JOURNAL OF MECHANICS OF CONTINUA AND MATHEMATICAL SCIENCES
www.journalimcms.org

ISSN (Online) : 2454 -7190 Vol.-20, No.-10, October (2025) pp 96-117 ISSN (Print) 0973-8975

UTILIZATION ROUGH CONCEPT TO SOLVE DE NOVO PROGRAMMING PROBLEM UNDER AMBIGUITY: REAL CASE STUDY

Iftikhar Ali Hussein[1], Hagazy Zahar[2], Naglaa Ragaa Saied[3]
Rabie Mosaad Rabie[4]

[1] Middle Technical University, Technical Engineering College-Baghdad, Baghdad, Iraq.

[2,3,4] Department of Operations Research and Management, Faculty of Graduate Studies for Statistical Research, Cairo University, Giza, Egypt.

E-mail: [1]eftekhar.ali@mtu.edu.iq, [2]hgsabry@cu.edu .eg
[3]naglaa777subkiii@yahoo.com , [4]Rabee.saleh86@gmail.com

Corresponding author E-mail: **Iftikhar Ali Hussein**

https://doi.org/10.26782/jmcms.2025.10.00007

(Received: July 10, 2025; Revised: September 17, 2025; October 01, 2025)

Abstract

Multi-objective Linear Programming (MOLP) traditionally optimizes multiple conflicting objectives simultaneously. This research extends the De Novo Programming (DNP) concept, which focuses on optimal system design, to situations with uncertainty in resource allocation and budget constraints. A novel mathematical model, Rough Interval Multi-Objective De Novo Programming (RIMODNP), has been introduced. This model incorporates the Rough Interval (RI) concept, where all problem coefficients are represented by lower and upper interval bounds, each having two terms (upper and lower).

The study outlines the mathematical formulation of the RIMODNP model, detailing the methodology used to transform its uncertain nature into deterministic sub-problems. It presents two primary approaches, Zeleny's and the Optimum-Path Ratio Method, for finding optimal designs. Applied to the Baghdad Water Department, the model optimizes resource allocation for increased water production, improved water quality, and reduced water loss while considering unknown constraints.

The results, obtained by solving the deterministic sub-problems, provide the decision-maker with a range of optimal system designs. The application to the Baghdad Water Department shows significant increases in profit and cost savings across different scenarios, highlighting the model's ability to offer robust and effective solutions under conditions of uncertainty.

Keywords: De Novo programming; Multi-objective linear programming; Resource allocation; Rough Interval; Tong-Shaoching method; Zeleny Approach; Optimal path-ratios.

I. Introduction

Multi-objective linear programming (MOLP) is a powerful branch of mathematical optimization focused on problems with several conflicting objectives that need to be optimized at the same time. Unlike traditional linear programming, which usually deals with just one goal, MOLP tackles situations where two or more objectives are at play. This makes MOLP a specific type of vector linear programming and part of the broader field of multi-objective optimization. A standard MOLP model is defined by its mathematical structure, including objective functions, constraints, and decision variables. Various techniques and algorithms, such as linear programming solvers and evolutionary algorithms, can be used to solve MOLP problems and find effective solutions [II].

While MOLP excels at optimizing within existing systems, the challenge often lies in designing optimal systems from scratch. To address this, Zeleny expanded the concept of multi-objective programming to De Novo Programming (DNP). DNP is essentially a tool for reshaping potential combinations within linear systems, specifically aiming for optimal system designs. It provides a strategic approach to finding the best possible system configuration, rather than just optimizing within given limitations [IX]. Despite Zeleny's de novo programming (DNP) approach being effective under conditions of certainty, it is not suitable for situations with uncertain data. To address this limitation, many researchers have developed various methods for solving DNP problems under uncertainty, including fuzzy (FDNP) and stochastic programming (SDNP). The two suggested models provide a single subjective solution [XI], [XV].

Building on this body of research, our study introduces a new model called Rough Interval Multi-Objective De Novo Programming (RIMODNP) to address the problem of resource allocation under uncertainty with an undetermined budget. This problem is solved using two distinct methods: Zeleny's approach and the Optimal-Path Ratios Method, with the latter two assuming the right-hand side of constraints is unknown. Our article is structured into four sections: the first is this introduction, followed by the materials and methods, then the application of the proposed mathematical model to a real case study (the Baghdad Water Department), and finally, the results, discussion, and conclusion.

II. Materials and Methods

i) Multi-Objective Linear Programming (MOLP)

The standard model of MOLP is characterized by its mathematical representation, which includes the objective functions, constraints, and decision variables involved in the optimization problem, as shown in problem (1) below:

$max\ or\ min\ Z_k = \sum_{j=1}^{n} C_{kj}\, Y_j, \qquad \mathrm{k} = 1,2,\dots,l,$

subject to :

$$\sum_{j=1}^{n} a_{ij}\, Y_{j_i} \leq=\geq b_i, \quad i = 1,2,\dots,m, \qquad (1)$$
$$Y_j \geq 0, j = 1,2,\dots,n$$

Where the parameters b_i $(i = 1,2, \dots, m)$ represents the given available resources as constants. The efficient solution concept results from the solution of the MOLP model [X], [VI].

ii) Multi-Objective De Novo Programming (MODNP)

DNP is used for reshaping feasible sets in linear systems; it is utilized as an approach to optimum system design. Given resource pricing and a budget, the MOLP problem is reformulated to get the MODNP formulation from the problem (2), it is necessary to convert b_i from constants to variables, and then determine their values in (2) as follows:

$$max \ \ Z_k = \sum_{j=1}^{n} C_{kj} \, Y_j, \qquad \text{k} = 1,2, \dots, l,$$

subject to :

$$\sum_{j=1}^{n} a_{ij} \, Y_j \le=\ge b_i, \quad i = 1,2, \dots, m,$$

$$\sum_{j=1}^{n} p_i \, b_i \quad \le \beta \quad , \quad X_j \ge 0, j = 1,2, \dots, n \qquad (2)$$

Where: $Y_i.b_i$: are decision variables for products and available resources, respectively; P_i, β: are the given values of both the unit price of resource i and total available budget, respectively.

Where ***Z***: maximize profit for single or multiple objective problems. Now, the problem is to allocate the budget so that the resulting portfolio of resources maximizes the value of the product mix (with given unit prices of ***m*** resources, and with given total available budget [XII].

iii) Rough Interval Linear Programming Model:

The Rough Interval Linear Programming (RILP) model is designed to address situations where data values are uncertain. It expands upon standard linear programming by using rough interval coefficients, which allow for the estimation of data through upper and lower interval bounds when exact figures are unavailable. This enables predictions even when precise knowledge of a data value is lacking, the general form of RILP is as follows:

$$Max \ or \ Min \ \ f = \sum_{j=1}^{n}([\underline{c}_j^L, \underline{c}_j^U], [\overline{c}_j^L, \overline{c}_j^U]) y_j$$

$$subject \ to: \sum_{j=1}^{n}([\underline{a}_{ij}^L \ \underline{a}_{ij}^U], [\overline{a}_{ij}^L, \overline{a}_{ij}^U]) y_j \le ([\underline{b}_i^L, \underline{b}_i^U], [\overline{b}_i^L, \overline{b}_i^U]) \qquad (3)$$

$$y_j \ge 0, \ j = 1,2, \dots, n, \quad i = 1,2, \dots, m$$

Where: $([\underline{c}_j^L, \underline{c}_j^U], [\overline{c}_j^L, \overline{c}_j^U])$,$([\underline{a}_{ij}^L, \underline{a}_{ij}^U], [\overline{a}_{ij}^L, \overline{a}_{ij}^U])$, and$([\underline{b}_i^L, \underline{b}_i^U], [\overline{b}_i^L, \overline{b}_i^U])$are rough interval coefficients of the objective function and constraints, and also, let y = $(y_1, y_2, \dots, y_n)^t$ represent the vector of all decision variables, see for more details [V].

iv) Proposed Mathematical Model

Rough Interval Multi-Objective Linear Programming *(RIMODNP)*

The proposed mathematical model can be formulated by assuming coefficients are a rough interval for problem (1), the model can be written as in problem (4):

$$Max\ or\ Min\ \ Z_1 = \sum_{j=1}^{n}([\underline{c}_j^L, \underline{c}_j^U], [\overline{c}_j^L, \overline{c}_j^U])Y_j$$
$$Max\ or\ Min\ \ Z_2 = \sum_{j=1}^{n}([\underline{c}_j^L, \underline{c}_j^U], [\overline{c}_j^L, \overline{c}_j^U])Y_j$$
$$\vdots$$
$$Max\ or\ Min\ \ Z_l = \sum_{j=1}^{n}([\underline{c}_j^L, \underline{c}_j^U], [\overline{c}_j^L, \overline{c}_j^U])Y_j$$
$$subject\ to:$$
$$\sum_{j=1}^{n}([\underline{a}_{ij}^L\ \underline{a}_{ij}^U], [\overline{a}_{ij}^L, \overline{a}_{ij}^U])Y_j \le ([\underline{b}_i^L, \underline{b}_i^U], [\overline{b}_i^L, \overline{b}_i^U]) \tag{4}$$
$$Y_j \ge 0,\ j = 1,2,\dots,n, \quad i = 1,2,\dots,m, k = 1,2,\dots,l$$

Where $([\underline{c}_j^L, \underline{c}_j^U], [\overline{c}_j^L, \overline{c}_j^U])$, $([\underline{a}_{ij}^L, \underline{a}_{ij}^U], [\overline{a}_{ij}^L, \overline{a}_{ij}^U]), and([\underline{b}_i^L, \underline{b}_i^U], [\overline{b}_i^L, \overline{b}_i^U])$are rough interval coefficients of objective functions and constraints, and also, x_j : represent the decision variables. By reshaping RIMOLP, we obtained Rough Interval Multi-Objective De Novo Programming (RIMODNP) as in problem (5) below:

$$Min\ or\ Max\ f^K(Y) = \sum_{i=1}^{m}\sum_{j=1}^{n}([\underline{C}_{ij}^{KL}, \underline{C}_{ij}^{KU}], [\overline{C}_{ij}^{KL}, \overline{C}_{ij}^{KU}])Y_{ij}$$
$$subject\ to$$
$$\sum_{j=1}^{n}([\underline{a}_{ij}^L, \underline{a}_{ij}^U], [\overline{a}_{ij}^L, \overline{a}_{ij}^U])Y_j \le b_i$$
$$\sum_{j=1}^{n}([\underline{P}_i^L, \underline{P}_i^U], [\overline{P}_i^L, \overline{P}_i^U])b_i \le ([\underline{\beta}^L, \underline{\beta}^U], [\overline{\beta}^L, \overline{\beta}^U]), Y_j \ge 0\,, \tag{5}$$
$$i = 1,2,\dots,m,\ j = 1,2,\dots,n, and\ \ k = 1,2,\dots,l\dots\dots$$

Where: $([\underline{C}_{ij}^{KL}, \underline{C}_{ij}^{KU}],[\overline{C}_{ij}^{KL}, \overline{C}_{ij}^{KU}])$: is a vector of rough interval coefficients for the multi-objective function $([\underline{a}_{ij}^L, \underline{a}_{ij}^U], [\overline{a}_{ij}^L, \overline{a}_{ij}^U])$: is a matrix of rough interval coefficients for constraints of the multi-objective function , $([\underline{P}_i^L, \underline{P}_i^U], [\overline{P}_i^L, \overline{P}_i^U])$: is a vector of rough interval coefficients of the unit price of resources i and $\left(\left[\underline{\beta}^L, \underline{\beta}^U\right], \left[\overline{\beta}^L, \overline{\beta}^U\right]\right)$: is a rough interval of the total available budget.

where $(i = 1,2,\dots,m;\ \ j = 1,2,\dots,n)$, $Y = (y_1, y_2, \dots, y_n)^t$ denote the vector of all decision variables.

$f^{RI(k)} = \left(\left[\underline{f}^{kL}, \underline{f}^{kU}\right] : \left[\overline{f}^{kL}, \overline{f}^{kU}\right]\right)$ Respectively and $k = 1,2,\dots,$ K is the number of objectives.

The conditions for the validity of the mathematical model (RIMODNP):[V] [VII],

- The rough interval $\left(\left[\underline{f}^{kL}, \underline{f}^{kU}\right] : \left[\overline{f}^{kL}, \overline{f}^{kU}\right]\right)$is called the surely (possibly) optimal range of model (5), if the optimal range is a subset of $\left(\left[\underline{f}^{kL}, \underline{f}^{kU}\right] : \left[\overline{f}^{kL}, \overline{f}^{kU}\right]\right)$.

- Let $\left[\underline{f}^{kL}, \underline{f}^{kU}\right] : \left[\overline{f}^{kL}, \overline{f}^{kU}\right]$ be surely optimal (possibly) optimal range of the model (5). Then the rough interval $\left(\left[\underline{f}^{kL}, \underline{f}^{kU}\right], \left[\overline{f}^{kL}, \overline{f}^{kU}\right]\right)$ is called the rough optimal range of model (5).
- The optimal solution of each corresponding MODNP model (5), whose optimal value belongs to $\left[\underline{f}^{kL}, \underline{f}^{kU}\right] : \left[\overline{f}^{kL}, \overline{f}^{kU}\right]$ is called a completely (rather) satisfactory solution of the model (5).
- $\left[\underline{P}_i^L, \underline{P}_i^U\right] \subseteq \left[\overline{P}_i^L, \overline{P}_i^U\right] \Rightarrow \overline{P}_i^L \leq \underline{P}_i^U \leq \underline{P}_i^L \leq \overline{P}_i^U$
- $\left[\underline{\beta}^L, \underline{\beta}^U\right] \subseteq \left[\overline{\beta}^L, \overline{\beta}^U\right] \Rightarrow \overline{\beta}^L \leq \underline{\beta}^U \leq \underline{\beta}^L \leq \overline{\beta}^U$

Finding the Optimal Design

i) Zeleny Approach is a method used to find the optimal system design for each sub-problem by calculating and replacing budget constraints with specific sub-problem constraints [IX].

Steps to Implement Zeleny's Approach (De Novo Programming)

Zeleny's approach for optimal system design, especially within the context of multi-objective de novo programming (MODNP), can be generally outlined as follows:

Step 1: Define the System and Objectives.
Step 2: Identify Available Resources and Their Costs**.**
Step 3: Formulate the Multi-Objective Problem**.**
Step 4: Determine the Budget Constraint for Each Sub-problem.
Step 5: Solve Each Sub-problem as a Single-Objective Optimization Problem (often via scalarization)
Step 6: Identify the Optimal System Design for Each Sub-problem.
Step 7: Present the Results to the Decision-Maker [IX].

ii) Optimum-Path Ratio Method

The optimum-path ratio for achieving the best performance for a given budget $\boldsymbol{B}$ is defined as: $\boldsymbol{\gamma_1} = \frac{\beta}{\beta^*}$ the given budget level $\leq \beta^*$. Optimal system design for $\boldsymbol{B}$: $Y = \boldsymbol{\gamma}_1 Y^*$, $b = \boldsymbol{\gamma}_1 b^*$, $Z = \boldsymbol{\gamma}_1 f^*$, the optimum-path ratio represents an effective and fast tool for the efficient optimal redesign of large-scale linear systems [IV].

It is possible to define six types of optimum-path ratios as shown in Table 1.:

Table1. Six types of optimum-path ratios

Ratio (1)	Ratio (2)	Ratio (3)	Ratio (4)	Ratio (5)	Ratio (6)
$\boldsymbol{\gamma_1} = \frac{\boldsymbol{\beta}}{\boldsymbol{\beta^*}}$	$\gamma_2 = \frac{\beta}{\beta^{**}}$	$\gamma_3 = \frac{\sum_i \alpha_i \beta_i^j}{\beta^{**}}$	$\gamma_4 = \frac{\beta}{\beta^{**}}$	$\gamma_5 = \frac{\sum_i \alpha_i \beta_i^j}{\beta^*}$	$\gamma_6 = \frac{\sum_i \alpha_i \beta_i^j}{\beta^{**}}$

Optimal System Design

A series of optimal system designs can be determined by examining the design configurations, which are discoverable through the optimum-path ratios listed in Table 1.

(i) $y^1 = \boldsymbol{\gamma}^1 x^{**}, \quad b^1 = \boldsymbol{\gamma}^1 b^{**} \quad and\ f^1 = \boldsymbol{\gamma}^1 f^{**}$ (6a)

(ii) $y^2 = \boldsymbol{\gamma}^2 x^{**}, \quad b^2 = \boldsymbol{\gamma}^2 b^{**} \quad and\ f^2 = \boldsymbol{\gamma}^2 f^{**}$ (6b)

(iii) $y^3 = \boldsymbol{\gamma}^3 x^{**}, \quad b^3 = \boldsymbol{\gamma}^3 b^{**} \quad and\ f^3 = \boldsymbol{\gamma}^3 f^{**}$ (6c)

(iv) $y^4 = \boldsymbol{\gamma}^4 x^{*}, \quad b^4 = \boldsymbol{\gamma}^4 b^{*} \quad and\ f^4 = \boldsymbol{\gamma}^4 f^{*}$ (6d)

(v) $y^5 = \boldsymbol{\gamma}^5 x^{*}, \quad b^5 = \boldsymbol{\gamma}^5 b^{*} \quad and\ f^5 = \boldsymbol{\gamma}^5 f^{*}$ (6e)

(vi) $y^6 = \boldsymbol{\gamma}^6 x^{nd}, \quad b^6 = \boldsymbol{\gamma}^6 b^{nd} \quad and\ f^6 = \boldsymbol{\gamma}^6 f^{nd}$ (6f)

The optimum system design above (y^i, b^i, f^i), i=1,….,6, Where: b^i: optimum portfolio of resources to be acquired at the current market prices, p, allows one to produce x^i and realize the multi-criteria performance f^i [IX].

III. Method for converting proposed model (RIMODNP) to MODNP

In this section, two methods are used to convert the uncertainty proposed mathematical model (RIMODNP) into four deterministic sub-problems (MODNP),

The Separation Method

Separation method (SM) is one of the methods that is used to convert the main model into two sub-models: (the lower model and the upper model) and then solve each sub-model separately. The results corresponding to the sub-model are $\overline{f}^{*U}$,$\underline{f}^{*L}$, and $\underline{x}_{ij}^{*}, \overline{x}_{ij}^{*}$ respectively.

The steps of the method can be summarized as follows [VI],[XI]:

Step 1: Converting the total interval into two sub-models.

Step 2: Solve the lower interval (the first bound) to get an optimal solution $\underline{x}_{ij}^{*}$, and optimal value $\underline{f}^{*L}$.

Step 3: Solve the upper interval (the second bound) to get an optimal solution $\overline{x}_{ij}^{*}$, and optimal value $\overline{f}^{*U}$.

Step 4: The optimal solution of the major problem is $X^{*I} = \left[\underline{x}_{ij}^{*}, \overline{x}_{ij}^{*}\right]$, $f^{*I} = [\underline{f}^{*L}, \overline{f}^{*U}]$.

i) Tong-Shaocheng Method (TSM)

TSM is a major important method used to obtain the best and the worst optimal value for the objective function. In this method, the main model is converted into two classical sub-models, LP (lower sub-model and the upper sub-model).

The process of converting to obtain the best and the worst values of the objective function can be summarized as follows [II], [VII]:

a) By solving sub-model (7) below to find "The Worst Optimal Solution"

$Max\ or\ Min\ \underline{f} = \sum_{j=1}^{n} \underline{c}_j\, x_j$

$subject\ to$: $\sum_{j=1}^{n} \overline{a}_{ij}\, x_j \ \leq = \geq [\underline{b}_i]$ (7)

$x_j \ \geq 0,\ j = 1,2, \dots, n, \quad i = 1,2, \dots, m$

b) By solving sub-model (8) below to find "The Best Optimal Solution"

$Max\ or\ Min\ \ \overline{f} = \sum_{j=1}^{n} \overline{c}_j\, x_j$

$subject\ to$: $\sum_{j=1}^{n} \underline{a}_{ij}\, x_j \ \leq = \geq [\overline{b}_i]$ (8)

$x_j \ \geq 0,\ j = 1,2, \dots, n, \quad i = 1,2, \dots, m$

And then solving each sub-model separately. The sub-model (7) has three possible solutions, as follows:

- The sub-model (7) has a Finite, bounded optimal range, if sub-model (7) and sub-model (8) have optimal solutions.
- Sub-model (7) is unbounded, then sub-model (8) is unbounded.
- Sub-model (7) is infeasible, then sub-model (8) is infeasible.

This method used for solving the problem to find the best and the worst optimal value for objective function, the major problem converted into two classical sub-problems LP (Lower problem and Upper problem) the steps of method can be summarized as follows: an operations converting to obtain of the best and the worst values of the objective function is as the following: [I]

By solving below problem (9) to find "The Worst Optimal Solution":

$Max\ or\ Min\ \ Z = \sum_{j=1}^{n} \underline{c}_j\, y_j$

$subject\ to$: $\sum_{j=1}^{n} \overline{a}_{ij}\, y_j \ \leq = \geq \underline{b}_i$ (9)

$y_j \ \geq 0,\ j = 1,2, \dots, n, \quad i = 1,2, \dots, m$

By solving below problem (10) to find "The Best Optimal Solution":

$Max\ or\ Min\ \ \overline{Z} = \sum_{j=1}^{n} \overline{c}_j\, y_j$

$subject\ to$: $\sum_{j=1}^{n} \underline{a}_{ij}\, y_j \ \leq = \geq [\overline{b}_i]$ (10)

$y_j \ \geq 0,\ j = 1,2, \dots, n, \quad i = 1,2, \dots, m$

and then solve each problem separately. The problem (9) has three possible solutions, as follows:

- Problem (9) has a Finite, bounded optimal range, if problem (9) and problem (10) have optimal solutions.
- If Problem (9) is unbounded, then Problem (10) is unbounded.
- If Problem (9) is infeasible, then Problem (10) is infeasible.

- **Approaches for Transforming RIMODNP into MODNP**

In order to transform the model RIMODNP into four sub-models, two methods **(SM & TSM)** will be used to achieve this purpose. Fig.1 represents the general flowchart for applying these methods.

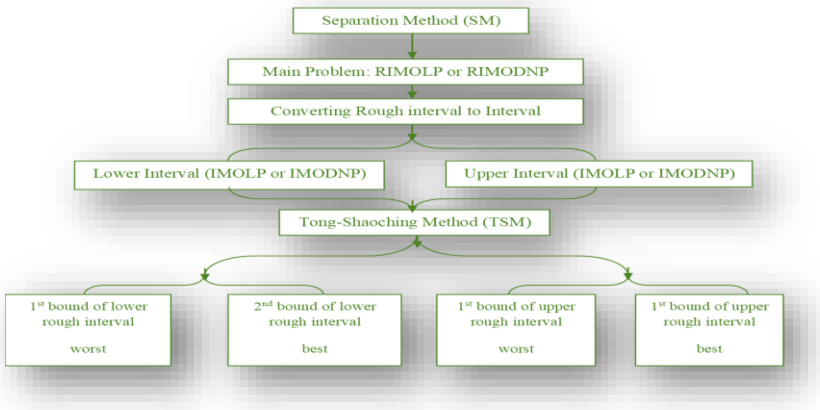

Fig 1. General flowchart to convert (RIMODNP) into (MODNP)

The solution steps of the proposed mathematical model

Fig2. Illustrates the formulation and solution procedures for the mathematical model presented in this study.

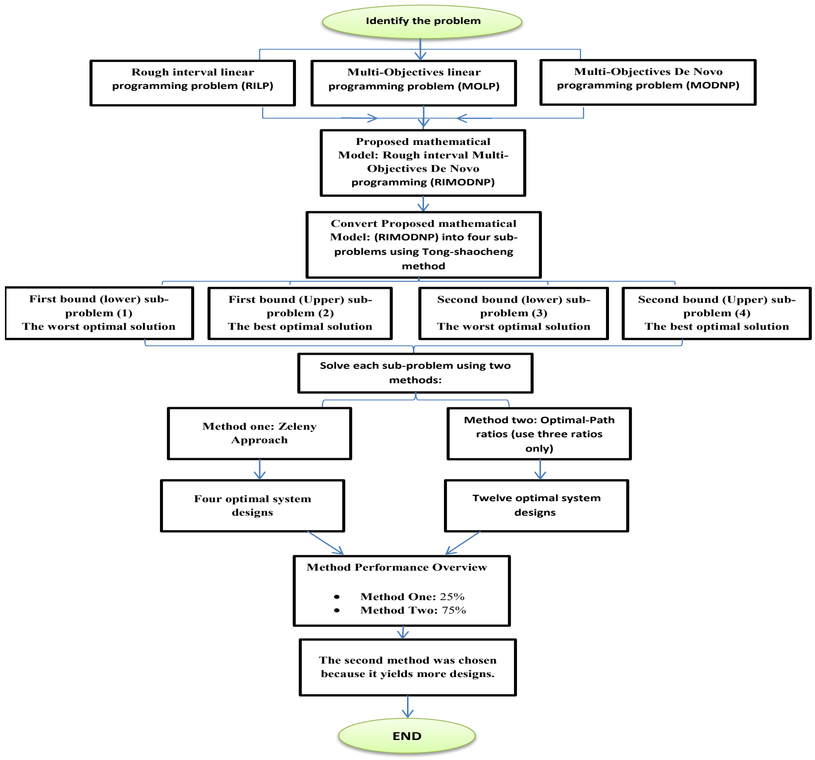

Fig. 2. The solution steps of the proposed mathematical model

III. Results and Discussion

Application Proposed mathematical Model: Real Case Study (Baghdad Water Department) the model (RIMODNP) is applied on drinking water filtration stations for the city of Baghdad, Fig.3. the distribution of water stations in Baghdad according to Municipalities: (AL-Karkh & AL-Rusafa)

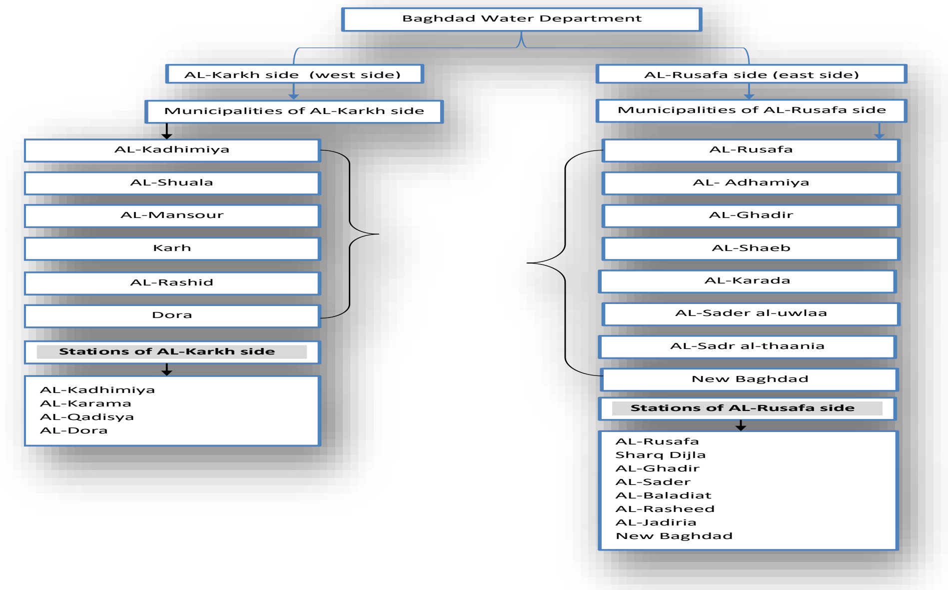

Fig.3 General Flowchart for distribution water stations in Baghdad

IV. Formulation general mathematical model (RIMODNP) of the case study:

This model was used to solve a case study, assuming the right-hand side of the constraints b_i is unknown. The general mathematical model (RIMODNP) (11), which was derived previously, can be applied to the case study as follows:

$$\boldsymbol{Max}\ \boldsymbol{f}^{(1)}(\boldsymbol{x}) = [850,890],[875,925]x_1 + [560,654],[620,675]x_2 + [615,650],[640,700]x_3 + [47,70],[66,80]x_4 + [86,100],[88,120]x_5 + [42,66],[49,75]x_6 + [55,66],[60,75]x_7 + [36,49],[38,53]x_8 + [28,39],[29,42]x_9 + [37.5,47],[38,50]x_{10} + [25,34.5],[28,41]x_{11} + [59,71.5],[67,89]x_{12} + [34.5,67],[45,77]x_{13}$$

$$\boldsymbol{Max}\ \boldsymbol{f}^{(2)}(\boldsymbol{x}) = [2.1,3.2],[2.8,3.5]x_1 + [2.5,4],[3.5,5]x_2 + [1.7,2.7],[2.5,4]x_3 + [1.9,4],[3.5,5]x_4 + [2.2,4],[3.5,4.5]x_5 + [2.9,3.6],[3,4.2]x_6 + [2.6,3.8],[2.9,4.5]x_7 + [1.6,2.9],[2.1,3.9]x_8 + [2.8,3.9],[2.9,4.2]x_9 + [3.2,3.7],[3.5,4]x_{10} + [2.5,3.4],[2.8,4.1]x_{11} + [2.9,4.1],[3.4,4.7]x_{12} + [3.4,4.4],[3.5,4.8]x_{13}$$

$$\boldsymbol{Min}\ \boldsymbol{f}^{(3)}(\boldsymbol{x}) = -[212,222.5],[218.75,231.25]x_1 - [140,163.5],[155,168.5]x_2 - [153.75,162.5],[160,175]x_3 - [11.75,17.5],[16.2,20]x_4 - [21.5,25],[22,30]x_5 - [10.5,16.5],[12.5,18.75]x_6 - [13.75,16.5],[15,18.75]x_7 - [9,12.25],[9.5,13.25]x_8 - [7,9.75],[7.25,10.5]x_9 - [9.25,11.75],[9.5,12.5]x_{10} - [6.25,8.625],[7,10.5]x_{11} - [14.75,17.875],[16.75,22.25]x_{12} - [8.625,16.75],[11.25,19.25]x_{13}$$

s.to

$[0.75,4.375],[1,6.25]x_1 +$
$[0.933,2.141],[1.68,3.175]x_2 + [0.775,1.125],[1.1,1.55]x_3 + [0.225,0.391],[0.3,0.4]x_4 + [0.266,0.333],[0.287,0.433]x_5 + [0.158,0.233],[0.191,0.55]x_6 +$
$[0.083,0.091],[0.083,0.091]x_7 + [0.017,0.066],[0.033,0.075]x_8 +$
$[0.25,0.33],[0.29,0.41]x_9 + [0.133,0.19],[0.15,0.208]x_{10} +$
$[0.308,0.425],[0.317,0.5]x_{11} + [0.191,0.275],[0.266,0.35]x_{12} +$
$[0.22,0.26],[0.24,0.28]x_{13} \le b_1$ (The amount of Alum consumed Ton/ $\boldsymbol{m^3}$ of water per day)

$[1,1.75],[1.4,2]x_1 + [1,1.6],[1.25,1.9]x_2 + [1.5,2.6],[1.85,3]x_3 + [0.1,0.5],[0.35,0.95]x_4 +$
$[0.175,0.3],[0.2,0.45]x_5 + [0.13,0.16],[0.14,0.18]x_6 + [0.13,0.15],[0.14,0.19]x_7 +$
$[0.14,0.16],[0.15,0.19]x_8 + [0.125,0.16],[0.145,0.2]x_9 + [0.2,0.35],[0.26,0.41]x_{10} +$
$[0.21,0.33],[0.27,0.4]x_{11} + [0.1,0.14],[0.125,0.18]x_{12} + [0.075,0.13],[0.09,0.2]x_{13} \le b_2$
(The amount of Chlorine consumed Ton/ $\boldsymbol{m^3}$ of water per day)

$[10,25],[16,30]x_1 + [12,96],[72,106]x_2 +$
$[10,22],[16,25]x_3 + [42,55],[50,60]x_4 + [28,87],[77,116]x_5 + [22,29],[25,38]x_6 +$
$[26,54],[44,63]x_7 + [50,129],[110,183]x_8 + [25,183],[163,282]x_9 +$
$[84,210],[173,224]x_{10} + [94,262],[210,270]x_{11} + [29,85],[45,154]x_{12} +$
$[48,60],[52,72]x_{13} \le b_3$ (The number of daily examinations: (Chlorine, Turbidity, Bacteriology, Hardness, conductivity, chemical)

$[1200,2500],[2000,4320]x_1 + [955,1500],[1100,2333]x_2 +$
$[894,1440],[1250,2055]x_3 + [164,250],[220,300]x_4 + [115,180],[155,250]x_5 +$
$[107,150],[115,210]x_6 + [94,120],[99,200]x_7 + [70,95],[85,100]x_8 +$
$[65,83],[75,95]x_9 + [94,125],[117,210]x_{10} +$
$[62,87],[73,180]x_{11} + [110,185],[166,225]x_{12} + [160,225],[185,310]x_{13} \le b_4$ (The amount of fuel consumed /Liters)

$[1,3],[2,4]x_1 + [1,2],[2,3]x_2 + [5,9],[7,15]x_3 + [1,2],[2,3]x_4 + [1,2],[2,3]x_5 +$
$[2,4],[3,5]x_6 + [1,2],[2,3]x_7 + [1,2],[2,3]x_8 + [2,4],[3,6]x_9 + [1,1],[1,1]x_{10} +$
$[1,1],[1,1]x_{11} + [1,1],[1,1]x_{12} + [2,4],[3,6]x_{13} \le b_5$ (The number of contracts)

$[23,23],[23,23]x_1 + [18.2,18.2],[18.2,18.2]x_2 + [17,17],[17,17]x_3 + [2,2],[2,2]x_4 +$
$[3,3],[3,3]x_5 + [2.2,2.2],[2.2,2.2]x_6 + [1.8,1.8],[1.8,1.8]x_7 + [1.2,1.2],[1.2,1.2]x_8 +$
$[1.3,1.3],[1.3,1.3]x_9 + [1.8,1.8],[1.8,1.8]x_{10} + [1.16,1.16],[1.16,1.16]x_{11} +$
$[2,2],[2,2]x_{12} + [3,3],[3,3]x_{13} \le b_6$ (The design capacity of the water production stations, measured m^3 /day)

$\left[\underline{P}_1^L,\underline{P}_1^U\right],\left[\overline{P}_1^L,\overline{P}_1^U\right]b_1 + \left[\underline{P}_2^L,\underline{P}_2^U\right],\left[\overline{P}_2^L,\overline{P}_2^U\right]b_2 + \left[\underline{P}_3^L,\underline{P}_3^U\right],\left[\overline{P}_3^L,\overline{P}_3^U\right]b_3 + \left[\underline{P}_4^L,\underline{P}_4^U\right],\left[\overline{P}_4^L,\overline{P}_4^U\right]b_4 +$
$\left[\underline{P}_5^L,\underline{P}_5^U\right],\left[\overline{P}_5^L,\overline{P}_5^U\right]b_5 + \left[\underline{P}_6^L,\underline{P}_6^U\right],\left[\overline{P}_6^L,\overline{P}_6^U\right]b_6 \le ([\underline{\beta}^L,\underline{\beta}^U],[\overline{\beta}^L,\overline{\beta}^U])$

$b_1 \ge 0, b_2 \ge 0, b_3 \ge 0, b_4 \ge 0, b_5 \ge 0, and\ b_6 \ge 0$
$x_1 \ge 0, x_2 \ge 0, x_3 \ge 0, x_4 \ge 0, x_5 \ge 0, x_6 \ge 0, x_7 \ge 0, x_8 \ge 0, x_9 \ge 0, x_{10} \ge 0, x_{11} \ge 0, x_{12} \ge 0, \& x_{13} \ge 0$ (11)

Transforming Model RIMODNP to MODNP

Transforming RIMODNP into IMODNP sub-problems using the SM method, and then converting IMODNP into four sub-problems using the Tong-Shaoching method, problem (11) was used to divided into four sub-problems were named:

- Sub-problem (12) 1st bound of lower rough interval.
- Sub-problem(13) 2nd bound of lower rough interval
- Sub-problem (14) 1st bound of upper rough interval

- Sub-problem (15) 2nd bound of upper rough interval

Results of Zeleny approach

The optimal system design for the initial lower bound of the rough interval problem in RIMODNP was determined using Zeleny's approach. This involved the budget constraint for sub-problem (12) being calculated, and the existing budget constraint was then replaced by the specific constraints of sub-problem (12).

The same methodology was applied to find the optimal system design for **sub-problems (13, 14, and 15)**. The results from applying Zeleny's approach to these sub-problems are presented in **Table 2**.

Table 2: Results from Zeleny's Approach for Sub-problems 12, 13, 14, and 15

	Optimal system design					
1st bound of lower rough interval Sub-problem 12		400.7102	$\underline{x}_1^{2L}$	274.0547	$\underline{x}_3^{3L}$	105.6345
	$\underline{b}_1^{1L}$	857.519828	$\underline{b}_1^{2L}$	1200.359586	$\underline{b}_1^{3L}$	132.043125
	$\underline{b}_2^{1L}$	641.13632	$\underline{b}_2^{2L}$	479.595725	$\underline{b}_2^{3L}$	274.6497
	$\underline{b}_3^{1L}$	38468.1792	$\underline{b}_3^{2L}$	10017.755	$\underline{b}_3^{3L}$	2323.959
	$\underline{b}_4^{1L}$	601065.3	$\underline{b}_4^{2L}$	1001775.5	$\underline{b}_4^{3L}$	152113.68
	$\underline{b}_5^{1L}$	801.4204	$\underline{b}_5^{2L}$	822.1641	$\underline{b}_5^{3L}$	950.7105
	$\underline{b}_6^{1L}$	7292.92564	$\underline{b}_6^{2L}$	6303.2581	$\underline{b}_6^{3L}$	1795.7865
	$\underline{f}_2^{1L}$	1001.7755	$\underline{f}_1^{2L}$	232946.495	$\underline{f}_3^{3L}$	16162.0785
	$\underline{\beta}_2^{1L}$	39999988	$\underline{\beta}_1^{2L}$	39999996	$\underline{\beta}_3^{3L}$	39999953
	$\underline{\beta}^{L}$	40000000				
	Optimal system design					
2nd bound of lower rough interval Sub-problem 13	$\underline{x}_2^{1U}$	797.5087	$\underline{x}_1^{2U}$	747.4105	$\underline{x}_3^{3U}$	205.1924
	$\underline{b}_1^{1U}$	744.0756	$\underline{b}_1^{2U}$	560.5512	$\underline{b}_1^{3U}$	191.4445
	$\underline{b}_2^{1U}$	797.5087	$\underline{b}_2^{2U}$	747.4016	$\underline{b}_2^{3U}$	307.7886
	$\underline{b}_3^{1U}$	9570.104	$\underline{b}_3^{2U}$	7474.016	$\underline{b}_3^{3U}$	2051.924
	$\underline{b}_4^{1U}$	761620.8	$\underline{b}_4^{2U}$	896881.9	$\underline{b}_4^{3U}$	183442
	$\underline{b}_5^{1U}$	797.5087	$\underline{b}_5^{2U}$	747.4016	$\underline{b}_5^{3U}$	1025.962
	$\underline{b}_6^{1U}$	14514.66	$\underline{b}_6^{2U}$	17190.24	$\underline{b}_6^{3U}$	3488.271
	$\underline{f}_2^{1U}$	3190.0348	$\underline{f}_1^{2U}$	665187.335	$\underline{f}_3^{3U}$	33343.765
	$\underline{\beta}_2^{1U}$	54999995	$\underline{\beta}_1^{2U}$	54999996	$\underline{\beta}_3^{3U}$	54999972
	$\underline{\beta}^{U}$	55000000				
	Optimal system design					
1st bound of upper rough interval	$\overline{x}_2^{1L}$	2395.0288	$\overline{x}_1^{2L}$	1802.2726	$\overline{x}_3^{3L}$	2591.8822
	$\overline{b}_1^{1L}$	7604.216	$\overline{b}_1^{2L}$	11264.20375	$\overline{b}_1^{3L}$	4017.41741

	$\bar{b}_2^{1L}$	4550.555	$\bar{b}_2^{2L}$	3604.5452	$\bar{b}_2^{3L}$	7775.6466
	$\bar{b}_3^{1L}$	253873.1	$\bar{b}_3^{2L}$	54068.178	$\bar{b}_3^{3L}$	64797.055
	$\bar{b}_4^{1L}$	5587602	$\bar{b}_4^{2L}$	7785817.632	$\bar{b}_4^{3L}$	5326317.921
	$\bar{b}_5^{1L}$	7185.086	$\bar{b}_5^{2L}$	7209.0904	$\bar{b}_5^{3L}$	38878.233
	$\bar{b}_6^{1L}$	43589.52	$\bar{b}_6^{2L}$	41452.2698	$\bar{b}_6^{3L}$	44061.9974
	$\bar{f}_2^{1L}$	8382.6008	$\bar{f}_1^{2L}$	1576988.525	$\bar{f}_3^{3L}$	-414701.152
	$\bar{\beta}_2^{1L}$	49999998.37	$\bar{\beta}_1^{2L}$	49999998.17	$\bar{\beta}_3^{3L}$	49999999.52
	$\bar{\beta}^{L}$	50000000				
Optimal system design						
2nd bound of upper rough interval Sub-problem 15	$\bar{x}_2^{1U}$	322.3201	$\bar{x}_1^{2U}$	241.5151	$\bar{x}_3^{3U}$	70.6547
	$\bar{b}_1^{1U}$	1023.366	$\bar{b}_1^{2U}$	1509.469	$\bar{b}_1^{3U}$	109.5149
	$\bar{b}_2^{1U}$	612.4082	$\bar{b}_2^{2U}$	483.0302	$\bar{b}_2^{3U}$	211.9642
	$\bar{b}_3^{1U}$	34165.93	$\bar{b}_3^{2U}$	7245.453	$\bar{b}_3^{3U}$	1766.369
	$\bar{b}_4^{1U}$	751972.8	$\bar{b}_4^{2U}$	1043345	$\bar{b}_4^{3U}$	145195.5
	$\bar{b}_5^{1U}$	966.9603	$\bar{b}_5^{2U}$	966.0604	$\bar{b}_5^{3U}$	1059.821
	$\bar{b}_6^{1U}$	5866.226	$\bar{b}_6^{2U}$	5554.847	$\bar{b}_6^{3U}$	1201.131
	$\bar{f}_2^{1U}$	1128.12	$\bar{f}_1^{2U}$	211325.7	$\bar{f}_3^{3U}$	11304.8
	$\bar{\beta}_2^{1U}$	649999981.92	$\bar{\beta}_1^{2U}$	64999985.3	$\bar{\beta}_3^{3U}$	64999957.07
	$\bar{\beta}^{U}$	65000000				

Table 2 presents the results of **Zeleny's approach**, showcasing the **optimal system design** for each sub-problem within the main problem (11). These results provide the decision-maker (DM) at the Baghdad Water Department with four distinct designs, allowing them to select the most suitable option.

- **Results of optimal-path ratios Method**

Optimal system design is achieved through the use of optimal path-ratios, which necessitates the following:

Sub-problem Ratios: The ratios for each sub-problem are calculated using the formulas provided in Table 1; the results are displayed in Table 3.

Table 3: Ratios for each sub-problem

Ratios	1st bound of lower rough interval sub-problem (12)	2nd bound of lower rough interval sub-problem (13)	1st bound of upper rough interval sub-problem (14)	2nd bound of upper rough interval sub-problem (15)
r^1	0.370195177	0.376027359	0.414136627	0.341815309
r^2	0.33333348	0.333333392	0.333333342	0.333333463
r^3	0.332999972	0.332999984	0.332999991	0.333

Optimal systems design for the optimal path-ratios:

After getting the values of ratios for each sub-problem, Optimal system designs are obtained by applying formulas (6a,6b, and 6c), the results are shown in Tables (4,5, 6, and 7):

Table 4: Designs for the 1st Lower Rough Interval Bound of Sub-problem (12)

Design 1		**Design 2**		**Design 3**	
$\underline{r}^{1L}$		$\underline{r}^{2L}$		$\underline{r}^{3L}$	
0.370195177		0.33333348		0.332999972	
$\underline{x}^{1L}$		$\underline{x}^{2L}$		$\underline{x}^{3L}$	
$\underline{x}_1^{2L**}$	148.3409834	$\underline{x}_1^{2L**}$	133.5700725	$\underline{x}_1^{2L**}$	133.4364854
$\underline{x}_2^{1L**}$	101.4537282	$\underline{x}_2^{1L**}$	91.35157069	$\underline{x}_2^{1L**}$	91.26020743
$\underline{x}_3^{3L**}$	39.10538242	$\underline{x}_3^{3L**}$	35.21150155	$\underline{x}_3^{3L**}$	35.17628554
$\underline{b}^{1L}$		$\underline{b}^{2L}$		$\underline{b}^{3L}$	
$\underline{b}_1^{1L**}$	810.6987619	$\underline{b}_1^{1L**}$	729.9745009	$\underline{b}_1^{1L**}$	729.2441442
$\underline{b}_2^{1L**}$	516.5635921	$\underline{b}_2^{1L**}$	465.127453	$\underline{b}_2^{1L**}$	464.662082
$\underline{b}_3^{1L**}$	17637.39603	$\underline{b}_3^{1L**}$	15881.17555	$\underline{b}_3^{1L**}$	15865.28606
$\underline{b}_4^{1L**}$	532457.5462	$\underline{b}_4^{1L**}$	479438.7876	$\underline{b}_4^{1L**}$	478959.0978
$\underline{b}_5^{1L**}$	952.9915932	$\underline{b}_5^{1L**}$	858.0987109	$\underline{b}_5^{1L**}$	857.2401629
$\underline{b}_6^{1L**}$	5698.033147	$\underline{b}_6^{1L**}$	5130.659004	$\underline{b}_6^{1L**}$	5125.525659
$\underline{f}^{1L}$		$\underline{f}^{2L}$		$\underline{f}^{3L}$	
$\underline{f}^{1L**}$	623.0106701	$\underline{f}^{1L**}$	560.9752035	$\underline{f}^{1L**}$	560.4139346
$\underline{f}^{2L**}$	169345.725	$\underline{f}^{2L**}$	152483.3476	$\underline{f}^{2L**}$	152330.7844
$\underline{f}^{3L**}$	42365.76032	$\underline{f}^{3L**}$	38147.24555	$\underline{f}^{3L**}$	38109.07833

Based on the calculations using the Optimal Path-Ratios method, six optimal system designs were derived for the first bound of the lower rough interval in sub-problem (12), as presented in Table 9. In the first design, the production capacities of the three drinking water purification projects, Al-Karkh, Al-Rusafa, and Sharq-Dijla water stations, increased by 148.34, 101.45, and 39.11 thousand cubic meters, respectively. This increase required resource allocations of 810.70, 516.56, 17,637.40, 532,457.55, 952.99, and 5,698.03 units. As a result, the design achieved a profit increase of 623.01 DI per 1000 cubic meters of water produced. Additionally, improvements in water quality contributed to a profit of 169,345.73 DI per unit, while reducing water loss led to cost savings of 42,365.76 DI per 1000 cubic meters.

Throughout the six optimal designs obtained for sub-problem (12), the first, second, and third designs corresponding to the weight ratios 0.3702, 0.3333, and 0.3330 demonstrated significant improvements across all three projects mentioned above. These designs achieved profit increases ranging from 560.41 to 623.01 DI. In terms of water quality enhancement, the gains ranged from 152,330.78 to 169,345.73 DI, and the reduction in water loss resulted in cost savings between 38,109.08 and 42,365.76 DI.

For the second bound of the lower rough interval in sub-problem (13), the results obtained using the Optimal Path-Ratios method are presented in Table 5.

Table 5. Designs for 2nd Lower Rough Interval Bound of Sub-problem (13)

Design 1		Design 2		Design 3	
$\underline{r}^{1U}$		$\underline{r}^{2U}$		$\underline{r}^{3U}$	
0.376027359		0.333333392		0.332999984	
$\underline{x}^{1U}$		$\underline{x}^{2U}$		$\underline{x}^{3U}$	
$\underline{x}_1^{2U**}$	281.046834	$\underline{x}_1^{2U**}$	249.1373051	$\underline{x}_1^{2U**}$	248.8877178
$\underline{x}_2^{1U**}$	299.8850902	$\underline{x}_2^{1U**}$	265.8367012	$\underline{x}_2^{1U**}$	265.5703843
$\underline{x}_3^{3U**}$	77.15795626	$\underline{x}_3^{3U**}$	68.39758705	$\underline{x}_3^{3U**}$	68.32906592
$\underline{b}^{1U}$		$\underline{b}^{2U}$		$\underline{b}^{3U}$	
$\underline{b}_1^{1U**}$	562.5662878	$\underline{b}_1^{1U**}$	498.6935697	$\underline{b}_1^{1U**}$	498.1939754
$\underline{b}_2^{1U**}$	696.6688586	$\underline{b}_2^{1U**}$	617.5703869	$\underline{b}_2^{1U**}$	616.9517011
$\underline{b}_3^{1U**}$	7180.668986	$\underline{b}_3^{1U**}$	6365.389336	$\underline{b}_3^{1U**}$	6359.01245
$\underline{b}_4^{1U**}$	692625.6748	$\underline{b}_4^{1U**}$	613986.2586	$\underline{b}_4^{1U**}$	613371.1634
$\underline{b}_5^{1U**}$	966.7217055	$\underline{b}_5^{1U**}$	856.9619416	$\underline{b}_5^{1U**}$	856.1034318
$\underline{b}_6^{1U**}$	13233.67108	$\underline{b}_6^{1U**}$	11731.14496	$\underline{b}_6^{1U**}$	11719.39263
$\underline{f}^{1U}$		$\underline{f}^{2U}$		$\underline{f}^{3U}$	
$\underline{f}^{1U**}$	496409.2029	$\underline{f}^{1U**}$	440047.8358	$\underline{f}^{1U**}$	439606.9931
$\underline{f}^{2U**}$	2307.216712	$\underline{f}^{2U**}$	2045.259666	$\underline{f}^{2U**}$	2043.210712
$\underline{f}^{3U**}$	124102.3007	$\underline{f}^{3U**}$	110011.959	$\underline{f}^{3U**}$	109901.7483

After applying the Optimal Path-Ratios method, six optimal system designs were obtained for Problem (13), as presented in Table 10. In Design 1, under the ratio of 0.376027359, the production capacities increased significantly across all three water treatment stations. Specifically, the Al-Karkh Water Station increased by 281.05 thousand m³/day, the Al-Rusafa Water Station by 299.89 thousand m³/day, and the Sharq-Dijla Water Station by 77.16 thousand m³/day. Achieving this required the allocation of resources in the amounts of 562.57, 696.67, 7,180.67, 692,625.67, 966.72, and 13,233.67 units across various resource categories.

As a result, the design met several objectives. The first objective, increasing water production capacity per 1000 m³, generated a profit of 496,409.20 DI. The second objective, enhancing water quality, resulted in profits of 2,307.22 DI. Regarding the third objective, reducing water loss contributed to a cost reduction of 124,102.30 DI per 1000 m³.

In Designs 2 and 3, associated with ratios 0.3333333392 and 0.332999984, it was observed that the values of the ratios—and consequently, the resulting designs—were nearly identical. The difference between the two designs is minimal and practically negligible.

For the first bound of the upper rough interval in Problem (14), the corresponding optimal system designs are displayed in Table 6. Below:

able 6: Designs for the 1st upper Rough Interval Bound of Sub-problem (14)

Design 1		Design 2		Design 3	
$\bar{r}^{1L}$		$\bar{r}^{2L}$		$\bar{r}^{3L}$	
0.414136627		0.333333342		0.332999991	
$\bar{x}^{1L}$		$\bar{x}^{2L}$		$\bar{x}^{3L}$	
$\bar{x}_1^{2L**}$	746.3870955	$\bar{x}_1^{2L**}$	600.757549	$\bar{x}_1^{2L**}$	600.1567596
$\bar{x}_2^{1L**}$	991.8691488	$\bar{x}_2^{1L**}$	798.3429541	$\bar{x}_2^{1L**}$	797.5445688
$\bar{x}_3^{3L**}$	1073.393352	$\bar{x}_3^{3L**}$	863.9607558	$\bar{x}_3^{3L**}$	863.0967493
$\bar{b}^{1L}$		$\bar{b}^{2L}$		$\bar{b}^{3L}$	
$\bar{b}_1^{1L**}$	9477.86359	$\bar{b}_1^{1L**}$	7628.612732	$\underline{b}_1^{1U**}$	7620.983715
$\bar{b}_2^{1L**}$	6597.505629	$\bar{b}_2^{1L**}$	5310.248978	$\underline{b}_2^{1U**}$	5304.938448
$\bar{b}_3^{1L**}$	154364.5764	$\bar{b}_3^{1L**}$	124246.0985	$\underline{b}_3^{1U**}$	124121.8458
$\bar{b}_4^{1L**}$	7744246.313	$\bar{b}_4^{1L**}$	6233246.075	$\underline{b}_4^{1U**}$	6227012.499
$\bar{b}_5^{1L**}$	22062.05611	$\bar{b}_5^{1L**}$	17757.4704	$\underline{b}_5^{1U**}$	17739.71198
$\bar{b}_6^{1L**}$	53466.6087	$\bar{b}_6^{1L**}$	43034.59825	$\underline{b}_6^{1U**}$	42991.56137
$\bar{f}^{1L}$		$\bar{f}^{2L}$		$\bar{f}^{3L}$	
$\bar{f}_1^{2L**}$	1955019.326	$\bar{f}_1^{2L**}$	1573570.371	$\bar{f}_1^{2L**}$	1571996.717
$\bar{f}_2^{1L**}$	8244.909268	$\bar{f}_2^{1L**}$	6636.223366	$\bar{f}_2^{1L**}$	6629.586791
$\bar{f}_3^{3L**}$	488754.8314	$\bar{f}_3^{3L**}$	393392.5926	$\bar{f}_3^{3L**}$	392999.1791

The results of solving sub-problem (14) using the Optimal Path-Ratios method, as presented in Table 11, reveal six optimal system designs. The first design, corresponding to a ratio of 0.414136627, demonstrates notable increases in production capacity across the three major water treatment stations. Specifically, the Al-Karkh station increases by 746.38 thousand m³/day, the Al-Rusafa station by 991.87 thousand m³/day, and the Sharq-Dijla station by 1,073.39 thousand m³/day. Achieving these enhancements requires the allocation of key resources in the following amounts: 9,477.86; 6,597.51; 154,364.58; 7,744,246.31; 22,062.06; and 53,466.61 units across various input categories.
As a result, the design meets three primary objectives:

- Increased water production capacity, yielding a profit of 1,955,019.33 DI per 1,000 m³;
- Improved water quality, generating a profit of 8,244.91 DI;
- Reduced water loss, leading to cost savings of 488,754.83 DI per 1,000 m³.

Table 7: Designs for the 2nd upper Rough Interval Bound of Sub-problem (15)

Design 1		Design 2		Design 3	
$\bar{r}^{1U}$		$\bar{r}^{2U}$		$\bar{r}^{3U}$	
0.34181531		0.33333346		0.333	
$\bar{x}^{1U}$		$\bar{x}^{2U}$		$\bar{x}^{3U}$	
$\bar{x}_1^{2U**}$	110.1739449	$\bar{x}_1^{2U**}$	107.4400742	$\bar{x}_1^{2U**}$	107.3325933
$\bar{x}_2^{1U**}$	73.32454541	$\bar{x}_2^{1U**}$	71.50506051	$\bar{x}_2^{1U**}$	71.4335283
$\bar{x}_3^{3U**}$	24.15085818	$\bar{x}_3^{3U**}$	23.55157562	$\bar{x}_3^{3U**}$	23.5280151
$\bar{b}^{1U}$		$\bar{b}^{2U}$		$\bar{b}^{3U}$	
$\bar{b}_1^{1U**}$	903.1958478	$\bar{b}_1^{1U**}$	880.7838274	$\bar{b}_1^{1U**}$	879.9027092
$\bar{b}_2^{1U**}$	446.8901874	$\bar{b}_2^{1U**}$	435.8009956	$\bar{b}_2^{1U**}$	435.3650292
$\bar{b}_3^{1U**}$	14758.81638	$\bar{b}_3^{1U**}$	14392.58917	$\bar{b}_3^{1U**}$	14378.19112
$\bar{b}_4^{1U**}$	663297.201	$\bar{b}_4^{1U**}$	646838.0571	$\bar{b}_4^{1U**}$	646190.9735
$\bar{b}_5^{1U**}$	1022.998943	$\bar{b}_5^{1U**}$	997.6141124	$\bar{b}_5^{1U**}$	996.6161196
$\bar{b}_6^{1U**}$	4314.462238	$\bar{b}_6^{1U**}$	4207.402605	$\bar{b}_6^{1U**}$	4203.193606
$\bar{f}^{1U}$		$\bar{f}^{2U}$		$\bar{f}^{3U}$	
$\bar{f}^{1U**}$	155998.759	$\bar{f}^{1U**}$	152127.7853	$\bar{f}^{1U**}$	151975.5998
$\bar{f}^{2U**}$	677.1359172	$\bar{f}^{2U**}$	660.3333776	$\bar{f}^{2U**}$	659.6727935
$\bar{f}^{3U**}$	38999.68974	$\bar{f}^{3U**}$	38031.94632	$\bar{f}^{3U**}$	37993.89993

The results of the second bound of the upper rough interval problem (sub-problem 15), obtained using the Optimal Path-Ratios method, led to the development of three new optimal system designs, as presented in Table 7. These designs represent viable options for the case study and can be offered to the decision maker (Baghdad Water Department) as potential candidates for final implementation. Furthermore, since three optimal designs were generated for each sub-problem outlined in Tables 4, 5, 6, and 7, the decision maker now has a total of twelve optimal system design alternatives. The final selection can be made based on available budget constraints and strategic priorities.

Based on the above results from the RIMODNP, the mathematical models under uncertainty can be compared as shown in the table below:

IV. Comparison of Zeleny's Approach and Optimal-Path Ratios Method

This comparison examines two distinct methodologies for determining optimal system designs within the context of a Rough Interval Multi-Objective De Novo Programming (RIMODNP) problem, specifically applied to the Baghdad Water Department.

Table 8: A comparison between Zeleny's Approach and Optimal-Path Ratios Method

Method	Results	Advantages	Disadvantages
Zeleny's approach is a method used to find the optimal system design for each sub-problem by calculating and replacing budget constraints with specific sub-problem constraints.	• Distinct Designs: For each sub-problem (12, 13, 14, and 15), Zeleny's approach yielded one distinct optimal system design. This means that for the entire problem with four sub-problems, the decision-maker (DM) is presented with four distinct design options in total. • Specific Values: The results in Table 2 show concrete numerical values for production capacities and resource allocations for each sub-problem's optimal design. For example, for sub-problem (12), the optimal system design includes specific values like 400.7102, 274.0547, and 105.6345 for production capacity, and various resource allocation figures.	• Clear Single Solution: Zeleny's approach provides a single, clear optimal system design for each sub-problem. This can be beneficial when the decision-maker prefers a straightforward recommendation without extensive analysis of multiple alternatives for each sub-problem. • Foundation for De Novo Programming: Zeleny's work is foundational in de novo programming, focusing on designing ideal systems from scratch rather than optimizing existing ones, which can lead to innovative solutions.	• Limited Alternatives: The primary drawback is that it offers only one optimal design per sub-problem. While four options are presented for the overall problem, each sub-problem itself lacks alternative solutions, potentially limiting the decision-maker's flexibility, especially under conditions of uncertainty. • Potential for Less Robustness under Uncertainty: As highlighted in external research (and implied by the text's mention of RIMODNP), Zeleny's original approach was primarily designed for certainty conditions. While adapted here for rough intervals, a single optimal solution might be less robust if conditions change significantly.
Optimal-Path Ratios Method The Optimal-Path Ratios method utilizes calculated ratios for each sub-problem to derive multiple optimal system designs.	* Multiple Designs per Sub-problem: This method generates multiple optimal system designs for each sub-problem. For instance: * Total Alternatives: This provides the decision-maker with a significantly larger set of alternatives, totaling twelve optimal system design options across all four sub-problems (3 designs per sub-problem for 4 sub-problems). * Detailed Impact of	* Enhanced Flexibility for Decision-Makers: The most significant advantage is the provision of multiple optimal design alternatives for each sub-problem. This allows the Baghdad Water Department to choose the most suitable option based on factors like available budget constraints and	• Increased Complexity: While providing more options is beneficial, it also introduces a higher degree of complexity for the decision-maker. Evaluating and comparing twelve different designs, each with numerous parameters, requires more effort and potentially more sophisticated decision-making tools. • Potential for

Method	Results	Advantages	Disadvantages
	Ratios: The method explicitly shows how different ratios (e.g., r1, r2, r3) lead to different design outcomes, including changes in production capacities, resource allocations, and the resulting profits from increased water production, improved water quality, and reduced water loss. For example, in sub-problem (12), Design 1 (ratio 0.370195177) led to a profit increase of 623.01 DI, while other designs with different ratios yielded slightly lower but comparable profits. * Near-Identical Designs for Similar Ratios: The text notes that for sub-problem (13), Designs 2 and 3, corresponding to very close ratios (0.3333333392 and 0.332999984), resulted in nearly identical designs, indicating a certain level of sensitivity or insensitivity to minute changes in ratio values.	strategic priorities, offering greater flexibility and adaptability. * Suitability for Uncertainty: The text explicitly states that this method is "more efficient than others in solving the proposed model because it provides alternatives to the decision-maker (DM)," implying its robustness and suitability for conditions of uncertainty, which is often characteristic of real-world water management. * Detailed Trade-off Analysis (Implicit): By presenting multiple designs based on different ratios, the method implicitly allows for a more nuanced understanding of the trade-offs between objectives and resource allocation strategies, even if these trade-offs aren't explicitly quantified in the provided text.	Redundancy: As observed in sub-problem (13), very similar ratios can lead to nearly identical designs. While not a major flaw, it might mean that some of the "alternatives" are not truly distinct enough to warrant individual consideration.

Table 9: Comparison of uncertainty modeling Approaches FDNP &SDNP with suggested model RIMODNP

Feature	Fuzzy De novo programming (FDNP)	Stochastic De novo programming (SDNP)	Rough interval De novo programming (RIMODNP)
Problem Basis	Uncertainty is vague and non-probabilistic	Uncertainty is random with known probabilities	Data is imprecise and incomplete

Model conversion	Uses a satisfaction level (α) to convert fuzzy numbers into single values.	Calculates the expected value across all scenarios.	Generates four separate deterministic models based on the upper & lower bounds of the rough intervals
Outcomes	Provides a single subjective solution that depends on the chosen (α) value.	Provides a single solution that is optimal "on average"; it does not account for worst-case or best-case scenarios.	Provides two bounds of solution, lower bound (worst), upper bound (best), giving the decision-maker a clear trade-off between risk and reward.
Advantage	Handles linguistic or vaguely defined data.	Optimal for problems with known probability distributions.	Most practical when the data is imprecise. It avoids subjective assumptions of FDNP and data requirements of SDNP, providing a robust, comprehensive picture of the solution space.

V. Conclusion

This research successfully addressed the problem of uncertainty in Multi-Objective De Novo Programming (MODNP) by introducing a novel model, Rough Interval Multi-Objective De Novo Programming (RIMODNP). By integrating the Rough Interval (RI) concept, the model effectively represents ambiguous data in resource allocation and budget constraints, which is common in real-world scenarios. The study's key contributions and findings are as follows:

i) **Effective Uncertainty Modeling:** The RIMODNP model provides a robust framework for handling imprecise and incomplete data. By representing coefficients with lower and upper interval bounds, it avoids the limitations of single-point solutions often found in Fuzzy or Stochastic De Novo Programming models, which rely on subjective assumptions or extensive probabilistic data. This approach offers a more practical and comprehensive picture of the solution space.

ii) **Generation of Robust and Actionable Solutions:** The proposed methodology of transforming the uncertain RIMODNP model into deterministic sub-problems using the **Separation Method (SM)** and the **Tong-Shaocheng Method (TSM)** proved highly effective. This process yielded a range of optimal system designs that directly address the multi-objective problem. The application to the Baghdad Water Department case study demonstrated that these solutions can lead to significant increases in profit, improved water quality, and reduced water loss.

iii) **Comparison of Solution Methodologies:** The research provided a valuable comparison between **Zeleny's approach** and the **Optimal-Path Ratios Method**. While Zeleny's approach provided a single, clear optimal design for each sub-problem, the Optimal-Path Ratios Method proved to be more advantageous for decision-makers. It generated multiple distinct alternatives (up to twelve in this case study) for the overall problem, offering greater flexibility and a clearer view of the trade-offs between different objectives and resource allocations. This makes it particularly suitable for environments with high uncertainty, as it empowers decision-makers to select the most fitting solution based on strategic priorities and budget constraints.

iv) **Practical Applicability:** The successful application of the RIMODNP model to a real-world case study highlights its immense practical value. The results are not just theoretical but provide concrete, implementable plans for the Baghdad Water Department, demonstrating the model's potential to be a powerful decision-support tool in water management and other industries facing similar resource allocation challenges under ambiguity.

Vi. Future Work

Based on the research findings and identified limitations, here are the key areas for future improvements and explorations:

i) **Hybrid Uncertainty Models:** Future research could explore the integration of different types of uncertainty into a single, comprehensive model. Combining the rough interval concept with elements of **stochastic programming (for random data)** or **fuzzy programming (for linguistic and vague data)** could lead to more sophisticated models that can handle a wider array of real-world ambiguities.

ii) **Advanced Solution Algorithms:** While the current methods were effective, exploring more advanced optimization algorithms could enhance the model's efficiency and scalability. Investigating **meta-heuristic algorithms**, such as **genetic algorithms** or **particle swarm optimization**, could provide faster and potentially more robust solutions, especially when dealing with larger, more complex datasets.

iii) **Sensitivity and Risk Analysis:** A more detailed **sensitivity analysis** is warranted to better understand how changes in the rough interval bounds (the degree of uncertainty) impact the optimal solutions. Additionally, incorporating a **risk analysis** component could help decision-makers evaluate the potential worst-case scenarios associated with each design alternative.

iv) **Broader Industrial Applications:** The RIMODNP model could be applied to other sectors that deal with resource allocation and system design under uncertainty, such as **supply chain management**, **production planning**, or **large-scale infrastructure projects**. This would validate the model's generalizability and demonstrate its utility across diverse industrial contexts.

VII. Acknowledgement

The authors would like to thank the employers of the Baghdad water department for providing information about stations purification of water capital, Baghdad.

Conflict of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

References

I. A. Hamzehee, A. Yaghoobi, and M. Mashinchi, "Linear programming with rough interval coefficients," *J. Intell. Fuzzy Syst.*, vol. 26, pp. 1179–1189, 2014. 10.3233/IFS-130804.

II. C. Sen, "Sen's Multi-Objective Optimization (MOO) technique – A review," *Int. J. Mod. Trends Sci. Technol.*, vol. 6, no. 9, pp. 208–210, 2020. 10.46501/IJMTST060931.

III. H. M. Saad and M. J. Mhawes, "The relationship and impact of the external auditor's fees on audit quality of financial statements," *Tech. J. Manag. Sci.*, vol. 2, no. 1, 2025.

IV. I. A. Hussein, H. Zaher, N. Saeid, and H. Roshdy, "A multi-objective de novo programming models: A review," *Int. J. Prof. Bus. Rev.*, vol. 9, no. 1, pp. 1–22, 2024. 10.26668/businessreview/2023.v9i1.4180 .

V. I. A. Hussein, H. Zaher, N. Saeid, and H. Roshdy, "Optimum system design using rough interval multi-objective de novo programming," *Baghdad Sci. J.*, vol. 20, no. 4, p. 8740, 2023. 10.21123/bsj.2023.8740.

VI. J. Chen, T. Du, and G. Xiao, "A multi-objective optimization for resource allocation of emergent demands in cloud computing," *J. Cloud Comput.*, vol. 10, no. 20, pp. 1–17, 2021. 10.1186/s13677-021-00237-7.

VII. M. Allahdadi and M. Nehi, "The optimal value bounds of the objective function in the interval linear programming problem," *Chiang Mai J. Sci.*, vol. 42, no. 2, pp. 501–511, 2015.

VIII. M. M. Luhandjula and P. Pandian, "A new method for finding an optimal solution of interval integer transportation problems," *Appl. Math. Sci.*, vol. 4, no. 37, pp. 1819–1830, 2010.

IX. M. Zeleny, "Multiple criteria decision making (MCDM): From paradigm lost to paradigm regained?," *J. Multiple Criteria Decis. Anal.*, vol. 18, pp. 77–89, 2011. 10.1002/mcda.473.

X. N. K. Bisuiki, S. Mazumader, and G. A. Panda, "Gradient-free method for multi-objective optimization problem," in *Optimization, Variational Analysis and Applications*, vol. 355, 2020, pp. 19–34. 10.1007/978-981-16-1819-2_2.

XI. T. Shaocheng, "Interval number and fuzzy number linear programming," *Fuzzy Sets Syst.*, vol. 66, no. 3, pp. 301–306, 1994. 10.1016/0165-0114(94)90097-3.

XII. W. C. Lo, C. H. Lu, and Y. C. Chou, "Application of multi-criteria decision making and multi-objective planning methods for evaluating metropolitan parks in terms of budget and benefits," *Mathematics*, vol. 8, no. 8, pp. 1–17, 2020. 10.3390/math8081304.

XIII. Y. H. Hasan Al-Karawi and H. Ghodrati, "The effect of the source of commitment and ethical style of auditors on independent auditors' whistle-blowing," *Tech. J. Manag. Sci.*, vol. 1, no. 1, 2024.

XIV. Y. Shi, "Studies on optimum-path ratios in multi-criteria de novo programming problems," *Comput. Math. Appl.*, vol. 29, no. 5, pp. 43–50, 1995. 10.1016/0898-1221(94)00247-I.

XV. Z. Y. Zhuang and A. Hocine, "Meta goal programing approach for solving multi-criteria de novo programing problem," *Eur. J. Oper. Res.*, vol. 265, no. 1, pp. 228–238, 2018. doi: 10.1016/j.ejor.2017.07.035.