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MAX PLUS ALGEBRA FOR URBAN TRAFFIC OPTIMIZATION IN MATARAM CITY, INDONESIA

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Abstract

This study applies a Max-Plus algebra-based model to optimize traffic signal timing and enhance intersection efficiency in urban settings, with a case study conducted at the Pejanggik–Bung Hatta intersection in Mataram, Indonesia. Primary data, including signal durations and traffic density, were gathered through direct field observations. A directed graph was developed to represent traffic movements and potential conflicts, after which the Welch-Powell algorithm and Max-Plus algebra were applied to design a synchronized and periodic signal schedule. The optimized system successfully reduced the total traffic light cycle time from 525 to 375 seconds, maintaining an equitable distribution of green times and achieving a 28.57% improvement in efficiency. An eigenvalue of 75 seconds was obtained, indicating a stable and recurring timing cycle. These results demonstrate the practical utility of the Max-Plus approach in managing urban traffic, offering a cost-effective and mathematically robust strategy to alleviate congestion and promote sustainable transportation planning. The methodology is adaptable to other intersections experiencing similar traffic issues and provides valuable guidance for policymakers and urban traffic engineers in developing responsive and inclusive traffic control solutions.

Keywords: Max-Plus Algebra, Mataram City, Urban Traffic, Sustainable Cities.

I. Introduction

Road intersections are a crucial component of urban transportation networks, where ineffective traffic light management often leads to congestion, fuel waste, and increased emissions. Consequently, proper analysis and optimization of signal durations are vital for ensuring efficiency and safety [XII].

The theoretical foundation for this work lies in max-plus algebra, an algebraic structure applied to graph representations and discrete event systems. Recently, research on algebraic structures has focused on graphs and their properties [XVII, XVIII]. Baccelli et al. [VIII] pioneered the development of max-plus system theory, while Butkovič [XIX] advanced applications in synchronized and max-linear systems. Additional terminologies and formalizations can be found in [XV, XXI]. Max-plus algebra has since been widely applied to discrete-event systems, such as production lines, game theory [IV], optimal controls [XIV], computer networks, and traffic systems [I, XIII]. Extensions include its use in discrete-time quantum models [XVI], statistical perspectives [XXII], and fuzzy systems, where fuzzy max-algebra was introduced to address uncertainty [XI].

Applications in traffic signal control have been widely investigated. Early studies modeled queuing systems and synchronization rules at intersections [XX], while later works applied max-plus algebra to intelligent scheduling of traffic flows [III]. The method has also been extended to predictive traffic signal models [VII] and periodic scheduling in transportation systems, such as high-speed railways [IX] and public transport [XXIV]. Comparative approaches, such as the power algorithm versus the Kleene star method, have been studied for railroad scheduling [VI]. More recently, van den Boom and De Schutter [XXII] introduced a stochastic max-plus predictive control framework, which enhances robustness under fluctuating traffic demand. These advances underscore the increasing importance of algebraic models in intelligent transportation systems (ITS) [V].

Beyond traffic, max-plus algebra has been successfully applied in service and healthcare systems [X, XXIII], production machines [II], and queueing networks [XXV], all emphasizing its versatility in modeling periodic and synchronized operations.

Despite these advances, practical field-tested applications remain limited. Many studies focus on theoretical models or stochastic/predictive extensions, while fewer works directly apply max-plus algebra to intersections in developing cities, where low-cost yet effective solutions are urgently needed. In Indonesia, traffic congestion and accidents at intersections are a pressing issue, with reported accidents in Mataram rising from 259 in 2020 to 321 in 2021. Observations suggest that unbalanced signal durations, perceived as excessively long red lights or too-short green lights, contribute to traffic violations and accident risks.

This study addresses that gap by applying a digraph-based max-plus algebra model to optimize traffic signal durations at the Pejanggik–Bung Hatta intersection in Mataram City. Unlike prior research emphasizing stochastic robustness, our work demonstrates a practical, field-based optimization using primary data, showing how algebraic methods can produce synchronized and periodic schedules that reduce waiting times, minimize flow conflicts, and enhance overall intersection efficiency. By situating this contribution within recent ITS research, the study highlights the practicality of max-plus algebra in supporting inclusive and sustainable urban traffic management, particularly in resource-constrained contexts of developing cities.

II. Methods

The data used in this research is primary data obtained by monitoring and recording data on traffic lights at the Pejanggik-Bung Hatta Road intersection in Mataram City. The data collection time was on November 18, 2022, between 3.46 PM and 4.39 PM. Vehicles are dominated by private vehicles of the 4-wheeled and 2-wheeled types, as well as public vehicles, such as public transport, trucks, and others.

The study focuses on a single isolated intersection as a baseline test case. Although the max-plus framework naturally supports coupled cycle constraints and is adaptable to arterial or corridor-level optimization, this initial implementation was limited to a single-node system to demonstrate applicability.

The steps of this research are as follows:

1. Collect data on the duration of one traffic light cycle at the intersection.
2. Create an illustration of the intersection flow that will be used for the study.
3. Represents traffic flows in a directed graph (digraph), with flows represented as nodes and two flows that cannot pass simultaneously represented as edges.
4. Analyzing traffic light duration and flow density.
5. Determine the effectiveness level (E) of the new traffic light duration with the formula:

$$E = \frac{T_{new} - T_{original}}{T_{original}} \times 100\% ,$$

where T_{new} is the optimized total signal cycle duration, and $T_{original}$ is the pre-optimization cycle duration. The value of E is the ratio of the difference between the duration of the old green light and the new green light, or the duration of the old red light and the new red light, as a result of point coloring using the Welch-Powell algorithm.

IV. Results and Discussion

The following illustrates the traffic flow at the Pejanggik-Bung Hatta intersection as seen in Figure 1.

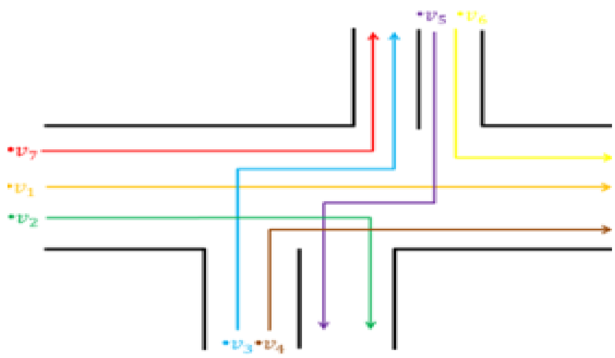

Fig. 1. Illustration of Traffic Flow at Pejanggik-Bung Hatta Road Intersection.

Figure 1 shows the layout of the traffic flow at the Pejanggik-Bung Hatta intersection. Each color-coded arrow represents a different traffic flow direction:

a. The orange line labeled v_1 represents the flow from Pejanggik Street, then straight ahead.
b. The green line labeled v_2 states that the flow from Pejanggik Street then changes direction to the right towards Bung Hatta Street.
c. The blue line labeled v_3 indicates the flow from Bung Hatta Street straight to Bung Hatta Canal Street.
d. The brown line labeled v_4 indicates the flow from Bung Hatta Street, then changes direction to the right towards Pejanggik Street.
e. The purple line labeled v_5 indicates the flow from Terusan Bung Hatta Street, then straight to Bung Hatta Street.
f. The yellow line labeled v_6 states that the flow from Terusan Bung Hatta Street then changes direction to the left towards Pejanggik Street.
g. The red line labeled v_7 indicates the flow from Pejanggik Street, then changes direction to the left towards Terusan Bung Hatta Street.

The visualization of Figure 1 helps to identify which flows can or cannot pass simultaneously, forming the foundation for the digraph model. The list of flows at the Pejanggik-Bung Hatta intersection that can pass simultaneously with other flows is as follows:

a. The v_1 current can pass simultaneously with v_2, v_4, v_6, v_7
b. The v_2 current can pass simultaneously with v_1, v_6, v_7
c. The v_3 current can pass simultaneously with v_4, v_5, v_6, v_7
d. The v_4 current can pass simultaneously with v_1, v_3, v_6, v_7
e. The v_5 current can pass simultaneously with v_3, v_6, v_7
f. The v_6 current can pass simultaneously with $v_1, v_2, v_3, v_4, v_5, v_7$
g. The v_7 current can pass simultaneously with $v_1, v_2, v_3, v_4, v_5, v_6$

The list of flows at the Pejanggik-Bung Hatta intersection that cannot pass simultaneously with other flows is as follows:

a. The v_1 flows cannot pass simultaneously with v_3, v_5
b. The v_2 flows cannot pass simultaneously with v_3, v_4, v_5
c. The v_3 flows cannot pass simultaneously with v_1, v_2
d. The v_4 flows cannot pass simultaneously with v_2, v_5
e. The v_5 flows cannot pass simultaneously with v_1, v_2, v_4

Representation of the Pejanggik-Bung Hatta intersection flow in digraph form can be seen in Figure 2.

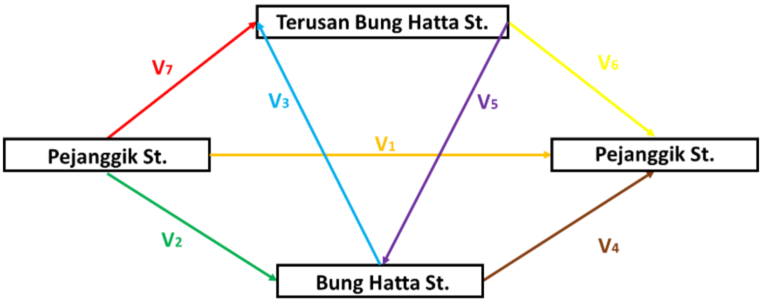

Fig. 2. Illustrative Digraph of the Flow of Pejanggik-Bung Hatta Road Intersection

Figure 2 presents a simplified digraph of the intersection based on conflict relationships among traffic flows. In this figure, nodes represent individual flows (v_1 to v_7), and edges denote conflicting flow pairs that cannot pass at the same time. Flows v_6 and v_7, which are compatible with all others, are excluded from the critical path analysis.

Since the flows v_6 and v_7 are flows that can pass simultaneously with all flows; they can be ignored. Assume that v_i with $i = 1,2,\ldots,5$ is a traveling flow, t_{ij} is the travel time required from v_i to v_j (red light duration for each flow v_i) with $i \neq j$ and $j = 1,2,\ldots,5$, and d_i is the time required for each flow v_i (green light duration for each flow v_i). Let us define $x_i(k)$ as the time when the flow v_i starts running/crossing on the k-th trip with $k \in \mathbb{N}_0$. Next, the following figure demonstrates the flow digraph of the Pejanggik-Bung Hatta intersection based on the definitions and assumptions that have been given.

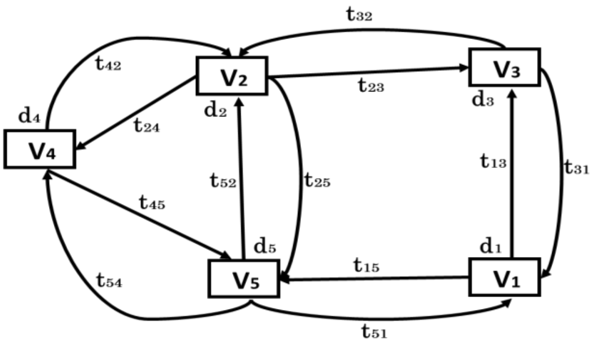

Fig. 3. Illustrative Digraph of the Flow of Pejanggik-Bung Hatta Road Intersection as Assumed

The flow digraph, as shown in Figure 3, is used as the basis for constructing the max-plus algebraic system. This includes red light durations as travel times for each flow and green light durations as service times for each flow.

Furthermore, Tables 1 and 2 present baseline signal timings and observed congestion loads, which inform the engineered adjustments in Table 3. The following summarizes the initial observed traffic light durations for each flow at the intersection in Table 1.

Table 1: Primary data of the traffic light duration of the Pejanggik-Bung Hatta intersection in seconds

Current	Traffic light duration		Total
	Red	Green	
v_1	75.5	29.5	105
v_2	65.5	39.5	105
v_3	77	28	105
v_4	77	28	105
v_5	75.5	29.5	105
Total	370.5	154.5	525

From Table 1, we get that the red and green times vary across flows, resulting in a total cycle time (traffic light duration) of 105 seconds for each direction. These values serve as the baseline before optimization. Furthermore, the time load durations for each flow, measured in seconds, are provided in Table 2.

Table 2: Primary data on time load duration due to traffic flow density at Pejanggik-Bung Hatta intersection

Current	Load duration time (in seconds)
v_1	2.34
v_2	1.64
v_3	4.16
v_4	2.83
v_5	2.80

Table 2 shows the typical duration of congestion for each flow. From Table 2, it is known that the longest load duration is v_3 which is 4.16. So, it is known that v_3 and v_2, namely $4.16 - 1.64 = 2.52$. To balance the current time, engineering is implemented. Engineering is done by finding the biggest difference between $4.16 - 2.34 = 1.82$ and $4.16 - 1.64 = 2.52$. Both values are rounded to 2.
From Table 3, the engineered (adjusted) traffic light durations were obtained according to the time load of each traffic flow, where red and green times are modified to match flow loads while maintaining the total cycle time of 105 seconds per direction. These changes are intended to create a more equitable distribution of wait and pass times.

Table 3: Engineered traffic light duration data for Pejanggik-Bung Hatta intersection in seconds.

Current	Traffic light duration (primary)		Traffic light duration (engineering)		Total
	Red	Green	Red	Green	
v_1	75.5	29.5	73.5	31.5	105
v_2	65.5	39.5	67.5	37.5	105
v_3	77	28	75	30	105
v_4	77	28	75	30	105
v_5	75.5	29.5	73.5	31.5	105
Total	370.5	154.5	364.5	160.5	

Furthermore, by using max-plus algebraic operations on the traffic light setting model at the Pejanggik-Bung Hatta Road intersection, the time when v_i walks/crosses the $k+1$ th trip is obtained as follows

$$\begin{aligned}
x_1(k+1) &= 73.5 \otimes x_1(k) \oplus 30 \otimes x_3(k) \oplus 31.5 \otimes x_5(k)\\
x_2(k+1) &= 67.5 \otimes x_2(k) \oplus 30 \otimes x_3(k) \oplus 30 \otimes x_4(k) \oplus 31.5 \otimes x_5(k)\\
x_3(k+1) &= 75 \otimes x_3(k) \oplus 31.5 \otimes x_1(k) \oplus 37.5 \otimes x_2(k)\\
x_4(k+1) &= 75 \otimes x_4(k) \oplus 37.5 \otimes x_2(k) \oplus 31.5 \otimes x_5(k)\\
x_5(k+1) &= 73.5 \otimes x_5(k) \oplus 31.5 \otimes x_1(k) \oplus 37.5 \otimes x_2(k) \oplus 30 \otimes x_4(k)
\end{aligned} \tag{1}$$

If (1) is written into max-plus algebraic matrix form, we get

$$x_i(k+1) = \begin{pmatrix} 73.5 & \varepsilon & 30 & \varepsilon & 31.5 \\ \varepsilon & 67.5 & 30 & 30 & 31.5 \\ 31.5 & 37.5 & 75 & \varepsilon & \varepsilon \\ \varepsilon & 37.5 & \varepsilon & 75 & 31.5 \\ 31.5 & 37.5 & \varepsilon & 30 & 73.5 \end{pmatrix} \otimes x(k) \tag{2}$$

with $x(k) = (x_1(k) \quad x_2(k) \quad x_3(k) \quad x_4(k) \quad x_5(k))^T$. Equation (2) is in the form of an equation

$$x(k+1) = A \otimes x(k) \tag{3}$$

The following are the eigenvalues and eigenvectors of the matrix A. Based on equation (3), the activity line $x(k)$ for $k = 0,1,2,3,\dots,35$ is obtained as follows

$$x(k) = \begin{pmatrix} 0\\0\\0\\0\\0 \end{pmatrix}, \begin{pmatrix} 73.5\\67.5\\75\\75\\73.5 \end{pmatrix}, \begin{pmatrix} 147\\135\\150\\150\\147 \end{pmatrix}, \begin{pmatrix} 220.5\\202.5\\225\\225\\220.5 \end{pmatrix}, \begin{pmatrix} 294\\270\\300\\300\\294 \end{pmatrix}, \begin{pmatrix} 367.5\\337.5\\375\\375\\367.5 \end{pmatrix},$$

$$\begin{pmatrix} 441\\405\\450\\450\\441 \end{pmatrix}, \begin{pmatrix} 514.5\\480\\525\\525\\514.5 \end{pmatrix}, \begin{pmatrix} 588\\555\\600\\600\\588 \end{pmatrix}, \begin{pmatrix} 661.5\\630\\675\\675\\661.5 \end{pmatrix}, \begin{pmatrix} 735\\705\\750\\750\\735 \end{pmatrix},$$

$$\begin{pmatrix} 808.5\\780\\825\\825\\808.5 \end{pmatrix}, \begin{pmatrix} 882\\855\\900\\900\\882 \end{pmatrix}, \begin{pmatrix} 955.5\\930\\975\\975\\955.5 \end{pmatrix}, \begin{pmatrix} 1029\\1005\\1050\\1050\\1029 \end{pmatrix}, \begin{pmatrix} 1102.5\\1080\\1125\\1125\\1102.5 \end{pmatrix},$$

$$\begin{pmatrix} 1176\\1155\\1200\\1200\\1176 \end{pmatrix}, \begin{pmatrix} 1249.5\\1230\\1275\\1275\\1249.5 \end{pmatrix}, \begin{pmatrix} 1323\\1305\\1350\\1350\\1323 \end{pmatrix}, \begin{pmatrix} 1396.5\\1380\\1425\\1425\\1396.5 \end{pmatrix}, \begin{pmatrix} 1470\\1455\\1500\\1500\\1470 \end{pmatrix},$$

$$\begin{pmatrix} 1543.5\\1530\\1575\\1575\\1543.5 \end{pmatrix}, \begin{pmatrix} 1617\\1605\\1650\\1650\\1617 \end{pmatrix}, \begin{pmatrix} 1690.5\\1680\\1725\\1725\\1690.5 \end{pmatrix}, \begin{pmatrix} 1764\\1755\\1800\\1800\\1764 \end{pmatrix}, \begin{pmatrix} 1837.5\\1830\\1875\\1875\\1837.5 \end{pmatrix},$$

$$\begin{pmatrix}1911\\1905\\1950\\1950\\1911\end{pmatrix}, \begin{pmatrix}1984.5\\1980\\2025\\2025\\1984.5\end{pmatrix}, \begin{pmatrix}2058\\2055\\2100\\2100\\2058\end{pmatrix}, \begin{pmatrix}2131.5\\2130\\2175\\2175\\2131.5\end{pmatrix}, \begin{pmatrix}2205\\2205\\2250\\2250\\2205\end{pmatrix},$$
$$\begin{pmatrix}2280\\2280\\2325\\2325\\2280\end{pmatrix}, \begin{pmatrix}2355\\2355\\2400\\2400\\2355\end{pmatrix}, \begin{pmatrix}2430\\2430\\2475\\2475\\2430\end{pmatrix}, \begin{pmatrix}2505\\2505\\2550\\2550\\2505\end{pmatrix}, \begin{pmatrix}2580\\2580\\2625\\2625\\2580\end{pmatrix}.$$

The system of max-plus algebraic equations (1) to (3) derived from this configuration is then solved using matrix representation. The matrix A incorporates travel times and service durations for each flow, enabling the calculation of state transitions over multiple cycles. The solution vector $x(k)$ for $k = 0$ to 35 (partially displayed) shows the periodic progression of start times for each flow. It can be seen that $x(31) = 75 \otimes x(30)$ thus obtained, $p = 31, c = 75$, and $q = 30$. So, the eigenvalue of the matrix A is $\lambda = \frac{c}{p-q} = \frac{75}{31-30} = 75$, and the eigenvector of matrix A is

$$\boldsymbol{v} = \bigoplus_{i=1}^{p-q} \left(\lambda^{\otimes(p-q-i)} \otimes x(q+i-1)\right)$$
$$= (2205 \quad 2205 \quad 2250 \quad 2250 \quad 2205)^T.$$

Furthermore, from the $\boldsymbol{v}$, the time when the flowv_i starts to walk/cross for the traffic system to take place periodically is $(0 \quad 0 \quad 45 \quad 45 \quad 0)^T$. For the start time, it means the time when the flows v_i with $i = 1,2,\ldots,5$, start running/crossing for the first trajectory is the 0^{th} second, $0, 45, 45$, and 0, respectively. This adjustment allows for synchronized operation with reduced conflict. For the next process, $k = 2,3,\ldots,n$. Each processor works periodically, with the period being the eigenvalue of the matrix A, which is 75 seconds.

Table 4 gives the duration of the traffic lights at the Pejanggik-Bung Hatta intersection based on the results of the analysis and calculations that have been carried out.

Table 4. Data from the analysis of the duration of traffic lights at the Pejanggik-Bung Hatta intersection in seconds.

Current	Traffic light duration (analysis result)		Total
	Red	Green	
v_1	43.5	31.5	75
v_2	37.5	37.5	75
v_3	45	30	75
v_4	45	30	75
v_5	43.5	31.5	75
Total	214.5	160.5	375

Compared to the original 525-second total duration (Table 1), as shown in Table 4, the optimized total new light duration is reduced to 375 seconds. Importantly, the green times are redistributed proportionally according to each flow's congestion load, contributing to a balanced and efficient intersection operation. The eigenvalue of 75 confirms a stable periodic system, representing an optimal steady-state cycle duration for synchronized traffic flow. From the adjustments result (new duration) data in Table 4, we obtain that the time load of each flow is more proportional to the effective level of

$$E = \frac{375 - 525}{525} \times 100\% \ = 28.57\%.$$

The increase in effectiveness confirms that the optimized model not only maintains safety by minimizing conflicts but also significantly improves the efficiency of traffic flow. Finally, Table 4 presents the optimized durations obtained through max-plus algebra, confirming a stable 75-second periodic cycle and a 28.57% increase in efficiency.

The narrative progression of the visual representation (Figures 1-3), data tabulation (Tables 1-4), and matrix analysis shows how max-plus algebra provides a theoretical and practical framework for traffic signal optimization. The application of the max-plus algebraic model at the Pejanggik-Bung Hatta intersection produces a more efficient traffic light timing arrangement compared to the original 525-second configuration. By adjusting the duration of green and red lights based on the load of each traffic flow, the model achieves a proportional and balanced distribution of time without changing the total cycle time per flow. The result is a consistent 75-second periodic cycle, where each flow starts at more synchronized intervals. These improvements are validated by the system's eigenvalues and eigenvectors, which confirm the stability and periodicity of the new configuration. The total cycle time is reduced to 375 seconds, enhancing vehicle flow productivity without reducing total green-light allocation.

The eigenvalue of 75 seconds represents the optimal interval for repeating non-conflicting flow patterns, allowing the system to operate predictably and efficiently. This reduction from 525 to 375 seconds indicates a significant gain in system performance, as it minimizes idle time and queuing while maintaining fairness across traffic directions. By aligning the signal timing with flow-specific congestion levels, the model ensures smooth transitions and improved throughput. Overall, this leads to a measurable increase in efficiency, quantified at 28.57%, demonstrating the practical value of the model in optimizing traffic light operations at congested intersections.

Beyond efficiency gains, the max-plus algebraic model supports sustainable urban mobility by reducing emissions, fuel consumption, and delays associated with prolonged idling. Through proportional distribution of green times based on flow density, it promotes inclusivity for all road users, including those in less dominant streams. As a low-cost, mathematically grounded approach, it is particularly suited for implementation in developing cities with limited infrastructure. Although this study focused on a single isolated intersection, the framework is extendable to multi-intersection corridors, where green-wave synchronization can further enhance network-wide mobility. Validation through queue length and delay measurements, as well as simulation benchmarking (e.g., VISSIM or SUMO), is recommended for future work to strengthen the empirical reliability of the results. By simulating nonsimultaneous flows and embedding traffic load data into a matrix structure, eigenvalues and eigenvectors define optimal cycle durations and start times, ensuring synchronized and conflict-free operation. This highlights the adaptability of the max-plus approach and its contribution to sustainable and inclusive traffic management strategies.

V. Conclusions

Based on the results of calculations and analysis using the max-plus algebraic model on traffic light settings at the Pejanggik-Bung Hatta intersection, a more efficient and proportional timing system is obtained compared to the original arrangement. This model can adjust the duration of green and red lights based on the traffic flow load, resulting in a system that works periodically with an optimal cycle duration of 75 seconds. The eigenvalues and eigenvectors of the system matrix support the stability and synchronization of traffic timing between flows, and show that this approach is effective in reducing vehicle waiting time and improving traffic flow. This is reinforced by the achievement of an effective level of 28.57%, which shows a significant efficiency improvement in traffic time distribution compared to the baseline condition. Thus, the max-plus algebraic model can be adapted by municipal authorities to optimize other similar intersections, especially in urban areas experiencing traffic congestion due to uncoordinated signal timings. For future work, this approach can be enhanced by: (i) Integrating real-time sensor data to dynamically adjust signal timing; (ii) Combining max-plus algebra with machine learning techniques for predictive traffic control under fluctuating demand; and (iii) Extending the methodology to multi-intersection corridors, enabling green-wave synchronization and providing a more comprehensive solution for arterial networks. By addressing these extensions, the max-plus algebraic model can evolve into a robust decision-support tool for sustainable, inclusive, and adaptive urban traffic management.

Conflict of Interest:

There was no relevant conflict of interest regarding this paper.

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