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QUADRIPARTITIONED PYTHAGOREAN NEUTROSOPHIC SUPER HYPERSOFT SET

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Abstract

The aim of this paper is to introduce the new concept of the quadripartitioned Pythagorean Neutrosophic Super Hypersoft Set and discuss some of its properties. The focus of this paper is to propose a new notion of QPNSHS and study some basic operations and results in QPNSHS. Further, we develop a systematic study on QPNSHS and obtain various properties induced by them. Some equivalent characterizations and interrelations among them are discussed with counterexamples.

Keywords: Quadri-partitioned Set, Neutrosophic Hypersoft Set, Super Hypersoft Set, Pythagorean Hypersoft Set, Pythagorean Neutrosophic Hypersoft Set, Pythagorean Neutrosophic Super Hypersoft Set, QPNSHS.

I. Introduction

In 1965, A. Zadeh laid the groundwork for FS [XIII]. Fuzzy sets are determined by the level of membership values. The IVF sets notion was presented [XII] to address the ambiguity surrounding membership values. We must consider membership and non-membership values in certain real-world issues in order to accurately represent an item in a dubious and ambiguous state. IFSs, which are helpful in some circumstances, were initially created by Atanassov [I]. IFS, which consider both truth and falsehood values, can be used to deal with inadequate data.

Smarandache was the first to propose the idea of the Neutrosophic set [IX]. The membership values for truth, doubt, and falsehood are shown by the neutrosophic set. The idea of QSVNS is actually stretched from Smarandache's four numerical-valued neutrosophic logic and Belnap's four-valued logic, where the indeterminacy is divided into two parts, namely, "unknown" i.e., neither true nor false, and "contradiction" i.e., both true and false. In the context of neutrosophic study, however, the QSVNS looks quite logical. Also, in their study, Chatterjee [II] et al. analyzed a real-life example for a better understanding of a QSVNS environment and showed that such situations occur very naturally.

The idea of a soft set was first presented by Molodstov [V] as a novel numerical technique for dealing with uncertain situations. He defines a soft set as a family of universal sets with parameter subsets. Soft sets are useful in many fields, including game theory, AI, and basic DM problems [III]. Many scholars have studied the foundations of soft set theory in recent years. Ali et al. [VIII] offered the subset and superset, whereas Maji et al. [IV] provided a theoretical analysis of soft sets. Smarandache suggested a fresh approach for dealing with uncertainty. He expanded the Softset$_{(SS)}$ to Hypersoft$_{(HS)}$ set and Super Hyper soft set is related to the Smarandache power set [X, XI].

[VI] Hemalatha and Francina Shalini defined the concept of the Pythagorean Neutrosophic Super Hypersoft set. We can relate to this more in our daily lives.

II. Preliminaries

Definition 2.1: [XIII] Consider U be the universal set and the power set of U is P(U). For t ≥ 1 let $(\mathfrak{S}_1, \mathfrak{S}_2, \mathfrak{S}_3 \ldots\ldots \mathfrak{S}_n)_{NHS}$ be t-distinct attributes, each of whose associated attributive values is the set $(\mathfrak{s}_1, \mathfrak{s}_2, \mathfrak{s}_3 \ldots\ldots \mathfrak{s}_t)_{NHS}$ with $(\mathfrak{s}_{y} \cap \mathfrak{s}_{z})_{NHS} = \emptyset$ for $y \neq z$ and $y, z \in \{1,2,, \ldots\ldots, t\}$. The connection between these sets is stated as $(\mathfrak{s}_1, \mathfrak{s}_2, \mathfrak{s}_3 \ldots\ldots \mathfrak{s}_t)_{NHS} = \lambda$, and $\mathfrak{F}$: $(\mathfrak{s}_1, \mathfrak{s}_2, \mathfrak{s}_3 \ldots\ldots \mathfrak{s}_t)_{NHS} \longrightarrow \mathbb{P}$ (U) and ($\mathfrak{F}$, $(\mathfrak{s}_1, \mathfrak{s}_2, \mathfrak{s}_3 \ldots\ldots \mathfrak{s}_t)_{NHS}$ =$\{(\lambda, < x_{NHS}, \mathcal{T}_{\mathfrak{F}(\lambda)}(x)_{NHS}, I_{\mathfrak{F}(\lambda)}(x)_{NHS}, F_{\mathfrak{F}(\lambda)}(x)_{NHS} >) : x \in U, \}$ where $\mathcal{T}$ represents truth membership, I represents indeterminacy, and F represents falsity membership such that $\mathcal{T}_{\mathfrak{F}(\lambda)}(x)_{NHS}, I_{\mathfrak{F}(\lambda)}(x)_{NHS}, F_{\mathfrak{F}(\lambda)}(x)_{NHS} \in [0,1]$ also $0 \leq \mathcal{T}_{\mathfrak{F}(\lambda)}(x)_{NHS}, I_{\mathfrak{F}(\lambda)}(x)_{NHS}, F_{\mathfrak{F}(\lambda)}(x)_{NHS} \leq 3$.

Definition 2.2: [XII] Let $\mathfrak{Z}$ represent the universal set and the power set of $\mathfrak{Z}$ is $\mathfrak{G}(\mathfrak{Z})$. For k$\geq$1, let $(\mathscr{b}_1, \mathscr{b}_2, \mathscr{b}_3 \ldots\ldots \mathscr{b}_k)_{PNSHS}$ be k-distinct attributes, each of whose associated attributive values is the set $(\mathcal{B}_1, \mathcal{B}_2, \mathcal{B}_3 \ldots\ldots \mathcal{B}_k)_{PNSHSS}$ with $(\mathcal{B}_m \cap \mathcal{B}_n)_{PNSHSS} = \emptyset$ as well as m $\neq$ n and m, n $\in \{1,2, \ldots, k\}$. Let $\mathfrak{G}(\mathcal{B}_1)_{PNSHSS}, \mathfrak{G}(\mathcal{B}_2)_{PNSHSS}, \mathfrak{G}(\mathcal{B}_3)_{PNSHSS}, \ldots\ldots, \mathfrak{G}(\mathcal{B}_k)_{PNSHSS} = \mathfrak{T}$ be the power sets of the set $(\mathcal{B}_1, \mathcal{B}_2, \mathcal{B}_3 \ldots\ldots \mathcal{B}_k)_{PNSHS}$ respectively.

Then $(\mathfrak{f}, \mathfrak{G}(\mathcal{B}_1)_{PNSHSS} \times \mathfrak{G}(\mathcal{B}_2)_{PNSHSS} \times \mathfrak{G}(\mathcal{B}_3)_{PNSHSS} \times, \ldots\ldots, \times \mathfrak{G}(\mathcal{B}_k)_{PNSHSS})$ is PNSHSS over $\mathfrak{Z}$.

Where

$\mathfrak{f}$: $(\mathfrak{G}(\mathcal{B}_1)_{PNSHSS} \times \mathfrak{G}(\mathcal{B}_2)_{PNSHSS} \times \mathfrak{G}(\mathcal{B}_3)_{PNSHSS} \times, \ldots\ldots, \times \mathfrak{G}(\mathcal{B}_k)_{PNSHSS}) \longrightarrow (\mathfrak{P}(\mathfrak{Z})_{PNSHSS})$ and

$$\mathfrak{f}(\mathfrak{G}(\mathcal{B}_1)_{PNSHSS} \times \mathfrak{G}(\mathcal{B}_2)_{PNSHSS} \times \mathfrak{G}(\mathcal{B}_3)_{PNSHSS} \times, \ldots\ldots, \times \mathfrak{G}(\mathcal{B}_k)_{PNSHSS}) = \{\mathfrak{T}, < x, \mathcal{T}_{\mathfrak{f}(\mathfrak{T})}(x), I_{\mathfrak{f}(\mathfrak{T})}(x), F_{\mathfrak{f}(\mathfrak{T})}(x) >: x \in \mathfrak{Z}, \mathfrak{T} \in (\mathfrak{G}(\mathcal{B}_1)_{PNSHSS} \times \mathfrak{G}(\mathcal{B}_2)_{PNSHSS} \times \mathfrak{G}(\mathcal{B}_3)_{PNSHSS} \times, \ldots\ldots, \times \mathfrak{G}(\mathcal{B}_k)_{PNSHSS})\}$$

Where $\mathcal{T}_{\mathfrak{f}(\mathfrak{T})}$ and $F_{\mathfrak{f}(\mathfrak{T})}$ are the dependent components. $I_{\mathfrak{f}(\mathfrak{T})}$ is an independent component. Also,

$$0 \leq \left(\mathcal{T}_{\mathfrak{f}(\mathfrak{T})}(x)\right)^2 + \left(I_{\mathfrak{f}(\mathfrak{T})}(x)\right)^2 + \left(F_{\mathfrak{f}(\mathfrak{T})}(x)\right)^2 \leq 2 \text{ and } \mathcal{T}_{\mathfrak{f}(\mathfrak{T})}(x) + F_{\mathfrak{f}(\mathfrak{T})}(x) \leq 1.$$

Definition2.3: [XIV]

Let X be an initial universe set, and E is a set of parameters. Consider a non-empty set A and $A \subseteq E$. Let $P(X)$ denote the set of all quadri-partitioned neutrosophic sets of X.

The collection (F, A) is termed the quadri-partitioned neutrosophic soft set (QNSS) over X, where F is a mapping given by $F : A \longrightarrow P(X)$, where $A = \{< x, \mathcal{T}_A(x), \mathcal{C}_A(x), \mathcal{U}_A(x), F_A(x) >: x \in U\}$ Where $\mathcal{T}_A, \mathcal{C}_A, \mathcal{U}_A, F_A : \rightarrow [0,1]$ and $0 \leq \mathcal{T}_A(x) + \mathcal{C}_A(x) + \mathcal{U}_A(x) + F_A(x) \leq 4$. Here $\mathcal{T}_A(x)$ is the truth membership, $\mathcal{C}_A(x)$ is contradiction membership, $\mathcal{U}_A(x)$ is ignorance membership, and $F_A(x)$ is the false membership.

III. Quadripartitioned Pythagorean Neutrosophic Super Hypersoft Set

Definition: 3.1

Let U be the universal set and P(U) be the power set of U. Consider $H_1, H_2, H_3 \ldots\ldots H_n$ for n$\geq$ 1 be n-distinct attributes, whose corresponding attribute values are respectively the set
$h_1, h_2, h_3 \ldots\ldots h_n$ with $h_i \cap h_j = \emptyset$ for i≠j and i,j $\in \{1,2,3, \ldots \ldots, n\}$
Let $\mathbb{P}(\mathfrak{h}_1), \mathbb{P}(\mathfrak{h}_2), \mathbb{P}(\mathfrak{h}_3), \ldots \ldots, \mathbb{P}(\mathfrak{h}_n) = \lambda$ be the power sets of the set $h_1, h_2, h_3 \ldots\ldots h_n$ respectively. QPNSHS
Then the pair $\left(F, \mathbb{P}(\mathfrak{h}_1), \mathbb{P}(\mathfrak{h}_2), \mathbb{P}(\mathfrak{h}_3), \ldots \ldots, \mathbb{P}(\mathfrak{h}_n)\right)$ is said to be QPNSHS over U.
where F: $\mathbb{P}(\mathfrak{h}_1), \mathbb{P}(\mathfrak{h}_2), \mathbb{P}(\mathfrak{h}_3), \ldots \ldots, \mathbb{P}(\mathfrak{h}_n) \longrightarrow$ P(U) and

$$F\left(\mathbb{P}(\mathfrak{h}_1), \mathbb{P}(\mathfrak{h}_2), \mathbb{P}(\mathfrak{h}_3), \ldots \ldots, \mathbb{P}(\mathfrak{h}_n)\right) = \{\lambda, < x, \mathcal{T}_{F(\lambda)}(x), \mathcal{C}_{F(\lambda)}(x), U_{F(\lambda)}(x), F_{F(\lambda)}(x) >: x \in U, \lambda \in \mathbb{P}(\mathfrak{h}_1), \mathbb{P}(\mathfrak{h}_2), \mathbb{P}(\mathfrak{h}_3), \ldots \ldots, \mathbb{P}(\mathfrak{h}_n)\}$$

$$0 \leq \left(\mathcal{T}_{F(\lambda)}(x)\right)^2 + \left(\mathcal{C}_{F(\lambda)}(x)\right)^2 + \left(\mathcal{U}_{F(\lambda)}(x)\right)^2 + \left(F_{F(\lambda)}(x)\right)^2 \leq 2$$

Definition: 3.2

Let $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS}$ and $(\lambda_{\mathfrak{b}}, \mathfrak{D})_{QPNSHS}$ be two QPNSHS over the universe U. Consider $\mathfrak{H}_1, \mathfrak{H}_2, \mathfrak{H}_3 \ldots\ldots \mathfrak{H}_n$ for n$\geq$ 1 be n-distinct attributes, whose corresponding attribute values are respectively the set
$\mathfrak{h}_1, \mathfrak{h}_2, \mathfrak{h}_3 \ldots\ldots \mathfrak{h}_n$ith $\mathfrak{h}_i \cap \mathfrak{h}_j = \emptyset$ for i≠j and i, j $\in \{1,2,3, \ldots \ldots, n\}$
Let $\mathbb{P}(\mathfrak{h}_1), \mathbb{P}(\mathfrak{h}_2), \mathbb{P}(\mathfrak{h}_3), \ldots \ldots, \mathbb{P}(\mathfrak{h}_n) = \lambda$ be the power set of the set $\mathfrak{h}_1, \mathfrak{h}_2, \mathfrak{h}_3 \ldots\ldots \mathfrak{h}_n$ respectively.
Then $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS}$ is the QPNSHS subset of $(\lambda_{\mathfrak{b}}, \mathfrak{D})_{QPNSHS}$ if
$\mathcal{T}(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \leq \mathcal{T}(\lambda_{\mathfrak{b}}, \mathfrak{D})_{QPNSHS}$,
$\mathcal{C}(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \leq \mathcal{C}(\lambda_{\mathfrak{b}}, \mathfrak{D})_{QPNSHS}$,
$\mathcal{U}(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \leq \mathcal{U}(\lambda_{\mathfrak{b}}, \mathfrak{D})_{QPNSHS}$
$F(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \leq F(\lambda_{\mathfrak{b}}, \mathfrak{D})_{QPNSHS}$

Definition:3.3

Consider $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS}$ be the QPNSHS over the universe U. Let $\mathfrak{H}_1, \mathfrak{H}_2, \mathfrak{H}_3 \ldots\ldots \mathfrak{H}_n$ for n $\geq$ 1 be n-distinct attributes, whose corresponding attributive values are respectively the set
$\mathfrak{h}_1, \mathfrak{h}_2, \mathfrak{h}_3 \ldots\ldots \mathfrak{h}_n$ with $\mathfrak{h}_i \cap \mathfrak{h}_j = \emptyset$ for i≠j and i, j $\in \{1,2,3, \ldots \ldots, n\}$
Let $\mathbb{P}(\mathfrak{h}_1), \mathbb{P}(\mathfrak{h}_2), \mathbb{P}(\mathfrak{h}_3), \ldots \ldots, \mathbb{P}(\mathfrak{h}_n) = \lambda$ be the power set of the set $\mathfrak{h}_1, \mathfrak{h}_2, \mathfrak{h}_3 \ldots\ldots \mathfrak{h}_n$ respectively. The complement of QPNSHS $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS}$ is denoted by $(\lambda_{\mathfrak{a}}, \mathfrak{C})^c_{QPNSHS}$ and is defined as

$$(\lambda_{\mathfrak{a}}, \mathfrak{C})^{c}_{QPNSHS} = \{\langle F_{F(\lambda)}(x), \mathcal{U}_{F(\lambda)}(x), \mathcal{C}_{F(\lambda)}(x), \mathcal{T}_{F(\lambda)}(x)\rangle : x \in U, \lambda \in \mathbb{P}(\mathfrak{h}_1), \mathbb{P}(\mathfrak{h}_2), \mathbb{P}(\mathfrak{h}_3), \ldots \ldots, \mathbb{P}(\mathfrak{h}_n)\}$$

Definition: 3.4

Let $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS}$ and $(\lambda_{\mathfrak{b}}, \mathfrak{D})_{QPNSHS}$ be two QPNSHS over the universe U. Consider $\mathfrak{H}_1, \mathfrak{H}_2, \mathfrak{H}_3 \ldots \mathfrak{H}_n$ for n≥ 1 be n-distinct attributes, whose corresponding attribute values are respectively the set

$\mathfrak{h}_1, \mathfrak{h}_2, \mathfrak{h}_3 \ldots \mathfrak{h}_n$ with $\mathfrak{h}_i \cap \mathfrak{h}_j = \emptyset$ for i≠j and i, j ∈ $\{1,2,3, \ldots \ldots, n\}$

Let $\mathbb{P}(\mathfrak{h}_1), \mathbb{P}(\mathfrak{h}_2), \mathbb{P}(\mathfrak{h}_3), \ldots \ldots, \mathbb{P}(\mathfrak{h}_n) = \lambda$ be the power set of the set $\mathfrak{h}_1, \mathfrak{h}_2, \mathfrak{h}_3 \ldots \mathfrak{h}_n$ respectively.

Then $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \cup (\lambda_{\mathfrak{b}}, \mathfrak{D})_{QPNSHS}$ is given as

$$(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \cup (\lambda_{\mathfrak{b}}, \mathfrak{D})_{QPNSHS} = \left\{\left\langle \begin{matrix} x, \widetilde{max}\left(\mathcal{T}(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS}, (\lambda_{\mathfrak{b}}, \mathfrak{D})_{QPNSHS}\right), \widetilde{max}\left(\mathcal{C}(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS}, \mathcal{C}(\lambda_{\mathfrak{b}}, \mathfrak{D})_{QPNSHS}\right), \\ \underbrace{min}\left(\mathcal{U}(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS}, \mathcal{U}(\lambda_{\mathfrak{b}}, \mathfrak{D})_{QPNSHS}\right), \underbrace{min}\left(F(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS}, F(\lambda_{\mathfrak{b}}, \mathfrak{D})_{QPNSHS}\right) \end{matrix} \right\rangle\right\}$$

Definition: 3.5

Let $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS}$ and $(\lambda_{\mathfrak{b}}, \mathfrak{D})_{QPNSHS}$ be two QPNSHS over the universe U. Consider $\mathfrak{H}_1, \mathfrak{H}_2, \mathfrak{H}_3 \ldots \mathfrak{H}_n$ for n≥ 1 be n-distinct attributes, whose corresponding attribute values are respectively the set

$\mathfrak{h}_1, \mathfrak{h}_2, \mathfrak{h}_3 \ldots \mathfrak{h}_n$ith $\mathfrak{h}_i \cap \mathfrak{h}_j = \emptyset$ for i≠j and i, j ∈ $\{1,2,3, \ldots \ldots, n\}$

Let $\mathbb{P}(\mathfrak{h}_1), \mathbb{P}(\mathfrak{h}_2), \mathbb{P}(\mathfrak{h}_3), \ldots \ldots, \mathbb{P}(\mathfrak{h}_n) = \lambda$ be the power set of the set $\mathfrak{h}_1, \mathfrak{h}_2, \mathfrak{h}_3 \ldots \mathfrak{h}_n$ respectively.

Then $\mathfrak{F}(\lambda_1) \cap \mathfrak{F}(\lambda_2)$ is given as

$$(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \cap (\lambda_{\mathfrak{b}}, \mathfrak{D})_{QPNSHS} = \left\{\left\langle \begin{matrix} x, \underbrace{min}\left(\mathcal{T}(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS}, \mathcal{T}(\lambda_{\mathfrak{b}}, \mathfrak{D})_{QPNSHS}\right), \quad \underbrace{min}\left(\mathcal{C}(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS}, \mathcal{C}(\lambda_{\mathfrak{b}}, \mathfrak{D})_{QPNSHS}\right), \\ \widetilde{max}\left(\mathcal{U}(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS}, \mathcal{U}(\lambda_{\mathfrak{b}}, \mathfrak{D})_{QPNSHS}\right), \widetilde{max}\left(F(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS}, F(\lambda_{\mathfrak{b}}, \mathfrak{D})_{QPNSHS}\right) \end{matrix} \right\rangle\right\}$$

Definition: 3.6

Consider $(\lambda_{\mathfrak{a}}, \mathfrak{C})$ be the QPNSHS over the universe U. Let $\mathfrak{H}_1, \mathfrak{H}_2, \mathfrak{H}_3 \ldots \mathfrak{H}_n$ for n≥ 1 be n-distinct attributes, whose corresponding attributive values are respectively the set

$\mathfrak{h}_1, \mathfrak{h}_2, \mathfrak{h}_3 \ldots \mathfrak{h}_n$ with $\mathfrak{h}_i \cap \mathfrak{h}_j = \emptyset$ for i≠j and i, j ∈ $\{1,2,3, \ldots \ldots, n\}$

Let $\mathbb{P}(\mathfrak{h}_1), \mathbb{P}(\mathfrak{h}_2), \mathbb{P}(\mathfrak{h}_3), \ldots \ldots, \mathbb{P}(\mathfrak{h}_n) = \lambda$ be the power sets of the set $\mathfrak{h}_1, \mathfrak{h}_2, \mathfrak{h}_3 \ldots \mathfrak{h}_n$ respectively. Then $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS}$ is empty QPNSHS if T $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} = 0, \mathcal{C}(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} = 0, \mathcal{U}(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} = 1, F(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} = 1 \ \forall x \in U, \ \lambda \in \mathbb{P}(\mathfrak{h}_1), \mathbb{P}(\mathfrak{h}_2), \mathbb{P}(\mathfrak{h}_3), \ldots \ldots, \mathbb{P}(\mathfrak{h}_n)$. It is denoted by $0_{QPNSHSS}$

Definition: 3.7

Consider $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS}$ be the QPNSHS over the universe U. Let $\mathfrak{H}_1, \mathfrak{H}_2, \mathfrak{H}_3 \ldots \mathfrak{H}_n$ for n ≥ 1 be n-distinct attributes, whose corresponding attributive values are respectively the set

$\mathfrak{h}_1, \mathfrak{h}_2, \mathfrak{h}_3 \ldots \mathfrak{h}_n$ with $\mathfrak{h}_i \cap \mathfrak{h}_j = \emptyset$ for i≠j and i, j ∈ $\{1,2,3, \ldots \ldots, n\}$

Let $\mathbb{P}(\mathfrak{h}_1), \mathbb{P}(\mathfrak{h}_2), \mathbb{P}(\mathfrak{h}_3), \ldots \ldots, \mathbb{P}(\mathfrak{h}_n) = \lambda$ be the power sets of the set $\mathfrak{h}_1, \mathfrak{h}_2, \mathfrak{h}_3 \ldots \mathfrak{h}_n$ respectively. Then $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS}$ is the universe QPNSHS if $\mathcal{T}$ $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} = 1, \mathcal{C}(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} = 1, \mathcal{U}(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} = 0, F(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} = 0 \ \forall x \in U, \ \lambda \in \mathbb{P}(\mathfrak{h}_1), \mathbb{P}(\mathfrak{h}_2), \mathbb{P}(\mathfrak{h}_3), \ldots \ldots, \mathbb{P}(\mathfrak{h}_n)$. 1_{QPNSHS}

Remark: $0^{c}_{QPNSHSS} = 1_{QPNSHS}\ and\ 1^{c}_{QPNSHS} = 0_{QPNSHS}$

Definition: 3.8
Let $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS}$ and $(\lambda_{\mathfrak{b}}, \mathfrak{D})_{QPNSHS}$ be two QPNSHS over the universe U. Consider $\mathfrak{H}_1, \mathfrak{H}_2, \mathfrak{H}_3 \ldots\ldots \mathfrak{H}_n$ for n$\geq$ 1 be n-distinct attributes, whose corresponding attribute values are respectively the set
$\mathfrak{h}_1, \mathfrak{h}_2, \mathfrak{h}_3 \ldots\ldots \mathfrak{h}_n$ with $\mathfrak{h}_i \cap \mathfrak{h}_j = \emptyset$ for i$\neq$j and i, j $\in \{1,2,3, \ldots\ldots, n\}$
Let $\mathbb{P}(\mathfrak{h}_1), \mathbb{P}(\mathfrak{h}_2), \mathbb{P}(\mathfrak{h}_3), \ldots\ldots, \mathbb{P}(\mathfrak{h}_n) = \lambda$ be the power sets of the set $\mathfrak{h}_1, \mathfrak{h}_2, \mathfrak{h}_3 \ldots\ldots \mathfrak{h}_n$ respectively. Then $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \backslash (\lambda_{\mathfrak{b}}, \mathfrak{D})_{QPNSHS}$ may be defined as,
$(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \backslash (\lambda_{\mathfrak{b}}, \mathfrak{D})_{QPNSHS} =$

$$\left\{\left\langle \begin{matrix} x, \underbrace{min}\left(\mathcal{T}(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS}, F(\lambda_{\mathfrak{b}}, \mathfrak{D})_{QPNSHS}\right),\ \underbrace{min}\left(\mathcal{C}(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS}, \mathcal{U}(\lambda_{\mathfrak{b}}, \mathfrak{D})_{QPNSHS}\right), \\ \overbrace{max}\left(\mathcal{U}(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS}, \mathcal{C}\left(F(\lambda_2)_{QPNSHS}\right)\right), \overbrace{max}\left(F(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS}, \mathcal{T}(\lambda_{\mathfrak{b}}, \mathfrak{D})_{QPNSHS}\right) \end{matrix} \right\rangle\right\}$$

Definition: 3.9
Let $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS}$ and $(\lambda_{\mathfrak{b}}, \mathfrak{D})_{QPNSHS}$ be two QPNSHS over the universe U. Consider $\mathfrak{H}_1, \mathfrak{H}_2, \mathfrak{H}_3 \ldots\ldots \mathfrak{H}_n$ for n$\geq$ 1 be n-distinct attributes, whose corresponding attribute values are respectively the set
$\mathfrak{h}_1, \mathfrak{h}_2, \mathfrak{h}_3 \ldots\ldots \mathfrak{h}_n$ with $\mathfrak{h}_i \cap \mathfrak{h}_j = \emptyset$ for i$\neq$j and i, j $\in \{1,2,3, \ldots\ldots, n\}$
Let $\mathbb{P}(\mathfrak{h}_1), \mathbb{P}(\mathfrak{h}_2), \mathbb{P}(\mathfrak{h}_3), \ldots\ldots, \mathbb{P}(\mathfrak{h}_n) = \lambda$ be the power sets of the set $\mathfrak{h}_1, \mathfrak{h}_2, \mathfrak{h}_3 \ldots\ldots \mathfrak{h}_n$ respectively. Then $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS}$ AND $(\lambda_{\mathfrak{b}}, \mathfrak{D})_{QPNSHS}$ is denoted by $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \wedge (\lambda_{\mathfrak{b}}, \mathfrak{D})_{QPNSHS}$ and is defined by
$(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \wedge (\lambda_{\mathfrak{b}}, \mathfrak{D})_{QPNSHS} = (\lambda_{\mathfrak{c}}, \mathfrak{C} \times \mathfrak{D})_{QPNSHS}$
Where $\lambda_{\mathfrak{c}}(\mathfrak{c}, \mathfrak{d})_{QPNSHS} = \lambda_{\mathfrak{a}}(\mathfrak{c})_{QPNSHS} \cap \lambda_{\mathfrak{b}}(\mathfrak{d})_{QPNSHS}\ \forall\ \mathfrak{c} \in \mathfrak{C}\ and\ \mathfrak{d} \in \mathfrak{D}$

Definition: 3.10
Let $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS}$ and $(\lambda_{\mathfrak{b}}, \mathfrak{D})_{QPNSHS}$ be two QPNSHS over the universe U. Consider $\mathfrak{H}_1, \mathfrak{H}_2, \mathfrak{H}_3 \ldots\ldots \mathfrak{H}_n$ for n$\geq$ 1 be n-distinct attributes, whose corresponding attribute values are respectively the set.
$\mathfrak{h}_1, \mathfrak{h}_2, \mathfrak{h}_3 \ldots\ldots \mathfrak{h}_n$ with $\mathfrak{h}_i \cap \mathfrak{h}_j = \emptyset$ for i$\neq$j and i, j $\in \{1,2,3, \ldots\ldots, n\}$
Let $\mathbb{P}(\mathfrak{h}_1), \mathbb{P}(\mathfrak{h}_2), \mathbb{P}(\mathfrak{h}_3), \ldots\ldots, \mathbb{P}(\mathfrak{h}_n) = \lambda$ be the power sets of the set $\mathfrak{h}_1, \mathfrak{h}_2, \mathfrak{h}_3 \ldots\ldots \mathfrak{h}_n$ respectively. Then $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS}$ OR $(\lambda_{\mathfrak{b}}, \mathfrak{D})_{QPNSHS}$ is denoted by $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \vee (\lambda_{\mathfrak{b}}, \mathfrak{D})_{QPNSHS}$ and is defined by
$(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \vee (\lambda_{\mathfrak{b}}, \mathfrak{D})_{QPNSHS} = (\lambda_{\aleph}, \mathfrak{C} \times \mathfrak{D})_{QPNSHS}$

Where,

$$(\lambda_{\aleph}, \mathfrak{c} \times \mathfrak{d})_{QPNSHS} = (\lambda_{\mathfrak{a}}, \mathfrak{c})_{QPNSHS} \cup (\lambda_{\mathfrak{a}}, \mathfrak{d})_{QPNSHS}\ \forall\ \mathfrak{c} \in \mathfrak{C}\ and\ \mathfrak{d} \in \mathfrak{D}$$

Theorem: 3.11
Let $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS}$ and $(\lambda_{\mathfrak{b}}, \mathfrak{C})_{QPNSHS}$ be two QPNSHS over the universe U. Then the following are true,

- **i.** $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \subseteq (\lambda_{\mathfrak{b}}, \mathfrak{C})_{QPNSHS}\ iff\ (\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \cap (\lambda_{\mathfrak{b}}, \mathfrak{C})_{QPNSHS} = (\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS}$
- **ii.** $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \subseteq (\lambda_{\mathfrak{b}}, \mathfrak{C})_{QPNSHS}\ iff\ (\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \cup (\lambda_{\mathfrak{b}}, \mathfrak{C})_{QPNSHS} = (\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS}$

Proof:

i. Suppose that $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \subseteq (\lambda_{\mathfrak{b}}, \mathfrak{C})_{QPNSHS}$, then $(\lambda_{\mathfrak{a}}(e)) \subseteq (\lambda_{\mathfrak{b}}(e))$ for all $e \in \mathfrak{C}$. Let $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \cap (\lambda_{\mathfrak{b}}, \mathfrak{C})_{QPNSHS} = (\lambda_{\mathfrak{c}}, \mathfrak{C})_{QPNSHS}$

Since $\left(\lambda_{\mathfrak{c}}(e)\right)_{QPNSHS} = (\lambda_{\mathfrak{a}}(e)) \cap \left(\lambda_{\mathfrak{b}}(e)\right) = (\lambda_{\mathfrak{a}}(e))$ for all $e \in \mathfrak{C}$, by definition $(\lambda_{\mathfrak{c}}, \mathfrak{C})_{QPNSHS} = (\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS}$.

Suppose that $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \cap (\lambda_{\mathfrak{b}}, \mathfrak{C})_{QPNSHS} = (\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS}$

Let $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \cap (\lambda_{\mathfrak{b}}, \mathfrak{C})_{QPNSHS} = (\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS}$

Since $(\lambda_{\mathfrak{c}}(e)) = (\lambda_{\mathfrak{a}}(e)) \cap \left(\lambda_{\mathfrak{b}}(e)\right) = (\lambda_{\mathfrak{a}}(e))$ for all $e \in \mathfrak{C}$, we know that $(\lambda_{\mathfrak{a}}(e)) \subseteq (\lambda_{\mathfrak{b}}(e))$ for all $e \in \mathfrak{C}$. Hence $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \subseteq (\lambda_{\mathfrak{b}}, \mathfrak{C})_{QPNSHS}$.

ii. The proof is similar to **i**.

Theorem: 3.12

Let $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS}$, $(\lambda_{\mathfrak{b}}, \mathfrak{C})_{QPNSHS}$, $(\lambda_{\mathfrak{c}}, \mathfrak{C})_{QPNSHS}$, and $(\lambda_{\mathfrak{d}}, \mathfrak{C})_{QPNSHS}$ be two QPNSHS over the universe U. Then the following are true,

i. $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \cap (\lambda_{\mathfrak{b}}, \mathfrak{C})_{QPNSHS} = \phi_{\mathfrak{C}}$, then $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \subseteq (\lambda_{\mathfrak{b}}, \mathfrak{C})^{c}_{QPNSHS}$

ii. if $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \subseteq (\lambda_{\mathfrak{b}}, \mathfrak{C})_{QPNSHS}$ and $(\lambda_{\mathfrak{b}}, \mathfrak{C})_{QPNSHS} \subseteq \mathfrak{F}(\lambda_{\mathfrak{c}}, \mathfrak{C})_{QPNSHS}$ then $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \subseteq (\lambda_{\mathfrak{c}}, \mathfrak{C})_{QPNSHS}$

iii. if $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \subseteq (\lambda_{\mathfrak{b}}, \mathfrak{C})_{QPNSHS}$ and $(\lambda_{\mathfrak{c}}, \mathfrak{C})_{QPNSHS} \subseteq (\lambda_{\mathfrak{d}}, \mathfrak{C})_{QPNSHS}$ then $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \cap (\lambda_{\mathfrak{c}}, \mathfrak{C})_{QPNSHS} \subseteq (\lambda_{\mathfrak{b}}, \mathfrak{C})_{QPNSHS} \cap (\lambda_{\mathfrak{d}}, \mathfrak{C})_{QPNSHS}$

iv. if $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \subseteq (\lambda_{\mathfrak{b}}, \mathfrak{C})_{QPNSHS}$ iff $(\lambda_{\mathfrak{b}}, \mathfrak{C})^{c}_{QPNSHS} \subseteq (\lambda_{\mathfrak{a}}, \mathfrak{C})^{c}_{QPNSHS}$

proof:

i. suppose that $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \cap (\lambda_{\mathfrak{b}}, \mathfrak{C})_{QPNSHS} = \emptyset_{\mathfrak{C}}$

then $(\lambda_{\mathfrak{a}}(e)) \cap \left(\lambda_{\mathfrak{b}}(e)\right) = \emptyset$

so $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \subseteq U \backslash \left(\lambda_{\mathfrak{b}}(e)\right) = \left(\lambda^{c}_{\mathfrak{b}}(e)\right) \forall\, e \in \mathfrak{C}$

Therefore, we have $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \subseteq (\lambda_{\mathfrak{b}}, \mathfrak{C})^{c}_{QPNSHS}$

Proof of (ii) and (iii) is obvious

iv. $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \subseteq (\lambda_{\mathfrak{b}}, \mathfrak{C})_{QPNSHS}$

$\Leftrightarrow (\lambda_{\mathfrak{a}}(e)) \subseteq \left(\lambda_{\mathfrak{b}}(e)\right) \forall\, e \in \mathfrak{C}$

$\Leftrightarrow \left(\lambda_{\mathfrak{b}}(e)\right)^{c} \subseteq \left(\lambda_{\mathfrak{a}}(e)\right)^{c} \forall\, e \in \mathfrak{C}$

$\Leftrightarrow \left(\lambda^{c}_{\mathfrak{b}}(e)\right) \subseteq (\lambda^{c}_{\mathfrak{a}}(e)) \forall\, e \in \mathfrak{C}$

$\Leftrightarrow (\lambda_{\mathfrak{b}}, \mathfrak{C})^{c}_{QPNSHS} \subseteq (\lambda_{\mathfrak{a}}, \mathfrak{C})^{c}_{QPNSHS}$

Definition: 3.13

Let I be an arbitrary index $\left\{\left(\lambda_{\mathfrak{a}_i}, \mathfrak{C}\right)\right\}_{i \in I}$ be a subfamily of QPNSH sets over the universe U

i. The union of these $QPNSHS$ sets is the $QPNSHS$ set $(\lambda_{\mathfrak{c}}, \mathfrak{C})_{QPNSHS}$ where $\lambda_{\mathfrak{c}}(e) = \cup_{i \in I}\left(\lambda_{\mathfrak{a}_i}(e)\right) for\ each\ e \in \mathfrak{C}$ we write $\cup_{i \in I}\left(\lambda_{\mathfrak{a}_i}, \mathfrak{C}\right)_{QPNSHS} = (\lambda_{\mathfrak{c}}, \mathfrak{C})_{QPNSHS}$

ii. The intersection of these $QPNSHS$ sets is the $QPNSHS$ set$(\lambda_{\mathfrak{c}}, \mathfrak{C})_{QPNSHS}$ where $\lambda_{\mathfrak{c}}(e) = \cap_{i\in I}\left(\lambda_{\mathfrak{a}_i}(e)\right)$ for each $e \in \mathfrak{C}$ we write $\cap_{i\in I}\left(\lambda_{\mathfrak{a}_i}, \mathfrak{C}\right)_{QPNSHS} = (\lambda_{\mathfrak{v}}, \mathfrak{C})_{QPNSHS}$

Theorem: 3.14

Let I be an arbitrary index set and $\left\{\left(\lambda_{\mathfrak{a}_i}, \mathfrak{C}\right)_{QPNSHS}\right\}_{i\in I}$ be a subfamily of QPNSHS over the universe U. Then

i. $\left(\cup_{i\in I}\left(\lambda_{\mathfrak{a}_i}, \mathfrak{C}\right)_{QPNSHS}\right)^c = \left(\cap_{i\in I}\left(\lambda_{\mathfrak{a}_i}, \mathfrak{C}\right)_{QPNSHS}\right)^c$

ii. $\left(\cap_{i\in I}\left(\lambda_{\mathfrak{a}_i}, \mathfrak{C}\right)_{QPNSHS}\right)^c = \left(\cup_{i\in I}\left(\lambda_{\mathfrak{a}_i}, \mathfrak{C}\right)_{QPNSHS}\right)^c$

Proof:

i. $\left(\cup_{i\in I}\left(\lambda_{\mathfrak{a}_i}, \mathfrak{C}\right)_{QPNSHS}\right)^c = (\lambda_{\mathfrak{c}}, \mathfrak{C})^c_{QPNSHS}$, By the definition

$(\lambda^c_{\mathfrak{c}}(e)) = U_E \setminus (\lambda_{\mathfrak{c}}(e)) = U_E \setminus \cup_{i\in I} (\lambda_{\mathfrak{a}}(e)) = \cap_{i\in I}\left(U_E \setminus (\lambda_{\mathfrak{a}}(e))\right) \forall\, e \in \mathfrak{C}$

On the other hand, $\left(\cap_{i\in I}\left(\lambda_{\mathfrak{a}_i}, \mathfrak{C}\right)_{QPNSHS}\right)^c = (\lambda_{\mathfrak{d}}, \mathfrak{C})_{QPNSHS}$

by the definition, $(\lambda_{\mathfrak{d}}(e)) = \left(\cap_{i\in I}\left(\lambda^c_{\mathfrak{a}_i}, \mathfrak{C}\right)_{QPNSHS}\right)^c = \cap_{i\in I}\left(U - \left(\lambda_{\mathfrak{a}_i}, \mathfrak{C}\right)_{QPNSHS}\right) \forall$ $e \in \mathfrak{C}$

ii. Proof is similar to (i).

Note: We denote $\emptyset_E$ by $\emptyset$ and U_E by U.

Theorem 3.15

Let $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS}$ be QPNSHS over the universe U. Then the following are true.

i. $(\emptyset, \mathfrak{C})^c_{QPNSHS} = (U, \mathfrak{C})_{QPNSHS}$

ii. $(U, \mathfrak{C})^c_{QPNSHS} = (\emptyset, \mathfrak{C})_{QPNSHS}$

Proof:

i. Let $(\emptyset, \mathfrak{C})_{QPNSHS} = (\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS}$

Then $(\lambda_{\mathfrak{a}}(e)) = \{< x, \mathcal{T}_{\mathfrak{F}(\lambda_{\mathfrak{a}}(e))}(x), \mathcal{C}_{\mathfrak{F}(\lambda_{\mathfrak{a}}(e))}(x), \mathcal{U}_{\mathfrak{F}(\lambda_{\mathfrak{a}}(e))}(x), F_{\mathfrak{F}(\lambda_{\mathfrak{a}}(e))}(x) >: x \in U, \}$
$= \{(x, 0,0,1,1): x \in U\} \forall\, e \in \mathfrak{C}$

Thus $(\emptyset, \mathfrak{C})^c_{QPNSHS} = (\lambda_{\mathfrak{a}}, \mathfrak{C})^c_{QPNSHS}$

Then $\forall\, e \in \mathfrak{C}$

$$\left(\lambda_{\mathfrak{a}}(e)\right)^c = \left\{< x, \mathcal{T}_{\mathfrak{F}(\lambda_{\mathfrak{a}}(e))}(x), \mathcal{C}_{\mathfrak{F}(\lambda_{\mathfrak{a}}(e))}(x), \mathcal{U}_{\mathfrak{F}(\lambda_{\mathfrak{a}}(e))}(x), F_{\mathfrak{F}(\lambda_{\mathfrak{a}}(e))}(x) >: x \in U, \right\}^c$$
$$= \{(x, 1,1,0,0): x \in U\} = U$$

Thus $(\emptyset, \mathfrak{C})^c_{QPNSHS} = (U, \mathfrak{C})_{QPNSHS}$

ii. Proof is similar to (i)

Theorem 3.16

Let $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS}$ be QPNSHS over the universe U. Then the following are true.

i. $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \cup (\emptyset, \mathfrak{C})_{QPNSHS} = (\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS}$

ii. $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \cup (U, \mathfrak{C})_{QPNSHS} = (\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS}$

Proof:

i. $(\lambda_a, \mathfrak{C})_{QPNSHS} = \{ e, < x, \mathcal{T}_{\mathfrak{F}(\lambda_a(e))}(x), \mathcal{C}_{\mathfrak{F}(\lambda_a(e))}(x), \mathcal{U}_{\mathfrak{F}(\lambda_a(e))}(x), F_{\mathfrak{F}(\lambda_a(e))}(x) >: \ x \in U \} \forall\, e \in \mathfrak{C}$

$(\emptyset, \mathfrak{C})_{QPNSHS} = \{(x, 0,0,1,1): x \in U\} \ \forall\, e \in \mathfrak{C}$

$(\lambda_a, \mathfrak{C})_{QPNSHS} \cup (\emptyset, \mathfrak{C})_{QPNSHS} =$

$\left\{e, \left(x, \widehat{max}\left(\mathcal{T}_{\mathfrak{F}(\lambda_a(e))}(x), 0\right), \widehat{max}\left(\mathcal{C}_{\mathfrak{F}(\lambda_a(e))}(x), 0\right), \underbrace{min}\left(\mathcal{U}_{\mathfrak{F}(\lambda_a(e))}(x), 1\right), \underbrace{min}\left(F_{\mathfrak{F}(\lambda_a(e))}(x), 1\right)\right) : x \in U\right\} \forall\, e \in \mathfrak{C}$

$= \{ e, < x, \mathcal{T}_{\mathfrak{F}(\lambda_a(e))}(x), \mathcal{C}_{\mathfrak{F}(\lambda_a(e))}(x), \mathcal{U}_{\mathfrak{F}(\lambda_a(e))}(x), F_{\mathfrak{F}(\lambda_a(e))}(x) >: \ x \in U, \} = \mathfrak{F}(\lambda_a, \mathfrak{C})$

ii. Proof is similar to (i).

Theorem: 3.17

Let $(\lambda_a, \mathfrak{C})_{QPNSHS}$ be QPNSHS over the universe U. Then the following are true.

i. $(\lambda_a, \mathfrak{C})_{QPNSHS} \cap (\emptyset, \mathfrak{C})_{QPNSHS} = (\lambda_a, \mathfrak{C})_{QPNSHS}$

ii. $(\lambda_a, \mathfrak{C})_{QPNSHS} \cap (U, \mathfrak{C})_{QPNSHS} = (\lambda_a, \mathfrak{C})_{QPNSHS}$

Proof: The proof is similar to the above theorem

Theorem 3.18

Let $(\lambda_a, \mathfrak{C})_{QPNSHS}$ and $(\lambda_b, \mathfrak{C})_{QPNSHS}$ be two PNSHSS over the universe U. Then the following are true.

i. $(\lambda_a, \mathfrak{C})_{QPNSHS} \cup (\emptyset, \mathfrak{D})_{QPNSHS} = (\lambda_a, \mathfrak{C})_{QPNSHS}\ iff\ \mathfrak{D} \subseteq \mathfrak{C}$

ii. $(\lambda_a, \mathfrak{C})_{QPNSHS} \cup (U, \mathfrak{D})_{QPNSHS} = (\mathrm{U}, \mathfrak{C})_{QPNSHS}\ iff\ \mathfrak{C} \subseteq \mathfrak{D}$

Proof:

i. We have for $(\lambda_a, \mathfrak{C})_{QPNSHS}$,

$$\lambda_a(e) = \left\{\left(x, \mathcal{T}_{\lambda_a}, \mathcal{C}_{\lambda_a}, \mathcal{U}_{\lambda_a}, F_{\lambda_a}\right) x \in U\right\} \forall e \in \mathfrak{C}$$

Also let $(\emptyset, \mathfrak{D})_{QPNSHS} = (\lambda_b, \mathfrak{D})_{QPNSHS}$ then

$\lambda_b(e) = \{(x, 0, 0, 1,1): x \in U\} \forall\, e \in \mathfrak{D}$

Let $(\lambda_a, \mathfrak{C})_{QPNSHS} \cup (\emptyset, \mathfrak{D})_{QPNSHS} = (\lambda_a, \mathfrak{C})_{QPNSHS} \cup (\lambda_b, \mathfrak{D})_{QPNSHS} = (\lambda_c, \mathfrak{L})_{QPNSHS}$

where

$\mathfrak{L} = \mathfrak{C} \cup \mathfrak{D}$ for all $e \in \mathfrak{L}$

$\lambda_c(e)$ may be defined as

$$\begin{cases} \left\{\left(x, \mathcal{T}_{\lambda_a(e)}, \mathcal{C}_{\lambda_a(e)}, \mathcal{U}_{\lambda_a(e)}, F_{\lambda_a(e)}\right) : x \in U\right\} if\ e \in \mathfrak{C} - \mathfrak{D} \\ \{(x, 0,0,1,1): x \in U\}\ if\ e \in \mathfrak{D} - \mathfrak{C} \\ \left\{(x, \widehat{max}(\mathcal{T}_{\lambda_a(e)}, 0), \widehat{max}(\mathcal{C}_{\lambda_a(e)}, 0), \underbrace{min}(\mathcal{U}_{\lambda_a(e)}, 1), \underbrace{min}(F_{\lambda_a(e)}, 1))\right\} if\ e \in \mathfrak{C} \cap \mathfrak{D} \end{cases}$$

$$= \begin{cases} \left\{\left(x, \mathcal{T}_{\lambda_a(e)}, \mathcal{C}_{\lambda_a(e)}, \mathcal{U}_{\lambda_a(e)}, F_{\lambda_a(e)}\right) : x \in U\right\} if\ e \in \mathfrak{C} - \mathfrak{D} \\ \{(x, 1,1,0,0): x \in U\}\ if\ e \in \mathfrak{D} - \mathfrak{C} \\ \left\{\left(x, \mathcal{T}_{\lambda_a(e)}, \mathcal{C}_{\lambda_a(e)}, \mathcal{U}_{\lambda_a(e)}, F_{\lambda_a(e)}\right): x \in U\right\} if\ e \in \mathfrak{C} \cap \mathfrak{D} \end{cases}$$

Let $\mathfrak{D} \subseteq \mathfrak{C}$

Then $\lambda_c(e) = \begin{cases} \left\{\left(x, \mathcal{T}_{\lambda_a(e)}, \mathcal{C}_{\lambda_a(e)}, \mathcal{U}_{\lambda_a(e)}, F_{\lambda_a(e)}\right) : x \in U\right\} if\ e \in \mathfrak{C} - \mathfrak{D} \\ \left\{\left(x, \mathcal{T}_{\lambda_a(e)}, \mathcal{C}_{\lambda_a(e)}, \mathcal{U}_{\lambda_a(e)}, F_{\lambda_a(e)}\right): x \in U\right\} if\ e \in \mathfrak{C} \cap \mathfrak{D} \end{cases}$

$= \lambda_a(e)\ \forall\, e \in \mathfrak{C}$

Conversely Let $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \cup (\emptyset, \mathfrak{D})_{QPNSHS} = (\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS}$
Then $\mathfrak{C} = \mathfrak{C} \cup \mathfrak{D} \Longrightarrow \mathfrak{D} \subseteq \mathfrak{C}$

ii. Proof is similar to (i)

Theorem:3.19

Let $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS}$ and $(\lambda_{\mathfrak{b}}, \mathfrak{D})_{QPNSHS}$ be two QPNSHS over the universe U. Then the following are true.

i. $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \cap (\emptyset, \mathfrak{D})_{QPNSHS} = (\emptyset, \mathfrak{C} \cap \mathfrak{D})_{QPNSHS}$

ii. $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \cap (U, \mathfrak{D})_{QPNSHS} = (\lambda_{\mathfrak{a}}, \mathfrak{C} \cap \mathfrak{D})_{QPNSHS}$

Proof:

i. We have for $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS}$

$$\lambda_{\mathfrak{a}}(e) = \left\{\left(x, \mathcal{T}_{\lambda_{\mathfrak{a}}}, \mathcal{C}_{\lambda_{\mathfrak{a}}}, \mathcal{U}_{\lambda_{\mathfrak{a}}}, F_{\lambda_{\mathfrak{a}}}\right) x \in U\right\} \forall e \in \mathfrak{C}$$

Also let $(\emptyset, \mathfrak{D})_{QPNSHS} = (\lambda_{\mathfrak{b}}, \mathfrak{D})_{QPNSHS}$ then
$\lambda_{\mathfrak{b}}(e) = \{(x, 0, 0, 1, 1) : x \in U\} \forall\ e \in \mathfrak{D}$
Let $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \cap (\emptyset, \mathfrak{D})_{QPNSHS} = (\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \cap (\lambda_{\mathfrak{b}}, \mathfrak{D})_{QPNSHS} = (\lambda_{\mathfrak{c}}, \mathfrak{L})_{QPNSHS}$

where

$\mathfrak{L} = \mathfrak{C} \cap \mathfrak{D}$ for all $e \in \mathfrak{L}$

$$\lambda_{\mathfrak{c}}(e) = \left\{\left(x, \underbrace{min}(\mathcal{T}_{\lambda_{\mathfrak{a}}(e)}, \mathcal{T}_{\lambda_{\mathfrak{b}}(e)}), \underbrace{min}(\mathcal{C}_{\lambda_{\mathfrak{a}}(e)}, \mathcal{C}_{\lambda_{\mathfrak{b}}(e)}), \overbrace{max}(\mathcal{U}_{\lambda_{\mathfrak{a}}(e)}, \mathcal{U}_{\lambda_{\mathfrak{b}}(e)}), \overbrace{max}(F_{\lambda_{\mathfrak{a}}(e)}, F_{\lambda_{\mathfrak{b}}(e)})\right) : x \in U\right\}$$

$$= \left\{\left(x, \underbrace{min}(\mathcal{T}_{\lambda_{\mathfrak{a}}(e)}, 0), \underbrace{min}(\mathcal{C}_{\lambda_{\mathfrak{a}}(e)}, 0), \overbrace{max}(\mathcal{U}_{\lambda_{\mathfrak{a}}(e)}, 1), \overbrace{max}(F_{\lambda_{\mathfrak{a}}(e)}, 1)\right) : x \in U\right\}$$

$$= \{(x, 0,0,1,1) : x \in U\}$$

$$= (\lambda_{\mathfrak{b}}, \mathfrak{D})_{QPNSHS} = (\emptyset, \mathfrak{D})_{QPNSHS}$$

Thus $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \cap (\emptyset, \mathfrak{D})_{QPNSHS} = (\emptyset, \mathfrak{D})_{QPNSHS} = (\emptyset, \mathfrak{C} \cap \mathfrak{D})_{QPNSHS}$

ii. Proof is similar to (i).

Theorem:3.20

Let $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS}$ and $(\lambda_{\mathfrak{b}}, \mathfrak{D})_{QPNSHS}$ be two PNSHSS over the universe U. Then the following are true.

i. $\left((\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \cup (\lambda_{\mathfrak{b}}, \mathfrak{D})_{QPNSHS}\right)^c \subseteq {(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS}}^c \cup {(\lambda_{\mathfrak{b}}, \mathfrak{D})_{QPNSHS}}^c$

ii. ${(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS}}^c \cap {(\lambda_{\mathfrak{b}}, \mathfrak{D})_{QPNSHS}}^c \subseteq \left((\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \cap (\lambda_{\mathfrak{b}}, \mathfrak{D})_{QPNSHS}\right)^c$

Proof:

i. Let $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \cup (\lambda_{\mathfrak{b}}, \mathfrak{D})_{QPNSHS} = (\lambda_{\mathfrak{c}}, \mathfrak{L})_{QPNSHS}$ where $\mathfrak{L} = \mathfrak{C} \cap \mathfrak{D}$ for all $e \in \mathfrak{L}$

$\lambda_{\mathfrak{c}}(e)$ may defined as

$$\begin{cases} \left\{\left(x, \mathcal{T}_{\lambda_{\mathfrak{a}}(e)}, \mathcal{C}_{\lambda_{\mathfrak{a}}(e)}, \mathcal{U}_{\lambda_{\mathfrak{a}}(e)}, F_{\lambda_{\mathfrak{a}}(e)}\right) : x \in U\right\} if\ e \in \mathfrak{C} - \mathfrak{D} \\ \left\{\left(x, \mathcal{T}_{\lambda_{\mathfrak{b}}(e)}, \mathcal{C}_{\lambda_{\mathfrak{b}}(e)}, \mathcal{U}_{\lambda_{\mathfrak{b}}(e)}, F_{\lambda_{\mathfrak{b}}(e)}\right) : x \in U\right\} if\ e \in \mathfrak{D} - \mathfrak{C} \\ \left\{\left(x, \overbrace{max}(\mathcal{T}_{\lambda_{\mathfrak{a}}(e)}, \mathcal{T}_{\lambda_{\mathfrak{b}}(e)}), \overbrace{max}(\mathcal{C}_{\lambda_{\mathfrak{a}}(e)}, \mathcal{C}_{\lambda_{\mathfrak{b}}(e)}), \underbrace{min}(\mathcal{U}_{\lambda_{\mathfrak{a}}(e)}, \mathcal{U}_{\lambda_{\mathfrak{b}}(e)}), \underbrace{min}(F_{\lambda_{\mathfrak{a}}(e)}, F_{\lambda_{\mathfrak{b}}(e)})\right) : x \in U\right\} \\ if\ e \in \mathfrak{C} \cap \mathfrak{D} \end{cases}$$

Thus $\left((\lambda_a,\mathfrak{C})_{QPNSHS} \cup (\lambda_b,\mathfrak{D})_{QPNSHS}\right)^c = (\lambda_c,\mathfrak{L})^c_{QPNSHS}$ where $\mathfrak{L} = \mathfrak{C} \cap \mathfrak{D}$ for all $e \in \mathfrak{L}$

$$\left(\lambda_c(e)\right)^c = \begin{cases} \left(\lambda_a(e)\right)^c if\ e \in \mathfrak{C} - \mathfrak{D} \\ \left(\lambda_b(e)\right)^c if\ e \in \mathfrak{D} - \mathfrak{C} \\ \left((\lambda_a,\mathfrak{C})_{QPNSHS} \cup (\lambda_b,\mathfrak{D})_{QPNSHS}\right)^c if\ e \in \mathfrak{C} \cap \mathfrak{D} \end{cases}$$

$$= \begin{cases} \left\{\left(x, \mathcal{T}_{\lambda_a(e)}, \mathcal{C}_{\lambda_a(e)}, \mathcal{U}_{\lambda_a(e)}, F_{\lambda_a(e)}\right) : x \in U\right\} if\ e \in \mathfrak{C} - \mathfrak{D} \\ \left\{\left(x, \mathcal{T}_{\lambda_b(e)}, \mathcal{C}_{\lambda_b(e)}, \mathcal{U}_{\lambda_b(e)}, F_{\lambda_b(e)}\right) : x \in U\right\} if\ e \in \mathfrak{D} - \mathfrak{C} \\ \left\{(x, \widetilde{max}(\mathcal{T}_{\lambda_a(e)}, \mathcal{T}_{\lambda_b(e)}), \widetilde{max}(\mathcal{C}_{\lambda_a(e)}, \mathcal{C}_{\lambda_b(e)}), \underbrace{min}(\mathcal{U}_{\lambda_a(e)}, \mathcal{U}_{\lambda_b(e)}), \underbrace{min}(F_{\lambda_a(e)}, F_{\lambda_b(e)})) : x \in U\right\} \\ if\ e \in \mathfrak{C} \cap \mathfrak{D} \end{cases}$$

Again $(\lambda_a,\mathfrak{C})^c_{QPNSHS} \cup (\lambda_b,\mathfrak{D})^c_{QPNSHS} = \left(\mathcal{R}_g, \mathcal{S}\right)$ say $\mathcal{S} = \mathfrak{C} \cup \mathfrak{D}\ and\ \forall\ e \in \mathcal{S}$

$$\mathcal{R}_g(e) = \begin{cases} \left(\lambda_a(e)\right)^c if\ e \in \mathfrak{C} - \mathfrak{D} \\ \left(\lambda_b(e)\right)^c if\ e \in \mathfrak{D} - \mathfrak{C} \\ \left((\lambda_a,\mathfrak{C}) \cup (\lambda_b,\mathfrak{D})\right)^c if\ e \in \mathfrak{C} \cap \mathfrak{D} \end{cases}$$

$$\begin{cases} \left\{\left(x, \mathcal{T}_{\lambda_a(e)}, \mathcal{C}_{\lambda_a(e)}, \mathcal{U}_{\lambda_a(e)}, F_{\lambda_a(e)}\right) : x \in U\right\} if\ e \in \mathfrak{C} - \mathfrak{D} \\ \left\{\left(x, \mathcal{T}_{\lambda_b(e)}, \mathcal{C}_{\lambda_b(e)}, \mathcal{U}_{\lambda_b(e)}, F_{\lambda_b(e)}\right) : x \in U\right\} if\ e \in \mathfrak{D} - \mathfrak{C} \\ \left\{(x, \underbrace{min}(\mathcal{T}_{\lambda_a(e)}, \mathcal{T}_{\lambda_b(e)}), \underbrace{min}(\mathcal{C}_{\lambda_a(e)}, \mathcal{C}_{\lambda_b(e)}), \widetilde{max}(\mathcal{U}_{\lambda_a(e)}, \mathcal{U}_{\lambda_b(e)}), \widetilde{max}(F_{\lambda_a(e)}, F_{\lambda_b(e)})) : x \in U\right\} \\ if\ e \in \mathfrak{C} \cap \mathfrak{D} \end{cases}$$

So, $\mathfrak{L} \subseteq \mathcal{S}\ \forall\ e \in \mathcal{S}$

$$\left(\lambda_c(e)\right)^c \subseteq \mathcal{R}_g(e)$$

Thus $\left((\lambda_a,\mathfrak{C})_{QPNSHS} \cup (\lambda_b,\mathfrak{D})_{QPNSHS}\right)^c \subseteq (\lambda_a,\mathfrak{C})^c_{QPNSHS} \cup (\lambda_b,\mathfrak{D})^c_{QPNSHS}$

ii. Let $(\lambda_a,\mathfrak{C})_{QPNSHS} \cap (\lambda_b,\mathfrak{D})_{QPNSHS} = (\lambda_c,\mathfrak{L})_{QPNSHS}$ where $\mathfrak{L} = \mathfrak{C} \cap \mathfrak{D}$ for all $e \in \mathfrak{L}$

$\lambda_c(e)$ may defined as

$$\left\{(x, \underbrace{min}(\mathcal{T}_{\lambda_a(e)}, \mathcal{T}_{\lambda_b(e)}), \underbrace{min}(\mathcal{C}_{\lambda_a(e)}, \mathcal{C}_{\lambda_b(e)}), \widetilde{max}(\mathcal{U}_{\lambda_a(e)}, \mathcal{U}_{\lambda_b(e)}), \widetilde{max}(F_{\lambda_a(e)}, F_{\lambda_b(e)})) : x \in U\right\}$$

Thus $\left((\lambda_a,\mathfrak{C})_{QPNSHS} \cap (\lambda_b,\mathfrak{D})_{QPNSHS}\right)^c = (\lambda_c,\mathfrak{L})^c_{QPNSHS}$ where $\mathfrak{L} = \mathfrak{C} \cap \mathfrak{D}$ for all $e \in \mathfrak{L}$

$$\left(\lambda_c(e)\right)^c = \left\{x, \underbrace{min}(\mathcal{T}_{\lambda_a(e)}, \mathcal{T}_{\lambda_b(e)}), \underbrace{min}(\mathcal{C}_{\lambda_a(e)}, \mathcal{C}_{\lambda_b(e)}), \widetilde{max}(\mathcal{U}_{\lambda_a(e)}, \mathcal{U}_{\lambda_b(e)}), \widetilde{max}(F_{\lambda_a(e)}, F_{\lambda_b(e)})) : x \in U\right\}^c$$

$$= \left\{x, \widetilde{max}(\mathcal{T}_{\lambda_a(e)}, \mathcal{T}_{\lambda_b(e)}), \widetilde{max}(\mathcal{C}_{\lambda_a(e)}, \mathcal{C}_{\lambda_b(e)}), \min(\mathcal{U}_{\lambda_a(e)}, \mathcal{U}_{\lambda_b(e)}), min(F_{\lambda_a(e)}, F_{\lambda_b(e)})) : x \in U\right\}$$

Again $(\lambda_a,\mathfrak{C})^c_{QPNSHS} \cap (\lambda_b,\mathfrak{D})^c_{QPNSHS} = \left(\mathcal{R}_g, \mathcal{S}\right)_{QPNSHS}$ say $\mathcal{S} = \mathfrak{C} \cap \mathfrak{D}\ and\ \forall\ e \in \mathcal{S}$

$$\mathcal{R}_g(e) = (\lambda_a(e))^c \cap (\lambda_b(e))^c$$

$$= \left\{x, \underbrace{min}(\mathcal{T}_{\lambda_a(e)}, \mathcal{T}_{\lambda_b(e)}), \underbrace{min}(\mathcal{C}_{\lambda_a(e)}, \mathcal{C}_{\lambda_b(e)}), \widetilde{max}(\mathcal{U}_{\lambda_a(e)}, \mathcal{U}_{\lambda_b(e)}), \widetilde{max}(F_{\lambda_a(e)}, F_{\lambda_b(e)})) : x \in U\right\}$$

We see that $\mathfrak{L} = \mathcal{S}\ and\ \forall\ e \in \mathcal{S}$

$$\mathcal{R}_{\mathfrak{g}}(e) \subseteq \left(\lambda_{\mathfrak{c}}(e)\right)^{c}$$

Thus $(\lambda_{\mathfrak{a}}, \mathfrak{C})^{c}_{QPNSHS} \cap (\lambda_{\mathfrak{b}}, \mathfrak{D})^{c}_{QPNSHS} \subseteq \left((\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \cap (\lambda_{\mathfrak{b}}, \mathfrak{D})_{QPNSHS}\right)^{c}$

Theorem: 3.21

Let $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS}$ and $(\lambda_{\mathfrak{b}}, \mathfrak{D})_{QPNSHS}$ be two PNSHSS over the universe U. Then the following are true.

i. $\left((\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \cup (\lambda_{\mathfrak{b}}, \mathfrak{C})_{QPNSHS}\right)^{c} \subseteq (\lambda_{\mathfrak{a}}, \mathfrak{C})^{c}_{QPNSHS} \cap (\lambda_{\mathfrak{b}}, \mathfrak{C})^{c}_{QPNSHS}$

ii. $(\lambda_{\mathfrak{a}}, \mathfrak{C})^{c}_{QPNSHS} \cap (\lambda_{\mathfrak{b}}, \mathfrak{C})^{c}_{QPNSHS} \subseteq \left((\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \cup (\lambda_{\mathfrak{b}}, \mathfrak{C})_{QPNSHS}\right)^{c}$

Proof:

i. Let $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \cup (\lambda_{\mathfrak{b}}, \mathfrak{C})_{QPNSHS} = (\lambda_{\mathfrak{c}}, \mathfrak{C})_{QPNSHS}$, for all $e \in \mathfrak{C}$

$$\lambda_{\mathfrak{c}}(e) = \lambda_{\mathfrak{a}}(e) \cup \lambda_{\mathfrak{b}}(e)$$

$= \left\{(x, \overbrace{max}(\mathcal{T}_{\lambda_{\mathfrak{a}}(e)}, \mathcal{T}_{\lambda_{\mathfrak{b}}(e)}), \overbrace{max}(\mathcal{C}_{\lambda_{\mathfrak{a}}(e)}, \mathcal{C}_{\lambda_{\mathfrak{b}}(e)}), \underbrace{min}(\mathcal{U}_{\lambda_{\mathfrak{a}}(e)}, \mathcal{U}_{\lambda_{\mathfrak{b}}(e)}), \underbrace{min}(F_{\lambda_{\mathfrak{a}}(e)}, F_{\lambda_{\mathfrak{b}}(e)})) : x \in U\right\}$

Thus $\left((\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \cup (\lambda_{\mathfrak{b}}, \mathfrak{C})_{QPNSHS}\right)^{c} = (\lambda_{\mathfrak{c}}, \mathfrak{C})^{c}_{QPNSHS}$, for all $e \in \mathfrak{C}$

$$\left(\lambda_{\mathfrak{c}}(e)\right)^{c} = \left(\lambda_{\mathfrak{a}}(e) \cup \lambda_{\mathfrak{b}}(e)\right)^{c}$$

$= \left\{(x, \overbrace{max}(\mathcal{T}_{\lambda_{\mathfrak{a}}(e)}, \mathcal{T}_{\lambda_{\mathfrak{b}}(e)}), \overbrace{max}(\mathcal{C}_{\lambda_{\mathfrak{a}}(e)}, \mathcal{C}_{\lambda_{\mathfrak{b}}(e)}), \underbrace{min}(\mathcal{U}_{\lambda_{\mathfrak{a}}(e)}, \mathcal{U}_{\lambda_{\mathfrak{b}}(e)}), \underbrace{min}(F_{\lambda_{\mathfrak{a}}(e)}, F_{\lambda_{\mathfrak{b}}(e)})) : x \in U\right\}^{c}$

$= \left\{(x, \underbrace{min}(\mathcal{T}_{\lambda_{\mathfrak{a}}(e)}, \mathcal{T}_{\lambda_{\mathfrak{b}}(e)}), \underbrace{min}(\mathcal{C}_{\lambda_{\mathfrak{a}}(e)}, \mathcal{C}_{\lambda_{\mathfrak{b}}(e)}), \overbrace{max}(\mathcal{U}_{\lambda_{\mathfrak{a}}(e)}, \mathcal{U}_{\lambda_{\mathfrak{b}}(e)}), \overbrace{max}(F_{\lambda_{\mathfrak{a}}(e)}, F_{\lambda_{\mathfrak{b}}(e)})) : x \in U\right\}$

Again $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \cap (\lambda_{\mathfrak{b}}, \mathfrak{C})_{QPNSHS} = \left(\mathcal{R}_{\mathfrak{g}}, \mathfrak{C}\right)_{QPNSHS}$ where $\forall\ e \in \mathfrak{C}$

$$\mathcal{R}_{\mathfrak{g}}(e) = (\lambda_{\mathfrak{a}}(e))^{c} \cap (\lambda_{\mathfrak{b}}(e))^{c}$$

$= \left\{(x, \underbrace{min}(F_{\lambda_{\mathfrak{a}}(e)}, F_{\lambda_{\mathfrak{b}}(e)}), \underbrace{min}(\mathcal{U}_{\lambda_{\mathfrak{a}}(e)}, \mathcal{U}_{\lambda_{\mathfrak{b}}(e)}), \overbrace{max}(\mathcal{C}_{\lambda_{\mathfrak{a}}(e)}, \mathcal{C}_{\lambda_{\mathfrak{b}}(e)}), \overbrace{max}(\mathcal{T}_{\lambda_{\mathfrak{a}}(e)}, \mathcal{T}_{\lambda_{\mathfrak{b}}(e)})) : x \in U\right\}$

Thus $\left((\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \cup (\lambda_{\mathfrak{b}}, \mathfrak{C})_{QPNSHS}\right)^{c} \subseteq (\lambda_{\mathfrak{a}}, \mathfrak{C})^{c}_{QPNSHS} \cap (\lambda_{\mathfrak{b}}, \mathfrak{C})^{c}_{QPNSHS}$

ii. $(\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \cap (\lambda_{\mathfrak{b}}, \mathfrak{C})_{QPNSHS} = (\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS}$

$= \left\{(x, \underbrace{min}(F_{\lambda_{\mathfrak{a}}(e)}, F_{\lambda_{\mathfrak{b}}(e)}), \underbrace{min}(\mathcal{U}_{\lambda_{\mathfrak{a}}(e)}, \mathcal{U}_{\lambda_{\mathfrak{b}}(e)}), \overbrace{max}(\mathcal{C}_{\lambda_{\mathfrak{a}}(e)}, \mathcal{C}_{\lambda_{\mathfrak{b}}(e)}), \overbrace{max}(\mathcal{T}_{\lambda_{\mathfrak{a}}(e)}, \mathcal{T}_{\lambda_{\mathfrak{b}}(e)})) : x \in U\right\}$

thus $\left((\lambda_{\mathfrak{a}}, \mathfrak{C})_{QPNSHS} \cap (\lambda_{\mathfrak{b}}, \mathfrak{C})_{QPNSHS}\right)^{c} = (\lambda_{\mathfrak{a}}, \mathfrak{C})^{c}_{QPNSHS}$

$$\lambda_{\mathfrak{c}}(e) = \lambda_{\mathfrak{a}}(e) \cap \lambda_{\mathfrak{b}}(e)$$

$= \left\{(x, \underbrace{min}(F_{\lambda_{\mathfrak{a}}(e)}, F_{\lambda_{\mathfrak{b}}(e)}), \underbrace{min}(\mathcal{U}_{\lambda_{\mathfrak{a}}(e)}, \mathcal{U}_{\lambda_{\mathfrak{b}}(e)}), \overbrace{max}(\mathcal{C}_{\lambda_{\mathfrak{a}}(e)}, \mathcal{C}_{\lambda_{\mathfrak{b}}(e)}), \overbrace{max}(\mathcal{T}_{\lambda_{\mathfrak{a}}(e)}, \mathcal{T}_{\lambda_{\mathfrak{b}}(e)})) : x \in U\right\}^{c}$

$$= \Big\{\big(x, \widetilde{max}(F_{\lambda_a(e)}, F_{\lambda_b(e)}), \widetilde{max}(\mathcal{U}_{\lambda_a(e)}, \mathcal{U}_{\lambda_b(e)}), \underbrace{min}(\mathcal{C}_{\lambda_a(e)}, \mathcal{C}_{\lambda_b(e)}), \underbrace{min}(\mathcal{T}_{\lambda_a(e)}, \mathcal{T}_{\lambda_b(e)})\big) : x \in U\Big\}$$

Again $(\lambda_a, \mathfrak{C})^c_{QPNSHS} \cup (\lambda_b, \mathfrak{C})^c_{QPNSHS} = (\mathcal{R}_g, \mathfrak{C})_{QPNSHS}$ where $\forall\, e \in \mathfrak{C}$

$$\mathcal{R}_g(e) = (\lambda_a(e))^c \cup (\lambda_b(e))^c$$

$$= \Big\{\big(x, \widetilde{max}(F_{\lambda_a(e)}, F_{\lambda_b(e)}), \widetilde{max}(\mathcal{U}_{\lambda_a(e)}, \mathcal{U}_{\lambda_b(e)}), \underbrace{min}(\mathcal{C}_{\lambda_a(e)}, \mathcal{C}_{\lambda_b(e)}), \underbrace{min}(\mathcal{T}_{\lambda_a(e)}, \mathcal{T}_{\lambda_b(e)})\big) : x \in U\Big\}$$

$$\big((\lambda_a, \mathfrak{C})_{QPNSHS} \cap (\lambda_b, \mathfrak{C})_{QPNSHS}\big)^c = (\lambda_a, \mathfrak{C})^c_{QPNSHS} \cup (\lambda_b, \mathfrak{C})^c_{QPNSHS}$$

IV. Conclusion

In this paper, we have studied the QPNSHS. Based on their definition and properties, we have newly introduced the quadri-partitioned neutrosophic Super Hypersoft set and some basic definitions on this topic; also, we have discussed some of its properties and theorems.

Conflict of Interest

The authors declare that they have no conflict of interest.

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