JOURNAL OF MECHANICS OF CONTINUA AND MATHEMATICAL SCIENCES
www.journalimcms.org

ISSN (Online) : 2454 -7190 Vol.-21, No.-08, August (2026) pp 271-285 ISSN (Print) 0973-8975

STOCHASTIC ANALYSIS OF A SINGLE UNIT SYSTEM WITH VARYING DEMAND AND MINOR-MAJOR FAILURES

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https://doi.org/10.26782/jmcms.2026.08.00015

(Received: May 30, 2026; Revised: August 03, 2026; Accepted: August 12, 2026)

Abstract

With the rapid increase of the societal requirements and the complexity of the industrial systems there is a great need to achieve high reliability without losing profitability. Small system failures might lead to huge loss of operational activities, downtimes and low productivity. In this paper, the authors propose a stochastic reliability model to analyze the availability and profitability of an industrial system under environmental factors and fluctuating demand. The proposed model assumes that it is a single unit system where there are no failures that can be tolerated. It is also equipped with an inspection mechanism to identify the kind of failure occurring in the system. The failures are called minor and major failures.Minor failures are caused by lubrication or greasing problems which can be repaired and major failures are caused by electrical short circuiting, water ingestion in the time of adverse weather and overloading as well as prolonged continuous operation . In case of major failures the system is either repaired or replaced depending upon the extent of damage. The two operating cases are studied where the demand is higher than or equal to the production and the demand is lower than the production. The regenerative point technique and Semi-Markov processes are used to evaluate key performance measures and profit thoroughly with respect to the effects of different parameters of the system.Data has been collected from a steel manufacturing plant, Rajpura, Punjab, India.The outcomes are very informative for the visited plant and provide suggestions for best management of inspection, maintenance, and operational strategies.

Keywords: Reliability, Inspection, Variation in demand, Regenerative point technique, Semi-Markov process, Innovation.

I. Introduction

The reliability analysis has become a significant pillar inmodern industrial engineering. in the planning, operation, and economic sustainability of manufacturing systems. Davis [VI], who created a quantitative foundation on the research of a breakdown of the system. The early works [II,VII] were summarized into systemized engineering approaches by classical works, which put much accent on the practical significance of reliability in engineering design and maintenance planning.The reliability modeling later was extended by other researchers [IV-XVI] by adding redundancy, inspection as well as maintenance optimization. One of the prevailing weaknesses of most reliability models is that it assumes a fixed or deterministic demand. The environment in real manufacturing settings is seldom a constant demand; the workload changes may be too hard on the running units and cause them to fail. Recent studies have followed this pattern and have explored the reliability and performance optimization of manufacturing plants, energy systems, steel industries and multi-state repairable systems [XVII-XXIV]. Malhotra [XIII-XV] introduced reliability, and availability analysis of a standby unit system with varying demand. Farande and Hanumant [IV] recently explored the failures of steam turbines in thermal power plants and the subsystems associated with them, and suggested strategies of maintenance and repair scheduling based on the techniques of artificial intelligence and machine learning. Yousuf, Irshad, and Umair [XXX] examined drivers and obstacles to energy efficiency in steel industries, and state that the poor practices of energy management play a major role in undermining performance. Equally, Rathi, Sahu, and Kumar [XXV] used SMED techniques in steel rolling mills in order to eliminate operational constraints and maximize production.

With reference to manufacturing systems [XVII-XX], Malhotra and Kaur [XVII-XXI]have performed a stochastic analysis and analysis of availability, though they did not take into account system stress due to high demand, as well as, the operational strategy of temporarily resting an overworked machine and then relocating workload to a second unit. This kind of situation has been seen in a steel manufacturing plant, Rajpura, Punjab, India; a real-world industrial environment with constant use and a heavy load that causes faster wear and tear and higher chances of failure. Thus, the current paper attempts to overcome these shortcomings by introducing environmental impacts, demand variability due to the market conditions, and load-based failure mechanisms into the overall reliability framework of an inspection. Failures are classified as minor and major: minor failures can be repaired, and major failures can be repaired or replaced, depending on its difficulty: electrical faults, water ingress, orexcessive load of the operation may cause the failure. Regenerative point techniques and semi-Markov processes are used to analyse the behavior of the system.

II. Mathematical Model

The model suggested is a single unit repairable system, which works within different levels of demand and environmental factors. There is only a single unit, so any failure will disrupt the operation of the system at once, and will have to be inspected and fixed. A system of inspection is incorporated where the failure is categorized and whether it needs repair or replacement. There are minor and major

failures that are based on the results of inspection. Small failures are caused by the problem of lubrication or greasing and only need to be repaired. The main failures include electrical short circuit, water leaks or overload and constant working, the failures can be repaired or replaced entirely depending on the extent. The system is considered to be under two demand conditions:
High-demand case: demands are equal to or above production.
Low-demand case: demand < production.
Here, failure detection is followed by mandatory inspection in both instances. The repairman identifies the fault in the unit, puts the system into the down state and puts the system back to its original state. Once the system has been repaired or replaced, it acts like a new one. The breakdown time is exponentially distributed whereas the inspection, repair and replacement time are distributed generically. System performance is assessed using regenerative point technique and semi-Markov processes.

State Definitions: The system states depicted by the transition diagram can be translated in the following way:

- States X_i, where i = 0, 1, 2, 3, 5, 6, 7, 8 all are Regenerative states that means these states are independent.
- States X_i, where i = 9,10, 11, 12, 13, 14 all are Non-Regenerative states means these states are dependent to previous states.
- State X_0 (Operative state, high demand), State X_1 (Operative state, low demand), State X_2 (Inspection state)
- State X_3(Minor Failure under repair when demand is less than production)
- State X_5 (Minor failure under repair when demand is greater than equal to production),
- States X_6, X_7, X_8 (Major failures under repair or replacement when demand is greater than equal to production). Here, Large failures are as X_6 due to electrical fault, X_7 due to water-induced failure and X_8 due to excessive load failure.
- States X_9, X_{11}, X_{13}, (Major Repair states)
- States X_{10}, X_{12}, X_{14} (Major Replacement states)
- State X_4 (Down state) When under inspection or major corrective action the system is totally inoperative.

Performance Evaluation: The measures obtained using the semi-Markov framework are the following: Mean Time to System Failure (MTSF). Steady-state availability Expected downtime Number of repair visits Replacement frequency Repairman busy periods Profit analysis The impact of system parameters on availability and profitability is analyzed with the help of graphical analysis. The model evidences that inspection-based maintenance has a significant impact on lessening the effects of failure and enhancing economic performance. The effect of the key parameters on the system performance is examined through the use of graphs which are developed based on real-life industrial data. The results indicate that the inspection-based maintenance has a great influence on increasing reliability, decreasing downtimes, and increasing profitability during the changes in demand conditions. The suggested

framework will offer real-life advice to industrial planners and maintenance engineers who are aimed at optimizing the use of resources and maintaining effective work in the high-demand environment.

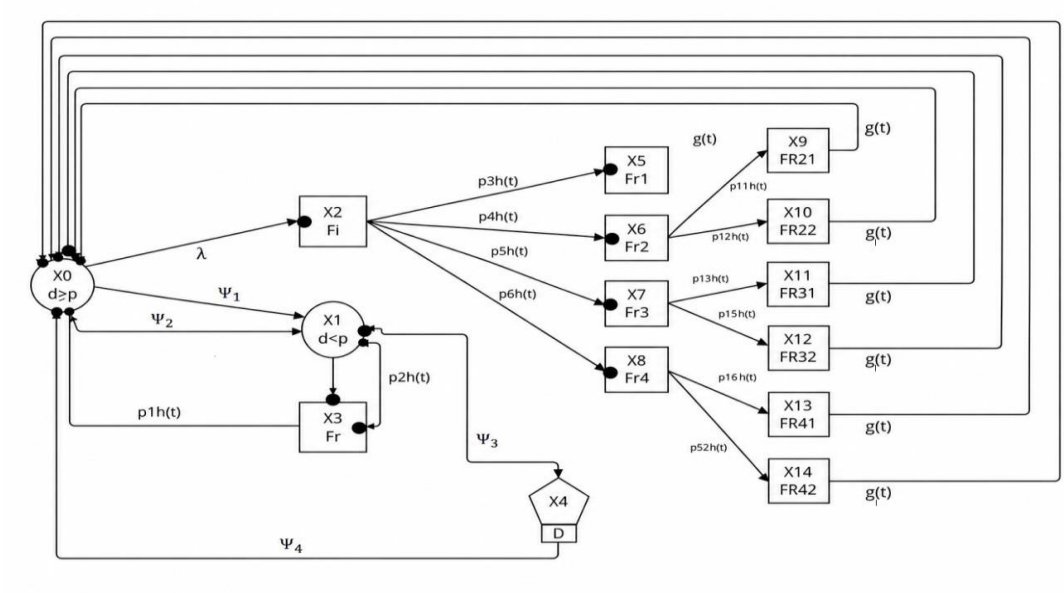


Fig. 1.State Transition Diagram of Model

III. Procedure

III.i. Table No. 1 Nomenclature: All the mathematical notations that are used in this paper are define in table no. 1[XVII]

λ	Failure rate of the operative unit
Ψ_1/Ψ_2	Rate of decrease/increase of demand < production / demand≥production
Ψ_3/Ψ_4	Rate of change from up to downstate (As demand < production)/ down to upstate (when demand increases)
$Op_{d\geq p}$/ $Op_{d<p}$	Operative unit when demand ≥ production / demand < production
D	Down State
p_i, i = 1, 2	Probability that during the repair time the demand ≥ production/demand< production
p_3	Probability of minor failure
p_i, i = 4, 5, 6	Probability of major failures
p_i, i = 41/ 51/ 61	Probability of major repair due to electrical fault / water-induced failure / excessive load failure.
p_i, i = 42/ 52/ 62	probability of major replacement due to due to electrical fault / water-induced failure / excessive load failure.
F_i	Failed unit under inspection
F_{r1}	Failed unit under minor repair
F_{ri}, i = 2, 3, 4	Unit under inspection due to major failure

FRi, i = 21,31, 41/ 22, 32,42	Failed unit under major continuous repair/replacement
q_{ij}(t),Q_{ij}(t)	p.d.f and c.d.f of regenerative state i to j [XVII]
g(t), h(t), g_1(t) / G(t), H(t), G_1(t)	p.d.f / c.d.f. for repair, inspection,replacement time [XVII]
M_i(t),m_{ij}	Probability that system up initially in regenerative state i Contribution to mean sojourn time in regenerative state i before transiting to regenerative state j without visiting to any other state[XVII]
'/*/ **	Symbol for derivative/ Laplace transforms / Symbol for Laplace Stieltjes transform[XVII]
© /Ⓢ	Symbol for Laplace convolution/for Stieltjes convolution[XVII]

III.ii. Transition Probabilities and Mean Sojourn Times for various states:

Here X_i, where i = 0, 1, 2, 3, 5, 6, 7, 8 are the regenerative states and States X_i, where i = 9, 10, 11, 12, 13, 14 are the non-regenerative states. Also States X_i, where i = 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13 and 14 are failed states. State X_4 is down state. The state transition diagram in figure 1 represents all possible states of the system at time t.
The transition probabilities [XVII] are:

$q_{01}(t) = \Psi_1 e^{-(\lambda+\Psi_1)t}$ $\quad$ $q_{02}(t) = \lambda e^{-(\lambda+\Psi_1)t}$

$q_{10}(t) = \Psi_2 e^{-(\lambda+\Psi_2+\Psi_3)t}$ $\quad$ $q_{13}(t) = \lambda e^{-(\lambda+\Psi_2+\Psi_3)t}$

$q_{14}(t) = \Psi_3 e^{-(\lambda+\Psi_2+\Psi_3)t}$ $\quad$ $q_{25}(t) = p_3 h(t)$

$q_{26}(t) = p_4 h(t)$ $\quad$ $q_{27}(t) = p_5 h(t)$

$q_{28}(t) = p_6 h(t)$ $\quad$ $q_{30}(t) = p_1\, g(t)$

$q_{31}(t) = p_2\, g(t)$ $\quad$ $q_{40}(t) = \Psi_4 e^{-\Psi_4 t}$

$q_{50}(t) = g(t)$ $\quad$ $q_{69}(t) = p_{41} h(t)$

$q_{6,10}(t) = p_{42} h(t)$ $\quad$ $q_{7,11}(t) = p_{51} h(t)$

$q_{7,12}(t) = p_{52} h(t)$ $\quad$ $q_{8,13}(t) = p_{61} h(t)$

$q_{8,14}(t) = p_{62} h(t)$ $\quad$ $q_{90}(t) = g(t)$

$q_{100}(t) = g_1(t)$ $\quad$ $q_{11,0}(t) = g(t)$

$q_{12,0}(t) = g_1(t)$ $\quad$ $q_{13,0}(t) = g(t)$

$q_{14,0}(t) = g_1(t)$

The non-zero elements $p_{ij} = \lim_{s\to 0} q_{ij}^*(s)$[XVII] are:

$p_{01} = \Psi_1/(\lambda + \Psi_1)$ $\quad$ $p_{02} = \lambda/(\lambda + \Psi_1)$ $\quad$ $p_{10} = \Psi_2/(\lambda + \Psi_2 + \Psi_3)$

$p_{13} = \lambda/(\lambda + \Psi_2 + \Psi_3)$ $\quad$ $p_{14} = \Psi_3/(\lambda + \Psi_2 + \Psi_3)$ $\quad$ $p_{25} = p_3$

$p_{26} = p_4$ $\quad$ $p_{27} = p_5$ $\quad$ $p_{28} = p_6$

$p_{30} = p_1$ $p_{31} = p_2$ $p_{40} = p_{50} = 1$

$p_{69} = p_{41}$ $p_{6,10} = p_{42}$ $p_{7,11} = p_{51}$

$p_{7,12} = p_{52}$ $p_{8,13} = p_{61}$ $p_{8,14} = p_{62}$

$p_{90} = p_{10,0} = p_{11,0} = p_{12,0} = p_{13,0} = p_{14,0} = 1$

The following are transition probabilities, it is validated that

$\sum_{i=1}^{2} p_{0i} = 1$ $\sum_{i=0,3,4} p_{1i} = 1$ $\sum_{i=5}^{8} p_{2i} = 1$ $\sum_{i=0}^{1} p_{3i} = 1$

$\sum_{i=9}^{10} p_{6i} = 1$ $\sum_{i=11}^{12} p_{7i} = 1$ $\sum_{i=13}^{14} p_{8i} = 1$

Also, $p_{40} = p_{50} = p_{90} = p_{10,0} = p_{11,0} = p_{12,0} = p_{13,0} = p_{14,0} = 1$

The mean sojourn times (μ_i)[XVII] for different states, are

$\mu_0 = 1/(\lambda + \Psi_1)$ $\mu_1 = 1/(\lambda + \Psi_2 + \Psi_3)$ $\mu_4 = 1/\Psi_4$

$\mu_2 = \mu_6 = \mu_7 = \mu_8 = \int_0^\infty \overline{H}(t)\, dt$

$\mu_3 = \mu_5 = \mu_9 = \mu_{11} = \mu_{13} = \int_0^\infty \overline{G}(t)\, dt = -g^*(0)$

$\mu_{10} = \mu_{12} = \mu_{14} = \int_0^\infty \overline{G_1}(t)\, dt = -g_1^*(0)$

Therefore, Mean time taken by the system to transit for any state j into state i is mathematically given by[XVII]:

$m_{ij} = \int_0^\infty t q_{ij}(t) dt = -q^{*\prime}_{ij}(0)$

Therefore,

$\sum_{i=1}^{2} m_{0i} = \mu_0$ $\sum_{i=0,3,4} m_{1i} = \mu_1$ $\sum_{i=5}^{8} m_{2i} = \mu_2$ $\sum_{i=0}^{1} m_{3i} = \mu_3$

$\sum_{i=9}^{10} m_{6i} = \mu_6$ $\sum_{i=11}^{12} m_{7i} = \mu_7$ $\sum_{i=13}^{14} m_{8i} = \mu_8$ $m_{40} = \mu_4$

$m_{20} = \mu_2$ $m_{50} = \mu_5$ $m_{90} = \mu_9$ $m_{10,0} = \mu_{10}$

$m_{11,0} = \mu_{11}$ $m_{12,0} = \mu_{12}$ $m_{13,0} = \mu_{13}$ $m_{14,0} = \mu_{14}$

IV. MEASURES OF SYSTEM EFFECTIVENESS

The measures of system effectiveness are given by:

IV.i. MEAN TIME TO SYSTEM FAILURE (MTSF)

The following recurring relations for $\phi_i(t)$ [XVII] are used for MTSF are acquired, using probabilistic arguments:

$\phi_0(t) = Q_{01}(t) \textcircled{S} Q_1(t) + Q_{02}(t)$

$\phi_1(t) = Q_{10}(t) \textcircled{S} Q_0(t) + Q_{13}(t) + Q_{14}(t) \textcircled{S} Q_4(t)$

$\phi_4(t) = Q_{40}(t) \textcircled{S} Q_0(t)$

Solving above equation for $\phi_O{}^{**}$(s), using Laplace –Steltjes Transform (L.S.T) we obtain

$\phi_O{}^{**}$(s) = N(s)/D(s)

Where, N(s) = $q_{02}{}^{*}$ (s) + $q_{01}{}^{*}$ (s) $q_{13}{}^{*}$ (s)

D(s) = 1 - $q_{01}{}^{*}$ (s) $q_{10}{}^{*}$ (s) - $q_{01}{}^{*}$ (s) $q_{14}{}^{*}$ (s)$q_{40}{}^{*}$ (s)

Now, when the system starts from the state '0',

then MTSF = $\lim_{s\to 0} \frac{(1-\phi_O{}^{**}(s))}{s}$

Applying L'Hospital rule and substituting the value of $\phi_O{}^{**}$(s) from equation, we have MTSF = N/D

Where N = $\mu_0 + p_{01}\mu_1 + p_{01}p_{14}\mu_4$, D = $p_{02} + p_{01}p_{13}$

IV.ii. AVAILABILITY

The recurring relations are obtained and solved for availabilities $A_i{}^{d}$(t), $A_0{}^{p}$ (t) [XVII] and their values in steady state are given by

$A_0{}^{d}$= N_1/D_1 , $A_0{}^{p}$= N_2/D_1

Where N_1 = (1- $p_{13}p_{31}$)μ_0 and

$D_1 = (1 - p_{13}p_{31})\mu_0 + p_{01}\mu_1 + p_{02}(1 - p_{13}p_{31})\mu_2 + p_{01}p_{13}\mu_3 + p_{01}p_{14}\mu_4 + p_{02}(1 - p_{13}p_{31})[p_{25}\mu_5 + p_{26}(p_{69}\mu_9 + p_{6,10}\mu_{10}) + p_{27}(p_{7,11}\mu_{11} + p_{7,12}\mu_{12}) + p_{28}(p_{8,13}\mu_{13} + p_{8,14}\mu_{14})] + p_{02}(1 - p_{13}p_{31})(p_{26}\mu_6 + p_{27}\mu_7 + p_{28}\mu_8)$ And $N_2 = p_{01}\mu_1$

IV.iii. BUSY PEIOD ANALYSIS OF REPAIRMAN (INSPECTION TIME ONLY)

In steady state, the total fraction of time taken by repairman in inspection I_0[XVII] are given by

I_0= $\lim_{s\to 0}(sI_0{}^{*}(s)) = N_3/D_1$

Where $N_3 = p_{02}(1 - p_{13}p_{31})(\mu_2 + \mu_6 p_{26} + \mu_7 p_{27} + \mu_8 p_{28})$, and D_1 is already specified.

IV.iv. BUSY PEIOD ANALYSIS OF REPAIRMAN (REPAIR TIME ONLY)

In steady-state, busy periods B_0[XVII] are given by

B_0 = $\lim_{s\to 0}(sB_0{}^{*}(s)) = N_4/D_1$

Where $N_4 = p_{02}(1 - p_{13}p_{31})(\mu_{11}p_{27}p_{7,11} + \mu_{13}p_{28}p_{8,13} + \mu_5 p_{25} + \mu_9 p_{26}p_{6,9}) + \mu_3 p_{01}p_{13}$ and D_1 is already specified.

IV.v. BUSY PEIOD ANALYSIS OF REPAIRMAN (REPLACEMENT TIME ONLY)

In steady-state, busy period of replacement BR_0 [XVII] is given by

$BR_0^*(s) = \lim_{s\to 0}(sBR^*(s)) = N_5/D_1$

Where $N_5 = p_{02}(1 - p_{13}p_{31})(\mu_{10}p_{26}p_{6,10} + \mu_{12}p_{27}p_{7,12} + \mu_{14}p_{28}p_{8,14})$

and D_1 is already specified.

IV.vi. EXPECTED NUMBER OF VISITS BY THE REPAIRMAN

The probability of expected number of visits by repairmanin steady-state V_0 [XVII] is given by

$V_0 = \lim_{s\to 0}(sV_0^*(s)) = N_6/D_1$

Where $N_6 = p_{02}(1 - p_{13}p_{31}) + p_{01}p_{13}$, and D_1 is already specified.

IV.vii. EXPECTED NUMBER OF REPLACEMENTS

The probability for the busy period RP_0 [XVII] when replacement isdone in steady state is given by,

$RP_0 = \lim_{s\to 0}(sRP_0^*(s)) = N_7/D_1$

Where $N_7 = p_{02}(1 - p_{13}p_{31})(p_{26}p_{6,10} + p_{27}p_{7,12} + p_{28}p_{8,14})$

and D_1 is already specified.

IV.viii. EXPECTED DOWNTIME

The expected downtime DT_0 [XVII] in steady state, is given by

$DT_0 = \lim_{s\to 0}(sDT_0^*(s)) = N_8/D_1$

Where $N_8 = \mu_4 p_{01}p_{14}$, and D_1 is already specified.

V. Profit Analysis

The profit in the steady state of the system model is as illustrated below:

Profit= Total Revenue -Total Cost/Expenses

$P = (C_0A_0^d + C_1A_0^P) - C_2I_0 - C_3B_0 - C_4BR_0 - C_5V_0 - C_6RP_0 - C_7DT_0$

Where, C_i (i = 0,1) = Revenue per unit up time when demand ≥/< production

C_i (i = 2,3,4) = Cost per unit up time for which the repairman is busy for inspection/repair or replacement

C_i (i = 5,6) = Cost per visit of the repairman/per unit replacement

C_7= Loss per unit time during the system in down state

Particular Case

The inspection rate (α_1), repair rate(α), replacement rate (α_2) follow exponentially for the particular case. Therefore, we consider

g(t) = $\alpha e^{-\alpha t}$ $g_1(t) = \alpha_2 e^{-\alpha_2 t}$ h(t) = $\alpha_1 e^{-\alpha_1 t}$

$\mu_0 = 1/(\lambda + \Psi_1)$ $\mu_1 = 1/(\lambda + \Psi_2 + \Psi_3)$ $\mu_4 = 1/\Psi_4$

$\mu_2 = \mu_6 = \mu_7 = \mu_8 = = \int_0^\infty \overline{H}(t)\, dt$

$\mu_3 = \mu_5 = \mu_9 = \mu_{11} = \mu_{13} = \int_0^\infty \overline{G}(t)\, dt = -g^*(0)$

$\mu_{10} = \mu_{12} = \mu_{14} = \int_0^\infty \overline{G_1}(t)\, dt = -g_1^*(0)$

α = 0.042 per hr, α_1 = 0.164 per hr, α_2 = 0.002 per hr, λ = 0.003 per hr

Ψ_1 = 0.0548 per hr, Ψ_2 = 0.1096 per hr, Ψ_3 = 0.0055 per hr, Ψ_4 = 0.1370 per hr, and various costs (in Indian Rupees),

C_0 = 1200000,C_1 = 600000, C_2 = 50000, C_3 = 30000, C_4 = 65000, C_5 = 300000, C_6 = 500000, C_7 = 10000.

VI. Results and discussion

The authors examined how different parameters can impact the single-unit standby system. The findings in this study were supported with only real-time data collected from a steel manufacturing plant, Gobindgarh, Punjab, India.

Table 2. Profit versus Revenue per unit time(C_1) for varying cost (C_3)

Revenue per unit time (C_1) (in INR)	Profit (in Thousands)		
	C_3=INR 1000	C_3=INR 1500	C_3=INR 2000
100	-87	-172	-257
200	-4	-89	-174
300	79	-6	-91
400	162	77	-8
500	245	160	75

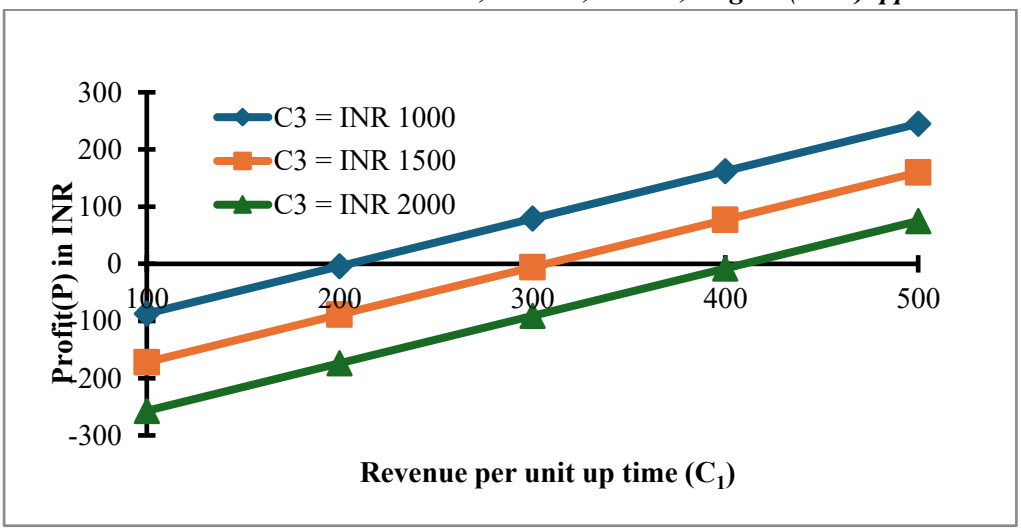


Fig. 2. Profit versus Revenue per unit time(C_1) for varying cost (C_3)

Fig. 2 and Table 2 shows that the system earns more profit at a lower value of C_3 = INR 1000 while the profit is less at higher value of C_3 = INR 1500 and INR 2000. This behaviour suggests that the system's profitability is positively affected by revenue per unit uptime and negatively affected by an increase in the cost parameter associated with this. So, it is crucial to have a better revenue rate and reduce operating expenses to achieve better system profitability.

Table 3. Profit(P) versus Failure rate (λ) for different values of Revenue per unit up Time (C_0)

Failure rate (λ) per hr	Profit (in Thousands)		
	C_0=INR 6000	C_0=INR 5000	C_0=INR 4000
0.01	2300	1500	500
0.012	1800	1000	0
0.014	1300	500	-500
0.016	800	0	-1000
0.018	300	-500	-1500

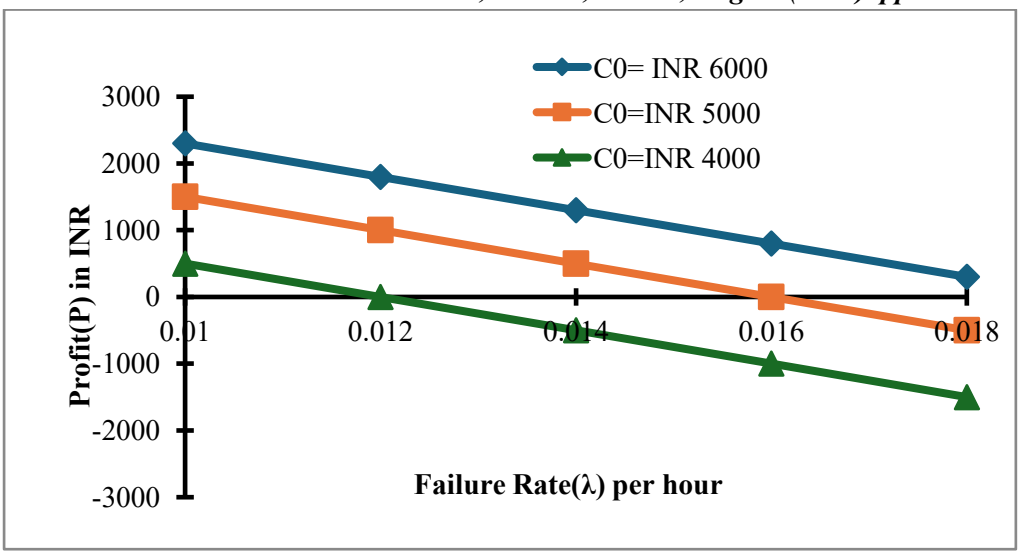


Fig. 3. Profit versus Failure rate (λ) for Revenue per unit up Time(C_0)

Fig. 3 and Table 3 shows the relationship between the failure rate (λ) and the profit of the system for various values of C_0. It is found that the profit drops off steadily as the failure rate rises for all three values of C_0 that are investigated. This is because the higher the failure rate, the more often the system will break down, causing more system downtime and less productivity. In addition the system generates more profit for C_0 = INR 6,000 than for C_0 = INR 5,000 and INR 4,000.

Table 4. Profit versus Repair rate (α) for varying cost (C_4)

Repair rate (α) per hr	Profit (in Thousands)		
	C_4 = INR 300	C_4 = INR 400	C_4 = INR 500
0.5	0	-500	-1000
1	500	0	-500
1.5	1000	500	0
2	1500	1000	500
2.5	2000	1500	1000
3	2500	2000	1500

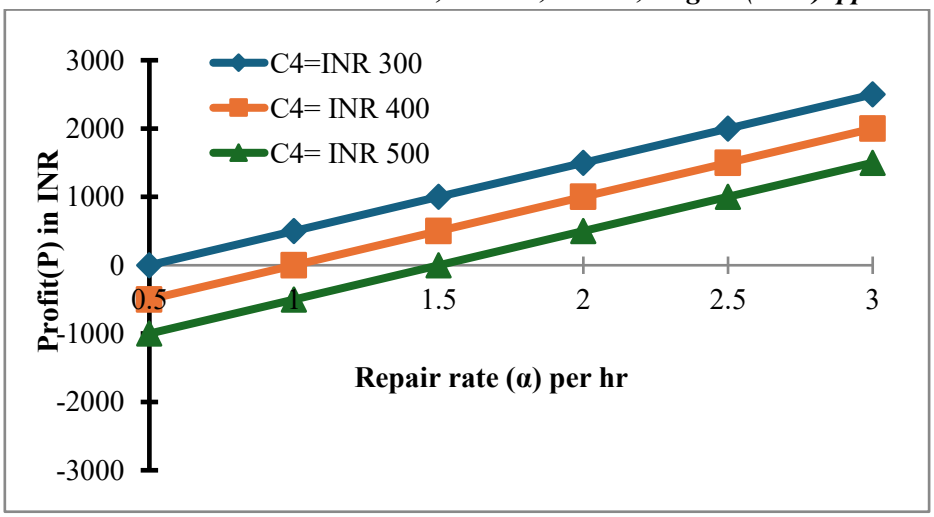


Fig. 4. Profit versus Repair rate (α) for varying cost (C_4)

Fig. 4 and Table 4 illustrates the profit of the system for a range of repair cost values (C_4) and repair rate values (α). As it can be observed that in all the three values of C_4 considered, the profit increases continuously with increase in repair rate. A higher repair rate means that time spent in restoring the failed system to its normal operating condition is decreased, thus reducing downtime and increasing production efficiency. Also, maximum profit is observed when C_4 = INR 300 and minimum profit is observed when C_4 = INR 500. These results indicate that faster repair activities positively influence system profitability, while an increase in repair cost has a negative impact on the overall profit.

VII. Conclusion

On visiting a steel manufacturing plant, astochastic reliability model has been developed that works under the different demand variations and environmental factors. The suggested inspection-based model efficiently separates between minor and major failures to provide the remedial measures in the form of repair or replacement approaches. The model considers the environmental stress factors like electrical faults, water ingress, and overload, which are part of real-world situations in a steel industry, and are not part of traditional reliability studies. Combining regenerative point methods with semi-Markov processes makes it possible to assess such key performance measures as Mean Time to System Failure (MTSF), system availability at various levels of demand, busy periods of repairmen, anticipated replacements, repair visits, downtime, and the total profit. The analytical and graphical findings indicate that the effectiveness of the inspection process, rate of repair and the intensity of failure are important factors that determine the availability and profitability of the systems. The greater the repair rate the better availability and MTSF is enhanced and the greater the failure rate the lesser the system lifetime and operational performance. In the study, it is found that maintenance through inspection

is very instrumental in reducing downtime and eliminating serious system degradation. Further, the separation of demand conditions (high demand and low demand) offers more information about the stress of workload and sustainability of the operations. In general, the model developed provides an effective decision-support mechanism in managing the maintenance planning and the industry at large because it strikes a balance between reliability and economic performance. The framework is specifically useful to those single-unit systems in which the failure toleration is low and the continuity of operation is very important. The proposed model is general, so can be used by any manufacturing plant where such a situation exists. This study can be further extended in the future with the use of multi-unit systems.

Conflict of Interest

There is no conflict of interest regarding this paper.

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