	8	2	0.984666	0.060406	0.034666	0.039594	AQL	7.950134
	10	6	1	0.965380	0.048606	0.015380	0.051394	AQL	5.975105
	12	6	1	0.975296	0.048606	0.025296	0.051394	AQL	5.975105
0.05	2	35	15	0.950638	0.042818	0.000638	0.007182	AQL	34.885713
	4	13	4	0.965870	0.040407	0.015870	0.009593	LQL	12.961970
	6	9	2	0.955238	0.030522	0.005238	0.019478	AQL	8.984597
	8	9	2	0.978377	0.030522	0.028377	0.019478	LQL	8.984597
	10	6	1	0.965380	0.048606	0.015380	0.001394	LQL	5.975105
	12	6	1	0.975296	0.048606	0.025296	0.001394	LQL	5.975105
0.01	2	53	22	0.953047	0.008228	0.003047	0.001772	LQL	52.992995
	4	18	5	0.954872	0.007741	0.004872	0.002259	LQL	17.997892
	6	13	3	0.967650	0.009989	0.017650	0.000011	LQL	12.997480
	8	11	2	0.962212	0.007486	0.012212	0.002514	LQL	10.998794
	10	11	2	0.978503	0.007486	0.028503	0.002514	LQL	10.998794
	12	11	2	0.986596	0.007486	0.036596	0.002514	LQL	10.998794

V. Comparison with the reference single sampling plan

To distinguish the intrinsic effect of the skip-lot mechanism from changes in the reference-plan design, a comparison with the single sampling plan (SSP) is made under the same lifetime distribution, termination ratio, quality ratio and risk constraints. For an SSP the ASN equals its sample size. For the proposed SkSP-R, ASN/n represents the average inspection intensity relative to a full reference-plan inspection. The table also gives an SSP design optimized under the same two-point constraints.

Table 5:

Setting	SkSP n	SkSP c	SSP n*	SSP c*	P_AQL (same n,c)	P_LQL (same n,c)	SkSP ASN	ASN/n	Reduction
$\theta=1.5, \delta=0.5,$ $\beta=0.25, r=2$	31	8	40	11	0.907430	0.209200	28.80240	92.91%	7.09%
$\theta=1.5, \delta=0.75,$ $\beta=0.25, r=2$	24	9	28	11	0.923172	0.188680	22.58644	94.11%	5.89%
$\theta=2, \delta=0.5,$ $\beta=0.25, r=2$	27	9	34	12	0.922789	0.207643	25.11130	93.00%	7.00%

$\theta=2, \delta=0.75, \beta=0.25, r=2$	19	9	24	12	0.931516	0.215893	17.57545	92.50%	7.50%
$\theta=1.5, \delta=0.5, \beta=0.10, r=6$	14	2	18	3	0.948551	0.091354	13.78672	98.48%	1.52%

For example, at $\theta=1.5$, $\delta=0.5$, $\beta=0.25$ and $\mu/\mu_0=2$, the selected SkSP-R plan is SkSP-R (31,8,0.5,2,1,2), with ASN=28.802405. Under the same $n=31$ and $c=8$, the reference SSP would inspect all 31 items whenever a lot is inspected, whereas the SkSP-R plan has $ASN/n=92.91\%$, corresponding to a 7.09% reduction in average inspection intensity. The separately optimized SSP satisfying the same two-point constraints requires $n=40$ and $c=11$. Thus, changes in ASN across Tables 1–4 should be interpreted as the combined effect of the reference-plan sample size, the lot-acceptance probability and the skip-lot inspection frequency, rather than as an isolated effect of a single distributional parameter.

VI. Real-Data Example

In this section, the proposed Type II GHLD is illustrated with following real lifetime data on clean upgradient groundwater monitoring wells in ($\mu g/L$) from Vinyl chloride data taken from Krishnamoorthy *et al.*, [XIII], the observations are 0.1, 0.1, 0.2, 0.2, 0.4, 0.4, 0.4, 0.5, 0.5, 0.5, 0.6, 0.6, 0.8, 0.9, 0.9, 1.0, 1.1, 1.2, 1.2, 1.3, 1.8, 2.0, 2.0, 2.3, 2.4, 2.5, 2.7, 2.9, 3.2, 4.0, 5.1, 5.3, 6.8, 8.0.

The shape parameter θ is estimated using ML method based on the Vinyl chloride data as $\hat{\theta} = 0.6809$. To test the goodness of fit, we apply the Kolmogorov-Smirnov test, it is observed that for the data set Kolmogorov-Smirnov statistic is 0.15704 with p-value is 0.4224. Thus, the data set is reasonably fitted for Type II GHLD. Figure 4 display Superimposed Empirical, Theoretical Density plots and Q-Q Plot, which shows that Type II GHLD well fitted for this data set.

The 25th percentile lifetime and termination-ratio setting used in the submitted example are retained. The specified design values are $\theta=0.6809$, $t_0/\mu_0=0.5$, $\beta=0.25$ and $\mu/\mu_0=4$. Using the two-point feasibility criterion, the selected plan parameters for the considered setup are SkSP-R ($n=23$, $c=2$, $i=2$, $f=0.5$, $k=1$, $m=2$).

Initiate the normal inspection plan with $n=23$ sample observations. If $d=2$ items fail before the termination time, the lot is accepted since $d \leq c$.

The inspection procedure is described as follows:

- Initiate the normal inspection plan with $n = 23$ sample observations for a pre assumed value $t_0 = 0.25$. Measure the observations that are failed before the duration of the experiment time, say $d = 2$. Hence the lot is accepted since $d \leq c=2$.
- When $i = 2$, Through this normal inspection two consecutive lots are not rejected based on time truncated life test. Now, terminate the normal inspection and follow Skipping Inspection.
- Test only a fraction $f = 0.5$ of lots that are chosen at random during the Skipping Inspection. This process is continued until a sampled lot is rejected.
- The clearance number for skipping inspection is $k = 1$. Following a rejection in the skipping inspection stage, the subsequent lot is handled according to the re-sampling procedure specified by the SkSP-R plan.

M. Rami Reddy et al

e. For this selected design, the failure probabilities at the AQL and LQL points are $p = 0.042936$ and 0.174103 , respectively.

The corresponding acceptance probabilities are

$$L(AQL)=0.961322 \text{ and } L(LQL)=0.240082,$$

The ASN at LQL is 21.359356. Hence, both risk constraints are satisfied:

$$L(AQL) \geq 0.95 \text{ and } L(LQL) \leq 0.25.$$

f. If the lot is failed to accept in re-sampling process, then directly apply the normal inspection as in Step (a).

g. All identified non-conforming items in the rejected lots are discarded or rectified with satisfactory units.

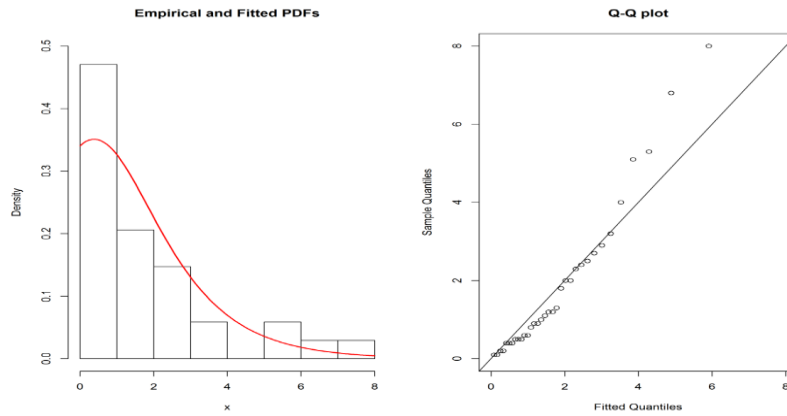


Fig. 4. Superimposed Empirical, Theoretical Density plots and Q-Q Plot of the fitted Type II GHLD for Vinyl chloride data.

VII. Summary and Conclusion

In this article, a SkSP-R plan for Type II GHLD is developed using a two-point optimization approach based on AQL and LQL. The numerical procedure evaluates the lot acceptance probability separately at the two design points and selects the minimum-ASN feasible integer plan. The tables provide the plan parameters, AQL and LQL acceptance probabilities, constraint residuals and ASN values for several distributional and quality settings. The results show that the proposed procedure can provide feasible plans while reducing the average inspection intensity through the skip lot mechanism under the specified risk constraints. The methodology can be extended in future work to shifted fractions non-conforming and to other probability distributions.

Conflict of Interest:

The authors declare no conflicts of interest regarding this paper.

M. Rami Reddy et al

References

- I. Aslam, M., Balamurali, S., Azam, M., & Jun, C.-H. (2014). Skip-lot sampling plan of type SkSP-2 with two-stage group acceptance sampling plan as reference plan, *Communications in Statistics — Simulation and Computation*, 43(4), 777–789. 10.1080/03610918.2012.715224
- II. Aslam, M., Balamurali, S., Jun, C.-H., & Ahmad, M. (2013). Optimal design of skip lot group acceptance sampling plans for the Weibull distribution and the generalized exponential distribution, *Quality Engineering*, 25(3), 1-10. 10.1080/08982112.2013.794834
- III. Aslam, M., Balamurali, S., Jun, C.-H. & Ahmad, M. (2010). Optimal designing of a skip-lot sampling plan by two-point method, *Pakistan Journal of Statistics*, 26(4), 585-592.
- IV. Aslam, M., Balamurali, S., Jun, C.-H., & Hussain, B. (2015). Design of SkSP-R variables sampling plans, *Revista Colombiana de Estadística*, 38(2), 413-429. 10.15446/rce.v38n2.51669.
- V. Aslam, M., Srinivasa Rao, G., Khan, N., & Jun, C.-H. (2014). SkSP-V sampling plan for the exponentiated Weibull distribution, *Journal of Testing and Evaluation*, 42(3), 687-694. 10.1520/JTE20130051.
- VI. Balamurali, S., & Jun, C.-H. (2011). A new system of skip-lot sampling plans having a provision for reducing normal inspection, *Applied Stochastic Models in Business and Industry*, 27(3), 348–363. 10.1002/asmb.844.
- VII. Balamurali, S., Aslam, M., & Jun, C.H. (2014). A new system of skip-lot sampling plans including resampling, *The Scientific World Journal*, 192412, 10.1155/2014/192412.
- VIII. Carr, W. E. (1982). Sampling Plan Adjustment for Inspection Error and Skip-Lot Plan. *Journal of Quality Technology*, 14(3), 105–116. 10.1080/00224065.1982.11978802.
- IX. Dodge, H. F. (1955). Skip-lot sampling plan. *Industrial Quality Control*, 11(5), 3-5.
- X. Hussain, J., Balamurali, S., & Aslam, M. (2020). Design of SkSP-R plan for popular statistical distributions, *Journal of Modern Applied Statistical Methods*, 19(1), eP2860. 10.22237/jmasm/1608552240.
- XI. Jilani, Sd., Rao, G. S., & Rao, A. V. (2024). Designing of multiple dependent state sampling plan for Type-II generalized half logistic distribution. *Research in Mathematics*, 11(1). 10.1080/27684830.2024.2326706.

- XII. Kantam, R. R. L., Ramakrishna, V., and Ravikumar, M. S. (2014). Estimation and testing in type II generalized half logistic distribution, *Journal of Modern Applied Statistical Methods*, 12(1), 198-206. 10.22237/jmasm/1398917760.
- XIII. Krishnamoorthy, K., Mathew, T., & Mukherjee, S. (2008). Normal-based methods for a gamma distribution. *Technometrics*, 50(1), 69-78. 10.1198/004017007000000353.
- XIV. P. Umamaheswari, K. P. Veerakumari, and S. Suganya. (2022). Construction and Selection of Skip Lot Sampling Plan of type Sksp-V for life tests based on Percentiles of Exponentiated Rayleigh Distribution. *Reliability: Theory & Applications*, 4(71), 123-131. 10.24412/1932-2321-2022-471-123-131.
- XV. P. Umamaheswari., K. Pradeepa Veerakumari, and S.Suganya. (2023). Selection procedure for skip lot sampling plan of type SkSP-R based on percentiles of exponentiated Rayleigh distribution, *Reliability: Theory & Applications*, 1(72), 432-439. 10.24412/1932-2321-2023-172-432-439.
- XVI. Parker, R. D., & Kessler, L. (1981). A Modified Skip-Lot Sampling Plan. *Journal of Quality Technology*, 13(1), 31–35. 10.1080/00224065.1981.11980983.
- XVII. Perry, R. L. (1973). Skip-Lot Sampling Plans. *Journal of Quality Technology*, 5(3), 123–130. 10.1080/00224065.1973.11980585.
- XVIII. Rao, G.S., Jilani, S.D. & Rao, A.V. (2021). Designing of multiple dependent state repetitive sampling plan for type-II generalized half logistic distribution. *Int J Syst Assur Eng Manag* 12, 1397–1409. 10.1007/s13198-021-01338-x.
- XIX. Rao, G. S., Jilani, S., & Rao, A. V. (2020). Group acceptance sampling plans for resubmitted lots under Type II generalized half logistic distribution, *Journal of Scientific Research*, 12(3), 311-325. 10.3329/jsr.v12i3.44406.
- XX. Reddy MR, Rao BS, Rosaiah K. (2024). Acceptance Sampling Plans based on Percentiles of Exponentiated Inverse Kumaraswamy Distribution. *Indian Journal of Science and Technology*. 17(16), 1681-1689. 10.17485/IJST/v17i16.222.
- XXI. Tripathi, H., Al-Omari, A. I., & Alomani, G. A. (2022). A SkSP-R plan under the assumption of Gompertz distribution, *Applied Sciences*, 12(12), 6131. 10.3390/app12126131.