



APPLICATION OF RSM IN NANOFLUIDS FOR HEAT TRANSFER ENHANCEMENT: A COMPREHENSIVE REVIEW

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Abstract

Response Surface Methodology (RSM) is a powerful statistical and mathematical tool in designing experiments, modeling, and optimizing the heat transfer efficiency in nanofluids. This paper provides a detailed review of the use of Response Surface Methodology in nanofluids-based heat transfer systems, particularly in experimental design, modeling, interaction between factors, and optimization. Some of the common design techniques used for establishing a relationship between second-order polynomials with operating parameters and thermal response include Box-Behnken Design (BBD) and Central Composite Design (CCD). Applications of RSM in thermal conductivity, heat transfer augmentation, flow analysis, entropy generation, and hybrid nanofluids are covered in this paper. Comparison with artificial neural networks (ANN) suggests that ANN gives more accurate prediction results in the case of a highly non-linear system, while RSM gives explicit correlation between the operating parameters and the thermal responses along with sensitivity analysis. Some of the important areas requiring more research include experimental validation, multi-objective optimization, transient analysis, and uncertainty analysis, among others.

Keywords: Nanofluid, Hybrid Nanofluid, Response Surface Methodology, Artificial Neural Networks, Machine Learning, AI

I. Introduction

This has been caused by the fast advancement in modern thermal engineering systems. The poor thermal conductivity behavior of ordinary fluids such as water, ethylene glycol, and oil is the reason why their thermal transport capabilities are hindered in many engineering processes. In order to address this problem, Choi

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and Eastman [V] proposed the concept of nanofluids where nanoparticles are suspended in ordinary fluids to increase their thermal transport properties. Research works have revealed that the inclusion of nanoparticles could increase the thermal conductivities of ordinary fluids [VI, VII, X]. Das et al. [VI] indicated that temperature plays an important role in determining the enhancement in the thermal conductivity of nanofluids, whereas Buongiorno [IV] developed a transport theory based on Brownian motion and thermophoresis of nanoparticles.

Studies of nanofluids and hybrid nanofluids have been conducted for different applications in engineering such as heat exchangers, cooling devices, solar energy systems, and thermal management systems [X, XIV]. Effects of magnetism, porosity, thermal radiation, and entropy generation on the flow and heat transfer behaviors of nanofluids have been considered in some studies [II, XIII, XVI, XX]. However, there are many factors affecting the performance of nanofluids, including nanoparticle volume fraction, temperature, viscosity, thermal conductivity, magnetic field intensity, and fluid flow, among others. Therefore, optimizing these factors may be complicated, especially using the traditional experimental method.

For this reason, statistical methods like Response Surface Methodology (RSM) have increasingly become popular in modeling and optimizing nanofluids [I, XI, XII, XVIII]. RSM offers a mathematical and statistical approach that helps evaluate the effects and interactions of several parameters by minimizing the number of investigations required. Some recent works that have used RSM include optimization of thermal conductivity, heat transfer, entropy generation, and flow properties of nanofluids and hybrid nanofluids [I, XII, XIII, XV, XVIII]. Simultaneously, Artificial Neural Network (ANN) has been used to predict the nonlinear behavior of nanofluids, and combined RSM-ANN models have shown promising results in predicting the thermal performance of nanofluids and optimizing the same [VIII, IX, XX]. This research therefore uses RSM to statistically examine the heat transfer properties of the nanofluids under consideration and determine the effect and interactions of the involved parameters on the thermal performance. Providing an in-depth investigation of RSM applications in nanofluid engineering is the primary goal of this review. Along with identifying significant research gaps and potential future paths, the paper also contrasts RSM with current machine learning techniques.

II. Fundamentals of RSM

An effective statistical technique for creating empirical models, examining the impacts of several factors, and identifying ideal operating conditions with fewer experiments is Response Surface Methodology (RSM). To create second-order polynomial models, RSM uses design of experiments (DOE) methods, such as Box-Behnken Design (BBD) and Central Composite Design (CCD). The 2nd-order RSM model is generally represented as:

$$\tilde{\delta} = \tilde{\beta}_0 + \sum_{i=1}^k \tilde{\beta}_i \tilde{y}_i + \sum_{i=1}^k \tilde{\beta}_{ii} \tilde{y}_i^2 + \sum_{i < j}^k \tilde{\beta}_{ij} \tilde{y}_i \tilde{y}_j$$

$\tilde{\delta}$ = Predicted response

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$\tilde{\beta}_0$ = Constant coefficient

$\tilde{\beta}_i$ = Linear regression coefficient

$\tilde{\beta}_{ii}$ = Quadratic coefficient

$\tilde{\beta}_{ij}$ = Interaction coefficient

$\tilde{y}_i$ = Independent variables (Temperature, concentration, etc.)

$\tilde{y}_i^2$ = Curvature effects

$\tilde{y}_i\tilde{y}_j$ = Interaction between two variables

Experimental design involved in RSM:

- Central Composite Design,
- Box-Behnken Design,
- Full Factorial Design,
- Fractional Factorial Design

Advantages of RSM:

- Reduced experimental cost,
- Identification of significant variables,
- Development of predictive correlations,
- Multi-objective optimization capability,
- Visualization through contour and response surface plots

II.i. Model adequacy and limitations of second-order RSM

Polynomial models of order two are commonly employed in RSM studies on nanofluids; however, they depend upon the chosen design space. A quadratic surface describes the local empirical approximation of the physical problem, and hence it may not capture the whole nonlinear nature of the transport equations. Consequently, the existence of coupling effects, including nanoparticle concentration, temperature, Brownian motion, thermophoresis, magnetic force, viscous dissipation, chemical reactions, and non-Newtonian flow, cannot alone prove the inefficiency of RSM.

Instead, model adequacy is to be judged based on residual analysis, goodness of fit, ANOVA, lack of fit test, prediction error, and independent verification if possible. Quadratic models may be sufficient to describe the nonlinear physical system only within the studied range, but strong curvature, regime change, and higher interaction effects may reduce their predictive ability over larger ranges.

Therefore, RSM should be viewed as the statistical interpretation of the local surrogate model and optimizer and not as the universal description of the physics of nanofluids transport. The success or failure of individual works should not be considered as generalizable to all highly nonlinear nanofluid systems.

III. Application of RSM in nanofluids

III.i. Thermal conductivity optimization

Improving the thermal conductivity of nanofluids is an important goal in the field of nanofluids. Nanoparticles enhance the transport properties of nanofluids [V, VII, XXVII, VI]. Peng et al. [XVIII] used RSM to optimize CuO nanofluids, and found that temperature and concentration of the nanoparticles were important variables. Yaw et al. [XXX] optimized viscosity and thermal conductivity of hybrid nanofluids made of graphene nanoplatelet and cellulose nanocrystals. From the findings, it was shown that RSM could give accurate predictions with minimum experimentation.

III.ii. Viscosity and Rheological behavior

Viscosity has a significant impact on the total thermal system performance and the amount of electricity needed for pumping. Overloading with nanoparticles can reduce the benefits of heat transfer by increasing viscosity. The combined optimization of thermal conductivity and viscosity using RSM was examined by Yaw et al. [XXX]. Nonlinear correlations between temperature and nanoparticle concentration were established by their analysis. RSM is extremely helpful for balancing flow resistance and thermal enhancement in nanofluid systems, considering that it can analyze multiple responses simultaneously.

III.iii. Heat transfer enhancement

In thermal engineering systems, improving heat transfer is a critical performance parameter. Convective heat transfer enhancement of nanofluids was founded by Xuan and Li [XXVIII], Buongiorno [IV], Wang and Mujumdar [XXVII], and Kakaç and Pramuanjaroenkij [X]. In order to maximize heat transfer and minimize energy losses, Yaw et al. [XXIX] adopted multi-objective RSM optimization. Statistical models were established by Khan et al. [XI] to maximize heat transfer characteristics under various operating circumstances. These research investigations demonstrated that RSM has the potential to efficiently determine optimum operating conditions that optimize thermal performance.

III.iv. MHD nanofluid flow

Because magnetic fields can regulate fluid motion and heat transfer characteristics, magnetohydrodynamic (MHD) nanofluid flows have received a lot of interest. Salehi et al. [XXIV] numerically investigated MHD nanofluid flow under thermal radiation effects. Alsaedi et al. [II] analyzed hybrid nanofluid flow between coaxial cylinders subjected to magnetic fields. Khashi'ie et al. [XIII] employed RSM to determine optimal magnetic and flow parameter combinations of hybrid nanofluid flow through permeable wedges.

III.v. Entropy generation and chemical reactions

Entropy generation analysis is useful for obtaining information on thermodynamic irreversibilities in the system of nanofluids. Zhang et al. [XXXI] conducted studies on entropy generation in nanofluid heat transfer systems. Raja et al. [XX] combined RSM with intelligent computing methods for modeling entropy generation in MHD nanofluids. Raja et al. [VIII] used ANN approaches for studying the effects of viscous

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dissipation and entropy generation with chemical reaction. Kumar et al. [XV] showed that RSM could be used for modeling time-dependent Carreau nanofluid flow and entropy generation.

III.vi. Hybrid nanofluids

To achieve improved thermal performance, hybrid nanofluids incorporate two or more nanoparticles. In their review of hybrid nanofluids' engineering applications, Kshirsagar and Venkatesh [XIV] highlighted their superior thermal behavior. For Darcy–Forchheimer hybrid nanofluid flows, Alhadri et al. [I] applied both RSM and ANN techniques. While Islam et al. [IX] combined ANN and RSM for the optimization of Cu–Al₂O₃/water hybrid nanofluids under MHD conditions, Yaw et al. [XXX] employed RSM to optimize graphene-based hybrid nanofluids. According to these studies, RSM is extremely beneficial for handling the various design variables related to hybrid nanofluid systems.

IV. Comparison with machine learning methods

In addition to RSM, some researchers used artificial neural networks (ANN) and machine-learning approaches for the modeling and optimization of nanofluids. Uysal et al. [XXV] used ANN to predict the thermal conductivity of the nanofluids, while Hosseini et al. [VIII], Alhadri et al. [I], and Islam et al. [IX] explored ANN and RSM for other nanofluids problems. These studies showed that the ANN method is capable of predicting with high accuracy, especially for nonlinear problems. However, the differences in the accuracy of prediction from the studies should be carefully compared due to the fact that different parameters are used by the studies (different datasets, input variables, response, etc.).

For the scientific comparison of ANN and RSM, the same observation dataset, input variables, response, division into training and testing sets, and independent test set should be used. Moreover, the performance of both models should be checked using various error metrics such as root mean square error (RMSE), mean absolute error (MAE), and prediction error on independent test data. It is also acceptable to calculate the coefficient of determination (R^2). **Table 1** shows the comparison study of ML and RSM.

Table-1: Comparison of RSM and Machine Learning methods in nanofluid investigations

Aspect	RSM	ANN / Machine Learning
Model type	Statistical/regression model	Data-driven nonlinear model
Mathematical representation	Explicit polynomial equation	Learned nonlinear mapping
Data requirement	Generally efficient for designed experimental datasets	Generally benefits from larger datasets
Interpretability	High	Generally lower
Interaction analysis	Explicit through model terms	Usually implicit
Experimental design	CCD, BBD, factorial and related DOE approaches	No specific DOE required
Nonlinear behavior	Effective within the adequacy	Can represent complex

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	of the selected response model	nonlinear relationships
Prediction accuracy	Depends on model adequacy and dataset	Depends on architecture, training data, and validation
Accuracy comparison	Should be compared using the same data and test set	Should be compared using the same data and test set
Recommended error metrics	RMSE, MAE, independent-test error, R^2	RMSE, MAE, independent-test error, R^2
Optimization	Strong interpretability and process optimization	Strong prediction; optimization generally requires an additional optimization framework
Physical interpretation	Relatively strong	Generally limited
Major limitation	May inadequately represent highly complex nonlinear relationships	Model complexity, data requirements, and limited interpretability

V. Summary table of literature

Table-2 represents the DOE structure and identifiability assessment of major RSM-based nanofluid studies.

Table-2

Model terms	Residual DF	Identifiability	Important limitation
Linear + quadratic + interaction	To be verified	Indeterminate from review data	DOE matrix absent
Full quadratic	9	Adequate	S range narrow
Linear/quadratic + interactions	Ample	Adequate	Type is categorical
Quadratic	To be calculated	Not verifiable	Matrix absent
Full quadratic	10	Adequate for selected factors	Other physical variables excluded
Full quadratic	10/group	Adequate within group	Cross-group interactions not estimable
Quadratic	To be calculated	Not yet verifiable	DOE matrix absent

Study	DOE type	Factors	Factor ranges	Levels	Runs
Peng et al. [XVIII]	RSM; exact design to be verified	T, concentration	25-40°C; 1000-4000 ppm	Not fully verified	Not verified
Khashi'ie et al. [XIII]	CCD	m, S, ϕ	0.20-0.30; 0.050-0.055; 0.01-0.03	3 each	19
Yaw et al. [XXIX, XXX]	CCD/factorial-type matrix	T, concentration, fluid type	20-50°C; 0.01-0.20%; GNP/GNP-CNC	4×4×2	32
Khan [XII]	CCD-RSM	Flow/thermal parameters	To be extracted	To be extracted	To be extracted
Islam et al. [IX]	CCD	Ra, Ha, ϕ	$10^4 - 10^6$; 0-80; 0-0.025	3 each	20
Manjunatha et al. [XVI]	Three separate CCDs	$\beta/K/C$; R/Du/Q; Kr/Sr/Sc	See original tables	3 each	20/group
Kumar et al. [XV]	RSM	Selected Carreau-flow parameters	To be extracted	To be extracted	To be extracted

VI. Critical analysis

Response Surface Methodology (RSM) has been proven as an effective optimization tool in the area of nanofluid research due to the ability of this technique to model even rather complex relationships between the operational parameters and responses based on a limited number of experiments or simulations. This effectiveness was shown by many authors such as Peng et al. [XVIII], Khashi'ie et al. [XIII], Yaw et al. [XXIX, XXX], and Kumar et al. [XV]. The predominance of the second-order polynomial model may limit the precision of modeling for nonlinearly and highly coupled nanofluids. The lack of experimental validation and independent datasets is another drawback of RSM application. Comparison of RSM with Artificial Neural Network (ANN) approach was recently conducted by Uysal et al. [XXV], Hosseini et al. [VIII], Alhadri et al. [I], and Islam et al. [IX]. The potential of ANN to

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model nonlinear relationships was demonstrated, while RSM is more interpretable mathematically.

VI.i. Comparative assessment of RSM factor domains and model adequacy

The suitability of second-order RSM models needs to be rigorously analyzed, taking into consideration the chosen factor range, response range, significance of linear, quadratic, and interaction terms, and model diagnostics. RSM methodology has been extensively used in relation to parameters like temperature, nanoparticle concentration, flow regime, magnetic field strength, and thermophysical properties. Nevertheless, the current literature does not contain enough data on the appropriateness of using second-order models to capture local nonlinear behavior within and outside the design space. Peng et al. [XVIII] have considered temperature and nanoparticle concentration as influential factors that affect the thermal conductivity, and Yaw et al. [XXX] found nonlinearity of these parameters. Hence, linear, quadratic, and interaction effects need to be considered separately for each.

Table-3: Comparative assessment of factor domains and second-order RSM model adequacy in selected nanofluid studies

Study	Peng et al. [XVIII]	Khashi'ie et al. [XIII]	Yaw et al. [XXIX]	Yaw et al. [XXX]	Kumar et al. [XV]
Factors	Temperature, concentration, etc.	Flow/MHD parameters	Operating parameters	Temperature, GNP/CNC concentration, etc.	Time-dependent Carreau-flow parameters
Factor ranges	Extract from paper	Extract	Extract	Extract	Extract
Response(s)	Thermal conductivity	Heat-transfer/flow responses	Heat transfer, energy/pressure responses	Thermal conductivity, viscosity	Entropy generation
RSM design	RSM/DOE	RSM	RSM	RSM	RSM
Significant linear terms	Extract	Extract	Extract	Extract	Extract
Significant quadratic terms	Extract	Extract	Extract	Extract	Extract
Significant interactions	Extract	Extract	Extract	Extract	Extract
Lack-of-fit	Extract	Extract	Extract	Extract	Extract
Residual diagnostics	Extract	Extract	Extract	Extract	Extract
Validation	Experimental	Numerical	Extract	Experimental	Numerical
Local-domain assessment	Local	Local	Local	Local	Local

study. The adequacy of the models needs to be analyzed in terms of lack of fit, residuals, prediction error, R^2 , and adjusted R^2 . A high value of R^2 does not prove the reliability of the models beyond the design domain. **Table 3** contains the comparative analysis.

VII. Design-of-experiments and identifiability considerations

An important question of methodology when studying nanofluids using RSM is whether the experiment design (DOE) used has enough independent data in order to establish individual and interaction effects. Temperature, nanoparticles concentration, magnetic field strength, and flow characteristics are usually considered to be independent RSM variables, but their chosen range of values and the combination in which experiments are conducted should provide enough variation to estimate regression coefficients properly. It is especially critical for nanofluids, since the temperature can affect the viscosity of the liquid, its base properties, Brownian motion, effective thermal conductivity, and heat transfer at once, whereas the nanoparticles concentration impacts thermal performance and hydraulic resistance.

Likewise, proper evaluation of the quadratic and interaction components necessitates enough sampling of the experimental design space. Hence, the factor ranks and regression coefficients must be viewed in context with respect to the experiment design, the extent of factor variation, number of experimental trials, lack of fit, and model adequacy. The articles under discussion need to be evaluated not just in light of the (R^2), level of significance, or optimal conditions, but also whether the CCD, BBD, or other design of experiment employed gives enough variation to separate main, quadratic, and interaction effects.

VIII. Research gaps

However, despite much work done in the area of the application of Response Surface Methodology (RSM) and machine learning in nanofluidics, there are still some open issues. First of all, the experiments to verify RSM models are still rare, as most of them are based on theoretical studies. Proper experimental validation is needed.

[XVIII, XIII, I]. Secondly, stability problems related to nanoparticle agglomeration and sedimentation are not considered in optimization procedures [XIV, XXX]. Thirdly, the problem of ternary and multicomponent nanofluids is underinvestigated despite good perspectives that such nanofluids have [XIV, I, XXX, IX]. Fourthly, the multi-objective optimization procedure needs more attention. The main objectives that need to be optimized simultaneously are the ones mentioned above, such as heat transfer, pressure drop, entropy generation, etc. [XXIX, XI]. Fifthly, integration of RSM with machine learning techniques needs to be studied further [VIII, I, IX, XX, XXI]. Sixthly, time-dependent processes in nanofluids are worth studying [XIX, XV]. Seventh, there is still a lack of progress in the optimization of thermodynamics with entropy generation and exergy analysis [XXXI, XX, XXI, XIV]. Eighth, there is still a lack of practical application in an industrial setting of RSM nanofluid optimization, especially in heat exchangers, cooling systems, solar collectors, and power generation systems [XXVII, X, XXIX]. Ninth, renewable energy applications, such as solar collectors, thermal energy storage, photovoltaic *T. Pradhan et al.*

cooling, and geothermal applications, need further investigation [XXIX, XI]. Tenth, complex physical phenomena, such as thermal radiation, chemical reactions, Brownian motion, thermophoresis, magnetic fields, non-Newtonian fluid mechanics, and changing thermophysical properties, have to be considered in further investigations [XXIV, II, XXX, XXI, XVI]. Eleventh, there is little investigation on uncertainty quantification and reliability analysis. Finally, CFD-RSM-AI frameworks need to be developed for physical modeling, optimization, and AI prediction.

IX. Future scope

The future scope of research on Response Surface Methodology (RSM) in nanofluids engineering is likely to lie in the integration of RSM with AI, including ANN, SVM, and Deep Learning, in order to enhance the prediction and optimization precision and minimize the experimental efforts. New advanced nanofluids like ternary, quaternary, bio-nanofluids, and ferrofluids demand optimization of several interrelated factors; thus, there are many applications of RSM available in these areas. Multi-objective optimization should take into consideration the heat transfer rate, thermal conductivity, viscosity, pressure drop, pumping power, entropy generation, and cost. RSM may be used in the optimization of renewable energy systems based on nanofluids; these include solar collectors, thermal energy storage, geothermal systems, and photovoltaic–thermal systems. The integration of RSM with CFD and AI would lead to increased computational efficiency and better prediction capabilities. Future models must consider complicated effects of Brownian motion, thermophoresis, radiation, magnetohydrodynamics, and non-Newtonian behavior of the flow.

X. Conclusion

- RSM is an efficient statistical and mathematical methodology for modeling and optimization of nanofluid thermal systems while minimizing experimental work.
- RSM can detect significant parameters and optimize responses like thermal conductivity, viscosity, heat transfer coefficient, flow behavior, and entropy production.
- Optimization of performance, pressure loss, pumping power, entropy generation, and economic considerations provides an integrated evaluation of nanofluid systems.
- There is great potential for RSM in nanofluid-based thermal energy storage, geothermal energy conversion, photovoltaic thermal hybrid systems, and solar collectors.
- Combining RSM with artificial intelligence and CFD methods can provide more accurate predictions while decreasing computational and experimental costs.
- Adding factors like radiation, chemical reactions, Brownian motion, thermophoresis, magneto-hydrodynamic (MHD), and non-Newtonian fluid flow behaviors can enhance the applicability of the model.

- Experimental work, uncertainty analysis, reliability-based optimization, digital twin technologies, and sustainable nanofluids are some important aspects for developing reliable and environment-friendly thermal management systems.

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Conflict of Interest:

The authors declare that there was no relevant conflict of interest regarding this paper.

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