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STRUCTURAL RELIABILITY ANALYSIS OF STEEL ELEMENTS WITH INTERVAL UNCERTAINTY OF DATA

Sergey Solovev[1], Evgeniy Ilichev[2], Anastasia Soloveva[3]

[1] Department of Industrial and Civil Construction, Vologda State University, Vologda 160000, Russia.

[2] Department of Civil Engineering Institute, Vologda State University, Vologda 160000, Russia.

[3] Department of Industrial and Civil Construction, Vologda State University, Vologda 160000, Russia.

Email: : [1] solovevsa@vogu35.ru, [2] ilichevea@vogu35.ru, [3] solovevaaa@vogu35.ru

Corresponding Author: **Sergey Solovev**

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Abstract

The paper presents an approach for structural reliability analysis of steel elements in cases of interval uncertainty of sample data. It is shown that epistemic uncertainty plays an important role in practical structural reliability analysis tasks, and this uncertainty must be effectively modeled, usually in the form of intervals. The p-box (probability box) is a suitable mathematical model for describing the strength of steel as a random variable during non-destructive testing of existing structures. An effective method of reliability analysis is the Interval Monte Carlo Simulation (IMCS) in the presence of random variables with mixed interval uncertainty. As a result, structural reliability will be expressed as a failure probability interval. If the range of failure probabilities turns out to be too wide or uninformative for decision-making, it is necessary to reduce epistemic uncertainty (collect additional data to narrow the intervals) or increase the area of the cross-sections for the structural elements.

Keywords: Structural Reliability, Failure Probability, Interval Uncertainty, Steel Structures, Safety, Reliability Index.

I. Introduction

I.i. Probabilistic methods in structural reliability analysis

Reliability is an indicator of the quality of a building or structure, reflecting its ability to perform its assigned functions during the estimated period of operation. There are various structural reliability measures: failure probability, failure rate, average operating time per failure, etc. As a rule, in civil engineering practice, the failure probability is used as a measure of the reliability of a structural object or its element.

Currently, the structural reliability is ensured through the use of reliability coefficients (or partial factors by Eurocode 0) based on the limit state method, introduced in 1955. This method of assessing the reliability of a structural object is called semi-probabilistic: the provisions of probability theory are used at certain stages of the design (for example, substantiation of the normative strength (characteristic value) of the material), followed by the introduction of reliability coefficients. Following such analysis, a structural system is categorized as either "reliable" or "unreliable." However, evaluating more reliable technical solutions under equivalent conditions often poses a challenge for engineers, especially when analyzing a structure as an integrated system or when comparing designs utilizing different materials. Implementing a single reliability metric, namely the probability of failure, will enable an effective techno-economic comparison of structural solutions for construction projects.

In paper [I], it is correctly noted that generally, structures are unique products, each one with distinctive characteristics. Hence, they are not like other technological products (e.g., machines), where a high grade of industrialization and QC & QA (quality control and quality assurance) policies could be applied. Thus, for the probabilistic analysis of a structural object, it is necessary to collect statistical data individually. And the amount of such data may not be enough to use classical probabilistic analysis. Therefore, new non-classical methods of reliability analysis (based on fuzzy set theory, non-parametric statistics, etc.) play a major role in this issue.

In studies [II, III], it is noted that in the case of accidents of steel structures, the main cause is the human factor, expressed in the appearance of a gross error in the design or design. Nevertheless, according to [III], about 25 percent of collapses occur due to insufficient maintenance of facilities and deterioration of technical condition. A survey of structures using a probabilistic reliability assessment will make it possible to effectively plan maintenance work for steel structures based on the current level of reliability.

I.ii. The problem of interval data in structural reliability analysis

The monograph [IV] presents the information for relevant uses of interval parameters in real cases:

Tolerances. Defined as the permissible interval for deviation of a parameter's numerical characteristic from its nominal (design) value. Tolerances are established for a wide range of parameters, including geometric properties of components (e.g., linear/angular dimensions, surface form/position) as well as mechanical, physical, and chemical characteristics (e.g., electrical resistance, material hardness, chemical composition).

- Interval-Valued Probabilities. This concept extends the application scope of traditional probability theory and mathematical statistics. While standard probability theory requires statistical stability (or uniformity) of relative frequencies, many real-world phenomena lack this property. For such cases, where the relative frequency of an event does not converge to a fixed value, interval-valued probability offers a solution by representing probability as a range bounded by the lower and upper limits of the relative frequency sequence over an increasing number of trials.

- Representation of numbers. In both manual and machine computing, we can, as a rule, effectively operate only with objects that have a finite description (finite constructive complexity), replacing an arbitrary real number with some approximation of it having a finite number of characters in the number system used. For example: 2/3≈0.6667. Thus, already at the beginning of the calculations, some unavoidable error is allowed — the error of representation. Another possible way to specify the error we are interested in, and even more preferable, is to give the user the narrowest possible boundaries, the lower and upper ones: 2/3 ∈ [0.6666, 0.6667]. Rounding errors can also be attributed here;

- Uncertainty in Physical Laws. Inherent limitations and ambiguities are present in the mathematical modeling of mechanical, physical, and chemical phenomena. These uncertainties arise from the natural complexity and idealizations involved in representing real-world behaviors through analytical models.

It is known from probability theory and mathematical statistics that the mathematical expectation and standard deviation of a random variable based on its sample statistical data can be estimated as a confidence interval. Let's assume that the strength reserve function for a steel element is a random variable, the mathematical expectation and standard deviation of which are estimated based on the test results of control samples. In the MathCAD software, we will simulate random numbers with a normal distribution with the parameters: m_g = 5 MPa, S_g = 2 MPa, and we will estimate confidence intervals for the reliability index β at every step of *n*.

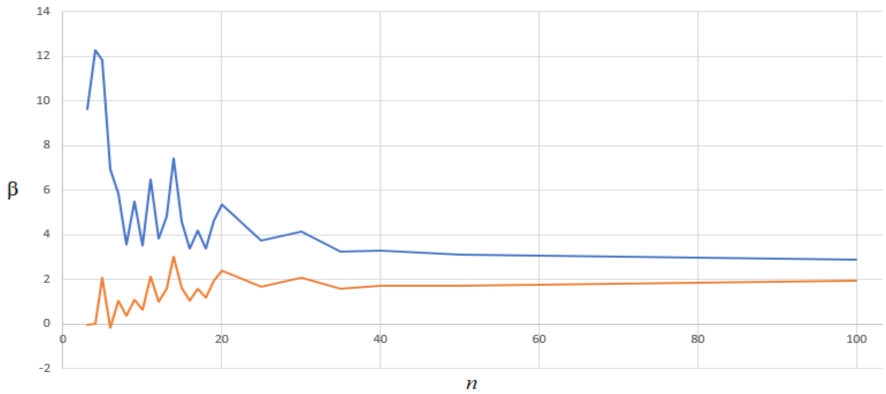


Fig. 1. Graph of changes in the confidence limits (blue line – the upper bound and orange line – the lower bound) of the reliability index β with an increase in the number of statistical tests *n*

As can be seen from Fig. 1, the boundaries of the reliability index are approaching the theoretical value of β=2.5, but remain quite wide even with 100 numerical tests. This reflects the need to take into account confidence limits and interval analysis methods in practical tasks of probabilistic assessment of the reliability of building structures.

II. Materials and Methods

II.i. Interval bounds for the cumulative distribution function

Let us consider the phenomena of uncertainties in probabilistic reliability analysis on the example of the yield strength of steel estimation. As part of the numerical experiment, we will simulate 120 values of tests of control samples of steel C345 on yield strength by Monte Carlo Simulation (MCS). Proceeding from the dependence $f_{y,n}=m_y - 1.645 \cdot S_y$, where m_y is the expected value of the yield strength of steel; S_y is a std. dev. of steel yield strength. Let CV be equal to 0.05 and $f_{y,n}$=345 MPa, we can obtain the next values for the MCS approach: m_y =375.92 MPa, S_y=18.80 MPa.

The dataset comprising 120 trials was partitioned into four distinct control groups, each containing 30 entries. The allocation was performed sequentially based on the order of the data, without any prior sorting or categorization. For this study, it is assumed that these four subsets are assigned to separate analytical teams tasked with identifying the most suitable probability distribution for subsequent reliability analysis of the steel structure. The goodness-of-fit for the chosen probability distribution, with its calibrated parameters, will be evaluated using the Kolmogorov-Smirnov (K-S) test statistic, which quantifies the maximum discrepancy between the empirical cumulative distribution function and the theoretical model under consideration. The development of a random variable model necessitates the selection of both a cumulative distribution function and its corresponding statistical parameters. A widely employed automated approach for this objective utilizes the Distribution Fitter tool within the MATLAB software environment, which facilitates the estimation of optimal distribution parameters from an empirical dataset.

Tables 1-2 present the data on choosing the PDFs for each group, and Figure 1 presents the empirical probability distribution functions of the sampled data population of each control group (CG). This illustrative fitting is not used in the future reliability analysis – it only demonstrates the ambiguity of distribution selection under small samples.

Table 1: CDF's fitting for steel yield strength simulation data (CG 1 and 2).

	Control group 1		**Control group 2**	
Distribution	Parameters. MPa	K-S statistics	Parameters. MPa	K-S statistics
Normal	m_y=377.86 S_y=16.20	**0.1143**	m_y=374.095 S_y=17.870	0.1117
Lognormal	μ=5.934 σ=0.043	0.1160	μ=5.923 σ=0.047	0.1045
Logistic	m_y=377.60 S_y=9.384	0.1320	m_y=373.557 S_y=9.174	**0.0731**
Generalized extreme value distribution	k= -0.220 σ =15.293 μ =371.784	0.1238	k= -0.123 σ =16.080 μ =366.909	0.1120
Birnbaum – Saunders	β=377.522 γ=0.042	0.1235	β=373.695 γ=0.046	0.1018

Table 2: CDF's fitting for steel yield strength simulation data (CG 3 and 4).

	Control group 3		**Control group 4**	
Distribution	Parameters. MPa	K-S statistics	Parameters. MPa	K-S statistics
Normal	m_y=369.333 S_y=19.983	0.1055	m_y=374.023 S_y=19.410	**0.1190**
Lognormal	μ=5.910 σ=0.054	**0.1005**	μ=5.923 σ=0.052	0.1197
Logistic	m_y=369.474 S_y=11.376	0.1235	m_y=373.938 S_y=11.414	0.1260
Generalized extreme value distribution	*k*= -0.353 σ =20.215 μ =363.078	0.1084	*k*= -0.331 σ =19.125 μ =367.743	0.1222
Birnbaum – Saunders	β=368.805 γ=0.054	0.1020	β=373.536 γ=0.051	0.1215

Data summarized in Table 1 reveal that a normal distribution is deemed the most appropriate in 50% of the cases, which is congruent with the properties of the parent population. Nevertheless, for control group 2, the hypothesis of normality is virtually the most inadequate of the distributions under investigation. Confirmation of the logistic probability distribution for yield strength values would require adjustments to the probabilistic reliability analysis conclusions for the steel structures, particularly when interval-based parameter estimates are considered.

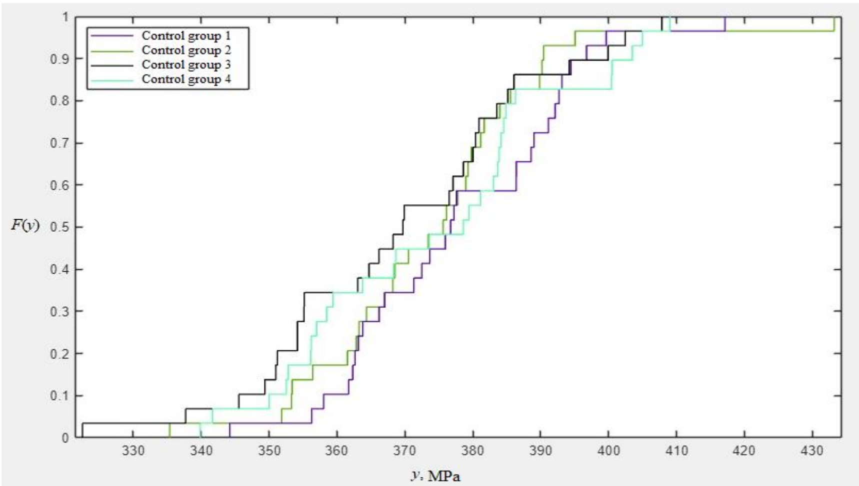


Fig. 2. Empirical CDFs of the yield strength of steel by control groups

Therefore, in the present study, we do not select any single distribution from Tables 1 and 2. Instead, the p-box (probability box) approach is adopted, which does not require a unique distribution function. The p-box directly accounts for both parameter uncertainty and model-form uncertainty. The sole purpose of this example is to motivate the need for a more robust approach when data are limited.

I.ii. Interval bounds for steel strength measurements

In the practice of inspecting existing structures, non-destructive devices, such as hardness meters, are used to determine the strength of steel. Ultrasonic hardness tester TKM-459S and tensile testing machine were used to estimate the yield and ultimate strength of steel samples in the laboratory (Fig. 3).

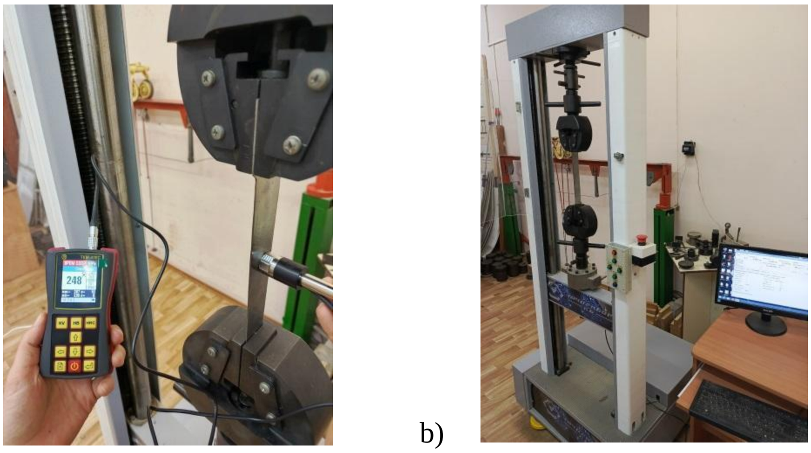
a)  b)

Fig. 3. a) Ultrasonic hardness tester TKM-459S; b) Tensile testing machine.

Based on the GOST 22761-77 "Metals and alloys. Method of measuring Brinell hardness by static action portable hardness meters", it is possible to estimate the relationship between the Brinell hardness and ultimate stress of steel (Fig. 3).

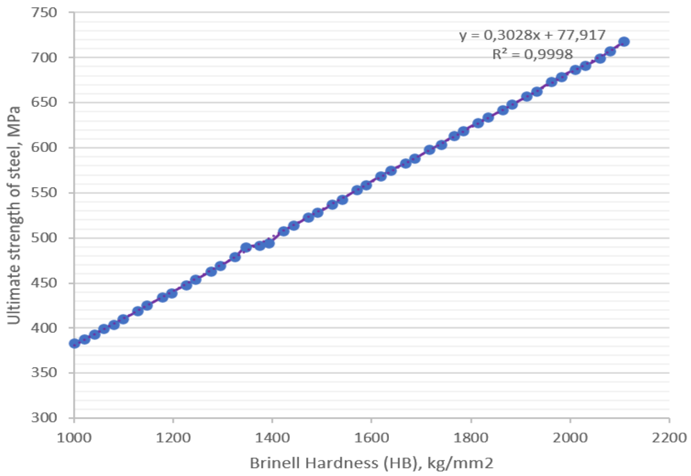


Fig. 4. Dependence between Brinell Hardness (HB) and Ultimate strength of steel.

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In analytical form, the dependence can be represented as:

$$f_u = 0.3028 \cdot \text{HB} + 77.917 \text{ MPa} \tag{1}$$

The dependence between yield strength f_y and ultimate strength f_u of steel can be presented as:

$$f_u = 0.892 \cdot f_y + 164.811 \text{ MPa} \tag{2}$$

As a result, it is possible to estimate the yield strength, f_y of steel, by a non-destructive method using the equation:

$$f_y = 0.3395 \cdot \text{HB} - 97.4148 \text{ MPa} \tag{3}$$

Equation (3) is recommended to be used preliminarily in the absence of information on the dependence of hardness and yield strength of steel, which can be clarified in the laboratory. If such a possibility exists, the expression will take the form:

$$f_y = \alpha \cdot \text{HB} - \beta \text{ MPa} \tag{4}$$

where α and β are factors which determined experimentally.

The least squares estimates for the parameters are presented as:

$$\hat{\alpha} = \frac{\sum_{i=1}^{n}\left(\text{HB}_i - \overline{\text{HB}}\right)\left(f_{y,i} - \overline{f}_y\right)}{\sum_{i=1}^{n}\left(\text{HB}_i - \overline{\text{HB}}\right)^2} \tag{5}$$

$$\hat{\beta} = \overline{f}_y + \hat{\alpha} \cdot \overline{\text{HB}} \quad \text{MPa} \quad (6)$$

where $\overline{\text{HB}}$ and $\overline{f}_y$ are the mean values of hardness and yield strength of steel.

Residual variance $\hat{\sigma}^2$ value is:

$$\hat{\sigma}^2 = \frac{1}{n-2}\sum_{i=1}^{n}\left(f_{y,i} - \hat{\alpha} \cdot \text{HB} - \hat{\beta}\right)^2 \quad \text{MPa}^2 \tag{7}$$

Covariance matrix can be presented as:

$$\Sigma = \begin{pmatrix} \text{Var}(\hat{\alpha}) & \text{Cov}\left(\hat{\alpha},\hat{\beta}\right) \\ \text{Cov}\left(\hat{\alpha},\hat{\beta}\right) & \text{Var}(\hat{\beta}) \end{pmatrix} \tag{8}$$

The confidence ellipse [VI] for the parameters α and β is given by the inequality:

$$\begin{pmatrix} \alpha - \hat{\alpha} \\ \beta - \hat{\beta} \end{pmatrix}^T \Sigma^{-1} \begin{pmatrix} \alpha - \hat{\alpha} \\ \beta - \hat{\beta} \end{pmatrix} \le 2\hat{\sigma}^2 F_{2,n-2}^{(1-\gamma)} \tag{9}$$

where $F_{2,n-2}^{(1-\gamma)}$ is the quantile of the Fischer distribution.

The hardness measurement error of steel should also be included in the calculation model. Let HB_m be the measured hardness, with total error δ (combining device precision and reproducibility). The true hardness lies in the interval [HB_m - δ, HB_m + δ]. The regression parameters (α, β) are confined to the joint confidence ellipsoid defined by $c = 2\hat{\sigma}^2 F_{2,n-2}^{(1-\gamma)}$. Then the possible f_y = α·HB – β lies in [$\mu(\text{HB}) - r(\text{HB})$, $\mu(\text{HB}) + r(\text{HB})$], where $\mu(\text{HB}) = \hat{\alpha} \cdot \text{HB} - \hat{\beta}$ and $\text{r(HB)} = \sqrt{c \cdot \left(\text{HB}^2 \text{Var}(\alpha)\right) + \text{Var}(\beta) - 2\text{HB} \cdot \text{Cov}(\alpha, \beta)}$. As a result:

$$\underline{f}_y = \min_{\text{HB} \in [\text{HB}_m \pm \delta]} \left(\mu(\text{HB}) - r(\text{HB})\right) \tag{10}$$

$$\overline{f}_y = \max_{\text{HB} \in [\text{HB}_m \pm \delta]} \left(\mu(\text{HB}) + r(\text{HB})\right) \tag{11}$$

Once the p-box for the yield strength of the steel is formed based on the experimental data, the algorithm for calculating the failure probability (as a measure of structural reliability) is reduced to the standard procedure [VII] based on the interval arithmetic and Interval Monte Carlo Simulation (IMCS).

III. Results and Discussion

The structural reliability analysis uses a mathematical model of the limit state:

$$P_f = \Pr\{g(\mathbf{X}) < 0\} \tag{12}$$

where P_f is the probability of failure; $g(\mathbf{X})$ is a limit state function; **X** is a vector of random variables consisting of $\{x_1, x_2, \ldots, x_n\}$ random variables.

For example, let us consider the task of reliability analysis of the tensile steel bar of the truss for the yield strength limit state. The mathematical model of the limit state will take the form:

$$g(\mathbf{X}) = (x_1 \cdot x_2) - x_3 < 0 \tag{13}$$

where x_1 is the yield strength of steel, x_2 is the cross-sectional area of the steel bar, and x_3 is the tensile force in the steel bar.

All parameters in the eq. (13) are random variables. Next, we need to determine the cumulative distribution functions and their parameters (for example, as presented in Table 3).

Table 3. Random variables parameters

Random variable	Designation	Cumulative distribution function $F(x)$	Expected value m_x	Standard deviation S_x	Confidence bounds
Yield strength of steel, f_y	x_1	Truncated lognormal	[250 , 255] MPa	[10 , 15] MPa	[200 , 300] MPa
Cross-sectional area of steel	x_2	Truncated normal	[18.2, 18.4] cm^2	[0.2, 0.3] cm^2	[17.7, 18.9] cm^2

Random variable	**Designation**	**Cumulative distribution function $F(x)$**	**Expected value m_x**	**Standard deviation S_x**	**Confidenc e bounds**
bar, *A*					
Tensile force, *N*	x_3	P-box based on the Chebyshev's inequality [VIII]	[300, 330] kN	[20, 30] kN	[200, 400] kN

One of the most effective ways to solve this problem is to use the Inverse transform sampling (or N.V. Smirnov's transform) [IX, X]. If we know the analytical form of the cumulative distribution function, we can sample the data as:

$$x_{i,j} = F^{-1}(u_j) \tag{14}$$

where u_i is a uniformly distributed random variable ranging from [0, 1].

In the case of p-boxes, we have two cumulative distribution functions – lower and upper (confidence bounds).

We will have pairs of x: $x_{i,j,\,low} = F_{low}^{-1}(u_j)$ and $x_{i,j,\,up} = F_{up}^{-1}(u_j)$.

So, using equation (14), we simulate j=1, 2, …, 1000000 values for every x_i, i=1, 2, 3. We can easily calculate the value of the $g(\mathbf{X})$ function by eq. (6) for every j=1, 2, …, 1000000. Then, the failure probability can be estimated as:

$$P_{f,up} = (1/N)\cdot(\Sigma I(g(\mathbf{X}_{\mathrm{up}})<0)) \tag{15}$$

$$P_{f,low} = (1/N)\cdot(\Sigma I(g(\mathbf{X}_{\mathrm{low}})<0)) \tag{16}$$

where I[.] this is an indicator function that has a value of 1 if I[.] is equal to "true", and the value is 0 if I[.] is equal to "false"; N is the number of random variable generations (N=1000000).

The interval Monte Carlo simulation described above implicitly assumes that the random variables x_1, x_2, and x_3 are independent when generating the vectors $\mathbf{X}_{\mathrm{low}}$ and $\mathbf{X}_{\mathrm{up}}$. However, in practice, the dependence structure among these variables is rarely known, and different dependence assumptions (independence, comonotonicity, countermonotonicity) can lead to different failure probabilities. According to the Fréchet–Hoeffding bounds [XI], for given marginal distributions, the joint distribution is not unique; the extremal failure probability is often attained under comonotonic or countermonotonic dependence, not under independence.

To obtain a robust (worst-case) estimate of the failure probability interval, we supplement the independent sampling scheme with two extreme dependence scenarios:

1. Comonotonic dependence (perfect positive correlation). All three variables are driven by the same uniform random number $U \sim U[0, 1]$:

$$x_i = F^{-1}(U) \tag{17}$$

where F^{-1} is the quantile function of the corresponding marginal p-box (lower or upper bound).

2. Mixed dependence (negative correlation between x_3 and the product x_1, x_2). We generate U, $V \sim U[0, 1]$ independently. Then set:

$$x_1 = F_1^{-1}(U),\ x_2 = F_2^{-1}(U),\ x_3 = F_3^{-1}(1-U) \tag{18}$$

This makes x_3 countermonotonic with x_1 and x_2, while x_1 and x_2 remain comonotonic. This scenario tends to maximize P_f, because a low product $x_1 \times x_2$ is combined with a high tensile force x_3.

For each scenario, we compute the failure probability using the same Monte Carlo scheme (equations (15)–(18)) with N=1000000 samples. The final lower and upper bounds of the failure probability are taken as the minimum and maximum over all considered dependence scenarios:

$$P_f^{low} = \min\left\{P_f^{low,ind}, P_f^{low,com}, P_f^{low,mix}\right\} \tag{19}$$

$$P_f^{up} = \max\left\{P_f^{up,ind}, P_f^{up,com}, P_f^{up,mix}\right\}, \tag{20}$$

This approach provides a conservative (enveloping) interval that accounts for the uncertainty in the dependence structure without requiring explicit specification of a copula. If the range of failure probabilities turns out to be too wide or uninformative for decision-making, it is necessary to reduce epistemic uncertainty (collect additional data to narrow the intervals) [XII] or increase the area of the cross-sections for the structural elements [XIII, XIV].

IV. Conclusions

The probability of failure is a reliability indicator that can quantitatively express the safety level of a structure or its individual element. In practical structural reliability analysis, epistemic uncertainty arising from a limited number of measurements or tests must be properly accounted for. The p-box (probability box) provides an effective mathematical model for describing the yield strength of steel as a random variable when using non-destructive testing of existing structures, as it avoids the need to select a single probability distribution from scarce data.

A key contribution is the calibration of the hardness-to-strength conversion, where the parameters (α, β) are treated via a joint confidence ellipse (incorporating their covariance) and the hardness measurement error is explicitly propagated. This yields a statistically consistent p-box for the yield strength.

The reliability analysis is performed using Interval Monte Carlo Simulation (IMCS) extended with three extreme dependence scenarios (independence, comonotonicity, and mixed countermonotonic dependence). The failure probability is then expressed as an interval that accounts not only for interval-valued parameters but also for the unknown dependence structure among random variables. This ensures that the computed bounds are conservative with respect to the Fréchet-Hoeffding limits.

If the resulting failure probability interval turns out to be too wide for decision-making, the engineer can either collect additional data to reduce epistemic uncertainty (i.e., narrow the input intervals) or increase the cross-sectional area of the structural elements. The proposed framework offers a rigorous yet practical tool for reliability assessment of steel structures under limited data.

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Conflict of Interest:

The authors declare that there is no conflict of interest regarding this article.

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