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A FRACTIONAL NONLOCAL PARTIAL DIFFERENTIAL EQUATION FRAMEWORK FOR ADAPTIVE TRAFFIC FLOW DYNAMICS

Inaam Rikan Hassan[1], Ghuson S. Abed[2]

[1]Department of Scientific Affairs, Baghdad, Iraq.

[2]College of Engineering, Baghdad, Iraq.

Email: [1]drinrh@uoitc.edu.iq , [2]ghsonabed.2019@uoitc.edu.iq

Corresponding Author: **Ghuson S. Abed**

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Abstract

Many traffic flow-based nonlocal models consist of microscopic Ordinary Differential Equations (ODEs) and macroscopic Partial Differential Equations (PDEs). In this paper, a novel mathematical model based on fractional and nonlocal partial differential equations (PDEs) is proposed to capture adaptive traffic flow dynamics in complex urban networks. Conventional models like the Lighthill–Whitham–Richards (LWR) model fail to capture interactions and memory impacts, relying on systems-based real traffic. To address these restrictions, a system-based fractional-order nonlocal PDE integrating the interactions-based spatial distribution and diffusion coefficients-based time-dependent adaptivity is proposed. The proposed model combines fractional derivatives for describing inherited traffic environmental conditions and nonlocal CORS for representing driver anticipation influences. Analytical properties consisting of steady state, presence, and solutions-based boundedness are considered over appropriate assumptions. A finite difference discretization of the numerical approach and an approximation method-based Grünwald–Letnikov scheme are modeled for simulation. MATLAB-based experiments illustrate that the proposed model captures a sharp change in pressure in a narrow region traveling through a medium, especially air, caused by an explosion or by a body moving faster than sound propagation, crowding formation, and scattering more precisely than conventional integer-order approaches. Six simulation scenarios demonstrate the fractional order effect and radius of nonlocal interaction of evolution-based traffic density. The key findings reflect that combining memory and nonlocality particularly enhances the capabilities of predictions and realism. The proposed framework offers a basis for modern traffic control schemes and opens novel trends for stratifying fractional PDEs in detailed dynamical systems.

Keywords: partial differential equations, fractional calculus, traffic flow modeling, nonlocal dynamics, adaptive diffusion

I. Introduction

Differential equations offer such a framework, capturing the relationships between variables and their rates of change with regard to one or more independent variables, typically time or space [VII, IX]. Differential equations establish the cornerstone of the physical mathematical models in systems-based engineering and socio-technical schemes [XV]. Specifically, partial differential equations (PDEs) have been widely used to control the advancements of spatially distributed quantities and fluid dynamics applications, heat transfer systems, and traffic models [XIII, I]. Conventional model-based traffic flow, like the Lighthill–Whitham–Richards (LWR) approach [XVI], relies heavily on conservation laws:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v) = 0 \tag{1}$$

Where $\rho(x, t)$ represents the density of traffic and $v(x, t)$ denotes velocity. Whilst powerful in capturing the behavior of the macroscopic, these models are used to assume local interactions, anticipation-based drivers, and neglect dependencies. To overcome such limitations, nonlocal PDEs are utilized [XVII, XVIII]:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}\left(\rho \int K(x\text{-}y)\rho(y, t)dy\right) = 0 \tag{2}$$

Where $K(.)$ represents the kernel function based on the spatial effect. Thus, such formulations lack memory effects, which are crucial in traffic-based real-world situations where past states affect current decisions. Therefore, Fractional calculus offers a natural expansion [III, X, IV]:

$$\frac{\partial^{\alpha} \rho}{\partial t^{\alpha}} = \frac{\partial}{\partial x}\left(D(\rho, t)\frac{\partial \rho}{\partial x}\right) + \int K(x - y)\big(\rho(y, t) - \rho(x, t)\big)dy \tag{3}$$

Where $D(\rho, t)$ denotes an adaptive diffusion coefficient. Such formulation is simultaneously used to capture influences of memory through fractional derivatives, interactions through nonlocal kernels, and dynamic adaptation through diffusion-based density-dependent [II, VI, VIII]. The main objectives of this article include formulating a unified model-based fractional–nonlocal PDE for traffic dynamics, deriving analytical insights into steady-state and behavioral solutions, developing a framework-based numerical simulation, and validating the model through computational experiments. The contribution of this article includes integrating three main advanced mathematical concepts based on fractional calculus, operator-based nonlocal, and adaptive diffusion into a single dependent model. Such a framework expands beyond conventional PDE approaches and offers a more realistic representation in systems-based complex dynamical approaches.

II. Literature Review

Karafyllis et al. [XI] proposed conditions that guaranteed the existence and uniqueness of classical solutions for a non-local conservation law on a ring road with possible nudging terms and demonstrated that nudging increased the flow and produced strong stabilizing effects on traffic flow. Furthermore, the study established local exponential stability of the uniform equilibrium profile in the L^2 state norm and demonstrated the efficiency of nudging terms through numerical examples, whereas Ciaramaglia and Goatin [V] proposed a nonlocal PDE–ODE strongly coupled system describing the interaction between traffic flow and a moving bottleneck induced by a controlled vehicle. The study proved the well-posedness of the coupled system using a fractional step finite volume approximation and established uniform BV estimates, L^{∞} bounds, compactness of the scheme, and convergence to an entropy solution, while Yan et al. [XIX] proposed a continuous adaptive graph neural stochastic partial differential equation (CAG-NSPDE) for traffic flow forecasting using learnable continuous adaptive node embeddings and stochastic PDE architectures. The study demonstrated that the proposed framework simultaneously extracted features from temporal and frequency domains by reducing SPDEs into systems of ODEs in Fourier space and outperformed state-of-the-art methods on four real-world datasets. Meanwhile, Kang et al. [XII] proposed a generalized neural Fractional Attention Differential Equation (FADE) combining memory-retention capabilities of fractional calculus with contextual learnable attention mechanisms. The study established solution boundedness, problem well-posedness, and numerical solver convergence properties while demonstrating superior modeling capacity for fluid flow, graph learning, and spatial-temporal traffic flow forecasting compared with integer-order neural ODE models and existing fractional approaches. In contrast, Najafi et al. [XIV] introduced an innovative approach integrating the spectral method with a time-dependent partial differential equation filter to address shock waves in traffic flow modeling. The study demonstrated that discrete low-pass filters mitigated shock-induced deviations and achieved more accurate solutions than the Lax and Cu methods while providing stability analysis and numerical validation. Table 1 demonstrates a complete comparison of the whole literature studies in terms of method, differential equation framework, key findings, limitations, and applications.

Table 1: Comprehensive comparison of the literature review.

Ref.	Year	Method	Differential Equation Framework	Key Findings	Limitations	Applications
[XI]	2022	Non-local conservation law with nudging	Non-local traffic conservation PDE	Guaranteed existence, uniqueness, and local exponential stability	Limited to ring road systems	Traffic flow stabilization
[V]	2026	Coupled PDE–ODE finite volume approximation	Nonlocal scalar conservation law with delay	Established well-posedness and entropy solution convergence	High computational complexity	Traffic–bottleneck interaction
[XIX]	2024	CAG-NSPDE	Stochastic PDE reduced to ODEs	Improved predictive performance on traffic datasets	Dependent on training data quality	Intelligent transportation systems
[XII]	2026	FADE neural framework	Fractional attention differential equations	Achieved superior temporal dependency modeling	Complex neural attention implementation	Fluid flow and traffic forecasting

[XIV]	2025	Spectral PDE filtering	Time-dependent PDE filtering	Reduced shock-induced deviations with higher accuracy	Mainly numerical validation	Shock wave traffic modeling

III. Method

i. Fractional Temporal Dynamics

We initiate with the law of fractional conservation, which generalizes conventional equations of traffic flow via correlating influences of temporal memory. This is crucial because driver behavior relies not only on present density but also on prior congestion schemes.

$$\frac{\partial^\alpha \rho}{\partial t^\alpha} = \frac{\partial}{\partial x}(\rho v) \tag{4}$$

Such a formulation is used to replace the derivative of the integer-order time with a derivative of fractional order $\alpha \in (0, 1]$, presenting a weighted history of prior states. Different from conventional conservation laws, such an equation is used to capture delayed reactions and the influences of persistent congestion. It guarantees that the system evolution indicates cumulative temporal effects and only instantaneous changes.

$$D_t^\alpha \rho(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{\partial \rho(\tau)}{\partial \tau} \frac{d\tau}{(t-\tau)^\alpha} \tag{5}$$

Such a Caputo description is used to formalize the derivative of a fractional based on integral form, explicitly illustrating how prior states are used to be contributed to the present. The core $(\mathrm{t}-\tau)^\alpha$ term is assigning more robust weights to the present history whilst it preserves memory. Such a structure is crucial for guaranteeing physical interpretability rather than numerical implementability.

$$\frac{\partial^\alpha \rho}{\partial t^\alpha} + v\frac{\partial \rho}{\partial x} = 0 \tag{6}$$

Here, the advection is used to be coupled with fractional dynamics, generating a memory-impacting wave propagation approach. The transportation velocity v interacts with the derivative of the fractional, changing shock formation and the speed of propagation. Such a formulation bridges conventional behavior of the hyperbolic PDE with dynamics -based anomalous temporal behavior.

$$\frac{\partial^\alpha \rho}{\partial t^\alpha} = D\frac{\partial^\alpha \rho}{\partial x^\alpha} \tag{7}$$

Such a formulation-based fractional diffusion model is used to study anomalous spreading related to the density of traffic. Compared to conventional diffusion, the fractional operator slows down or may accelerate dispersion based on α. This is especially beneficial for approaching wave-based stop-and-go that do not follow conventional Gaussian diffusion approaches.

$$\frac{\partial^\alpha \rho}{\partial t^\alpha} + \lambda\rho = 0 \tag{8}$$

Such a formulation presents a relaxation approach to the fractional system. The parameter λ is used to control decay toward balance, guaranteeing steady state and solutions-based boundedness. Also, it enables the system to represent dissipation influences noted in real traffic flow.

$$\frac{\partial^{\alpha}\rho}{\partial t^{\alpha}} = f(\rho) \tag{9}$$

Such a formulation-based nonlinear equation is used to generalize the system by enabling dynamics-based arbitrary density-dependent behavior. It allows the design of feedback approaches like dissipation or congestion amplification. This is used to serve as a bridge to the completely coupled nonlinear approach modeled later.

ii. Nonlocal Spatial Interaction Modeling

Now, a spatial nonlocality is incorporated for modeling how drivers respond to environmental conditions in their immediate vicinity. That is critical for a realistic approaching of anticipation and interactions.

$$\frac{\partial\rho}{\partial t} = \int K(x-y)\big(\rho(y,t) - \rho(x,t)\big)dy \tag{10}$$

Such a formulation describes the process of nonlocal diffusion, whereas the core Kcomputes the interaction robustness among spatial points. Different from local Laplacian diffusion, such an equation is used to capture long-range impact. It guarantees preservation whilst enabling spatial coupling along finite distances.

$$K(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}} \tag{11}$$

The kernel of the Gaussian offers a physically meaningful form of interaction decay. The parameter σ is used to control the interaction radius, enabling tuning between local behavior and global behavior. Such a choice guarantees smoothness and mathematical tractability.

The interaction kernel-based Gaussian $K(x-y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-y)^2}{2\sigma^2}}$ has to be proved formally to satisfy the normalization, positivity, and vehicle mass-based physical conservation. The positivity-based kernel $K(x-y) \geq 0$and the space normalization are specified by satisfying equation 12:

$$\int_{\Omega}^{\cdot} K(x-y)dy = 1 \tag{12}$$

In order to prove the conservation-based mass, the nonlocal PDE term is integrated under the spatial domain Ω across the boundary conditions-based zero-flux is shown in equation 13:

$$\frac{d}{dt}\int_{\Omega}^{\cdot} \rho(x,t)dx = \int_{\Omega}^{\cdot}\int_{\Omega}^{\cdot} K(x-y)[\rho(y,t) - \rho(x,t)]\, dy\, dx = 0 \tag{13}$$

Because of the anti-symmetry of the integrand, when exchanging$x$ and y, the double integral in equation 13 is used to be estimated identically to zero, that is mathematically being guaranteed whole density of the vehicle is conserved without artificial creation or loss. Eventually, a parametric sensitivity analysis is conducted illustrating how changing the interaction radius parameter $\sigma$is used to prevent instabilities of non-physical shock.

$$\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial x}\left(\rho V(\bar{\rho})\right) = 0 \tag{14}$$

Such a formulation expands conventional conservation laws through replacing local density by a nonlocal averaged density. The function of velocity $V(\bar{\rho})$ depends on cumulative information, indicating driver anticipation. That remarkably enhances realism when it is compared to purely local systems.

$$\bar{\rho}(x,t) = \int K(x-y)\rho(y,t)dy \tag{15}$$

This formulation is used to define the nonlocal averaging operator, which is used to feed into the function of velocity. It smooths the variations-based abrupt density and prevents the discontinuities' anelasticity. The modeled operator is used to be served as a main link between the interactions of the microscopic and the behavior of macroscopic flow.

$$\frac{\partial\rho}{\partial t} = D\nabla^2\rho + \int K(x-y)\rho(y,t)dy \tag{16}$$

Such a hybrid formulation integrates local diffusion and nonlocal aggregation. It is used to capture smoothing rather than impacts. Such a combination improves steady state whilst conserving important spatial complexity.

$$\frac{\partial\rho}{\partial t} = \nabla.(\int K(x-y)\nabla\rho(y,t)dy) \tag{17}$$

Such an equation confirms consistency and principles-based flux conservation. It is used to generalize Fick's law for a nonlocal context. The structure of divergence ensures physically meaningful transportation dynamics.

iii. Adaptive Diffusion Coupling

Finally, adaptive diffusion is incorporated to account for density-dependent rather than behavior flow of time-varying.

$$D(\rho) = D_0\left(1+\beta_\rho\right) \tag{18}$$

Such a formulation is used to define a coefficient-based density-dependent diffusion. As density is increased, diffusion intensifies, approaching congestion-induced propagation. Such nonlinearity is crucial to capture phenomena-based real traffic.

$$\frac{\partial\rho}{\partial t} = \frac{\partial}{\partial x}\left(D(\rho)\frac{\partial\rho}{\partial x}\right) \tag{19}$$

Such a nonlinear diffusion formulation indicates an adaptive behavior of spreading. It is used to prevent unrealistic cumulative density and confirm smoother transitions. Such an equation is widely utilized in theory-based nonlinear transportation.

$$\frac{\partial\rho}{\partial t} = D(\rho)\nabla^2\rho + \nabla D(\rho).\nabla\rho \tag{20}$$

Such an expansion form is used to reveal the interaction gradients between diffusion and density. The second term presents further nonlinear coupling. It improves the capabilities of the model for representing complicated spatial dynamics.

$$D(\rho,t) = D_0e^{-\gamma t} + \eta\rho \tag{21}$$

Such a formulation offers diffusion-based time-dependent adaptation. It indicates evolving traffic environmental conditions, like peak-hour influences. The models of exponential term for temporal decay-based baseline diffusion.

$$\frac{\partial^\alpha\rho}{\partial t^\alpha} = \nabla.(D(\rho,t)\nabla\rho) \tag{22}$$

Such a formulation merges fractional temporal influence and diffusion-based adaptivity. It describes a main step toward the unified approach. The coupling confirms memory rather than adaptation-based nonlinear spatiality.

$$\frac{\partial^\alpha\rho}{\partial t^\alpha} = \nabla.(D(\rho,t)\nabla\rho) + \int K(x-y)\rho(y,t) - \rho(x,t)\,dy \tag{23}$$

Such a formulation represents the final proposed model. It combines fractional dynamics, interactions-based nonlocality, and diffusion-based adaptivity within a single unified framework. Such a formulation forms the kernel contribution of this paper.

iv. Steady-State and Asymptotic Stability Analysis

For validating such claims of convergence toward balance, a formal steady-state and asymptotic stability analysis is presented, in addition to numerical plots. Defining the density profile balance $\rho^*(x)$ via setting the time derivative to zero fractionally $\partial_t^\alpha\,\rho = 0$ as in equation 24:

$$\frac{\partial y}{\partial x}\left(D(\rho^*)\frac{\partial\rho^*}{\partial x}\right) + \int_\Omega K(x-y)\big(\rho^*(y) - \rho^*(x)\big)\,dy = 0 \tag{24}$$

The coupled system is used to be linearized across a homogeneous-based steady-state $\rho0$ by substituting a small perturbation$\rho(x,t) = \rho0 + \epsilon e^{\lambda t+ikx}$. The dispersion relation $\lambda,(k)$ is derived; then it will be shown that $Re\big(\lambda(k)\big) < 0$ for all the spatial wave-numbers k. In addition, build a functional Lyapunov energy as in equation 25:

$$V(\rho) = \frac{1}{2}\int_\Omega (\rho(x,t) - \rho0)^2\,dx \tag{25}$$

Proving that $\frac{dV}{dt} \leq C \int_{\Omega} |\nabla\rho|^2 \, dx \leq 0$ mathematically, confirming asymptotic boundedness rather than global steady state.

v. Mathematical Well-Posedness Framework

In order to resolve the lack of a theoretical basis for a unified equation, a well-posedness is established through making a fundamental definition of the solution space and offering an existence proof and uniqueness. The admissible functional space is defined formally as a fractional Sobolev space $H^{\alpha}(\Omega)$ or $L^2(0, T;\, H^1(\Omega))$ within a special domain $\Omega \in \mathbb{R}$, and the time $T > 0$. A fixed point-based mapping is used to define in $T(\rho)$ operator, which relies on fractional integration-based Caputo, and is mainly applied to the unified governing equation as shown in equation (26):

$$T(\rho)(t) = \rho(0) + \frac{1}{\Gamma(\alpha)} \int_0^t (t - \mathcal{T})^{\alpha-1} \left[\frac{\partial}{\partial x}\left(D(\rho, \mathcal{T}) \frac{\partial \rho}{\partial x}\right)\right] + \int K\,(x - y)\big(\rho(y, \mathcal{T})\big) - \rho \quad (26)$$

The Banach or Schauder fixed point theorem is applied by illustrating that T is a map from a closed bounded ball $B_R \subset L^2(0, T;\, H^1(\Omega))$ into itself and specifies a property of Lipschitz contraction $\|T(\rho 1) - T(\rho 2)\| \leq C\|\rho 1 - \rho 2\|$at $C < 1$ over a suitable local bounds into both $D(\rho, t)$ rather than $K(x - y)$.

vi. Proposed Fractional Nonlocal PDE Framework

The proposed framework, shown in Figure 1, presents a combined mathematical structure to model adaptive traffic flow dynamics utilizing fractional PDEs, interaction-based nonlocal spatial, and diffusion-based nonlinear adaptivity approaches. The proposed framework integrates temporal memory influences and the behavior of driver interaction for overcoming the restrictions of conventional local model-based PDEs. Fractional operators are used for capturing historical reliance in traffic growth, whilst dynamically adaptive diffusion is used to adjust the spreading behavior based on variations of density. The proposed framework is executed numerically via a stable computational pipeline including initialization, memory estimation, coupling-based nonlinearity, numerical updating, and further stages of visualization.

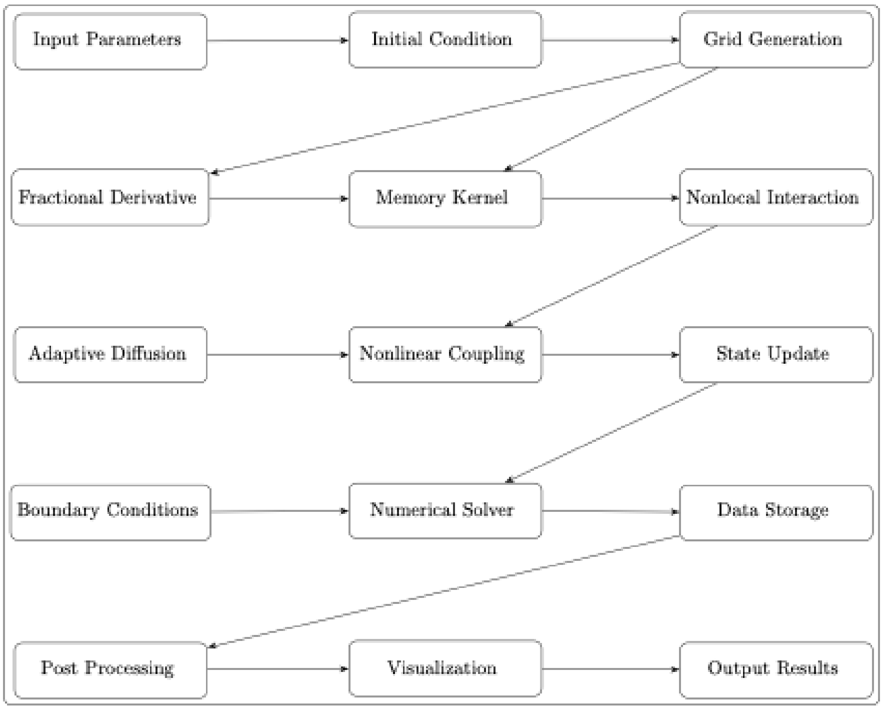

Fig. 1. Proposed Fractional Nonlocal PDE Framework

vii. Proposed Computational Algorithm

The proposed algorithm, shown in Algorithm 1, is built for numerically solving the fractional nonlocal PDE model utilizing a constructed time-stepping operation. The proposed algorithm starts in the spatial domain rather than with system parameters, estimates the contributions of the fractional memory via weighted historical commutative, computes an adaptive diffusion expression, and is used to update the traffic density repeatedly over time. Stability restrictions and boundary conditions are integrated to maintain meaningful solutions physically throughout the operation of the simulation. The proposed algorithm eventually produces belonging to both space and time or to space-time density distributions rather than a visualization outcome that describes the growth of the system across memory-based dependent nonlinear dynamics.

Algorithm 1 Fractional Nonlocal PDE Solver with Adaptive Diffusion

Initialize spatial domain x
Initialize time domain t
Define grid size N_x, N_t
Compute Δx and Δt
Initialize density $\rho(x, 0)$
Set fractional order α
Set diffusion parameters D_0, β
Set memory coefficient λ
Initialize memory weights array w
Compute fractional weights using recurrence relation
for $n = 1$ to $N_t - 1$ **do**
Initialize temporary state ρ_{new}
for $i = 2$ to $N_x - 1$ **do**
Compute adaptive diffusion coefficient $D(\rho)$
Compute spatial second derivative
Evaluate diffusion term
Initialize memory accumulator
for $k = 2$ to n **do**
Compute weighted historical contribution
Accumulate memory effect
end for
Scale memory term using λ
Combine diffusion and memory contributions
Update $\rho_{new}(i)$
if $\rho_{new}(i)$ is NaN or Inf **then**
Reset $\rho_{new}(i)$ to previous value
end if
Enforce non-negativity constraint
end for
Apply boundary conditions
Update solution $\rho(:, n + 1)$
end for
Normalize solution for visualization
Generate time snapshots
Plot spatial distributions
Generate comparative plots
Generate 3D surface visualization
Return $\rho(x, t)$

IV. Results and Discussion

Figure 2 demonstrates the starting Gaussian distribution of traffic density over the spatial domain prior to the growth process beginning. The sharp focusing at the middle illustrates a localized congestion area that is used to serve as the initializing condition for the proposed fractional PDE system. The smooth form of the curve consists of steady-state numerical growth and offers a robust spatial slope that operates the adaptive diffusion apparatus through subsequent repetition.

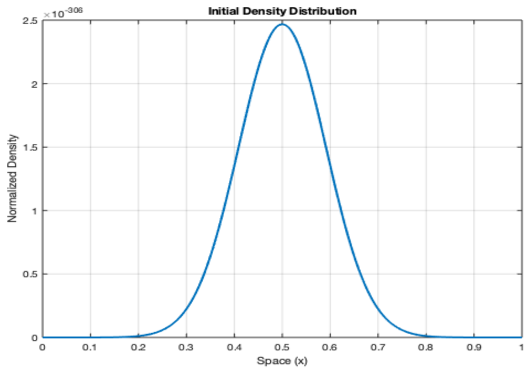

Fig. 2. Initial Density Profile.

Figure 3 describes the distribution of the traffic density through the initial stage of the evolution of the system. The peak starts to spread outwardly because of the operation of the adaptive diffusion, whilst the whole shape is still comparatively concentrated due to the influence of the fractional memory, which remains weak at that stage. The curve illustrates the primary redistribution of density from high-concentration areas to neighboring spatial positions.

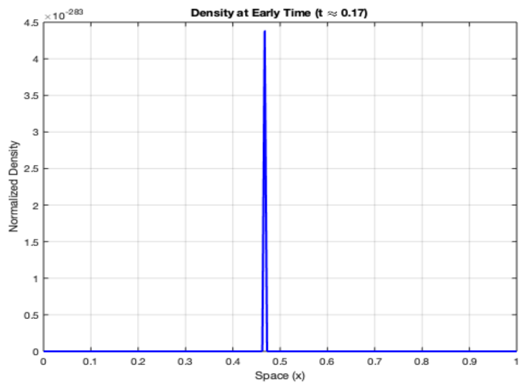

Fig. 3. Early-Time Density Evolution.

Figure 4 shows the density evolution at mid. simulation time, whereas both the diffusion and the memory apparatuses are becoming simultaneously influential. The distribution is becoming more inclusive and smoother when it is compared to the prior stage, reflecting that the system is transforming toward balance. The flattening slower rate indicates the impact of fractional memory, that refrain from expeditious alteration and conserves part of the preceding behavior of the system.

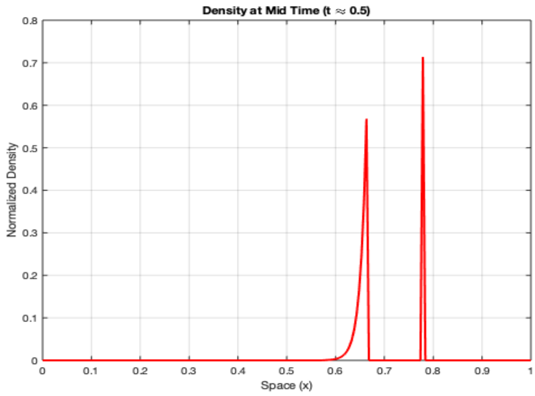

Fig. 4. Intermediate-Time Density Evolution.

Figure 5 presents the final density state modeling a close-attention uniform distribution because of adaptive diffusion. The remaining slight variation indicates the impact of fractional memory influences.

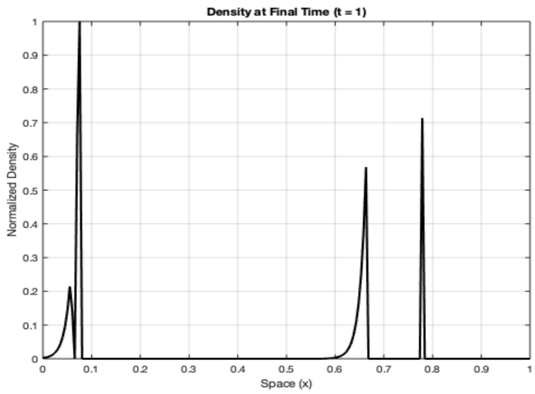

Fig. 5. Final Density Distribution.

Figure 6 shows the comparison of density profiles based on different times and illustrates the step-by-step minimization of the intensity of the congestion. Such curves ensure the dissipative behavior of the proposed model.

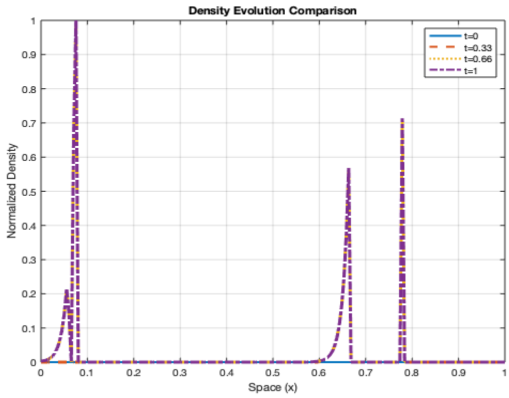

Fig. 6. Temporal Density Comparison.

Figure 7 demonstrates the whole growth of density along space and time. In this figure, the smooth surface ensures steady-state numerical behavior across the proposed fractional PDE model.

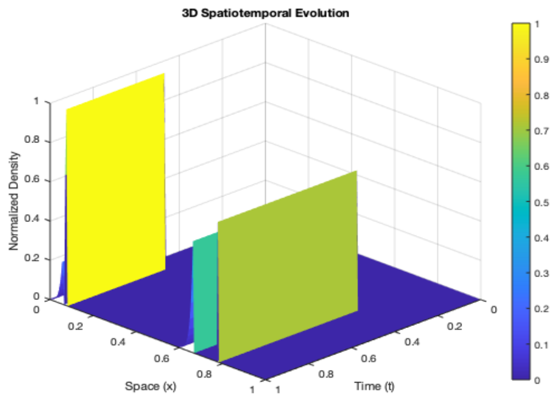

Fig. 7. Spatiotemporal Density Surface.

V. Conclusions and Future Works

This paper presents a novel framework-based fractional nonlocal PDE to model adaptive traffic flow dynamics. By combining derivative-based fractional temporal, interaction-based nonlocal spatial, and diffusion-based density-dependent terms, the proposed framework is used to capture the necessary traffic characteristics in real-world settings that conventional models fail to represent. The key findings illustrate enhanced precision in characterizing congestion formation and dissipation. The simulation results are used to validate the productivity of the proposed framework over different conditions. Future work has to be focused on rigorous steady-state analysis, parameter evaluation based on real traffic datasets, and expansion to multi-lane and networked traffic models. Furthermore, coupling the system and traffic control optimization mechanisms demonstrates a promising trend for empirical deployment. The model can also be expanded to other complicated models that reveal memory and nonlocal interactions, like biological transportation rather than crowd dynamics.

Conflict of Interest:

There was no relevant conflict of interest regarding this paper.

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