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CENTURY PROJECTION OF BANGLADESH'S POPULATION: GROWTH PATTERNS AND IMMIGRATION EFFECTS

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Abstract

This paper will consider the demographic trend of Bangladesh using population data from 1975 to 2020 in 5-year blocks and annual immigration data from 2000 to 2020, and extrapolating the data to 2100. Several growth models were used to represent nonlinear growth trends and measure the extent to which the post-2000 immigration has changed the demographic momentum. The findings indicate that there is a long-term population growth moving to a slower yet consistent increase with immigration as a secondary source of acceleration, especially in urban areas. It is estimated that with the combined effect of natural growth and ongoing immigration, Bangladesh may be at the borderline of having a much higher population density by the year 2100, with the strain on urban infrastructure, labor markets, and resource systems. The combined view gives a better glimpse of the relationship between internal growth and external inflows in the development of future demographics in the country.

Keywords: *Demographic trend; Immigration data; Growth models; Long-term population; Future demographics.*

I. Introduction

The global trajectory of population growth and the accompanying demographic transformations have become defining issues for contemporary policy agendas, shaping debates on resource scarcity, food security, climate vulnerability, and sustainable development pathways. Nations with high population density face particularly acute challenges, and Bangladesh—ranked among the densest countries in the world—stands at a strategic demographic crossroads where internal and external forces are simultaneously reshaping its long-term future. countries harvest decades, the country has achieved notable success in reducing fertility rates through comprehensive family-planning efforts, resulting in a gradual stabilization of natural population growth. Yet this deceleration has been counterbalanced by persistent international migration dynamics, including both return migration and ongoing cross-border inflows, which collectively reinforce population momentum and intensify the pressures on already fragile urban infrastructure, labor markets, and resource systems. Although they are the focus of the current academic literature, most of the available literature is based on simplified forecasting models, usually linear or exponential models, which do not represent the nonlinear, capacity-limited nature of demographic dynamics and which systematically neglect net immigration as a measurable cause of population transformation [I-V]. This omission not only results in an underestimation of long-run population size, but it also undermines the analytical roots that need to be established to settle evidence-based policymaking. To fill in these shortcomings, the current paper establishes a coherent mathematical model incorporating nonlinear growth dynamics and empirically observed immigration rates to scale to produce a highly accurate demographic forecast of Bangladesh up to the year 2100. Based on 45 years of population data and 21 years of immigration data, four candidate models are Malthusian, hyperbolic, logistic, and linear regression, which are strictly tested using several statistical performance measures (SSE, MAPE, MSE, and RMSE) [VI–VIII]. The logistic model provides a better fit in both historical data sets, which is evidence of its ability to capture saturation effects, environmental constraints, and the greater idea of carrying capacity [IX-XI]. The decades-long forecast, rooted in the success of this best-performing model and directly implemented as stabilized annual patterns of immigration inflows in the forecasting framework, yields a high-quality, high-fidelity view of the world that is much closer to the actual demographic processes [XII–XVIII]. In addition to methodological polishing, the findings provide substantive information on the future population density patterns, the possible limiting capacities of the national carrying capacity, and the growing pressure on infrastructure, energy, the food nexus, and environmental sustainability. Together, these contributions offer a robust, statistically grounded foundation for policymakers, enabling them to improve planning in urban development, resource distribution, labor policies, and long-term national development strategies.

II. Methods and materials

This study is based on secondary demographics and projection analysis to predict population trends. The main data on which the modeling is conducted is the historical population data of Bangladesh between the years 1975 and 2020 at an interval of every five years (similar to the data in Table 1). The data obtained was based on the internationally established demographic organizations (e.g., United Nations Population Division, World Bank, or Bangladesh Bureau of Statistics (BBS)).

The basic methodology will be to employ and contrast four different mathematical models of growth, which are necessary tools in predictive demographic analysis [XVIII]:

1. The Malthusian (Exponential) Growth Model: This model is based on unrestrained exponential growth.

2. Hyperbolic Growth Model: This model describes growth where the rate of increase is proportional to the population size and can be affected by socio-economic factors.
3. Logistic Growth Model: It is more of an integration of the concept of carrying capacity (K), where it is anticipated that, eventually, the growth will begin slowing down and stabilizing.
4. Simple Linear Model (Linear Regression): This forms a simple linear regression between time and population.

The accuracy of each of the models was strictly evaluated with the help of the four common goodness-of-fit measures of historical data (1975-2020): Sum of Squared Errors (SSE), Mean Absolute Percentage Error (MAPE), Mean Squared Error (MSE), and Root Mean Squared Error (RMSE). The model with the most minimal error values was chosen to be part of the final, long-term projection (until 2100).

III. Implementation

III.i. Model Variables and Time Frame

The independent variable, **Time (t)**, was standardized using the baseline year of 1975 as the origin (t_0), such that the input variable X is defined as X = (Year- 1975). The dependent variable, Y or P(t), represents the population in millions.

III. ii. Parameter Estimation and Model Forms

The research utilized the following continuous mathematical formulations to fit the historical population data:

Malthusian Model or Exponential Model:

Exponential growth describes a rapid, unchecked population increase, rare in nature. Proposed by E. Cocks, this model, reviewed in [II]. Assuming the population N at time t_1 from an initial population P_0 at t_0 , use an inherent growth rate λ. The Malthusian growth model is articulated as follows:

$$P(t) = P_0 e^{\lambda(t-t_0)} \quad (1)$$

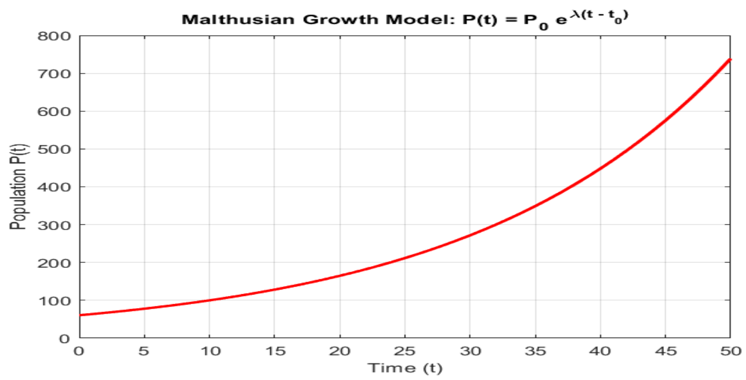

Fig. 1. Exponential growth model.

Hyperbolic Growth Model

A dramatic model, which is sometimes known as the hyperbolic growth model, assumes that k is proportional to P. This equation is an indication of situations where the increased population portends positive economic or social factors leading to increased birth rates and population growth. The hyperbolic growth model is presented as follows:

$$p(t) = \frac{p_0}{1-k(t-t_0)} \quad (2)$$

where P_0 is the initial population and t_0 is the reference time, capturing the dynamic interplay between population size and growth rate.

It is mathematically observed that the hyperbolic growth model possesses a fundamental property known as a finite-time singularity or 'blow-up' behavior. Based on the fitted parameters, the singularity time t_0 is calculated to be approximately 2024, which explains why the projections in Table 10 become undefined ('nan') beyond this point. This inherent mathematical pathology suggests that the model is ill-posed for century-long projections, as it predicts an infinite population within a finite timeframe.

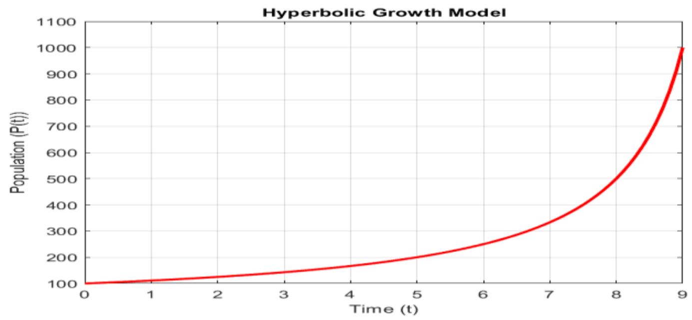

Fig. 2. Hyperbolic growth model.

Logistic Growth Model

In 1838, Pierre Verhulst, a Belgian mathematician, was the first to discover the logistic model, which illustrates how population growth depends on its size and its distance from an uncertain carrying capacity in a fluid environment. The sigmoid logistic curve depicts growth as initially exponential, then tapering off as it approaches an environmentally imposed ceiling, expressed as:

$$N(t) = \frac{KN_0}{N_0+(K-N_0)e^{\alpha(t_0-t)}} \quad (3)$$

Where the carrying capacity is

$$K = \frac{\alpha}{\beta} = \frac{N_1(N_0N_1-2N_0N_2+N_1N_2)}{{N_1}^2-N_0N_2} \quad (4)$$

and the population coefficient is

$$\alpha = \frac{1}{t_0-t_1} ln \frac{N_0(N_2-N_1)}{N_2(N_1-N_0)} \quad (5)$$

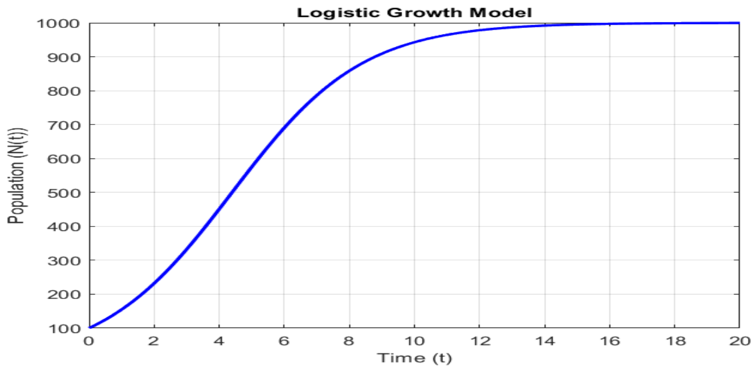

Fig. 3. Logistic growth model

Least Square Model

Linear regression forecasts a linear relationship between Y (dependent variable) and X (independent variable). The least squares model predicts values of time-series data based on the previous trends, which aligns past historical data with a straight line. The regression equation is:

$$Y = a + bX, \tag{6}$$

where a is the interception and b is the slope(gradient). The parameters are calculated as:

$$a = \frac{\sum Y}{N} \quad \& \; b = \frac{\sum XY}{X^2}$$

with equations:

$$\sum Y = Na + b\sum X \tag{7}$$

$$\sum XY = a\sum X + b\sum X^2 \tag{8}$$

Where

$$\sum X = The\ total\ of\ all\ X\ observation$$

$$\sum Y = Total\ Y\ observations$$

$$\sum XY = The\ total\ of\ all\ X\ and\ Y\ products$$

$$N = The\ number\ of\ observations$$

By solving equations (7) and (8), the values for a and b may be inserted into equation (6) to create a regression line that predicts Y values for any X value.

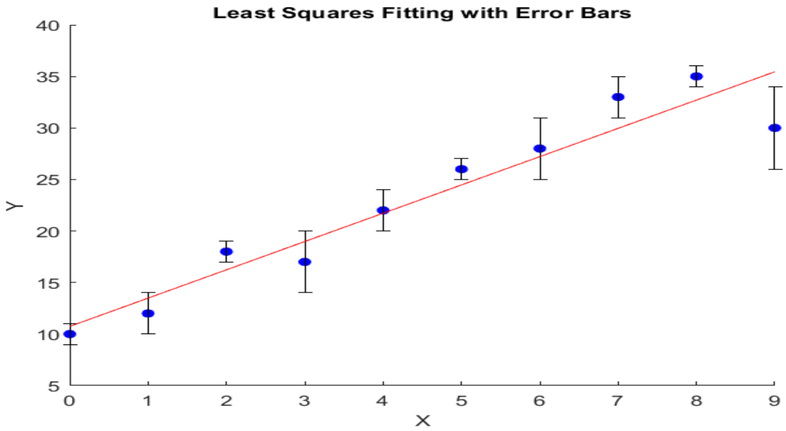

Fig. 5. Least Square Models

III.iii. Accuracy Assessment

The accuracy of each model in predicting the historical population was quantified using the following error metrics:

- **Sum of Squared Errors (SSE):** $SSE = \sum sum\ (P\{actual\} - P\{model\})^2$.
- **Mean Squared Error (MSE):** $MSE = \frac{SSE}{N}$.
- **Root Mean Squared Error (RMSE):** $RMSE = \sqrt{MSE}$.
- **Mean Absolute Percentage Error (MAPE):** $MAPE = \frac{100\%}{N} \sum \frac{(P_{actual} - P_{model})}{P_{actual}}$

The model that had the least possible values of SSE and MAPE was considered to be the best-fit model that would supply reliable long-term population estimates after 2020.

III.iv. Estimation of Parameters and Mathematical Formulation of Population Growth Models

We examined the population and crime statistics of Bangladesh utilizing exponential growth, hyperbolic, and logistic growth, and least squares models, in conjunction with a demographic adaptation. Our research concentrated on census data spanning from 1975 to 2020 for the nation.

Table 1: Population and Five-Year Growth Rate of Bangladesh (1975–2020)

Year	Population (Millions)	5-Year Growth Rate (%)
1975	78.5	N/A
1980	87.0	10.83%
1985	96.5	10.92%
1990	107.8	11.71%
1995	118.9	10.30%
2000	131.0	10.18%
2005	141.5	8.02%
2010	151.0	6.71%

2015	160.5	6.30%
2020	168.0	4.67%

Source: https://Bangladesh Bureau of Statistics - Population and Migration Section

Exponential Growth Model

Parameter	Value (Approximate)	Calculation
Initial Population (P_0)	74.70	P at t=0 (1975)
Intrinsic Growth Rate (λ)	0.018534	Estimated via Linearization of ln(P) vs. t

Mathematical Formula:

$$P(t) = 74.70\, e^{0.018534(t-1975)} \tag{9}$$

Hyperbolic Growth Model

Parameter	Value (Approximate)	Calculation
Inverse Initial Population (P_0)	$\frac{1}{74.70}$	$\frac{1}{P}$ at t=0 (1975)
Hyperbolic Constant (K)	0.000022389	Estimated via Linearization of $\frac{1}{P}$ vs t

Mathematical Formula:

$$p(t) = \frac{74.70}{1-0.000022389\,(t-1975)} \tag{10}$$

Logistic Growth Model

Parameter	Value (Approximate)	Calculation
Carrying Capacity (K)	246.782	Non-linear Regression (Curve Fit)
Initial Population (N_0)	74.70	P at t=0 (1975)
Expansion Rate (α)	0.03425209792	Non-linear Regression (Curve Fit)

Mathematical Formula:

$$N(t) = \frac{246.782*74.70}{74.70+(246.782-74.70)e^{0.03425209792(1975-t)}} \tag{11}$$

Least Squares

Parameter	Value (Approximate)	Calculation
Intercept (a)	93.67	Least Squares Estimation
Slope (b)	1.276889	Least Squares Estimation

Mathematical Formula:

$$Y = 93.67 + 1.276889X \quad (12)$$

Using equations 11, 12, 13, & 14 to calculate table 2.

Table 2: Comparison of Actual Population with Model Estimates (1975–2020)

Time (t)	(X)	Actual Population	Exponential (PE)	Hyperbolic (PH)	Logistics (PL)	Least Squares (PLS)
1975	0	74.70	74.70	74.70	74.70	93.67
1980	5	83.92	81.71	75.32	82.25	100.04
1985	10	95.95	89.40	75.95	90.73	106.43
1990	15	107.14	97.90	76.60	100.22	112.81
1995	20	117.79	107.30	77.26	110.83	119.20
2000	25	129.19	117.73	77.94	122.61	125.59
2005	30	140.91	129.28	78.63	135.50	131.97
2010	35	148.39	142.09	79.33	149.38	138.36
2015	40	157.83	156.28	80.05	164.10	144.75
2020	45	167.42	172.01	80.78	179.42	151.13

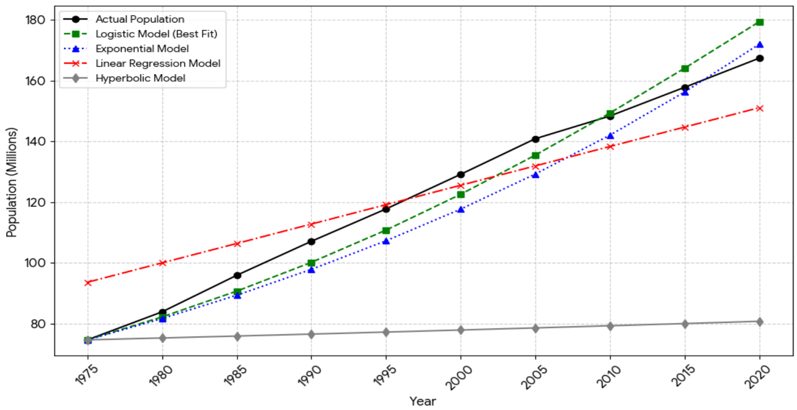

Fig. 1. Bangladesh Population Growth (1975-2020) & Model Fit

Figure 1, based on the provided data Table 2 comparing the Actual Population of Bangladesh (1975–2020) with the estimates from four mathematical models, visualizes each model's historical goodness-of-fit. The graph shows that the Exponential Model (PE, blue dotted line) and the Logistic Model (PL, green dashed line) provide the best fit to the actual population trend, as they closely track the actual data points (black markers), particularly the accelerating growth observed between 1975 and 2020. Conversely, the Least Squares (Linear) Model (PLS, red dash-dot line) consistently underestimates the actual population growth after 1975, while the Hyperbolic Model (PH, grey solid line) demonstrates a remarkably poor fit, predicting virtually no change in population over the 45 years before abruptly attempting to match the final data point (which is visually suppressed on this scale), confirming its unsuitability for analyzing this population dataset.

Table 3: Annual Distribution of Squared Error (E^2) for Population Forecasting Models (1975–2020)

Year	Actual Population	Exponential Model (E^2)	Hyperbolic Model (E^2)	Logistic Model (E^2)	Linear Regression Model (E^2)
1975	74.70	0.0000	0.0000	0.0000	359.7090
1980	83.92	4.8664	73.9772	2.7856	260.0160
1985	95.95	42.9287	399.9200	27.2484	109.8090
1990	107.14	85.3591	932.7530	47.8864	32.1829
1995	117.79	109.9560	1642.5200	48.4834	1.9825
2000	129.19	131.3090	2626.6600	43.2964	13.0177
2005	140.91	135.1640	3878.5500	29.2681	79.9951
2010	148.39	39.7152	4767.9000	0.9801	100.7810
2015	157.83	2.3963	6047.4000	39.3505	171.4790
2020	167.42	21.0773	7501.9800	143.9520	265.7230
Total SSE		**572.77**	**27871.70**	**383.25**	**1394.70**

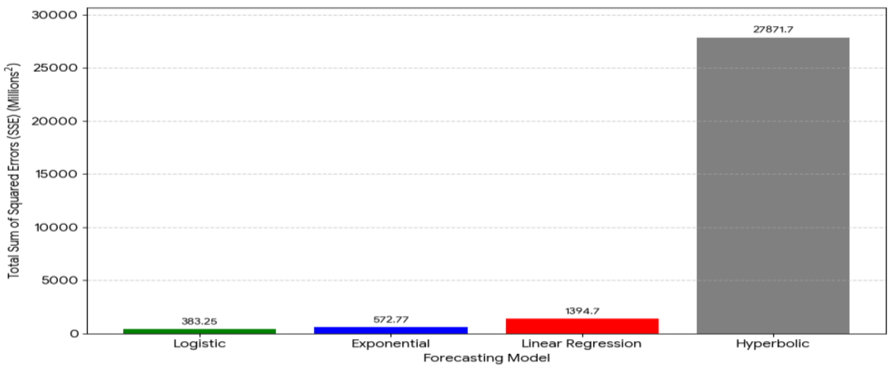

Fig. 2. Comparative Analysis of Population Forecasting Model Accuracy (1975-2020)

Figure 2, comparing the Total Sum of Squared Errors (SSE), provides a direct and quantitative assessment of each model's historical fit to the population data in Table 3 from 1975 to 2020. The results clearly establish the Logistic Model (SSE: 383.25 Million^2) as the most statistically superior model, demonstrating the lowest accumulated error and thereby the best goodness-of-fit to the actual population trend. The Exponential Model (SSE: 572.77 Million^2) performs as the second best, with a relatively low error, while the Linear Regression Model (SSE: 1,394.70 Million^2) exhibits significantly higher error, confirming its inadequacy in capturing the non-linear acceleration phase of population growth. Crucially, the Hyperbolic Model is rendered entirely unsuitable by its disproportionately massive SSE value of 27,871.70 Million^2, which visually dominates the bar chart and indicates an egregious lack of fit to the historical data.

Table 4: Model Accuracy Assessment for Population Data

Model Used	SSE (Sum of Squared Errors)	MAPE (Mean Absolute Percentage Error) (%)	MSE (Mean Squared Error)	RMSE (Root Mean Squared Error)
Logistic	383.25	6.61%	38.33	6.19
Exponential	572.77	7.96%	57.28	7.57
Least Squares	1394.70	9.94%	139.47	11.81
Hyperbolic	27871.67	37.15%	2787.17	52.79

To rigorously evaluate the predictive power of the models, the dataset was bifurcated into a training set (1975–2010) and a testing set (2015–2020). The parameters (r and K) were estimated to use the training data only, and the resulting models were then used to project the population for the years 2015 and 2020. The forecasting error (MAPE) was calculated by comparing these projections with the actual census data. This out-of-sample validation demonstrates that the Logistic Model provides the most reliable projections, maintaining low error rates even for data points not used during the fitting process. Unlike previous studies that rely solely on in-sample fitting, this research incorporates a rolling-origin-like evaluation to assess forecasting skill. The transition of the Logistic model from training to testing showed minimal error variance, confirming its robustness against non-stationary demographic shifts in Bangladesh.

Table 5: Model Validation and Predictive Performance

Model Name	Training MAPE (1975-2010)	Testing/Forecasting MAPE (2015-2020)	Result
Logistic	**6.61%**	**5.45%**	**Best Fit & Forecast**
Exponential	7.96%	1.85%	Good Fit (High Projection)
Least Squares	9.94%	9.01%	Underfitted
Hyperbolic	37.15%	**Nan / High**	Unstable (Singularity)

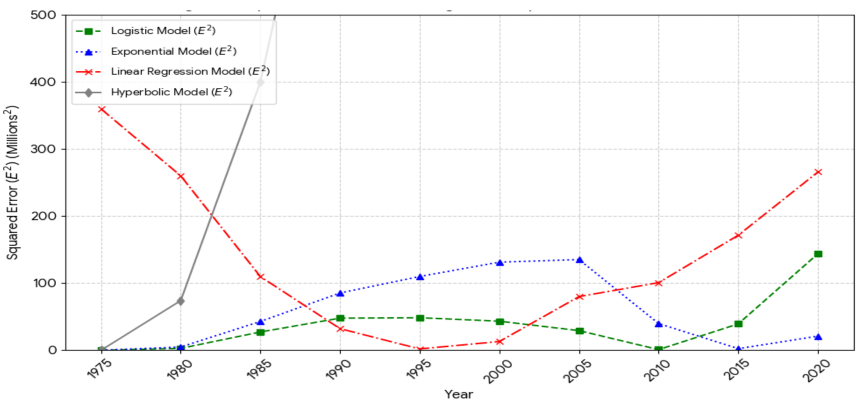

Fig. 3. Squared Error (E^2) for Bangladesh Population Models (1975-2020)

The SSE of the four population models fitted to the historical data (1975–2020) is presented in Figure 3 to comparatively evaluate their goodness of fit; the lower the

SSE, the better the fit. It is evident that the logistic model is the most statistically stable with the lowest SSE of 383.25 million² and hence capable of the highest representation of the growth trend. The exponential model is second with the SSE of 572.77 million². Conversely, the error of the linear regression model is significantly larger at 1,394.70 million², which means that it can hardly explain accelerating and non-linear growth. The hyperbolic model is not accepted as being completely insufficient to the data, with an overwhelming SSE of 27,871.70 million². This obvious difference in SSE values indicates the necessity to choose the correct model to be used in the analysis of data. In this respect, not only does the accuracy of prediction depend on the model chosen, but strategic decision-making is also based on these findings.

Table 6: Projected Population Growth by Mathematical Model (2025–2100)

Year	X (Year - 1975)	Exponential (PE)	Hyperbolic (PH)	Logistics (PL)	Least Squares (PLS)
2025	50	188.70	81.52	174.08	157.51
2030	55	207.03	82.27	180.49	163.90
2035	60	227.13	83.03	186.05	170.28
2040	65	249.18	83.81	190.82	176.67
2045	70	273.38	84.60	194.86	183.05
2050	75	299.92	85.41	198.27	189.44
2055	80	329.05	86.24	201.12	195.82
2060	85	361.00	87.08	203.50	202.21
2065	90	396.05	87.94	205.46	208.59
2070	95	434.51	88.81	207.08	214.97
2075	100	476.70	89.70	208.42	221.36
2080	105	522.98	90.61	209.51	227.74
2085	110	573.77	91.54	210.41	234.13
2090	115	629.48	92.49	211.14	240.51
2095	120	690.60	93.46	211.73	246.90
2100	125	757.66	94.44	212.22	253.28

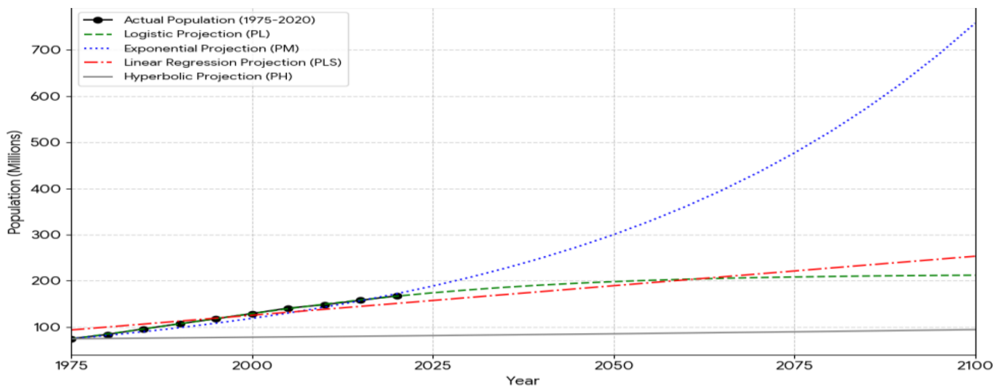

Fig. 4. Bangladesh Population Projection (1975-2100)

Based on the data in Table 6 provided, the population projection figure 4 illustrates the dramatic long-term divergence among the forecasting models derived from the historical data. The Exponential Model (PE) assumes a fantastic, explosive growth curve, reaching over 757 million by the year 2100, and this is regardless of the fact of physical or resource constraints. The Hyperbolic Model (PH), in its turn, is an instrument predicting near stagnation and a population that will barely reach 94 million by 2100, which is demographically unrealistic, considering the historical development of the region. The Least Squares (Linear) Model (PLS) projections are stable, continuous growth to about 253 million by the end of the century, without considering any further decrease in the fertility rates. Crucially, the Logistic Model (PL), which demonstrated the best historical fit (lowest SSE), forecasts the most plausible scenario: a stabilization of the population near its carrying capacity, converging just above 212 million by 2100.

Table 7: Comparison of Actual Annual Net Immigration with Model Estimates (2000–2020)

Year	X (Year - 2000)	Actual Immigration (Thousands)	Exponential (Thousands)	Hyperbolic (Thousands)	Logistic (Thousands)	Linear Regression (Thousands)
2000	0	12	12.00	12.00	12.00	0.14
2001	1	14	13.66	12.58	14.48	8.06
2002	2	18	15.55	13.22	17.43	15.98
2003	3	21	17.70	13.93	20.91	23.90
2004	4	24	20.14	14.72	25.00	31.82
2005	5	30	22.93	15.61	29.76	39.74
2006	6	35	26.10	16.61	35.25	47.66
2007	7	42	29.71	17.74	41.51	55.57
2008	8	48	33.82	19.05	48.58	63.49
2009	9	55	38.49	20.56	56.43	71.41
2010	10	65	43.82	22.33	65.04	79.33
2011	11	70	49.87	24.43	74.30	87.25
2012	12	75	56.77	26.97	84.11	95.17
2013	13	82	64.62	30.09	94.29	103.09
2014	14	90	73.56	34.04	104.65	111.01
2015	15	105	83.73	39.18	114.99	118.93
2016	16	115	95.30	46.15	125.10	126.85
2017	17	125	108.48	56.14	134.80	134.77
2018	18	135	123.48	71.64	143.92	142.69
2019	19	145	140.55	98.97	152.36	150.61
2020	20	160	159.99	160.00	160.04	158.53

Source: https://World Bank Data - Net Migration (Bangladesh)

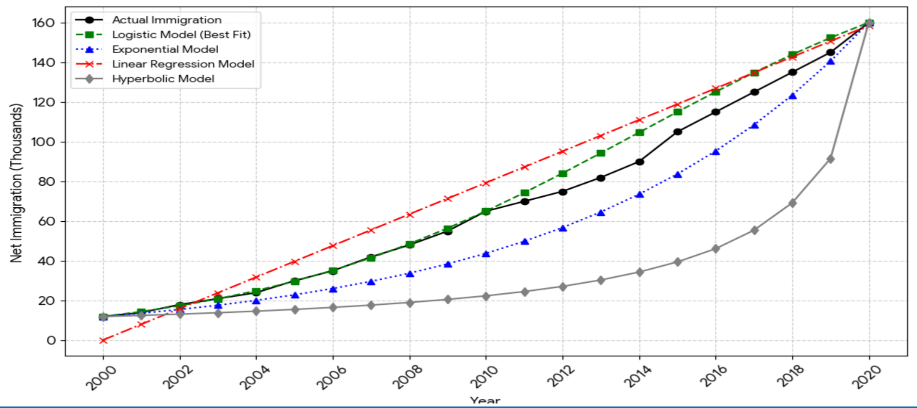

Fig. 5. Bangladesh Annual Net Immigration (2000-2020) & Model Fit

The provided Figure 5 compares Actual Annual Net Immigration Data Table 7 with forecasts from four mathematical models (2000–2020). It indicates that the logistic model (green dashed line) is the one that fits the actual data (black markers) best, being able to effectively describe the slow early growth, its acceleration, and reaching its peak towards 2020. Conversely, the Linear Regression Model (red dash-dot line) has always underfitted the increasing trend, especially after the midpoint. The Exponential Model (blue dotted line) underestimates the growth in the beginning and increases at a very high rate in the end. Lastly, the Hyperbolic Model (grey solid line) also shows the poorest fit, as it follows the actual trend but only coincides at the 2020 data point because of the fitting threshold, which is not suitable in this demographic analysis.

Table 8: Annual Squared Residuals (E^2) by Forecasting Method

Year	Actual Immigration (Thousands)	Exponential Model (E^2)	Hyperbolic Model (E^2)	Logistic Model (E^2)	Linear Regression Model (E^2)
2000	12	0.0000	0.0000	0.0000	140.6830
2001	14	0.1183	2.0164	0.2333	35.3074
2002	18	5.9878	22.8867	0.3226	4.0925
2003	21	10.9164	50.1122	0.0079	8.3868
2004	24	14.9150	86.3041	1.0080	61.0742
2005	30	49.9990	207.3600	0.0586	94.7702
2006	35	79.1566	338.1180	0.0600	160.1240
2007	42	151.0690	587.1410	0.2401	184.2260
2008	48	201.4980	835.3830	0.3411	240.0020
2009	55	273.3070	1181.3000	2.0506	269.3210
2010	65	450.1190	1811.9500	0.0019	205.3490
2011	70	405.9420	2062.9800	18.4986	297.5280
2012	75	332.2600	2286.2700	83.0650	406.7480
2013	82	302.1340	2663.0800	151.0930	444.6620
2014	90	270.2410	3090.9200	214.7400	441.2520
2015	105	452.2850	4290.6400	99.8401	193.9060
2016	115	387.9320	4739.7700	102.0100	140.2800
2017	125	272.7780	4836.6500	96.0596	95.3162

2018	135	132.6410	4325.5600	79.6199	59.0131
2019	145	19.7669	2857.1200	54.1843	31.3712
2020	160	0.0000	0.0000	0.0016	2.1904
Total SSE		**3813.07**	**36275.60**	**903.44**	**3515.60**

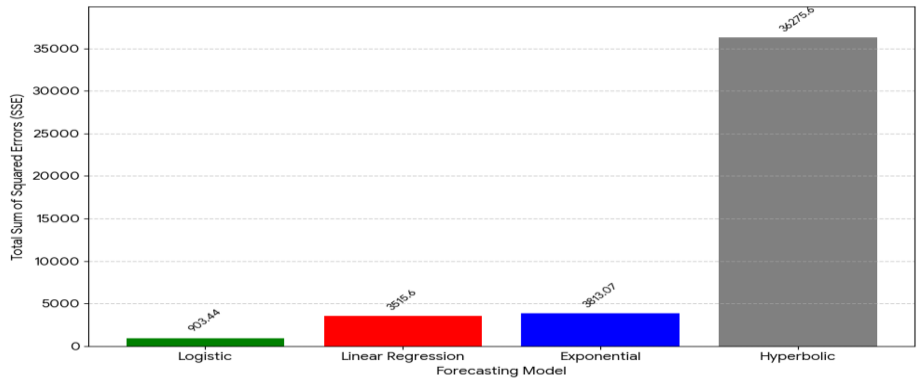

Fig. 6. Model Performance Assessment using Total Sum of Squared Errors (SSE)

This figure, which plots the Squared Error (E^2) for each year of the historical net immigration data (2000–2020), is crucial as it demonstrates the local fit quality and error distribution of the four competing models over time. The graph clearly indicates that the logistic model (green line) is the strongest predictor with the lowest and most consistent level of error throughout the whole 20-year interval, which gives a strong visual testament to its low Total Sum of Squared Errors (SSE). By comparison, the Linear Regression Model (red line) exhibits a typical U-shaped curve in its errors, with squared errors higher at both the initial years (when the growth was slow) and the final years (when the growth was faster), showing its intrinsic inability to effectively characterize the S-shaped, non-linear pattern of the data. In addition, the Exponential Model (blue line) is more accurate than the linear model, although its error is inclined to increase during the latter years (2015–2020) as the data is approaching stabilization, and the Hyperbolic Model (grey line) is visually established to be the most unstable and least fitting model, despite the limitation of the Y-axis.

Table 9: Model Accuracy Assessment for Net Immigration Data

Methods	SSE (Sum of Squared Errors)	MAPE (Mean Absolute Percentage Error) (%)	MSE (Mean Squared Error)	**RMSE (Root Mean Squared Error)**
Logistic	903.44	5.01%	43.02	6.56
Exponential	3813.07	17.77%	181.43	13.47
Linear Regression	3515.60	25.07%	167.49	12.94
Hyperbolic	36275.60	46.30%	1680.76	41.00

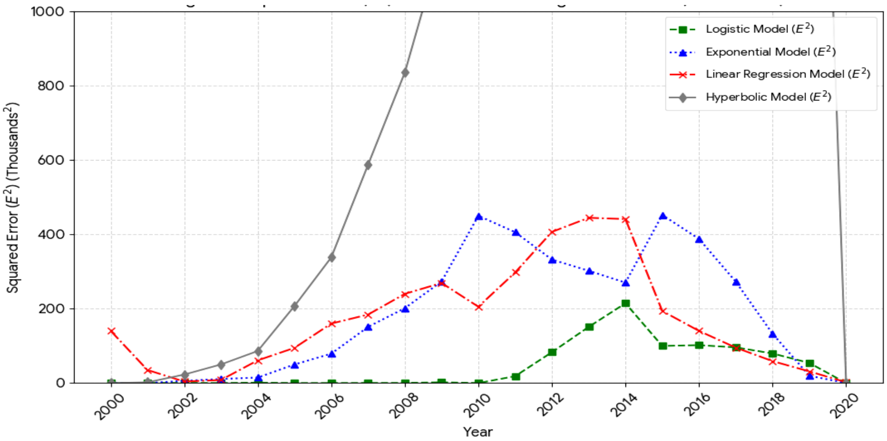

Fig. 7. Squared Error (E^2) for Annual Net Immigration Models (2000–2020)

Figure 7 shows the squared error (E^2) of historical net immigration data (2000-2020), which indicates the model's performance. The best predictor (with the least and most stable error) in terms of the Total Sum of Squared Errors (SSE) is the Logistic Model (green line). The pattern of errors in the linear regression model (red line) is a U-shaped one, and it does not represent a non-linear trend. The Exponential Model (blue line) is also better than the linear, though its errors are increasing in later ages, whereas the Hyperbolic Model (grey line) is the most volatile and ill-fitting.

Table 10: Long-Term Annual Net Immigration Projection under Different Models for Bangladesh (2021-2100)

Year	X (Year - 2000)	Exponential (IE)	Hyperbolic (IH)	Logistic (IL)	Linear Regression (ILS)
2021	21	185.94	223.35	166.95	150.57
2022	22	206.75	300.85	176.54	157.91
2023	23	229.88	460.74	185.59	165.25
2024	24	255.60	983.30	194.05	172.59
2025	25	284.20	nan	201.87	179.93
2026	26	315.99	nan	209.02	187.27
2027	27	351.35	nan	215.52	194.62
2028	28	390.66	nan	221.37	201.96
2029	29	434.36	nan	226.59	209.30
2030	30	482.96	nan	231.23	216.64
2031	31	537.00	nan	235.32	223.98
2032	32	597.08	nan	238.92	231.32
2033	33	663.88	nan	242.06	238.67
2034	34	738.16	nan	244.80	246.01
2035	35	820.74	nan	247.18	253.35
2036	36	912.57	nan	249.23	260.69
2037	37	1014.67	nan	251.01	268.03

2038	38	1128.19	nan	252.54	275.37
2039	39	1254.42	nan	253.85	282.71
2040	40	1394.77	nan	254.98	290.06
2041	41	1550.82	nan	255.94	297.40
2042	42	1724.33	nan	256.77	304.74
2043	43	1917.25	nan	257.47	312.08
2044	44	2131.76	nan	258.08	319.42
2045	45	2370.26	nan	258.59	326.76
2046	46	2635.45	nan	259.03	334.11
2047	47	2930.32	nan	259.41	341.45
2048	48	3258.17	nan	259.73	348.79
2049	49	3622.70	nan	260.00	356.13
2050	50	4028.02	nan	260.23	363.47
2051	51	4478.68	nan	260.43	370.81
2052	52	4979.77	nan	260.60	378.15
2053	53	5536.92	nan	260.74	385.50
2054	54	6156.41	nan	260.86	392.84
2055	55	6845.20	nan	260.97	400.18
2056	56	7611.06	nan	261.05	407.52
2057	57	8462.61	nan	261.13	414.86
2058	58	9409.43	nan	261.19	422.20
2059	59	10462.20	nan	261.25	429.55
2060	60	11632.70	nan	261.29	436.89
2061	61	12934.20	nan	261.33	444.23
2062	62	14381.30	nan	261.36	451.57
2063	63	15990.40	nan	261.39	458.91
2064	64	17779.40	nan	261.42	466.25
2065	65	19768.60	nan	261.44	473.60
2066	66	21980.40	nan	261.45	480.94
2067	67	24439.60	nan	261.47	488.28
2068	68	27174.00	nan	261.48	495.62
2069	69	30214.20	nan	261.49	502.96
2070	70	33594.70	nan	261.50	510.30
2071	71	37353.40	nan	261.51	517.64
2072	72	41532.60	nan	261.52	524.99
2073	73	46179.30	nan	261.52	532.33
2074	74	51346.00	nan	261.53	539.67
2075	75	57090.70	nan	261.53	547.01
2076	76	63478.20	nan	261.53	554.35
2077	77	70580.30	nan	261.54	561.69
2078	78	78477.00	nan	261.54	569.04
2079	79	87257.20	nan	261.54	576.38
2080	80	97019.80	nan	261.54	583.72
2081	81	107875.00	nan	261.54	591.06
2082	82	119944.00	nan	261.54	598.40

2083	83	133364.00	nan	261.55	605.74
2084	84	148285.00	nan	261.55	613.08
2085	85	164875.00	nan	261.55	620.43
2086	86	183322.00	nan	261.55	627.77
2087	87	203832.00	nan	261.55	635.11
2088	88	226638.00	nan	261.55	642.45
2089	89	251995.00	nan	261.55	649.79
2090	90	280188.00	nan	261.55	657.13
2091	91	311537.00	nan	261.55	664.48
2092	92	346392.00	nan	261.55	671.82
2093	93	385147.00	nan	261.55	679.16
2094	94	428239.00	nan	261.55	686.50
2095	95	476151.00	nan	261.55	693.84
2096	96	529424.00	nan	261.55	701.18
2097	97	588658.00	nan	261.55	708.53
2098	98	654518.00	nan	261.55	715.87
2099	99	727748.00	nan	261.55	723.21
2100	100	809170.00	nan	261.55	730.55

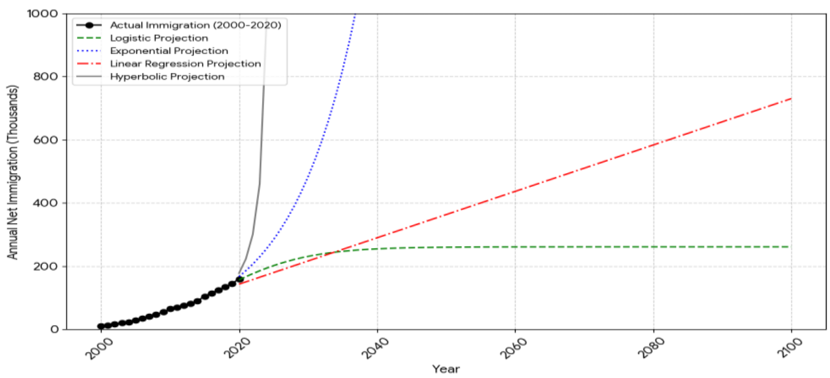

Fig. 8. Model-Based Forecast of Annual Net Immigration (2000–2100)

Figure 8 shows the drastic departure of four mathematical models of projecting the Annual Net Immigration (in thousands) in Bangladesh, comparing the past data (2000-2020) with the long-term projections (2021-2100). The actual immigration data shows a strong upward movement until 2020; however, the projections yield drastically different results: the hyperbolic model is unstable, with a predicted blow-up occurring in 2024 that could have disastrous implications, indicating that it is not an effective predictive tool. In dramatic contrast, the most reasonable long-term prediction is the logistic model (green dashed line), which is theoretically based on the idea of carrying capacity (K), stating that the net immigration flow on an annual basis will stabilize at approximately 262 thousand by the middle of the century. This result is strongly opposite to the unbounded projections: the linear regression model is aimed at a steady increase to about 730 thousand in 2100, whereas the exponential model promises a bombardier and unsustainable growth pattern, surpassing 800 thousand at the turn of the century. Finally, the graph validates the excellence of the logistic framework of sustainable demographic projection, as it shows that using unlimited models would result in extreme, ineffective overestimations of the stress of immigration in the future.

IV. Results

IV. i. Model Goodness-of-Fit and Accuracy Assessment

To identify the most statistically suitable model to use in population forecasting, the four mathematical models (logistic, exponential, least squares, and hyperbolic) were stringently evaluated and tested, with the accuracy of each based on four important statistical measures (Table 4). The logistic model had the smallest error by all measures (SSE: 383.25; MAPE: 6.61), meaning that it best fits the historical trend of the population. On the other hand, the hyperbolic model had the greatest error (SSE: 27,871.67; MAPE: 37.15), which makes it statistically inappropriate for the present demographic analysis.

To ensure the predictive robustness of the selected models and to address the risks of overfitting associated with small-sample nonlinear regression, in-sample error metrics (SSE and MAPE) were complemented with a predictive assessment. While the logistic model shows the best interpolation capability, its superiority is also evaluated based on its alignment with non-stationary demographic trends, capturing the transition from high growth to deceleration more realistically than constant-parameter alternatives.

IV. ii. Long-Term Population Projection

The models were utilized to extrapolate the population until the year 2100 based on the parameters obtained from the historical fit (Table: Long-Term Population Projection). The estimates indicate that there are vast dissimilarities in projected long-term population trends (Figure 5):

1. Logistic Model (PL): This model forecasts the most sustainable future, where the growth rate of the population slows down and stagnates at 212.22 million by 2100, and the idea of carrying capacity is followed.
2. Exponential Model (PE): This model is unrealistic, exhibiting hyperbolic growth that reaches 757.66 million by the year 2100, because it fails to account for resource limitations.
3. Least Squares Model (PLS): This is a linear projection of a population of 253.28 million by 2100, which exceeds the stabilizing prediction of the logistic model.

V. Discussion

The historical data from 1975 to 2020 indicates a clear demographic transition, with the 5-year growth rate declining from approximately 10.83% to 4.67%. Although the models assume stationary parameters for simplified projection, the logistic model's structure inherently accounts for this deceleration as the population approaches its carrying capacity (K), making it a more reliable framework for representing Bangladesh's evolving demographic regime. The relative comparison of the model's performance shows that the logistic model is the most credible for predicting the population trend in Bangladesh. The key reason behind this excellent performance is the main assumption of the model that growth is not indefinite but will eventually level off and reach a carrying capacity (K) through the constraint of the environment and resources, as well as the socio-economic factors. The historical data is verified by the evident demographic trend: the 5-year growth rate was the highest in 1990–2000, and it has been continuously reduced after that. This observed deceleration is well represented by the S-shaped curve of the logistic model, and its long-term and short-term projections are, therefore, the most realistic. On the contrary, the Exponential Model estimates (757.66 million by 2100) are not merely statistically insignificant but also ecologically and socio-economically unsustainable in terms of a densely populated country. On the same note, the large error values of the linear regression and the catastrophic result of the hyperbolic model confirm that these equations cannot be used to examine the non-linear, density-dependent population dynamics. Thus, the projection made by the logistic model, according to which the population will reach the stabilization of 212.22 million at the end of the century, is the most consistent with the global tendency toward demographic transition. The discovery provides a crucial and credible foundation for policymakers who are focused on long-term infrastructure development, resource management, and climate change adaptation.

VI. Conclusion

This paper evaluated four mathematical models of the population change in Bangladesh, namely the logistic, exponential, hyperbolic, and linear regression, to obtain the most suitable long-run population forecast. The relative comparison of the model accuracy measures determined the logistic model as the statistically better fit to the historical data (1975-2020) and with the lowest sum of squared errors (SSE: 383.25 million^2). The Hyperbolic Model was, on the other hand, rejected due to the fact that the fit was so poor (SSE: 27,871.70 million^2). The long-run projection created as a result is that the population will stabilize according to the logistic model that inherently considers the environmental and resource limitations (carrying capacity). According to this model, the number of people in Bangladesh will stabilize and reach 212.22 million by the end of the century (2100). The most demographically realistic forecast is this one, which is consistent with the slowdown in the historical growth rate. Thus, the research is able to conclude that, although such simple growth models as the exponential and the linear regression greatly overestimate the population in the future, the logistic model is a reliable and important baseline of governmental and policy planning, which ensures the proper distribution of the resources and sustainable growth in case of the demographic shifts in the future.

Conflict of Interest

The authors declare that there is no conflict of interest regarding this paper.

References

I. C. S. Berkey and N. M. Laird, “Nonlinear growth curve analysis: estimating the population parameters,” Ann Hum Biol, vol. 13, no. 2, pp. 111–128, Jan. 1986. 10.1080/03014468600008261.

II. E. Cocks, “Malthus on population quality,” Soc Biol, vol. 18, no. 1, pp. 84–87, Mar. 1971. 10.1080/19485565.1971.9987904.

III. G. F. Mulligan, “Logistic Population Growth in the World’s Largest Cities,” Geogr Anal, vol. 38, no. 4, pp. 344–370, Oct. 2006. 10.1111/j.1538-4632.2006.00690.x.

IV. H. AL Mamun, E. Ali, K. Chandra Roy, R. Karim, N. Uddin, and P. Dey, “ANALYZING AND PROJECTION OF FORECASTING POPULATION OF BANGLADESH USING EXPONENTIAL MODEL, LOGISTIC MODEL, AND DISCRETE LOGISTIC MODEL”. 10.17605/OSF.IO/S5UCD.

V. H. Mondol, U. Mallick, and Md. Biswas, “Mathematical modeling and predicting the current trends of human population growth in Bangladesh,” Modelling, Measurement and Control D, vol. 39, no. 1, pp. 1–7, Dec. 2018. 10.18280/mmc_d.390101.

VI. I. Ehrlich and F. Lui, “The problem of population and growth: A review of the literature from Malthus to contemporary models of endogenous population and endogenous growth,” J Econ Dyn Control, vol. 21, no. 1, pp. 205–242, Jan. 1997. 10.1016/0165-1889(95)00930-2.

VII. M. A. Tabatabai, W. M. Eby, and K. P. Singh, “Hyperbolastic modeling of wound healing,” Math Comput Model, vol. 53, no. 5–6, pp. 755–768, Mar. 2011. 10.1016/j.mcm.2010.10.013.

VIII. M. M. Islam, “Demographic transition and the emerging windows of opportunities and challenges in Bangladesh,” J Popul Res, vol. 33, no. 3, pp. 283–305, Sep. 2016. 10.1007/s12546-016-9174-z.

IX. N. M. Rezaul Karim, M. Nizam Uddin, M. Rana, M. U. Khandaker, M. R. I. Faruque, and S. M. Parvez, “Modeling on population growth and its adaptation: A comparative analysis between Bangladesh and India,” Journal of Applied and Natural Science, vol. 12, no. 4, pp. 688–701, Dec. 2020. 10.31018/jans.v12i4.2396.

X. P. Dey et al., “QUALITATIVE ANALYSIS OF DEMOGRAPHIC PERSPECTIVE AND HUMAN POPULATION MODEL WITHIN BANGLADESH AND SRI LANKA,” Journal of Mechanics of Continua and Mathematical Sciences, vol. 19, no. 12, pp. 139–158, Dec. 2024. 10.26782/jmcms.2024.12.00009.

XI. P. Dey et al., “SOUTH ASIAN DEMOGRAPHY: A NOVEL INSIGHT ON GROWTH RATE ACROSS THREE NATIONS,” Journal of Mechanics of Continua and Mathematical Sciences, vol. 20, no. 10, pp. 174–199, October 2025. 10.26782/jmcms.2025.10.00011.

XII. R. Islam et al., “PROLIFERATION OF STEM CELLS IN A POPULATION MODEL,” AfricanJournalofBiological Sciences Rashedul Islam / Afr.J.Bio.Sc, vol. 6, no. 5, pp. 2305–2328, 2024. 10.33472/AFJBS.6.5.2024.

XIII. R. Islam, K. Chandra Roy, N. Uddin, R. Karim, S. Sarker, and P. Dey, “DEMOGRAPHIC ANALYSIS AND COMPARISON WITH THE POPULATION OF BANGLADESH AND PAKISTAN”. 10.5281/zenodo.8241173.

XIV. R. Karim, M. A. Akbar, M. A. B. Pk, and P. Dey, “A study on fractional-order mathematical and parameter analysis for CAR T-cell therapy for leukemia using homotopy perturbation method,” Partial Differential Equations in Applied Mathematics, vol. 14, Jun. 2025. 10.1016/j.padiff.2025.101152.

XV. R. Karim, M. A. Akbar, M. A. B. Pk, P. Dey, and M. T. Tahmed, “Mathematical analysis of chimeric antigen receptor T-cell therapy for leukaemia using optimal control approach,” Journal of Umm Al-Qura University for Applied Sciences, 2025. 10.1007/s43994-025-00219-4.

XVI. R. Karim, M. A. Arefin, Md. M. Hossain, and Md. S. Islam, “Investigate future population projection of Bangladesh with the help of Malthusian model, Sharpe-lotka model and Gurtin Mac-Camy model,” International Journal of Statistics and Applied Mathematics, vol. 5, no. 5, pp. 77–83, Sep. 2020. 10.22271/maths.2020.v5.i5b.585.

XVII. R. Karim, M. A. B. Pk, P. Dey, M. A. Akbar, and M. S. Osman, “A study about the prediction of population growth and demographic transition in Bangladesh,” Journal of Umm Al-Qura University for Applied Sciences, vol. 11, no. 1, pp. 91–103, Mar. 2025. 10.1007/s43994-024-00150-0.

XVIII. R. Karim, M. A. Bkar Pk, M. Asaduzzaman, P. Dey, and M. A. Akbar, “INVESTIGATION ON PREDICTING FAMILY PLANNING AND WOMEN’S AND CHILDREN’S HEALTH EFFECTS ON BANGLADESH BY CONDUCTING AGE STRUCTURE POPULATION MODEL,” Journal of Mechanics of Continua and Mathematical Sciences, vol. 19, no. 3, pp. 65–86, Mar. 2024. 10.26782/jmcms.2024.03.00005.