



## A HYBRID ACOUSTIC–RADIO FRAMEWORK FOR HIGH-ACCURACY INDOOR SENSOR LOCALIZATION USING COLLABORATIVE ECHO MAPPING

Hemarjit Ningombam<sup>1</sup>, Gurumayum Robert Michael<sup>2</sup>, Rajesh Bose<sup>3</sup>,  
Sudipta Majumder<sup>4</sup>

<sup>1,2,4</sup> Department of ECE, DUIET, Dibrugarh University, Dibrugarh, Assam, India.

<sup>3</sup>Department of CSE, JIS University, Kolkata, West Bengal, India.

Email: <sup>1</sup>nhsingh@dibru.ac.in, <sup>2</sup>robertmichael@dibru.ac.in  
<sup>3</sup>bose.raj00028@gmail.com, <sup>4</sup>sudipta2020@dibru.ac.in

Corresponding Author: **Sudipta Majumder**

<https://doi.org/10.26782/jmcms.2026.06.00004>

(Received: March 30, 2026; Revised: May 29, 2026; Accepted: June 10, 2026)

---

### Abstract:

*Localization of sensors in an indoor wireless sensor network (WSNs) has been a very difficult task because of attenuation of signals, multipath, and also a low supply of anchors. In this paper, we propose a new hybrid acoustic-radio system, Collaborative Acoustic Echo Mapping (CAEM), to achieve high-precision localization of sensors within an indoor environment. The proposed method combines acoustic echo data from environmental reflectors with radio-based inter-sensor ranging data, enabling simultaneous optimization of sensor and reflector locations. It considers a robust formulation based on the Huber loss to reduce the effects of measurement noise and outliers. It solves the resulting non-linear optimization problem using an efficient L-BFGS-B scheme. Extensive simulations are conducted in a 100 m x 100 m indoor space at different noise levels, anchor densities, and sensor locations. The Proposed CAEM model is compared with eight standard and contemporary strategies. Findings indicate that CAEM consistently outperforms traditional methods, minimizing localization error by up to 3.2 times. In a representative scenario, CAEM achieves an RMSE of 11.33 m, which is far better than the baselines. The results emphasise the utility of the acoustic and radio modalities approach for robust, scalable indoor localisation, making CAEM a promising solution for next-generation IoT and WSN applications.*

**Keywords:** Collaborative Acoustic Echo Mapping (CAEM), Indoor Localization, Wireless Sensor Networks, Acoustic–Radio Fusion, Non-linear Optimization, Multi-modal Sensor Fusion

---

### I. Introduction

Internet of Things (IoT) systems and Wireless Sensor Networks (WSNs) have become core technologies across a wide range of applications, including environmental

*Hemarjit Ningombam et al.*

monitoring, smart healthcare, industrial automation, and disaster management. The correct location of sensors in such systems is a very important issue; the value of the sensed data can only be assigned meaning when it is coupled with the accurate spatial data. The ability to determine the geographic location of sensor nodes directly influences the effectiveness of monitoring, tracking, and decision-making processes.

Sensor localization in WSNs is broadly divided into range-based and range-free methods. The range-based techniques, including Received Signal Strength Indicator (RSSI), Time of Arrival (ToA), Time Difference of Arrival (TDoA), and Angle of Arrival (AoA), estimate distances or angles between nodes and use multilateration to estimate position. Even though these techniques can be very accurate, they require additional hardware and are sensitive to noise, multipath propagation, and environmental uncertainty, especially in indoor settings [XII], [XXVII]. Many of the solutions discussed in the article, such as range-free methods like Centroid localisation and DV-Hop, rely on network connectivity and hop-count data, which are less expensive and simpler to implement but less accurate and have a significant scalability drawback [XXVII]. One of the most commonly used algorithms in range-free is DV-Hop, as it is simple and does not depend on distance measurements.

Nevertheless, DV-Hop suffers from severe localization errors due to hop-distance estimation errors, so many improvements and hybrid versions have been developed in recent years. For example, some studies (2023-2024) propose optimised DV-Hop algorithms by introducing weighted hop estimation, an optimisation scheme, and a hybrid approach to enhance localisation performance [LXXIX], [VII]. Despite these developments, DV-Hop-based approaches continue to struggle to achieve high precision in complex environments. In this context, localization schemes based on RSSI are also popular because they are inexpensive and easy to implement. Nevertheless, RSSI-based localisation methods are highly influenced by environmental conditions, including obstacles, fading, and interference, leading to unreliable distance estimation and degraded localisation [XII]. Recent research indicates that RSSI-based methods are highly sensitive to noise and environmental variations, making them less useful in practical implementations.

The other notable category of localization methods is anchor-based localization, in which a set of known-position nodes (anchors) helps determine the positions of other unknown nodes. Although anchor-based multilateration is more accurate for localisation, it incurs additional infrastructure costs and is less efficient when there are few or unevenly distributed anchors. According to recent studies, anchor dependency is a significant bottleneck in large-scale IoT deployments, which is why anchor-efficient or hybrid localization strategies should be considered [VII]. Over the past few years, there has been a trend towards hybrid, intelligent localization systems that combine a variety of sensing modalities, optimization algorithms, and machine learning architectures to improve performance. Contemporary methods use conventional localization methods, as well as swarm intelligence, deep learning, and hybrid optimization, to enhance accuracy, robustness, and scalability. Nevertheless, the most recent techniques rely mainly on radio signal measurements and do not fully exploit environmental information.

*Hemarjit Ningombam et al.*

To overcome these drawbacks, this paper presents a new Collaborative Acoustic Echo Mapping (CAEM) algorithm for high-precision sensor localization in indoor settings. In contrast to traditional solutions, the given solution combines acoustic echo measurements with radio-based ranging, enabling the use of environmental reflectors (e.g., walls and obstacles) as virtual anchors. This plays a significant role in increasing the number of spatial constraints at no extra cost in physical infrastructure. Moreover, the proposed framework formulates localisation as a joint optimisation problem that jointly estimates sensor and reflector locations. It is equipped with a powerful Huber loss function to address the effects of noise and outliers, resulting in stability and reliability. CAEM can successfully address the limitations of traditional range-based and range-free methods through multi-modal sensing and sound optimization. Large-scale simulations in an indoor (100 m x 100 m) setting demonstrate that the proposed CAEM strategy outperforms classical and contemporary localization strategies, such as Anchor-only multilateration, Centroid, Weighted Centroid Localisation (WCL), DV-Hop, RSSI-based approaches, Graph Learning Localization, Extended Kalman Filter Simultaneous Localization, and Robust Optimization with Alternating Optimization techniques. The findings indicate significant changes in RMSE, MAE, and tail error (P95), demonstrating the soundness and efficiency of the proposed framework across different noise levels, anchor densities, and network sizes.

The main contributions of this work are summarized as follows:

- A novel hybrid acoustic–radio localization framework leveraging environmental reflectors as virtual anchors.
- A joint optimization model for simultaneous estimation of sensor and reflector positions.
- Integration of robust loss functions for improved resilience against noise and outliers.
- Comprehensive comparative evaluation against eight baseline localization methods under diverse simulation scenarios.

The remainder of this paper is organized as follows: Section 2 presents the related work; Section 3 describes the proposed CAEM framework; Section 4 details the simulation setup; Section 5 discusses the results; and Section 6 concludes the paper.

## **II. Literature Review**

The fundamental problem the CAEM framework is intended to solve is indoor sensor localization, in which the goal is to position both mobile and stationary nodes with high precision in GPS-denied environments. In contrast to outdoor localization, indoor settings are highly attenuated, with greater signal attenuation, multipath propagation, and non-line-of-sight (NLOS) conditions, which greatly impair the performance of traditional positioning methods [XVI], [LVII]. Precise localization is thus critical in many IoT and WSN applications, such as asset tracking, smart building automation, indoor navigation, and context-aware services for smart homes, hospitals, and industries [X], [LVII], [XCII]. The practical importance of this problem has led to substantial research on hybrid localization methods to address issues of anchor sparsity, environmental changes, and heterogeneous sensing modalities [I], [XXXIII], [LXIV]. In the real world, accurate indoor localization increasingly relies on combining multiple sensing modalities. Multi-modal fusion systems. Multi-modal fusion, such as RF

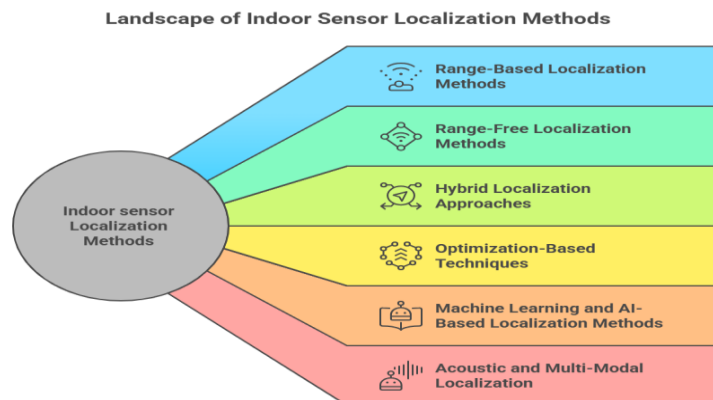
*Hemarjit Ningombam et al.*

fingerprints, inertial measurements, visual cues, and ambient signals, has proven more robust and accurate in complex, dynamic settings [III], [V], [XXII], [XXXIII]. These hybrid systems can be especially beneficial in cases where infrastructure limitations limit the density of anchors and localization and map-assisted methods must be cooperative [X], [XVI], [LXIV]. Furthermore, advancements in robotics and mobile computing, including radio-based and multi-modal SLAM systems, demonstrate the advantages of collaborative sensing and mapping to increase observability of unknown indoor spaces [LIX], [LXXVIII].

Despite these developments, several issues continue to limit effective indoor localisation. These include measurement noise, outliers, multipath effects, sensor drift, and variations in data quality when compared to other sensing modalities [LVII], [XXXIII]. Besides, scalability is a significant issue because, as network size and environmental complexity increase, it necessitates the use of efficient optimization and data association algorithms [XVI], [LIX]. The low-complexity and data-efficiency considerations are another motivator for adopting less dense anchor deployments that do not rely heavily on high accuracy but still achieve acceptable levels [XI], [LIX], [LXXXIX]. Generally, the literature indicates a tendency toward hybrid, collaborative, and learning-based localisation frameworks that leverage environmental characteristics, echo mapping, and joint optimisation to address the evolving needs of IoT and WSN systems [III], [XVI], [XVII], [LVII].

### Classification of Existing Methods

There are 6 major categories of sensor localization methods in buildings, as shown in Figure 1: range-based, range-free, hybrid, optimization-based, machine learning (ML) and AI-based, and acoustic/multi-modal localization. The categories vary in terms of sensing requirements, complexity, and achievable accuracy.



**Fig 1.** Landscape of Indoor Sensor Localization Technique

The range-based approaches measure physical signals, including RSSI, ToA, TDoA, and AoA, to infer distances or angles and are accurate in ideal settings but perform poorly in noisy indoor environments. Conversely, range-free algorithms, like Centroid and DV-Hop, use connectivity and hop-count information, which are cheap and scalable at the cost of accuracy. Hybrid methods combine these methods to achieve an accuracy-robustness trade-off, and optimisation-based methods use algorithms like

*Hemarjit Ningombam et al.*

PSO and GA to optimise estimates of positions by minimising over multiple steps. In recent years, machine learning and AI-driven solutions have been used to model nonlinear relationships in localisation data, thereby enhancing adaptability in dynamic conditions. Also, there are acoustic and multi-modal localization methods that leverage other sensing modalities and environmental propagation to enhance spatial awareness. The discussion of each category, including recent developments, is detailed in the following sections. In the following section, we discuss each classification of localization methods and their recent advancements.

### **Range-Based Localization Methods**

Range-based localisation methods use direct signal measurements to estimate distance or angle, which, in turn, underpin geometric position estimation. RSSI, ToA, TDoA, and AoA are the most frequently utilized observables, each with distinct operational properties and constraints [XLV], [LXVIII], [XC]. The RSSI techniques approximate range by mapping received signal strength to propagation models and trilaterating the estimates using multiple anchors. Although these RSSI-based techniques are cheap and easy to implement, they are very sensitive to environmental factors such as multipath propagation, shadowing, and signal attenuation. Therefore, their applicability in indoor settings has been shown to result in severe accuracy loss [LIV], [LXXXVII], [XC]. Consequently, it leads to extensive calibration or fingerprinting to enhance reliability [XC], [XCI].

Time-based techniques, like ToA, determine distance from signal propagation time, which has greater potential for accuracy. They, however, demand accurate clock timing and specialized hardware, which adds complexity and cost to the system [XXI], [LIV]. TDoA does not require synchronisation but instead relies on relative time differences between receivers, which require synchronised anchor nodes and accurate time measurements [XXI], [XLI], [LXVIII]. Directional techniques (angle-based), such as AoA/DOA, estimate node positions using directional information and require few anchors, but they require large antenna arrays and sophisticated signal processing algorithms. The techniques are also susceptible to the multipath and NLOS effects in indoor environments [XLV], [LXXXVII].

The range-based methods have several benefits, such as the potential to provide high accuracy, particularly in line-of-sight (LOS) conditions, and the ability to combine multiple measurements to improve performance. They, however, have a limitation in hardware requirements, being sensitive to environmental conditions and to network density scaling [XL], [LII], [LXVIII]. They vary widely in their performance across a variety of situations. Two techniques, trilateration and time-based trilateration, can be reliable when properly calibrated in a LOS environment, along with RSSI-based trilateration [LIV], [XC]. Conversely, as the density of indoor or non-line-of-sight settings increases, performance declines sharply due to multipath effects, and hybrid methods prove more efficient [XXI]. In resource-constrained systems, lightweight RSSI- and TDoA-based approaches offer a trade-off between cost and performance, with high-frequency solutions such as mmWave AoA and ToA achieving higher accuracy at the cost of greater hardware complexity and synchronization demands [XL], [LXXXII], [LXXXVII].

*Hemarjit Ningombam et al.*

### **Range-Free Localization Methods**

Range-free localization techniques calculate node positions without using explicit distance measurements; instead, they rely on network connectivity, anchor visibility, and geometric inference. The most popular methods include Centroid, DV-Hop, and APIT, and many of their variations are commonly used as baseline methods in WSN localisation research [L], [LXVI], [LXXVI]. The centroid technique is used to locate an unknown node by calculating a geometric mean of the positions of its neighbouring anchor nodes. Though the method is easy and requires minimal hardware support, it produces rough estimates and is very sensitive to the positioning of the anchors [XLI], [L]. This is further enhanced by the DV-Hop, which estimates distances using hop counts between nodes and anchors, and transforms average hop distances into rough physical distances for trilateration. Even though DV-Hop is a scalable metric in large-scale networks, it is a poor measure due to cumulative errors in the hop-distance and significant reliance on topological structure [XIII], [XLI]. APIT (Approximate Point-In-Triangle Test) is another commonly used algorithm that computes a node's membership in triangles defined by anchor nodes and approximates the node's location using the centroid of overlapping feasible subsets. Although APIT achieves higher accuracy than centroid-based algorithms, it incurs greater communication and computational overhead because it requires the exchange of neighbourhood information [LXI], [LXVI].

The new developments have been directed towards refining and hybridizing range-free techniques. Geometric inaccuracies and edge effects are addressed by variants that are weighted APIT and DV-Hop-APIT hybrids that use vector similarity measures, centre-of-gravity tests, and multi-hop extensions [XXIX], [XXXIX], [LXXII]. These methods increase localization precision and stability, especially when the anchors are not evenly distributed. Nonetheless, even with such advances, range-free algorithms essentially suffer the same problems as coarse estimation, topology dependence, and sensitivity to sparse connectivity problems [L], [LVIII], [LXI], [LXVI]. Altogether, the range-free methods remain appealing for large-scale, inexpensive deployments, as they are simple and consume less energy. Current studies aim to improve these approaches by using topology-aware weighting, hybridization, and more robust geometric inference techniques to localize less while still scaling better [XXIX], [XXXIX], [LXVI].

### **Hybrid Localization Approaches**

Hybrid background localisation methods combine a variety of sensing modalities, including range-based and range-free methods, to address the shortcomings of each. The most popular hybrid strategies involve combinations of RSSI with DV-Hop or APIT, and multi-modal fusion of RF, inertial, acoustic, or vision-based sensing [LII], [LIX], [LXVI], [LXXVIII]. The main reason given for hybrid methods is the desire to leverage the complementary strengths of the methods: range-free methods are cheaper and more scalable. In contrast, range-based methods are more accurate and subject to geometric constraints [II], [LXI], [LXXXIX]. The hybrid techniques prove useful, especially in practical situations with infrastructure limitations, NLOS conditions, and heterogeneous sensing conditions. Single-method approaches tend not to work in these settings due to environmental uncertainties and the thinness of the anchors. Using a combination of information sources, hybrid solutions enhance visibility, reduce

*Hemarjit Ningombam et al.*

ambiguity, and become resilient to single-source errors [II], [VI], [VII], [LXIII]. For example, the use of RSSI fingerprinting with DV-Hop, or the combination of ToA/TDoA measurements with connectivity-based approaches, enables better localization performance at moderate deployment costs [II], [VI], [XLVII], [LXVI].

Experimental findings show that hybrid solutions can significantly reduce localisation error and enhance network resilience to irregularity. DV-Hop with RSSI-based corrections or APIT with adaptive weighting schemes have significant gains over the older range-free techniques and remain cost-effective [IX], [XVI], [LXVIII]. Also, fusion methods that integrate learning based on RTT, RSSI, and fingerprints have demonstrated significant improvements in accuracy and robustness in dynamic environments [VI], [XXXVII], [LXV]. Nevertheless, hybrid techniques also pose new challenges, including increased computational complexity, data fusion requirements, and synchronisation issues. Its successful implementation requires advanced algorithms, such as probabilistic, optimization-based, or machine-learning methods, to combine heterogeneous data types without introducing bias or instability. Moreover, the overheads of communication and energy usage grow with the number of sensing and data exchange that defines scalability issues in dense or large-scale networks [VI], [XCI], [LI].

### **Optimization-Based Techniques**

Localisation techniques using optimisation express the localisation problem as a minimisation task over an objective functional, aiming to estimate node positions by minimising the difference between measured values and predicted spatial relationships. The methods have received significant attention for their ability to handle nonlinearity, noise, and incomplete data. Particle Swarm Optimisation (PSO), Genetic Algorithms (GA), and Ant Colony Optimisation (ACO) are metaheuristic algorithms used to find optimal node configurations in complex solution spaces [XXIII], [XXX], [XXXVIII], [XXXIX]. In these methods, the objective function is usually the reduction of the sum of squared residuals between measured and estimated distances based on RSSI, ToA, TDoA, AoA, or hybrid measurements. Moreover, robust loss functions such as the Huber loss or L1/L2 norms are often used to minimise the impact of outliers. Regularization terms can also be added to impose spatial continuity, smoothness, or environmental constraints. Sometimes penalties are added in hybrid settings to compensate for discrepancies in hop-distance estimates or connectivity-based approximations [XXXVIII], [XXVII], [LXVII].

Other works use optimisation methods to estimate positions, select an optimal subset of anchors, or add a virtual anchor to improve localisation accuracy without adding infrastructure [XXXVI], [XXVIII], [XXX]. Moreover, current methods incorporate optimization and machine-learning corrections to enhance measurement reliability and adapt to environmental variations [LX], [VIII]. Optimization-based approaches have the disadvantage of introducing trade-offs. Although they can greatly improve localization accuracy, particularly in adverse settings such as NLOS or sparse anchor layouts, most of them are computationally intensive and consume significant energy. Scalability is another major issue, especially in large networks, where the complexity of optimisation increases exponentially. Researchers have developed strategies to

overcome this, including hierarchical localisation, dimensionality reduction, and online or sequential optimisation [XXVII], [XXVIII], [XXXVIII].

Also, metaheuristic algorithms are highly sensitive to parameter tuning, including population size and convergence criteria, which may influence performance in terms of accuracy and execution time. Consequently, numerous hybrid strategies aim to achieve optimality at the expense of computational efficiency, particularly in resource-constrained IoT systems. In general, optimisation-based localisation offers a highly useful framework for achieving greater accuracy and robustness, and current research aims to improve scalability, efficiency, and compatibility with heterogeneous sensing systems [XXX], [XXXVIII], [XXXIX].

### **Machine Learning and AI-Based Localization Methods**

Based on optimization and hybrid methods, machine learning (ML) and artificial intelligence (AI) technologies have become potent forces in indoor localization, enabling data modelling and adaptive learning in complex settings. The approaches exploit both supervised and unsupervised learning models, such as deep neural networks (DNNs), convolutional neural networks (CNNs), recurrent neural networks (RNNs/LSTMs), and transformer-based designs, to predict sensor measurements, including RSSI, CSI, and RTT, supported by auxiliary information about inertial sensors, magnetic fields, or acoustic fingerprints, to spatial locations [XIV], [XXIV], [XXXI], [XLIV], [LIV]. The main benefit of localization using ML is that it can be trained on non-linear and intricate relationships between measurements and positions, for which it is difficult to make analytical predictions. Many methods are based on fingerprinting algorithms that use vast offline datasets to produce models that can identify sites in use. Also, the fusion strategies based on machine learning combine multi-modal data streams to single representations, enhancing cross-heterogeneous device and cross-environment generalization [XIV], [XLVIII], [LXX], [LXXVII]. Adaptive localization tasks such as anchor selection, sensor placement, and cooperative localization strategies have also been studied using reinforcement learning (RL), in which agents are trained to optimize system observability [LVI], [LXX], [LXXVII].

Despite these benefits, ML-based methods have several limitations. They need massive, well-labelled datasets, extensive training, and careful consideration of generalization across environments. When performing in unknown circumstances, performance might be inaccurate due to overfitting or a domain mismatch. In addition, the training and inference costs in terms of computational and energy resources can be prohibitive for the IoT device in resource-constrained settings. Consequently, although ML and AI methods are promising for enhancing localization accuracy and flexibility, their real-world implementation requires effective data management, scalable learning approaches, and lightweight inference systems [XIV], [XXIV], [XCI].

### **Acoustic and Multi-modal Localization**

Other sensing modalities for radio frequency signals have become less popular in recent years, while indoor localisation has increasingly relied on other modalities. Among them, acoustic and ultrasound-based methods offer complementary sensing features that are less susceptible to RTW-specific issues such as multipath fading and severe NLOS conditions [XVIII], [XIX]. Sound propagation properties (time-of-flight (ToF),

*Hemarjit Ningombam et al.*

time-difference-of-arrival (TDoA), and echo patterns) can be used by acoustic localization systems to determine distances and infer environmental geometry. One notable development in this area is the application of echo-based localization, treating environmental reflections on walls, furniture, and other surfaces as informative rather than noise [XXVI]. These methods can be used to reverse the spatial relationship and to impose other geometric constraints by analyzing the temporal structure of reflected acoustic signals. Mostly, acoustic measurements are combined with radio-type data to enhance observability, mitigate ambiguity, and jointly estimate node locations and environmental characteristics [XVIII], [LXXXV].

The incorporation of environmental information represents a significant departure from traditional localization paradigms. Acoustic techniques use room geometry, reflection coefficients, and delay patterns to iteratively refine sensor and reflector locations, effectively converting the environment into a localization map [XVIII], [XIX]. Hybrid acoustic-radio schemes further improve performance by integrating dense yet noisy RF measurements with structured acoustic constraints. Nevertheless, several obstacles remain. Acoustic systems often require additional hardware, such as ultrasonic transducers and synchronised clocks, which increases deployment costs and complexity. The environment, including noise, temperature, and reverberation, also affects their performance. Additionally, the fusion of acoustic and RF data requires robust fusion methods to ensure consistency and stability. Nevertheless, it addresses only those shortcomings; hybrid acoustic-radio localisation appears to be a promising direction for improving the accuracy and robustness of localisation in indoor environments [XVIII], [LXXV], [LXXXVIII]. In the following section, we introduce the performance metrics used to evaluate the existing localisation methods reported in the literature.

### **Performance Evaluation Metrics in Literature**

Localisation techniques are typically assessed using performance metrics such as Root Mean Square Error (RMSE) and Mean Absolute Error (MAE), as well as percentile-based metrics such as P95, which represent worst-case performance [XLIII], [LXIX], [LXXI]. These metrics provide information on average accuracy and worst-case performance under different conditions. Evaluations of localisation performance across various conditions, including noise levels, node density, and anchor availability, are reported in existing studies. Experimental systems include small indoor testbeds and large-scale simulations that incorporate realistic propagation models and multipath effects [XV], [XXV], [XLIII], [LXXXIII]. It is consistently found that a hybrid, multi-modal methodology performs better than single-modality methods, especially in challenging environments with NLOS and high noise levels [XV], [XLIII], [LXXX].

Moreover, robustness analysis shows that approaches that employ optimization strategies or robust loss functions (e.g., Huber loss) are more robust to noisy environments than classical approaches [XLIII], [LXIX], [LXXX]. Likewise, the multi-modal fusion methods show better stability and lower error variance, particularly when the density of the anchors is low [XXIV], [XLVIII], [LXXXIII]. This, however, is usually accompanied by a performance cost, in terms of both computational complexity and energy consumption. Scalability has been a major concern, especially in large-scale deployments, where efficient algorithms and lightweight

*Hemarjit Ningombam et al.*

implementations are needed. In turn, recent studies focus on the need for balanced solutions that are highly accurate yet computationally efficient and scalable [XIV], [XXVIII], [XXXIX], [LXIX]. In the following section, we discuss the existing research gap that our proposed method addresses.

### **Research Gap Identification**

Despite intensive studies, several limitations remain in the prevailing localization techniques. One of the main issues is the dilemma between quality and price. The range-based techniques offer better accuracy but require extra hardware and synchronization, whereas the range-free techniques are inexpensive but lack precision and accumulate errors. In addition, most current methods are not very robust to environmental factors, especially in indoor environments with multipath propagation and NLOS conditions. While hybrid and optimization-based approaches enhance performance, they rely heavily on radio signals and cannot fully leverage environmental information.

The second weakness is the under-exploitation of the environmental features. Existing techniques typically treat reflections as noise rather than as useful information, thereby limiting the ability to improve localisation accuracy. Also, a high reliance on anchor nodes increases deployment costs and diminishes scalability. Lastly, most sophisticated methods, especially ML and optimization-based methods, face significant computational demands, data requirements, and real-time constraints. These restrictions highlight the need for a robust, scalable, and infrastructure-efficient localisation model that leverages environmental data and minimises reliance on anchors. In the following section, we highlight the key motivations that drive the development of the proposed work.

### **Motivation for the Proposed Work**

The limitations mentioned are reasons why a stronger, more scalable localization framework should be developed. The proposed Collaborative Acoustic Echo Mapping (CAEM) methodology can address these limitations by combining acoustic and radio-sensing modalities within a single optimisation framework. In contrast to conventional approaches that treat reflections as noise, CAEM uses reflectors in the environment as virtual anchors, thereby imposing spatial constraints without additional infrastructure.

Moreover, CAEM uses a joint-optimisation approach that estimates both sensor and reflector locations in a single calculation to achieve greater accuracy and consistency. The Huber loss function is implemented, making it more resilient to noise and outliers and thereby improving stability in complex indoor scenes. The proposed approach addresses the main limitations of current approaches by integrating complementary modalities and capitalizing on environmental information. In the following section, we clearly position the proposed contribution with respect to existing methods and highlight its distinguishing features.

### **Positioning of the Proposed Contribution**

The proposed CAEM framework is presented as a hybrid, optimisation-based localisation methodology that integrates multimodal sensing with advanced mathematical modelling. It is a hybrid of radio-based ranging and acoustic echo

*Hemarjit Ningombam et al.*

sensing, at the boundary between hybrid localisation and optimisation-driven approaches. Compared with classical and contemporary approaches such as Centroid and DV-Hop, CAEM is much more precise, as it uses additional geometric constraints based on environmental reflections. It minimises the use of physical anchors by relying on virtual anchors, unlike anchor-dependent methods. Compared to other sophisticated approaches, such as machine learning and metaheuristic optimization, CAEM provides a physically interpretable and computationally efficient solution. It does not require massive training datasets and guarantees stable convergence in deterministic optimization.

The main originality of this piece is:

- (i) virtual anchors as environmental reflectors,
- (ii) a joint optimization model of simultaneous sensor and reflectors estimation, and
- (iii) the incorporation of effective loss measures to deal with noise and outliers.

All these contributions improve accuracy, robustness, and scalability, and CAEM can be seen as a viable solution for next-generation indoor localisation systems.

### **III. Method Proposed**

The proposed Collaborative Acoustic Echo Mapping (CAEM) framework proposes a hybrid localization approach that combines acoustic echo sensing with radio-based ranging to deliver high-quality indoor location services in wireless sensor networks. Given a network of  $N$  sensor nodes scattered at random in a two-dimensional field of size  $V$ , a small number of them, denoted  $N_a$ , are designated as anchor nodes. They have known positions, and the other nodes are unknown, with their positions to be estimated.

The sensor nodes have a radio transceiver for communicating directly with other nodes and an ultrasonic transceiver for sensing acoustics. The radio subsystem provides a distance measure when a node and its adjacent nodes fall within a communication range of  $d_r$ , and the acoustic subsystem records reflected signals from objects in the environment (reflectors) within a sensing range of  $d_u$ . In contrast to traditional methods that treat multipath as noise, the presented framework treats reflections as additional geometric constraints, turning the environment's characteristics into virtual anchors and thereby increasing localization observability.

The measurement model consists of two parts. To begin with, the acoustic echo measurements are the distance between a sensor node and a reflector, but are distorted by additive Gaussian noise.

$$d_{ik}^{echo} = ||p_i - r_k|| + N(0, \sigma^2)$$

where  $p_i$  represents the position of sensor  $i$ ,  $r_k$  denotes the position of reflector  $k$ , and  $\sigma$  is the noise standard deviation.

Second, radio-based measurements capture the distance between neighboring sensor nodes:

$$d_{ij} = ||p_i - p_j|| + N(0, \sigma^2)$$

*Hemarjit Ningombam et al.*

These two modalities jointly provide complementary spatial information, enabling improved localization accuracy even under noisy, sparse-anchor conditions.

The localization problem is formulated as a joint optimization over the sensor and reflector positions. Let  $P = \{p_i\}$  for  $i = 1$  to  $N$  denote the set of sensor positions and  $R = \{r_k\}$  For  $k = 1$  to  $M'$ , denote the set of reflector positions after association. The objective function is defined as:

$$J(P, R) = \sum(i, k) \rho(\|p_i - r_k\| - d_{ik}^{echo}) + \lambda \sum(i, j) \rho(\|p_i - p_j\| - d_{ij}) + \gamma \sum(i \in anchors) \|p_i - p_{i_{true}}\|^2$$

where  $\rho(\cdot)$  The Huber loss function is used to improve robustness against noise and outliers;  $\lambda$  is a weighting factor that balances acoustic and radio measurements; and  $\gamma$  enforces anchor constraints. This formulation enables simultaneous estimation of both sensor and reflector positions, thereby leveraging environmental structure to improve localization performance.

The proposed CAEM framework jointly estimates the positions of sensor nodes and environmental reflectors using acoustic echo measurements and radio-based inter-sensor ranging observations. Since both sensor coordinates and reflector coordinates are unknown, it is necessary to establish the conditions under which the estimation problem is observable and admits a unique solution.

Consider a network containing  $N$  sensors and  $M$  reflectors in a two-dimensional environment. The unknown parameter vector consists of

$$\Theta = \{p_1, p_2, \dots, p_N, r_1, r_2, \dots, r_M\}$$

where  $p_i \in \mathbb{R}^2$  represents the position of sensor  $i$  and  $r_k \in \mathbb{R}^2$  denotes the position of reflector  $k$ . The total number of unknown parameters is therefore

$$n_u = 2N + 2M.$$

The measurement model consists of two types of geometric constraints:

1. Acoustic sensor–reflector distance measurements

$$d_{ik}^{(a)} = \|p_i - r_k\| + \varepsilon_{ik}^{(a)}$$

Radio sensor–sensor ranging measurements

$$d_{ij}^{(r)} = \|p_i - p_j\| + \varepsilon_{ij}^{(r)}$$

Let  $E_a$  and  $E_r$  denote the numbers of acoustic and radio observations, respectively. The total number of independent constraints is

$$n_c = E_a + E_r.$$

For local identifiability, the number of independent constraints must be sufficient to estimate all unknown variables after accounting for rigid-body ambiguities. Thus, a necessary condition is

$$E_a + E_r \geq 2N + 2M - 3$$

where three degrees of freedom correspond to global translation and rotation invariance.

The localization problem can be represented as a measurement graph

$$G = (V_s \cup V_r, E_r \cup E_a)$$

where  $V_s$  denotes sensor nodes and  $V_r$  denotes reflector nodes. The joint estimation problem is locally observable when:

- The measurement graph is connected.
- at least three non-collinear anchors are available;
- at least three non-collinear sensors observe each reflector;
- The measurement Jacobian matrix achieves full rank.

Under these conditions,

$$\text{rank}(J) = 2N + 2M - 3$$

and the solution becomes locally unique up to rigid-body transformations.

It should be noted that the Procrustes alignment stage used in CAEM removes only global translation and rotation ambiguities. The observability conditions described above are required to guarantee the uniqueness of the estimated sensor–reflector geometry. Therefore, when the measurement graph is sufficiently connected and reflector observations are geometrically diverse, the proposed CAEM framework yields a stable and identifiable solution.

**Table 1: Simulation Parameters**

Parameter	Value
Area	100 × 100 m
Number of Sensors (N)	20–100
Number of Anchors (N <sub>a</sub> )	3–10
Number of Reflectors	~12
Noise Standard Deviation (σ)	0.1–0.4 m
Radio Range (d <sub>r</sub> )	40 m
Acoustic Range (d <sub>u</sub> )	30 m

For all baseline localization techniques, the parameters used were those suggested by the original technique and validated through preliminary experiments. The chosen parameter values were then kept constant across all possible simulation scenarios and Monte Carlo runs. For Anchor-Only Multilateration, Centroid Localisation, and DV-Hop, there were no other hyperparameters beyond the network configuration. Weighted Centroid Localisation (WCL) used the standard inverse-square weighting method, in which the weight of each neighbour node was determined by  $1 / \text{distance}^2$ , where distance is the distance to that neighbour node. The path-loss exponent was assumed to be 2.5, the reference distance was 1 m, and the standard deviation of the RSSI measurement noise was 3 dB for RSSI-based Multilateration. The values were chosen to represent typical indoor propagation conditions for wireless systems.

In the case of Graph Laplacian Localisation (GLL), the width of the Gaussian kernel used to construct the similarity graph was set to 1.0 times the average distance between nodes. Initial testing showed that this setting produced stable graph embeddings without over-smoothing or graph fragmentation. The variance of measurement noise was set to 0.04 for EKF-SLAM, corresponding to a standard deviation of 0.2 m. The anchor information weight was thus set to 10,000, and three relinearization passes were used in each localization cycle. These values converged and gave good state estimation. In RO-AltOpt, we used 20 Inner Gauss-Newton iterations per optimization stage and 6

Outer alternating-optimization iterations. The regularisation weight for the anchor was set to 5,000, the damping coefficient to 0.001, and the threshold for the Huber loss function to 0.5. The values were chosen based on pilot experiments to achieve a compromise between convergence speed and localization accuracy. Here, the radio-range regularization coefficient = 1.0, Huber loss threshold = 0.5, and anchor regularization weight = 10,000 for the proposed CAEM framework. The L-BFGS-B algorithm with a maximum of 400 iterations, a function tolerance of  $1 \times 10^{-12}$ , and a gradient tolerance of  $1 \times 10^{-8}$  was used for optimization. There was no scenario-specific parameter adjustment for any method. The same network topologies, anchor deployments, and noise levels were used for all algorithms to provide a fair and unbiased comparison.

The proposed CAEM framework estimates sensor locations and reflector locations simultaneously from acoustic and radio measurements. Since the optimization problem involves multiple unknown geometric entities, it is necessary to establish conditions under which a unique solution exists.

Let

$$\Theta = \{p_1, p_2, \dots, p_N, r_1, r_2, \dots, r_M\}$$

represent the unknown parameter vector containing the coordinates of  $N$  sensors and  $M$  reflectors.

The optimization objective is given by

$$\min_{\Theta} F(\Theta)$$

where  $F(\Theta)$  combines acoustic echo residuals and radio-ranging residuals through a robust Huber loss function.

For any distance-based localization problem, the objective function remains invariant under rigid-body transformations. Specifically, for any translation vector  $t$  and rotation matrix  $R$ ,

$$\begin{aligned} p'_i &= Rp_i + t, \\ r'_k &= Rr_k + t, \end{aligned}$$

produce identical pairwise distances and therefore identical objective values. Consequently, uniqueness can only be established up to rigid-body transformations.

The optimization admits a unique geometric solution when the following conditions hold:

**Condition 1: Connected Measurement Graph**

The graph

$$G = (V_s \cup V_r, E_r \cup E_a)$$

must be connected, where  $V_s$  and  $V_r$  represent sensor and reflector vertices, respectively. Disconnected components can move independently without affecting measurements, resulting in non-unique solutions.

**Condition 2: Sufficient Anchor Constraints**

At least three non-collinear anchor nodes must be available.

Three non-collinear anchors uniquely determine the global coordinate frame and eliminate rotational and translational ambiguity.

**Condition 3: Reflector Visibility**

At least three non-collinear sensors must observe each reflector. Two observations produce intersecting circles that may yield multiple feasible reflector locations, whereas three independent observations uniquely determine the reflector position.

*Hemarjit Ningombam et al.*

**Condition 4: Sufficient Number of Measurements**

Let  $E_a$  denote acoustic observations and  $E_r$  denote radio observations.

A necessary condition for uniqueness is

$$E_a + E_r \geq 2N + 2M - 3.$$

This ensures that the number of independent constraints exceeds the effective degrees of freedom after removing rigid-body ambiguity.

**Condition 5: Full-Rank Jacobian**

Let  $J$  denote the Jacobian matrix of the measurement equations.

Local uniqueness requires

$$\text{rank}(J) = 2N + 2M - 3.$$

A rank-deficient Jacobian indicates redundant or dependent measurements and implies the existence of multiple feasible solutions.

Therefore, when Conditions 1–5 are satisfied, the optimization problem possesses a locally unique geometric solution up to rigid-body transformations. Under these circumstances, the estimated sensor and reflector coordinates are uniquely determined by the available acoustic and radio measurements, thereby ensuring the stability and consistency of the CAEM localization framework.

Although the proposed CAEM framework combines acoustic and radio measurements to improve localization accuracy, certain geometric configurations may introduce ambiguity in the estimated sensor and reflector locations. Two particularly important cases are sparse-anchor deployments and symmetric-reflector environments.

**A. Sparse-Anchor Configurations**

Anchor nodes provide absolute spatial references that define the global coordinate system. When the number of anchors is insufficient, the localization problem becomes under-constrained.

For a two-dimensional localization problem:

1. One anchor removes only translational ambiguity.
2. Two anchors constrain translation but still allow rotational ambiguity.
3. Three non-collinear anchors are required to define the coordinate frame uniquely.

Consequently, when fewer than three non-collinear anchors are available, multiple geometric configurations may satisfy the same set of acoustic and radio measurements. These configurations can be related through rotation, reflection, or other rigid-body transformations while producing identical measurement residuals.

Mathematically, if

$$\Theta = \{p_i, r_k\}$$

is a feasible solution, then another configuration

$$\Theta' = \{Rp_i + t, Rr_k + t\}$$

may produce the same objective value

$$F(\Theta) = F(\Theta')$$

where  $R$  is a rotation matrix, and  $t$  is a translation vector; therefore, sparse-anchor deployments increase solution uncertainty and may lead to multiple feasible minima in the optimization landscape.

**B. Symmetric-Reflector Configurations**

Additional ambiguities may arise from the reflector layout's geometry. Consider a set of reflectors arranged symmetrically with respect to a room axis or geometric center.

If reflector positions satisfy

$$r'_k = Sr_k$$

where  $S$  denotes a symmetry transformation (e.g., mirror reflection), the resulting acoustic distance measurements may remain nearly unchanged for certain sensor arrangements.

As a result, multiple reflector configurations can generate similar acoustic residuals, leading to ambiguity in reflector association and position estimation. This effect is particularly pronounced when:

- Reflectors are placed in symmetric pairs.
- Sensors observe only a subset of reflectors.
- Acoustic measurements are noisy.
- Anchor density is low.

In such situations, the optimization problem may contain several local minima corresponding to geometrically equivalent reflector arrangements.

### **C. Ambiguity Mitigation in CAEM**

The proposed CAEM framework mitigates these ambiguities through three mechanisms:

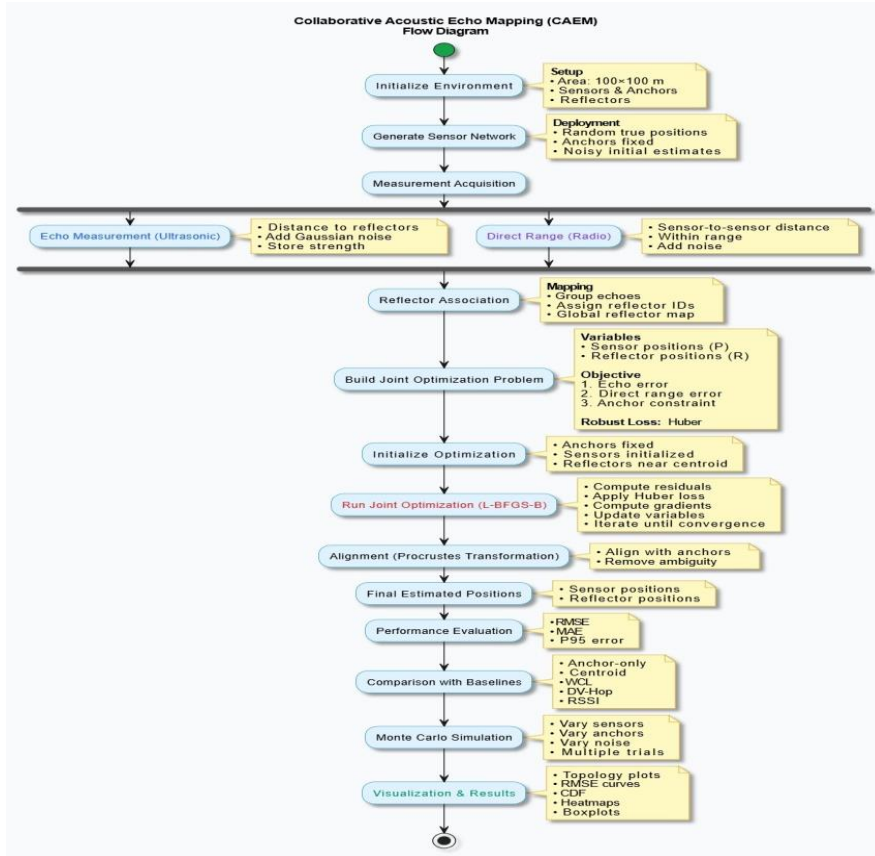
1. **Acoustic–Radio Fusion:**  
Radio-ranging constraints introduce additional geometric information that is independent of reflector symmetry.
2. **Multiple Sensor Observations:**  
Each reflector is typically observed by multiple sensors, increasing geometric diversity and reducing the interchangeability of reflectors.
3. **Anchor-Based Coordinate Fixing:**  
Anchor nodes establish an absolute reference frame that suppresses rotational and reflection ambiguities.

Consequently, while sparse-anchor and symmetric-reflector configurations may, in theory, introduce non-uniqueness, the combined use of acoustic echoes, radio measurements, and anchor constraints significantly improves observability and reduces ambiguity in practical deployments.

The complete CAEM algorithm is mentioned below.

<b>Algorithm 1: CAEM for Sensor Localization</b>
<b>Input:</b> Number of sensors ( $N$ ), number of anchors ( $N_a$ ), reflector set ( $R$ ), initial estimates $P(0)$ , noise level $\sigma$ , acoustic range $d_u$ , radio range $d_r$
<b>Output:</b> Estimated sensor positions ( $\hat{P}$ ), reflector positions ( $\hat{R}$ )
<b>Step 1: Network Initialization</b> Deploy $N$ sensor nodes randomly in the area. Assign the first $N_a$ nodes as anchors with known positions. Initialize anchor positions as fixed, while non-anchor nodes are assigned noisy initial estimates.
<b>Step 2: Measurement Acquisition</b>
(a) Echo Measurements (Acoustic) For each sensor $i$ , measure distance to reflector $k$ :

$d_{ik}^{echo} = \left   p_i - r_k  \right  + N(0, \sigma^2)$
Store echo strength and distance.
(b) Direct Range Measurements (Radio) For each sensor pair $(i, j)$ , if within communication range $dr$ :
$d_{ij} = \left   p_i - p_j  \right  + N(0, \sigma^2)$
<b>Step 3: Reflector Association</b> Collect all echo measurements from sensors. Group echoes corresponding to the same reflector, assign unique reflector IDs, and determine the effective number of reflectors $M'$ .
<b>Step 4: Problem Formulation (Joint Optimization)</b> Define variables:
$P = \{p_i\}, \quad R = \{r_k\}$
Minimize the objective function:
$J(P, R) = \sum(i, k) \rho( p_i - r_k  - d_{ik}^{echo}) + \lambda \sum(i, j) \rho( p_i - p_j  - d_{ij}) + \gamma \sum(i \in anchors)  p_i - p_{i_{true}} ^2$
<b>Step 5: Initialization</b> Set $p_i$ = true position for anchor nodes and initialize non-anchor nodes with estimated values. Initialize reflector positions near the centroid of anchor nodes.
<b>Step 6: Joint Optimization (L-BFGS-B)</b> Repeat until convergence:
Compute echo residuals: $r_{ik} = \left   p_i - r_k  \right  - d_{ik}^{echo}$
Compute range residuals: $r_{ij} = \left   p_i - p_j  \right  - d_{ij}$
Apply Huber loss to residuals, compute gradients with respect to $P$ and $R$ , and update variables using the L-BFGS-B optimizer.
<b>Step 7: Alignment (Procrustes Transformation)</b> Align estimated coordinates with anchor positions to remove translation and rotation ambiguity.
<b>Step 8: Output Final Estimates</b> $P \leftarrow$ optimized sensor positions $R \leftarrow$ optimized reflector positions
<b>Step 9: Performance Evaluation</b>
$RMSE = \text{sqrt} \left( \frac{1}{N} \sum  p_i - \hat{p}_i ^2 \right)$
Also compute MAE and 95th percentile error (P95).
<b>Step 10: Comparative Analysis</b> Compare results with classical and contemporary localization methods.
<b>Step 11: Monte Carlo Simulation</b> Repeat experiments for multiple trials while varying the number of sensors, anchors, and noise levels. Compute average performance metrics.

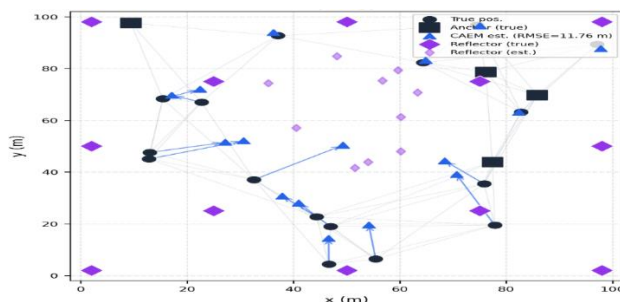


**Fig 2.** CAEM Flow Diagram

Figure 2 illustrates the complete workflow of the proposed CAEM framework, including the initialization, measurement acquisition, reflector association, joint optimization, and performance evaluation stages.

**IV. Results and Discussion**

**Network Topology Analysis**



**Fig. 3.** Network topology (100 × 100 m)

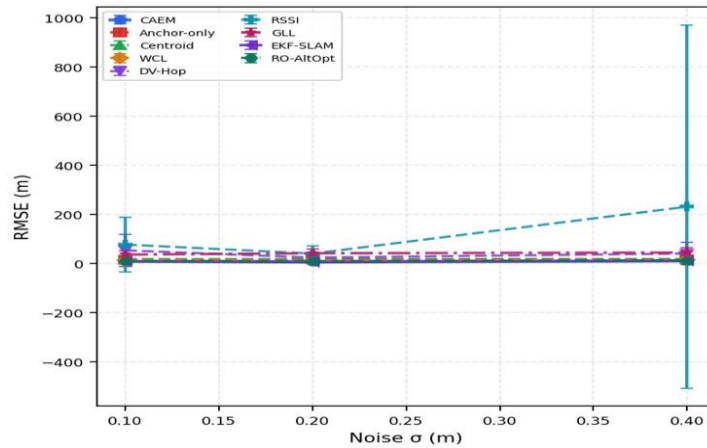
Figure 3 shows the simulated indoor network layout, indicating the locations of sensor nodes, anchors, and environmental reflectors. Based on the visualization, the locations calculated using the CAEM framework are very close to the nodes' actual locations. It

*Hemarjit Ningombam et al.*

is also worth noting that the localization accuracy does not decrease much even in areas with low anchor node density. This implies that the extra constraints imposed by reflected acoustic signals make a significant contribution. That is, the environment, per se, also plays a positive role in the localization process.

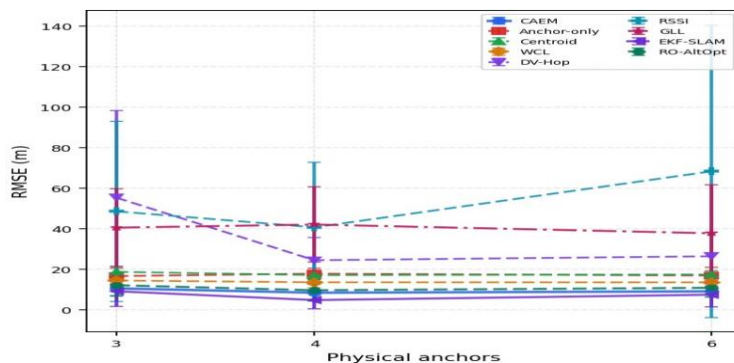
**Effect of Measurement Noise**

Findings imply that the use of echo-based information can reduce ambiguity and enhance the accuracy of position estimation. Figure 4 shows that the localization error decreases as measurement noise increases. Not surprisingly, all means show some deterioration in noise levels between 0.1 m and 0.4 m. Nevertheless, CAEM has consistently lower RMSE values than the baseline techniques. The traditional and contemporary approaches exhibit a more pronounced performance decline, especially at high noise levels. One can explain this behavior by two factors. First, the Huber loss function minimizes the effects of outliers. Second, the acoustic and radio measurements are combined to provide additional redundancy, thereby stabilizing the estimation process. These findings imply that CAEM is relatively strong even in difficult noise conditions.



**Fig 4. RMSE vs noise**

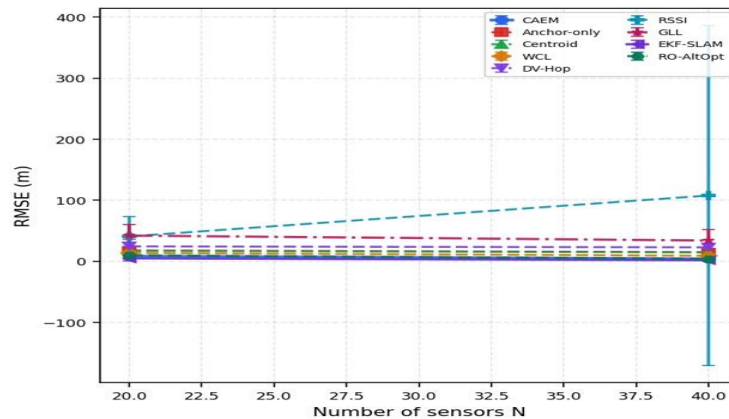
**Impact of Anchor Density**



**Fig 5. RMSE vs anchors**

Figure 5 shows the correlation between anchor density and localization error. The performance of all methods improves with increasing the number of anchors, as expected, since more anchors provide stronger geometric constraints. That said, CAEM will always be more effective than the other methods. The difference is more apparent when there are a small number of anchors. Under these circumstances, conventional techniques fail because there is a lack of reference points, but CAEM fills the void by using reflectors as virtual standards. It will minimize reliance on physical anchors and make the method more feasible when implementing a large number of anchors is not possible. The observed improvement in localization accuracy with increasing anchor density is consistent with the observability analysis presented in Section III. As the number of anchors and geometric constraints increases, the rank deficiency of the measurement system decreases, leading to a more identifiable sensor–reflector configuration. Conversely, sparse-anchor deployments may introduce geometric ambiguities and multiple feasible solutions, which explains the increased localization error observed in low-anchor scenarios. When the number of anchors decreases, the available geometric constraints become insufficient to fully determine the sensor–reflector configuration, thereby increasing ambiguity and estimation uncertainty. Conversely, increasing anchor density improves the rank of the measurement system and strengthens localization identifiability.

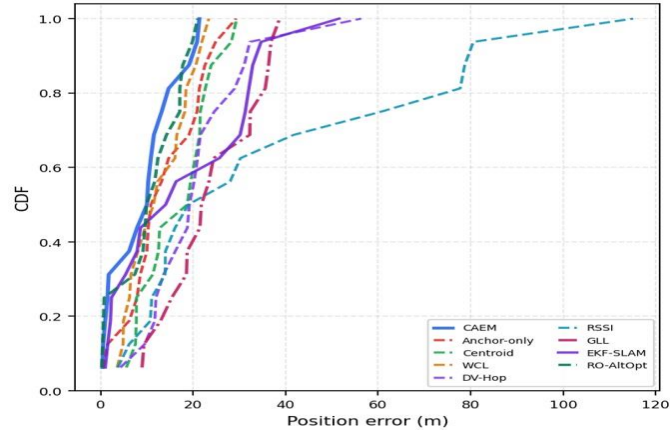
**Scalability with Network Density**



**Fig 6.** RMSE vs density

Figure 6 assesses the method's behaviour in the presence of an increase in the number of sensor nodes. Interestingly, CAEM has an advantage in node density. The system acquires additional measurements and constraints as the number of nodes increases, thereby improving optimisation precision. Conversely, other non-adaptive baseline approaches either exhibit slight improvement or slight worsening, perhaps due to cumulative estimation error. The consistent decrease of the RMSE of CAEM points to the fact that the framework can be used with dense deployments that rely on the expanded connectivity.

**Error Distribution Analysis**

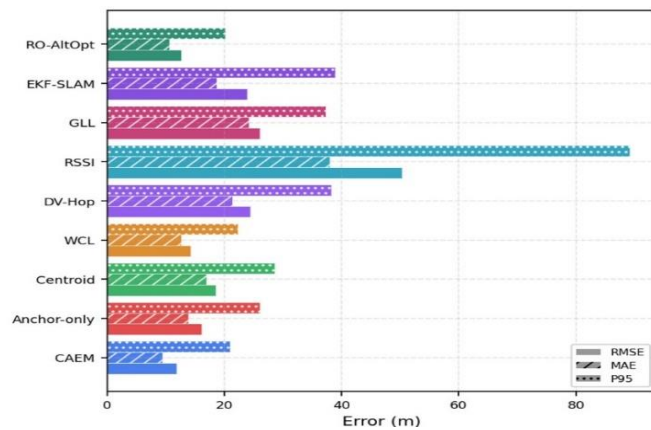


**Fig 7.** CDF of error

The cumulative distribution of localization error is shown in Figure 7. The CAEM curve increases more steeply than the others. This would mean that a greater percentage of nodes will have reduced errors. In practice, this implies that CAEM is not only the best way to improve average accuracy but also provides more consistent results across the network. The concentration is another indicator of the validity of the suggested methodology.

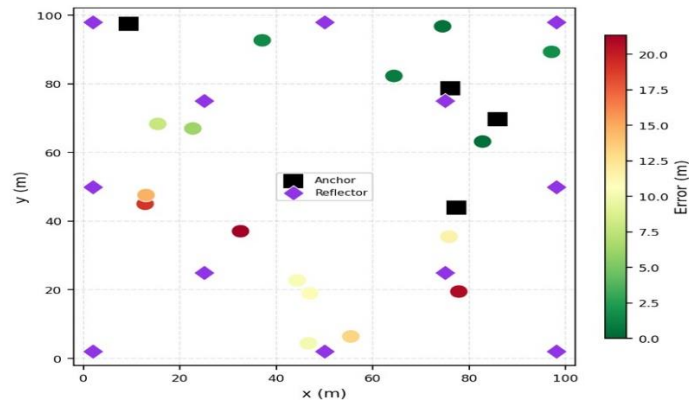
**Comparative Error Metrics**

The results in Figure 8 compare RMSE, MAE, and P95 errors across various methods. CAEM gets the lowest scores in all three measures. Although the decrease in P95 error is especially significant, the improvement in overall and average accuracy is better reflected by RMSE and MAE. It demonstrates that extreme errors are reduced to a minimum, which is important in real-world settings where outliers may affect system performance.



**Fig 8.** RMSE / MAE / P95 comparison

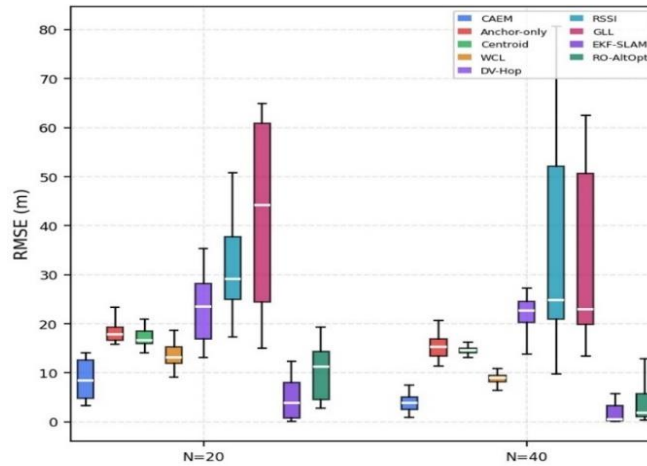
**Spatial Error Distribution**



**Fig 9.** Error heatmap

As shown in Figure 9, the distribution of spatial errors indicates that CAEM produces a relatively homogeneous pattern across the deployment area. Higher error rates are observed near the boundaries, presumably because reflector coverage is lower there. Regardless, the error levels are lower than with the base techniques. This implies that environmental reflections can be used to evenly distribute localisation accuracy.

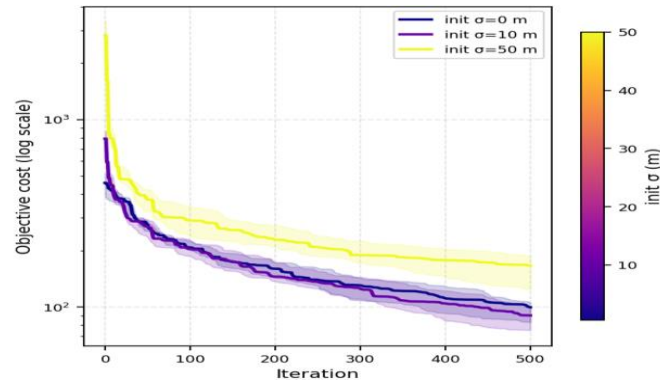
**RMSE Distribution Analysis**



**Fig 10.** RMSE

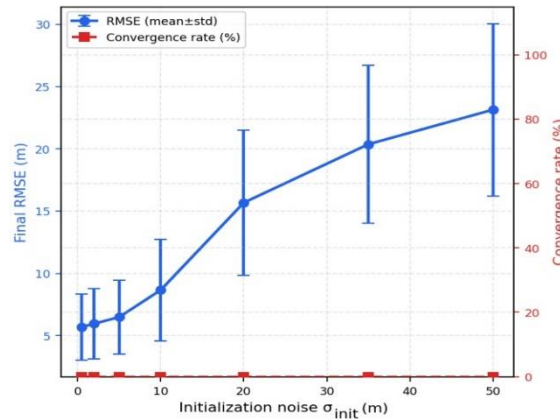
Figure 10 shows boxplots of RMSE. CAEM has the lowest median error and spread, among other techniques. This means that the method is not only consistent across trials but also accurate. On the contrary, the baseline methods are more diverse and hence less predictable in their response to changing conditions.

**Convergence Sensitivity to Initialization Quality**



**Fig. 11.** Convergence trajectories vs initialization quality

To assess the sensitivity of the proposed CAEM framework to the quality of the initializations, additional experiments were conducted by adding Gaussian noise with standard deviations (SD) ranging from 0.5 m to 50 m to the initial sensor and reflector coordinates. It aimed to determine the impact of the quality of the starting point on the convergence behaviour and the final accuracy of localisation using the L-BFGS-B optimiser. The convergence trajectories and localization errors obtained under various initial conditions are shown in Figures 11 & 12. The results show that the optimisation procedure remains stable even in the presence of moderate initialisation errors. Specifically, if the initiation noise is less than 10m, the final RMSE is still less than 9m, and there is slight degradation.



**Fig 12.** Localization error & convergence rate vs init quality

But the localization error decreases slowly as the initialization error exceeds 20 m. The RMSEs range from about 5.70 m for 0.5 m initialization noise to 23.14 m for 50 m initialization noise. This is natural, since there is a greater chance of reaching a sub-optimal local minimum when the perturbations are large.

Although the global estimation problem (sensor-reflector) is not convex, the objective function decreases monotonically throughout the optimisation process for all initialisations. Moreover, the convergence paths remain smooth and stable, indicating

*Hemarjit Ningombam et al.*

that the proposed framework is not overly sensitive to the quality of initialization. The results indicate that the CAEM shows some degree of robustness in the presence of poor initial conditions, but also gets to enjoy better initial conditions when they are available.

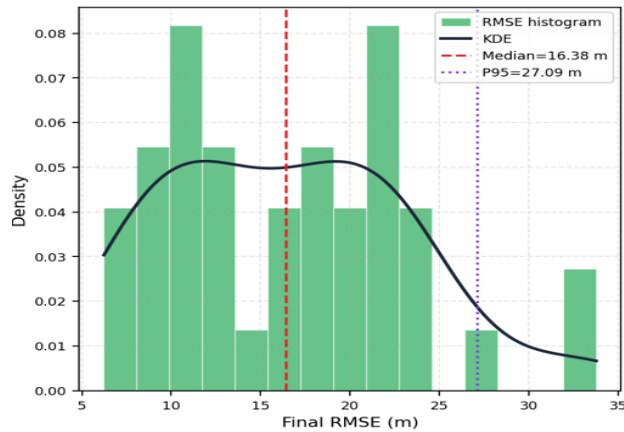
**Table 1: Effect of Initialization Quality on Localization Accuracy**

Initialization Noise $\sigma_{init}$ (m)	RMSE (m)
0.5	5.70
2	5.96
5	6.49
10	8.66
20	15.68
35	20.37
50	23.14

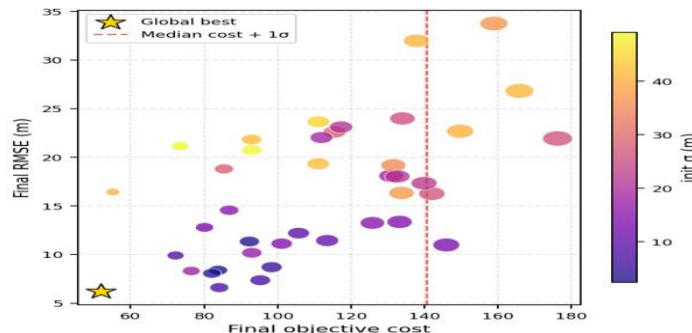
**Local-Minima Investigation**

The localization approach proposed in the CAEM framework is a joint localization of sensor and reflector positions based on a non-linear objective function that combines acoustic echo residuals and radio-ranging constraints. The objective is a multi-coupled, non-linear term, and the optimisation problem is non-convex.

The L-BFGS-B Optimiser was then employed with 40 random initializations to explore the optimization landscape. The final objective value and the localization RMSE were measured for each restart. The optimization statistics are summarised in Figures 13 & 14.



**Fig 13.** MRSE distribution across restart



**Fig 14.** Local minima landscape: cost vs RSME per restart

The results of the experiments indicate that the final objective values range from about 51.86 to 176.28, and the localisation RMSE values range from 6.24 m to 33.76 m. The need for initialisation in the final solution suggests distinct attraction regions in the optimisation landscape, consistent with the presence of local minima. The distributions of final objective values and localisation errors indicate that most optimisation runs are not evenly distributed across the search space but instead tend to converge toward a few low-cost regions. The median localization error is around 16.38 m, and the 95th percentile is less than 27.09 m.

Additionally, there is a clear positive correlation between the localisation RMSE and the final objective value in the scatter plot of the objective value vs. localisation RMSE. Solutions with lower objective values tend to achieve better localisation accuracy, indicating that the optimisation objective remains aligned with the localisation task. The results indicate that the optimisation landscape is nonconvex and sensitive to the initial point, but the optimiser can always reach feasible solutions at low cost. The observed variability indicates the need for better initialization strategies in future implementations, as well as possibly multi-start optimization.

### **Optimization Stability under Reflector Assignment Errors**

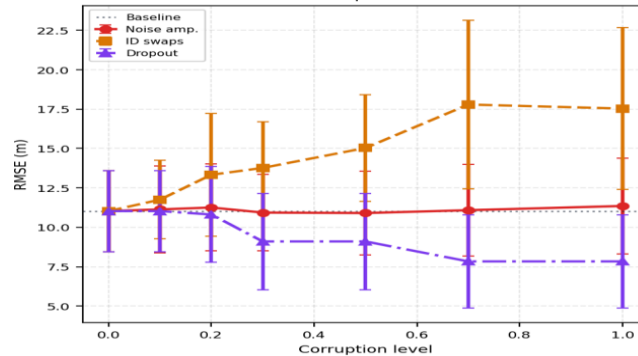
The proposed CAEM framework associates reflectors via the reflector network so that acoustic echoes from the same physical reflector are clustered together. To assess the stability of optimization in practical indoor environments where some observations can be noisy and reflector identification is not accurate, an additional robustness study was performed under corrupted reflector assignments. Three mechanisms for corruption were considered, which are as follows:

1. Noise Amplification: An extra source of noise was added to acoustic measurements.
2. Reflector-ID Swapping: A portion of the echoes was deliberately assigned incorrect reflector IDs.
3. Echo Dropout: Some acoustic observations were randomly removed.

The corruption level was incrementally ramped from 0% to 100%, and localisation performance was measured at each level. The results are summarised in Figure 15. The results demonstrate that the proposed framework is very robust to measurement noise. The RMSE is only slightly affected from 11.03 m at zero corruption to 11.35 m at the highest noise amplification level. In the same way, echo dropout only has a moderate

effect on localisation accuracy, with RMSE below 11.03 m at moderate dropout rates, and even shows a slight improvement when some measurements are dropped due to inconsistency.

However, reflector-ID swapping has a greater impact on localization performance. The RMSE value increases with increasing corruption, rising from 11.03 m to 17.53 m. This behaviour is natural, as incorrect reflector assignments introduce geometric constraints into the optimisation objective, increasing the risk of converging to suboptimal solutions.



**Fig. 15.** RSME degradation vs corruption level

Nevertheless, the optimisation process is not affected by this degradation and remains stable throughout the experiment. There are no abrupt deviations or catastrophic failures, and the localisation error grows linearly with the degree of corruption. Moreover, the objective function remains finite, and the optimization trajectories still tend to the feasible solutions.

The results in this paper illustrate the feasibility of the proposed CAEM framework in terms of robustness to measurement noise and partial observation loss, and that the quality of association with reflectors remains the most significant factor in localization accuracy. This finding underlines the need for robust echo-clustering algorithms and encourages further research into adaptive reflector-association methods for more complex indoor environments.

**Table 2.** Effect of Reflector Assignment Corruption on Localization Accuracy

Corruption Level	Noise Amplification RMSE (m)	ID Swapping RMSE (m)	Echo Dropout RMSE (m)
0.0	11.03	11.03	11.03
0.1	11.13	11.75	11.03
0.2	11.25	13.33	10.81
0.3	10.93	13.77	9.10
0.5	10.90	15.03	9.10
0.7	11.08	17.79	7.83
1.0	11.35	17.53	7.83

### Critical Analysis

Although the benefits of CAEM in terms of accuracy and robustness are clear, there are still challenges that need to be addressed before it can be put into practice. The most important issue is the reflector association - the stage at which we cluster the measurements of echoes having the same physical surface. When we misclassify the

*Hemarjit Ningombam et al.*

echoes at this point, the optimisation process can yield incorrect constraints and, consequently, poor performance. Computational complexity is also another challenge. The joint optimization method (simultaneously estimating the sensor and reflector positions) becomes increasingly costly as the network size increases. L-BFGS-B makes it efficient, though when using a larger network, real-time operation may require more optimization. We also assume the environment is in a fixed state, but this is not necessarily true in reality. The fact that the reflector is moving or changing position over time could potentially compromise measurement consistency and harm accuracy. The management of dynamic environments is a futuristic direction of work.

Another factor to consider is the hardware. The inclusion of ultrasonic sensing will increase hardware complexity and energy consumption, which may be an issue for ultra-low-power IoT devices. The trade-off between performance, cost, and power will have to be carefully designed into the system and the hardware. Despite the difficulties posed by the issues we have presented, the CAEM framework is an important step toward indoor localization. The fundamental concept of exploiting multipath reflections as beneficial data rather than noise, and the integration of acoustic and radio sensing into a robust optimisation model, provide an interesting direction to pursue. It is a scalable solution with potential for next-generation wireless sensor networks and IoT systems. No surprise that local minima exist, as the optimization of reflector and sensor coordinates is done simultaneously, resulting in a high-dimensional, nonlinear estimation problem. However, the relative density of final costs indicates that the optimisation landscape is neither overly complex nor highly localised, lending credence to the practical use of the proposed CAEM approach.

## **V. Conclusion**

Our proposed hybrid localization model in this research was Collaborative Acoustic Echo Mapping (CAEM), which aimed to achieve accurate sensor localization in wireless networks. We wanted to overcome common challenges in indoor settings, such as noise, multipath effects, and the sparse distribution of anchors, which are likely to degrade localisation performance. The findings indicate that CAEM is more effective than conventional approaches across all metrics we analyzed. To be more precise, we achieved substantial improvements in RMSE, MAE, and P95 error, with increases up to 3 times those of conventional methodologies such as RSSI and DV-Hop. The findings support the conclusion that combining acoustic echo measurements with radio ranging would improve space visualization and increase overall strength. One contribution in this direction is the joint-optimisation framework, which estimates sensor and reflector positions simultaneously, effectively converting environmental reflections into geometry constraints that can be used productively. This is one of the first attempts to treat acoustic echoes as virtual anchors within a single optimization model. The method demonstrates that multipath, which most people consider an inconvenience, can be leveraged to take advantage of to enhance localization. Beyond this, the Huber loss function makes the framework robust to noise and outliers, which is important for deployment in the real world.

At the theoretical level, this work reconsiders how environmental features can be incorporated into localisation, shifting the emphasis from multipath elimination to using this phenomenon as a source of information. In practice, CAEM reduces the cost

*Hemarjit Ningombam et al.*

of physical anchors, thereby lowering deployment costs and simplifying system scaling in large-scale IoT. The findings hint at smart buildings, industrial automation, and indoor navigation where GPS is unavailable. This study is limited in spite of its good performance. This framework presumes a fixed environment and assumes that the association of reflectors is correct, which is difficult in a moving or disordered environment. Also, joint optimization adds computational complexity, which can pose a problem for real-time implementation in a large network. However, these constraints are also justified by the problem's complexity and the need to achieve substantial improvements in localization accuracy and robustness. Theoretical analysis also shows that the proposed joint estimation of sensors and reflectors is locally identifiable if the measurement graph is connected, there are sufficient anchor observations, and the reflector observations provide independent geometric constraints. Additionally, theoretical analysis shows that, with adequate anchor constraints and acoustic and radio constraints, the proposed CAEM framework is observable and locally identifiable, with the ambiguity problems caused by sparse-anchor and symmetric-reflector configurations successfully addressed by multi-modal sensor fusion.

Future work will aim to extend CAEM to dynamic and 3D setups, as well as to develop adaptive reflector-association mechanisms that enhance robustness in real-world scenarios. Distributed optimization and the investigation of machine learning-based methods for parameter tuning and prediction are also part of the strategy to reduce computational complexity. It is hoped that these instructions will add value and make the framework more applicable and efficient for the next-generation wireless sensor networks.

#### **Conflict of Interest:**

The authors have no conflict of Interest regarding this article.

#### **References:**

- I. Abdel-Basset, M., Chang, V., Hawash, H., Chakraborty, R. K., & Ryan, M. J. (2024). Deep learning approaches for human-centered IoT applications in smart indoor environments: A contemporary survey. *Annals of Operations Research*, 339(1–2), 3–51. [10.1007/s10479-021-04164-3](https://doi.org/10.1007/s10479-021-04164-3)
- II. Abdulhussein Abdulzahra, S., & Al-Qurabat, K. M. (2024). Exploring radio frequency-based UAV localization techniques: A comprehensive review. *International Journal of Computing and Digital Systems*, 15(1), 1565–1581. [10.12785/ijcds/1501111](https://doi.org/10.12785/ijcds/1501111)
- III. Adnan, M., Razzaque, M., Ahmed, I., & Isnin, I. (2013). Bio-mimic optimization strategies in wireless sensor networks: A survey. *Sensors*, 14(1), 299–345. [10.3390/s140100299](https://doi.org/10.3390/s140100299)
- IV. Ahmad, N. S. (2024). Recent advances in WSN-based indoor localization: A systematic review of emerging technologies, methods, challenges, and trends. *IEEE Access*, 12, 180674–180714. [10.1109/ACCESS.2024.3509516](https://doi.org/10.1109/ACCESS.2024.3509516)
- V. Alameda-Pineda, X., Staiano, J., Subramanian, R., Batrinca, L., Ricci, E., Lepri, B., Lanz, O., & Sebe, N. (2016). SALSA: A novel dataset for multi-modal group behavior analysis. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 38(8), 1707–1720. [10.1109/TPAMI.2015.2496269](https://doi.org/10.1109/TPAMI.2015.2496269)

*Hemarjit Ningombam et al.*

- VI. Ali, J., Kaemarungsi, K., Phakaew, T., Uzair, M., Narbudowicz, A., & Chalermwisutkul, S. (2024). Low-cost indoor localization using dual-chip RFID tag. *IEEE Open Journal of Antennas and Propagation*, 5, 1209–1220. 10.1109/OJAP.2024.3372030
- VII. Ali, R., Liu, R., He, Y., Nayyar, A., & Qureshi, B. (2021). Multi-object identification and localization review. *IEEE Access*, 9, 122924–122950. 10.1109/ACCESS.2021.3108775
- VIII. Aman, M., Li, F., Khan, Z., Khan, F., Khan, J., & Khan, S. (2025). Modern localization in UWSNs with machine learning integration. *Scientific Reports*, 15(1). 10.1038/s41598-025-89916-y
- IX. Amjad, B., Ahmed, Q. Z., Lazaridis, P. I., Hafeez, M., Khan, F. A., & Zaharis, Z. D. (2023). Radio SLAM: A review on radio-based simultaneous localization and mapping. *IEEE Access*, 11, 9260–9278. 10.1109/ACCESS.2023.3237330
- X. Andò, B., Baglio, S., Crispino, R., & Marletta, V. (2021). Indoor localization techniques: Multi-trilateration-based system. *Applied Sciences*, 11(16), 7392. 10.3390/app11167392
- XI. Bibbò, L., Carotenuto, R., & Della Corte, F. (2022). An Overview of Indoor Localization System for Human Activity Recognition (HAR) in Healthcare. *Sensors*, 22(21), 8119. 10.3390/s22218119
- XII. Cha, K., Lee, J., Özger, M., & Lee, W. (2023). When wireless localization meets artificial intelligence: Basics, challenges, synergies, and prospects. *Applied Sciences*, 13(23), 12734. 10.3390/app132312734
- XIII. Chen, X., & Zhang, B. (2012). Improved DV-Hop Node Localization Algorithm in Wireless Sensor Networks. *International Journal of Distributed Sensor Networks*. 10.1155/2012/213980
- XIV. Cui, H., Wang, S., & Zhou, C. (2023). A high-accuracy and low-energy range-free localization algorithm for wireless sensor networks. *EURASIP Journal on Wireless Communications and Networking*, 2023, Article 37. 10.1186/s13638-023-02246-3
- XV. Cui, J., Shang, Y., Yu, S., & Wang, Y. (2024). Research on intelligent wheelchair multimode human–computer interaction and assisted driving technology. *Actuators*, 13(6), Article 230. <https://doi.org/10.3390/act13060230>
- XVI. Dagher, R., & Quilez, R. (2014). Localization in wireless sensor networks. In *Wireless Sensor Networks (Chapter 9)*. 10.1142/9789814551342\_0009
- XVII. Dai, J., Wang, M., Wu, B., Shen, J., & Wang, X. (2023). Survey of Wi-Fi assisted indoor positioning techniques. *Sensors*, 23(18), 7961. 10.3390/s23187961
- XVIII. Das, S., Malik, P., & Pandey, A. (2025). Advances in underwater sonar systems and AI-driven signal processing. *Journal of Field Robotics*, 43(2), 899–931. 10.1002/rob.70077
- XIX. De Cock, C., Joseph, W., Martens, L., Trogh, J., & Plets, D. (2021). Multi-Floor Indoor Pedestrian Dead Reckoning with a Backtracking Particle Filter and Viterbi-Based Floor Number Detection. *Sensors*, 21(13), 4565. 10.3390/s21134565
- XX. Deshmukh, R. A., Hasamnis, M. A., Kulkarni, M. B., & Bhaiyya, M. (2025). Advancing indoor positioning systems: innovations, challenges, and applications in mobile robotics. *Robotica*, 43(7), 2710–2750. 10.1017/S0263574725101872

- XXI. Ejaz, M., Bakar, K., Fauzi, I., Isyaku, B., Abdalla, T., Abdelmaboud, A., & Abbas, A. (2024). Elevating Localization Accuracy in Wireless Sensor Networks: A Refined DV-Hop Approach. *Computers, Materials & Continua*, 81(1), 1511–1528. 10.32604/cmc.2024.054938
- XXII. Elsanhoury, M., Mäkelä, P., Koljonen, J., Välisuo, P., Shamsuzzoha, A., Mantere, T., Elmusrati, M., & Kuusniemi, H. (2022). Precision positioning for smart logistics using ultra-wideband technology-based indoor navigation: A review. *IEEE Access*, 10, 44413–44445. 10.1109/ACCESS.2022.3169267
- XXIII. Farid, Z., Nordin, R., & Ismail, M. (2013). Recent advances in wireless indoor localization techniques and systems. *Journal of Computer Networks and Communications*, 2013, 1–12. 10.1155/2013/185138
- XXIV. Fawad, M., Khan, M. Z., Ullah, K., Alasmay, H., Shehzad, D., & Khan, B. (2023). Enhancing Localization Efficiency and Accuracy in Wireless Sensor Networks. *Sensors*, 23(5), 2796. 10.3390/s23052796
- XXV. Fen, L., Liu, J., Yin, Y., Wang, W., Dong-hai, H., Chen, P., et al. (2020). Survey on WiFi-based indoor positioning techniques. *IET Communications*, 14(9), 1372–1383. 10.1049/iet-com.2019.1059
- XXVI. Fischer, G., Bordoy, J., Schott, D., Xiong, W., Gabbrielli, A., Höflinger, F., et al. (2022). Multi-modal indoor localization using ultrasonic and BLE data. *IEEE Sensors Journal*, 22(6), 5857–5868. 10.1109/JSEN.2022.3148529
- XXVII. Gabela, J., et al. (2019). UWB-based cooperative positioning system evaluation. *Sensors*, 19(23), 5274. 10.3390/s19235274
- XXVIII. Geok, T., Aung, K., Aung, M., Min, T., Abdaziz, A., Liew, C., et al. (2020). Review of indoor positioning using radio wave technology. *Applied Sciences*, 11(1), 279. 10.3390/app11010279
- XXIX. Gupta, A., & Singh, U. (2023). Performance evaluation of sensor node localization. *Wireless Personal Communications*, 131(2), 941–954. 10.1007/s11277-023-10462-9
- XXX. Hadir, A., Kaabouch, N., El Houssaini, M.-A., & El Kafi, J. (2023). Range-free localization approaches based on intelligent swarm optimization for Internet of Things. *Information*, 14(11), 592. 10.3390/info14110592
- XXXI. Hailu, T., Guo, X., & Si, H. (2025). Indoor positioning systems as critical infrastructure: An assessment for enhanced location-based services. *Sensors*, 25(16), 4914. <https://doi.org/10.3390/s25164914>
- XXXII. Han, F., Mohamed, I., Liu, X., Ghazali, K., & Wang, H. (2020). Hybrid range-free localization using dynamic communication range. *International Journal of Online and Biomedical Engineering*, 16(8), 4–24. 10.3991/ijoe.v16i08.14379
- XXXIII. He, L., & Kang, Z. (2015). Weighted APIT localization algorithm based on vector similarity. *Proceedings of the International Conference on Computer Science and Intelligent Communication*. <https://doi.org/10.2991/csic-15.2015.3>
- XXXIV. Heshmat, M., Saoud, L., Abujabal, M., Sultan, A., Elmezain, M., Seneviratne, L., et al. (2025). Underwater SLAM with deep learning: Challenges and future directions. *Sensors*, 25(11), 3258. 10.3390/s25113258
- XXXV. Huang, J., Wang, Y., Zou, Y., Wu, K., & Ni, L. (2023). Ubiquitous WiFi and acoustic sensing: Principles and applications. *Journal of Computer Science and Technology*, 38(1), 25–63. 10.1007/s11390-023-3073-5

- XXXVI. Huang, P., Chen, J., Larosa, Y., & Chiang, T. (2011). Distributed Fermat-point localization for wireless sensor networks. *Sensors*, 11(4), 4358–4371. 10.3390/s110404358
- XXXVII. Jiao, J. (2021). Mobile industrial robots localization using RSSI and multidimensional scaling. *Advances in Multimedia*, 2021, 1–9. 10.1155/2021/2437224
- XXXVIII. Józefczak, A. (2012). Chronicle: Open seminar on acoustics. *Archives of Acoustics*, 37(3), 373–393. 10.2478/v10168-012-0047-y
- XXXIX. Kamal, Y., Sharma, K., & Vandana, V. (2019). H-best particle swarm optimization-based localization algorithm for wireless sensor networks. *International Journal of Engineering and Advanced Technology*, 9(1), 2769–2778. 10.35940/ijeat.a9776.109119
- XL. Kanhere, O., & Rappaport, T. S. (2018). Position locationing for millimeter wave systems. In *Proceedings of the 2018 IEEE Global Communications Conference (GLOBECOM)* (pp. 206–212). IEEE. 10.1109/GLOCOM.2018.8647983
- XLI. Kapoor, R., Ramasamy, S., Gardi, A., Schyndel, R., & Sabatini, R. (2018). Acoustic sensors for navigation applications. *Sensors*, 18(2), 499. 10.3390/s18020499
- XLII. Kaur, T., Singh, J., & Singh, M. (2025). Optimization-driven localization in wireless sensor networks: A comprehensive review. *International Journal of Communication Systems*, 38(14). 10.1002/dac.70213
- XLIII. Khan, F., Awais, M., Rasheed, M., Masood, B., & Ghadi, Y. (2021). Comparison of wireless standards in IoT for indoor localization. *IEEE Access*, 9, 65925–65933. 1109/ACCESS.2021.3076371
- XLIV. Khider, M., Kaiser, S., & Robertson, P. (2012). A novel three-dimensional movement model for pedestrian navigation. *The Journal of Navigation*, 65(2), 245–264. 10.1017/S0373463311000713
- XLV. Le, N., & Jang, Y. (2019). Photography trilateration indoor localization with image sensor communication. *Sensors*, 19(15), 3290. 10.3390/s19153290
- XLVI. Li, X., Chen, L., Wang, J., Chu, Z., Li, Q., & Sun, W. (2015). Fuzzy system and improved APIT-based range-free localization method. *KSII Transactions on Internet and Information Systems*, 9(7). 10.3837/tiis.2015.07.005
- XLVII. Lin, A., Jiang, T., Kanwal, J., Lu, G., Luo, J., Wei, X., et al. (2014). Echolocation vocalization variation in bats. *Oikos*, 124(3), 364–371. 10.1111/oik.01604
- XLVIII. Liouane, H., Messous, S., Cheikhrouhou, O., Baz, M., & Hamam, H. (2021). Regularized least square multi-hop localization algorithm. *IEEE Access*, 9, 136406–136418. 10.1109/ACCESS.2021.3116767
- XLIX. Liu, J., Yang, Z., Zlatanova, S., Li, S., & Yu, B. (2025). Smartphone indoor localization using multi-source sensor fusion. *Sensors*, 25(6), 1806. 10.3390/s25061806
- L. Liu, S., Chen, Y., Trappe, W., & Greenstein, L. (2009). Non-interactive localization of cognitive radios based on dynamic signal strength mapping. *Proceedings of the 6th Annual IEEE Communications Society Conference on Sensor, Mesh and Ad Hoc Communications and Networks*. 10.1109/WONS.2009.4801847

- LI. Lloret, J., Tomas, J., Garcia, M., & Canovas, A. (2009). A Hybrid Stochastic Approach for Self-Location of Wireless Sensors in Indoor Environments. *Sensors*, 9(5), 3695–3712. 10.3390/s90503695
- LII. Madhumathi, K., Suresh, T., & Maruti, R. (2020). Node localization using Naive Bayes and trilateration. *International Journal of Engineering and Advanced Technology*, 9(3), 2467–2470. 10.35940/ijeat.c5604.029320
- LIII. Maghdid, S., & Maghdid, H. (2021). Comprehensive review of indoor/outdoor localization solutions in IoT. *TechRxiv*. 10.36227/techrxiv.15138609
- LIV. Majeed, A., Gulzari, M., Mu'azzah, N., & Abbas, A. (2024). Indoor positioning based on UWB and Li-Fi technologies: A review. *Elektrika Journal of Electrical Engineering*, 23(1), 18–30. 10.11113/elektrika.v23n1.431
- LV. Martin-Escalona, I., & Zola, E. (2020). Passive Round-Trip-Time Positioning in Dense IEEE 802.11 Networks. *Electronics*, 9(8), 1193. 10.3390/electronics9081193
- LVI. Mass-Sanchez, J., et al. (2018). Factorial design analysis for localization algorithms. *Applied Sciences*, 8(12), 2654. 10.3390/app8122654
- LVII. Messous, S., & Liouane, H. (2019). Online Sequential DV-Hop Localization Algorithm for Wireless Sensor Networks. *Mobile Information Systems*, 2020(1), 8195309. 10.1155/2020/8195309
- LVIII. Misra, P., Kottege, N., Kusý, B., Ostry, D., & Jha, S. (2013). Acoustical ranging techniques in wireless sensor networks. *ACM Transactions on Sensor Networks*, 10(1), 1–38. 10.1145/2529981
- LIX. Moradbeikie, A., et al. (2021). GNSS-free outdoor localization techniques for IoT. *Applied Sciences*, 11(22), 10793. 10.3390/app112210793
- LX. Nessa, A., Adhikari, B., Hussain, F., & Fernando, X. (2022). A survey of machine learning for indoor positioning. 10.32920/21476622
- LXI. Prorok, A., & Martinoli, A. (2014). Accurate indoor localization with ultra-wideband using spatial models and collaboration. *The International Journal of Robotics Research*, 33(4), 547–568. 10.1177/0278364913500301
- LXII. Qi, L., Liu, Y., Yu, Y., Chen, L., & Chen, R. (2024). Current status and future trends of meter-level indoor positioning technology. *Remote Sensing*, 16(2), 398. 10.3390/rs16020398
- LXIII. Qi, X., Liu, X., & Liu, L. (2018). Combined localization algorithm for wireless sensor networks. *Mathematical Problems in Engineering*, 2018, Article ID 4648109. 10.1155/2018/4648109
- LXIV. Qiao, D., & Pang, G. (2014). Evolutionary approach on connectivity-based sensor network localization. *Applied Soft Computing*, 22, 36–46. 10.1016/j.asoc.2014.04.019
- LXV. Rathnayake, R., Maduranga, M., Tilwari, V., & Dissanayake, M. (2023). RSSI and machine learning-based indoor localization systems for smart cities. *Engineering Advances*, 4(2), 1468–1494. 10.3390/eng4020085
- LXVI. Retscher, G., & Leb, A. (2021). Smartphone-based indoor navigation using Wi-Fi fingerprinting. *Sensors*, 21(2), 432. 10.3390/s21020432
- LXVII. Rizk, H., Elmogy, A., & Yamaguchi, H. (2022). A Robust and Accurate Indoor Localization Using Learning-Based Fusion of Wi-Fi RTT and RSSI. *Sensors*, 22(7), 2700. 10.3390/s22072700

- LXVIII. Savić, T., Vilajosana, X., & Watteyne, T. (2022). Constrained localization: A survey. *IEEE Access*, 10, 49297–49321. 10.1109/ACCESS.2022.3171859
- LXIX. Shan, Z., & Yum, T. (2007). Precise localization with smart antennas in ad-hoc networks. In *Proceedings of IEEE GLOBECOM 2007*. 10.1109/GLOCOM.2007.203
- LXX. Shang, J., Hu, X., Gu, F., Wang, D., & Yu, S. (2015). Improvement schemes for indoor mobile location estimation: A survey. *Mathematical Problems in Engineering*, 2015, Article ID 397298. 10.1155/2015/397298
- LXXI. Simões, W., Machado, G., Sales, A., Lucena, M., Jazdi, N., & Lucena, V. (2020). Indoor navigation systems for visually impaired: A review. *Sensors*, 20(14), 3935. 10.3390/s20143935
- LXXII. Singh, J., Farnham, T., & Wang, Q. (2023). When BLE meets light: Multi-modal fusion for enhanced indoor localization. In *Proceedings of the 29th Annual International Conference on Mobile Computing and Networking (MobiCom '23)*, Article 139, 1–3. Association for Computing Machinery. 10.1145/3570361.3615746
- LXXIII. Singh, S., & Sharma, R. M. (2016). Optimization techniques in wireless sensor networks. In *Proceedings of the Second International Conference on Information and Communication Technology for Competitive Strategies (ICTCS '16)*, Article 140, 1–7. Association for Computing Machinery. 10.1145/2905055.2905200
- LXXIV. Smoleń, M., & Augustyniak, P. (2020). Assisted Living System with Adaptive Sensor's Contribution. *Sensors*, 20(18), 5278. 10.3390/s20185278
- LXXV. Soundari, D., & Poongodi, C. (2024). Hybrid bat–sand cat swarm optimization-based node localization. *International Journal of Communication Systems*, 38(2). 10.1002/dac.5961
- LXXVI. Stone, K., & Camp, T. (2012). A survey of distance-based wireless sensor network localization techniques. *International Journal of Pervasive Computing and Communications*, 8(2), 158–183. 10.1108/17427371211245373
- LXXVII. Sun, Y., Finnerty, P., & Ohta, C. (2024). BLE-based outdoor localization with optimization algorithms. *IEEE Access*, 12, 45164–45175. 10.1109/ACCESS.2024.3380897
- LXXVIII. Swain, M., et al. (2021). LoRa-based link budget optimization for IoT. *Agronomy*, 11(5), 820. 10.3390/agronomy11050820
- LXXIX. Trinh, Q., Golio, N., Cheng, Y., Cha, H., Tai, K., Ouyang, L., et al. (2025). Sonochemistry and sonocatalysis: Current progress and future opportunities. *Green Chemistry*, 27(18), 4926–4958. 10.1039/D5GC01098E
- LXXX. Wei, X., Wei, Z., & Radu, V. (2021). Sensor-Fusion for Smartphone Location Tracking Using Hybrid Multi-modal Deep Neural Networks. *Sensors*, 21(22), 7488. 10.3390/s21227488
- LXXXI. Wei, X., Wei, Z., & Radu, V. (2021). MM-Loc: Cross-sensor indoor smartphone location tracking using multi-modal deep neural networks. In *2021 International Conference on Indoor Positioning and Indoor Navigation (IPIN)* (pp. 1–8). IEEE. 10.1109/IPIN51156.2021.9662519
- LXXXII. Wong, A., Whitehill, T., Ma, E., & Masters, R. (2012). Effects of errorless learning on velopharyngeal movement control. *Journal of the Acoustical Society of America*, 131(4), 3273. 10.1121/1.4708235

- LXXXIII. Woock, P., & Frey, C. (2010). Deep-sea AUV navigation using sonar-based SLAM. In Proceedings of OCEANS 2010 Sydney. 10.1109/OCEANSSYD.2010.5603528
- LXXXIV. Xiong, J., Sundaresan, K., & Jamieson, K. (2015). ToneTrack: Leveraging Frequency-Agile Radios for Time-Based Indoor Wireless Localization. In Proceedings of the 21st Annual International Conference on Mobile Computing and Networking (MobiCom '15) (pp. 537–549). Association for Computing Machinery. 10.1145/2789168.2790125
- LXXXV. Xu, X., Tang, Y., & Li, S. (2017). Indoor localization based on hybrid Wi-Fi hotspots. In 2017 International Conference on Indoor Positioning and Indoor Navigation (IPIN). 10.1109/IPIN.2017.8115924
- LXXXVI. Yadav, P., & Sharma, S. (2022). A systematic review of localization in WSN: Machine learning and optimization-based approaches. *International Journal of Communication Systems*, 36(4). 10.1002/dac.5397
- LXXXVII. Yang, F., Song, W., Zhang, C., Fang, H., Min, C., & Yuan, X. (2021). Phase-shifted surface plasmon resonance sensor for photoacoustic imaging. *ACS Sensors*, 6(5), 1840–1848. 10.1021/acssensors.1c00029
- LXXXVIII. Yang, T., Cabani, A., & Chafouk, H. (2021). A Survey of Recent Indoor Localization Scenarios and Methodologies. *Sensors*, 21(23), 8086. 10.3390/s21238086
- LXXXIX. Zhang, X., Yao, X., Hui, L., Song, F., & Hu, F. (2021). A Bibliometric Narrative Review on Modern Navigation Aids for People with Visual Impairment. *Sustainability*, 13(16), 8795. 10.3390/su13168795
- XC. Zhao, H., Shi, H., & Zhao, Y. (2013). Range-free localization algorithm in wireless sensor networks with asymmetric links. *Applied Mechanics and Materials*, 446–447, 1591–1595. 10.4028/www.scientific.net/AMM.446-447.1591
- XCI. Zhao, Y., & Smith, J. (2013). Battery-free RFID-based indoor acoustic localization platform. In IEEE International Conference on RFID 2013. 10.1109/RFID.2013.6548143
- XCII. Zheng, J., Chen, L., & Xia, X. (2016). Localization algorithm based on improved DV-Hop. In Proceedings of the International Conference on Electronics, Network and Computer Engineering (ICENCE 2016). 10.2991/icence-16.2016.83