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QUADRATIC CONVERGENCE METHOD FOR COMPLEX AND REAL ROOT-FINDING WITHOUT HIGHER-ORDER DERIVATIVES

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Abstract

When it comes to dealing with nonlinear equations, numerical methods play a crucial role. Still, many of these methods come with limitations such as guaranteeing actual convergence, high computational costs, or strong dependence on derivatives. Traditional techniques, in particular, tend to struggle when the first derivative is close to zero or when they require second or third derivatives, which adds layers of complexity.

The study presents a new iterative approach to overcome these challenges. It achieves a reliable second-order convergence and can handle both real and complex rootseven in situations where the first derivative approaches zero. The method starts with an initial guess, $w_0 \in C$*, and improves it step-by-step, gradually zeroing in on a solution. Its flexibility allows it to be applied to a broad range of equations.*

One of the key advantages is that it doesn't depend on higher-order *derivatives, which helps in maintaining a balance between computational efficiency and accuracy.. Interestingly, the method also manages to find complex roots even when the initial guess is entirely real, something many other methods struggle with.*

To evaluate how well the method works, experiments were conducted using Python version 3.10.12. The results shown in tables and graphs illustrate how the method converges over a set number of steps. Overall, this technique offers a reliable and practical alternative to conventional numerical methods, particularly for tackling nonlinear problems involving complex solutions.

Keywords: Nonlinear, Complex root, Iterative numerical methods, Second-order convergence, innovation.

I. Introduction

Many sciences and engineering applications, such as stability analyses in control systems and modelling [XV, XVI] in computational physics, rely heavily on nonlinear equations, but these equations are often very difficult to deal with. When these equations are more complex or have more dimensions, solutions to equations become more difficult to solve analytically (in a closed form), so researchers are more reliant on numerical methods. The current paper aims to overcome this ongoing hurdle by proposing an iterative solution that is specifically tailored to obtaining the real and compound roots of nonlinear equations. Specifically, our areas of interest are dominant convergence despite being in situations where the second derivative happens to be zero, which usually proves problematic for methods based on derivatives. Despite much literature devoted to numerical methods, the main disadvantages cannot be ignored, like excessive computation cost, sensitivity to initial guesses, and the cumbersome requirement of higher-derivative information. All the above challenges will indicate the necessity of continuous innovation. This piece of work is pursuing a new direction with the contingent on a quadratic formulation that prevents the complexity of higher-order derivatives. The aim is to combine low computational expense and method robustness with rapid convergence. We directly solve a critical issue of nonlinear equation solving in which the first derivative approaches the zero value or is unreliable, and solve them with stability, even with a change in starting points. Traub [XXVI] was the pioneer most responsible for iterative methods in numerical analysis, making the form of these methods and their convergence behaviour the subject of his book. His observations have simplified the workings of iterative algorithms and also emphasized the need to come up with methods that can perform a variety of non-linear equation solutions. After Traub, Ostrowski [XX] had already contributed significantly by looking at the way precision and computational work collaborate in root-finding problems. His findings were used in modern versions of iterative procedures that seek to provide both practical and correct analysis.

Since their establishment, scientists have come up with mechanisms that are specific to specific challenges. One more such method was proposed by King [XIII], who proposed the use of two evaluations at the starting point and at what was erroneously termed as a Newton point of fourth order. His approach showed that by only a small exertion of computation, more could be obtained in the way of much accelerated convergence. The situation is almost the same with Hansen and Patrick [VI], who explored a one-parameter family of flexible iterative techniques with familiar names as Laguerre and Halley, and determined that a large number converged with order three near simple roots. Their findings highlighted the fact that some of their approaches perform better with large initial approximations and some others with small ones. Later, Neta [XVIII] introduced a family of algorithms that resorted to an intelligent compromise between speed and efficiency of the sixth-order methods. His algorithms would need only three function calls and a single derivative per step, and they are very accurate with only a small extra cost. His studies showed that with the modulation of the proportion of the number of calls that ought to be made, together with the utilization of derivatives, there could be both precision and effectiveness. Non-iterative methods have also come into the limelight as well in addition to iterative methods. As an example, Paniconi and colleagues [XXI] have reviewed first and second-order methods of linearizing the nonlinear Richards' equation governing unsaturated flow. These were occasionally more effective than classical iterative schemes like Newton, but could be less well behaved in other circumstances. However, their work broadened the repertoire of nonlinear solutions to problems, proving that non-iterative approaches can enable iterative approaches if they are used intelligently. However, still another extension of the mathematical landscape was introduced by other analytical techniques, which was greatly noted by Ji-Huan [IX], who has reviewed emerging techniques like the homotopy perturbation method, the variational iteration method, and the modified Lindstedt-Poincar method. The procedures show the flexibility and strength of analytical approximations, especially in systems where the perturbation assumptions are not so harsh.

Weerakoon and Fernando [XXVIII] in this continuum went a step further and applied a trapezoidal approximation to the indefinite integral of the derivative and thus achieving convergence of order three and being superb on many test functions compared to conventional Newton iterations. Systems of nonlinear equations have also been the target of researchers to represent real-world situations where several variables are interacting. Abbasbandy [I] presented an iterative method, which is an extension of the Newton approach, enhanced with the help of Adomian decomposition, to deal with two-variable nonlinear problems with better convergence behavior. Similarly Ramos [XXIII] studied iterative schemes in initial- and boundary-value problems in ordinary and partial differential equations, showing profound relationships between long-established higher-order iteration methods, such as the Picard scheme, Banach scheme of fixed points, and even the variational iteration method; he recognized that a substantial number of these schemes are special cases of a general scheme of quasilinearizations that gave coherence to a previously diffuse subject. Successive developments have resulted in the ever-increasing palette of higher-order methods. As an example, Newton-Raphson schemes have been refined by Chun [XI], who incorporates an equivalent of the additions proposed by Abbasbandy, resulting in faster convergence in numerical experiments. At the same time, iterations described by Noor et al. [XIX] were proposed to always converge quadratically and require a rather small number of evaluations of the given function, enhancing the interest in finding efficient algorithms related to the root-finding process. An important aspect of designing an iteration method is how this method behaves at the edges. The (technically) ongoing methodological analysis by Melman [XVII] of the so-called double-step Newton method to attack the largest or smallest real zeros in an arbitrary strictly real-rooted polynomial revealed not only that the method does produce unexpected overshoot phenomena, but also simple strategies to achieve overshoot amelioration. In like manner, Sharma and Guha [XXIV] shifted Ostrowski's fourth-order ideas into an extended one-parameter family of 6th order methods, leading to a further increase in speed of convergence with only modest extra per-step cost.

$$p_{n+1} = p_n - \frac{h(p_n)}{h'(p_n)} \tag{1}$$

$$t_{n+1} = w_n - \frac{g(w_n)}{g'(t_n)}\frac{g(t_n)}{g(t_n)-2g(w_n)} \tag{2}$$

$$\widetilde{t_{n+1}} = t_{n+1} - \frac{g(t_{n+1})}{g'(t_n)}\frac{g(t_n)+ag(w_n)}{g(t_n)+(a-2)g(w_n)} \tag{3}$$

Moreover, Parhi and Gupta [XXII] based their idea on the third-order Weerakoon-Fernando technique to establish a sixth-order point-wise convergent procedure with only two and two first derivatives and without using any second derivatives, once again indicating how researchers are customer to be careful about the high-order convergence and the minimum usage of the function's derivative. In addition to solutions of single equations, in more general terms, the field addresses more general and more involved questions, such as nonlinear ordinary and partial differential equations, optimization problems on large scales, and matrix equations. Maheshwari [XIV] had illustrated a special formulation technique both on transcendental and higher-order-convergence transformation of partial-differential forms, none of which require second derivatives.

$$w_n = t_n - \frac{h(t_n)}{h'(t_n)} \tag{4}$$

$$t_{n+1} = t_n + \frac{1}{h'(t_n)}\left[\frac{\{h(t_n)\}^2}{h(w_n)-h(t_n)} - \frac{\{h(w_n)\}^2}{h(t_n)}\right] \tag{5}$$

Besides the above, Cordero et al. [IV] studied real dynamical behaviours of iterative methods, and showed that some of them can take irregular or chaotic dynamical patterns, and Noor et al. [XIX] generalized some iterative methods to solve a range of real-world problems, including population dynamics and particle motion on inclined planes. In seeking greater efficiency, Wang et al. [XXVII] proposed a general n-point method of Newton type with so-called self-accelerating parameters that was consistent with the Kung and Traub conjecture of optimality at 2n convergence order. The scientists are also still testing the frontiers of specialized areas. Al-Jawary et al. [III] have used iterative schemes, especially Tamimi-Ansari, Daftardar-Jafari, and Banach contraction, to second-order nonlinear ordinary differential equations in physics, and they have compared the same to established solvers such as Runge-Kutta and have found them more accurate. At the same time, Dehghan and Shirilord [V] presented and proved theorems on families of algorithms of orders of complexity that solve systems of nonlinear equations at trivial additional computational expense, and Kansal et al. [XXI] presented similar results. Based on these developments, Ivanov and Yang [VIII] developed iterative methods of nonlinear matrix equations, an area of substantial importance to control theory and other high-end engineering disciplines.

Abdullah et al. [II] have designed methods whose convergence order is between four and eight, taking a weight-function approach under the guidance of the Kung and Traub conjecture. Finally, Inderjeet and Bhardwaj [XXV] conducted head-on tests of a new Newton Raphson procedure with a number of typical iterative procedures; test cases pegged on fluid dynamics, heat-movement and construction mechanics were habitually seen to take lesser attempts with higher significant benefit under their variant, indicating the way that even so judgmentary stratagem may be refreshed to face current scientific and industry requirements. According to Siwach and Malhotra [XXV], the importance of iterative methods [VII] like Newton-Raphson, Halley, and Cauchy methods of solving nonlinear equations was discussed in the light of applications in various fields of science and engineering. They indicated that though these approaches have gained a lot of acceptance, they are quite demanding in terms of computations. Their paper brings the constant attempt to improve and streamline numerical algorithms in terms of efficiency and convergence rate. Based on this very large literature, what is clear is how higher-order methods can significantly decrease the number of iterations that lead to high precision. Nonetheless, common methods tend to have such drawbacks as the requirement to compute second or higher partial derivatives, vulnerability to inappropriate initial guesses, or unstable behavior under certain pathological conditions (such as when multiple roots exist or when the first derivative is zero). Moreover, the efficiency of many algorithms is impressive on real roots, but they may not have been fully optimised [X, XI, XII], and not fully evaluated, on complex roots essential in signal processing, in quantum mechanics, and control theory. To fill out such gaps, the present paper will present an iterative procedure specifically designed to find real and complex values of solutions to a nonlinear equation, explicitly taking into consideration those cases where the second derivative fails or is not dependable. By basing the process on quadratic principles, the error avoids the computational weight usually put on higher-order differentiations.

Our key objectives include enhanced computational efficiency through fewer function evaluations, robust convergence for a wide range of initial guesses, broad applicability across diverse problem domains, and cost-effectiveness due to the elimination of second-derivative requirements.

Almost all the above methods give desired results only after evaluating the second or higher-order derivative; hence, the computational cost is very high. In the present investigation, we propose a new iterative method that takes a comprehensive approach to identifying intricate and authentic roots of nonlinear equations across various orders. Its chief advantages over existing practices are:

- ❖ The algorithms solve problems quickly and efficiently.
- ❖ Fewer iterations are needed to reach convergence.
- ❖ The method is cost-effective because it dispenses with higher-order derivatives.
- ❖ Starting from either a real or complex initial guess, the researcher can locate all real or complex rootseven in situations where some traditional techniques (e.g., Newton’s method) may fail.

We validated our approach on a variety of standard benchmark problems, compiling the numerical outcomes in tables and illustrating the results with Python 3.10.9 plots. This suite of tests confirms that the method remains dependable across diverse scientific and engineering scenarios.

In support of these results, we start with a detailed convergence proof defining precisely when the method attains quadratic convergence. We then conduct numerical experiments on both traditional test functions and tougher nonlinear instances taken from applications in the real world. The authors compare to known and state-of-the-art iterative algorithms in demonstrating unambiguous improvement in the number of iterations, total function evaluations, and resulting error sizes. These advances are particularly valuable for problems where excessive use of derivatives may impose instability or excessive computational expense. Significantly, our approach accommodates complex roots with ease—a crucial characteristic in problems with complex eigenvalues or resonant frequencies. As a whole, with the provision of rigorous convergence using a few derivative calls, this work provides a useful and economical contribution to computational mathematics. The rest of the paper walks through the derivation of the algorithm, its theoretical foundation, detailed experiment results, discussion of the observed performance, and concluding remarks that reinforce the need for ongoing advances in higher-order iterative techniques.

II. Formulation of Method

Let h(y) = 0 be a nonlinear equation.

h(y) be a differential function in some interval $I \subset R \subset C$.

Consider a cubic equation.

$$h(y) = m_0 y + m_1 y^3 \quad (6)$$

Let y_n be the n^{th} approximation from equation (6), we obtain

$$h(y_n) = m_0 y_n + m_1 y_n^2 \quad (7)$$

Differentiating equation (6) w.r.t 'y', we get

$$h'(y) = m_0 + 3 m_1 y^2 \quad (8)$$

Putting $y = y_n$ in equation (8), we get

$$h'(y_n) = m_0 + 3 m_1 y_n^2 \quad (9)$$

Let y_{n+1} be an exact root of the equation (6), so

$$h(y_{n+1}) = m_0 y_{n+1} + m_1 y_{n+1}^3 \quad (10)$$

$$y_{n+1}^2 = -\frac{m_0}{3m_1} \quad (11)$$

Equations (7) and (9) are solved to get the values of m_0 and m_1.

$$m_0 = \frac{3h(y_n) - y_n h'(y_n)}{2y_n}$$

$$m_1 = \frac{y_n h'(y_n) - h(y_n)}{2y_n^3} \quad (12)$$

Substituting these values in equation (11)

$$y_{n+1} = y_n\sqrt{1 - \frac{2\,h(y_n)}{y_n h'(y_n) - h(y_n)}}) \quad (13)$$

Equation (14) is the proposed method.

Dealing with Multiple (or Near-Multiple) roots: The proposed method is mostly for simple roots. To deal with the multiple roots, the algorithm can be adjusted by the inclusion of the multiplicity correction factor.

$$y_{n+1} = y_n\sqrt{1 - \frac{2\,mh(y_n)}{y_n c - h(y_n)}}) \quad (14)$$

Where m is the multiplicity of the root, the author found that the convergence rate remains preserved.

III. Convergence Analysis

Theorem Let r ∈ D be a simple root of a sufficiently differentiable function h: D → R in an open interval D. The method defined by equation (14) is of second order.

Proof: Let 'r' be a simple root of h(y) = 0 and substituting $y_n = c_n + r$ in equation (13), we get

$$c_{n+1} + r = \sqrt{1 - \frac{2\,h(c_n+r)}{(c_n+r)h'(c_n+r) - h(c_n+r)}} \qquad (15)$$

Note that g(r) = 0 and with the help of Taylor's series expand $h(c_n + r)$ and $h'(c_n + r)$ about the point 'r', we get

$$h(c_n + r) = h'(r)\, c_n + \frac{c_n^2}{2} h''(r) + O(c_n^3) \qquad (16)$$

$$h'(c_n + r) = h'(r) + h''(r)\, c_n + \frac{c_n^2}{2} h''(r) + O(c_n^3) \qquad (17)$$

Substituting the values of $h(c_n + r)$ and $h'(c_n + r)$ in equation (15), we get

$$c_{n+1} = \frac{1}{2}\left(\frac{h''(r)}{h'(r)} - \frac{3}{r}\right) c_n^2 + O(c_n^3) \qquad (18)$$

On omitting c_n^3 and the higher power of c_n, we have

$$c_{n+1} = Lc_n^2, \text{ where } L = \frac{1}{2}\left(\frac{h''(r)}{h'(r)} - \frac{3}{r}\right)$$

Hence, the proposed method has a second-order of convergence.

Although our method does not involve dealing with second derivatives, division by small $h'(y_n)$ is still a burning issue. When a large variation or irregular steps are identified, the authors follow the damping strategy by multiplying the developed formula by $0 \le \lambda \le 1$. For threshold protection, if $|h'(y_n)| < \in$, where ∈ is a very small number near 10^{-8}, the authors suggest either skipping the iteration or adjusting the denominator to ∈.

Non-linear equations emerge across an array of applied-science disciplines, including astrophysical dynamics, traffic-flow modelling, and quantum-mechanical systems, and researchers have proposed many numerical schemes to tackle them. Yet rigorous treatments that reliably locate complex roots remain comparatively scarce. The present study addresses this gap by introducing a broadly applicable algorithm that delivers those complex solutions while retaining the dual advantages of rapid, quadratic convergence and low computational overhead. Because the procedure is derivative-light and scalable, practitioners can adapt it to equations of any degree, whether algebraic or transcendental. To substantiate its versatility, we apply the method to a spectrum of representative test problems, implement the computations in Python, and visualise the iterative behaviour through informative plots; a selection of these illustrative examples appears in the following section.

IV. Numerical Examples

In this section, a series of numerical examples is presented to demonstrate the performance and reliability of the proposed method when applied to various nonlinear equations. The selected functions include polynomials, trigonometric expressions, and logarithmic functions spanning both real-valued and complex-valued domains. Particular attention is given to cases where the initial guess is real but the resulting root is complex, as well as to scenarios where the first derivative at the initial point is zero. These examples serve to highlight the method's stability, convergence behaviour, and its ability to handle challenging root-finding conditions.

Table 1: Complex root obtained when the initial guess is a real number

f(w)	w_0	α	Number of Iterations
$w^3 + w^2 + 3$	0.5	0.4319+1.195i	8
$w^4 - w + 1$ $w^3 + 3w^2 + 24w + 19$	0.5 2	-0.7271+0.9341i -1.0714+4.5842i	9 6
$w^4 + w^3 + w^2 + 0.5w + 1$ $\sin w - 2$	-2 1	0.3206-0.8324i 1.5708+1.317i	7 10
$\tan w - i$	1	1.3287+7.4174i	18
$w^3 + 1$ $w^3 + \sin w + 1$	1 1.5	0.5+0.866i 0.3595+1.262i	6 5

Table 1 shows the outcome of using the proposed iterative method to solve a variety of nonlinear equations, highlighting cases where complex roots were achieved even when purely real initial guesses were used.

For every function f(w), the tables show the initial guess 'w_0', root 'α', and number of iterations to converge. The findings identify how real-valued inputs can give rise to complex-valued solutions and mirror the behaviour of complex roots in nonlinear systems. For example, with w_0 = 1, the function $\tan w - i$ yielded a complex root, α =1.3287 + 7.4174i in 18 iterations, while the function $w^3 + 1$ converged to a root 0.5 + 0.866i in just 6 iterations.

In general, the table underscores the ability of iterative methods to traverse the intricate solution space even from actual-valued initial values.

Table 2: Complex root obtained when the initial guess is a complex number

f(w)	w_0	α	Number of Iterations
$w^4 + 2w^2 + 5$	1+2i	00.7862+1.272i	5
$w^5 + w^2 + 1$	0.1+1i	-0.2179+1.167i	6
$w^6 - w^2 + 3$	0.5+0.5i	1.0429+0.5019i	5
$w^3 - w + 1$	0.5+0.5i	0.6624+0.5623i	4
$\sin w - (1 + i)$	1+1i	0.6662+1.0613i	5
$\cos w - i$	1i	-1.5708+0.8814i	7
$\ln w - (0.2 + i)$ $\tan w + \tanh w - (1 + 1i)$	0.5+1i 1+0.5i	0.6599+1.0278i 1.9564+1.9564i	5 9

Table 2 describes the behaviour of the algorithm when used in a variety of nonlinear equations, where the initial guesses 'w_0' and the computed roots 'α'are the complex plane. Functions used for testing cover a variety of algebraic and transcendental forms such as polynomials, trigonometric functions, logarithmic functions, and their combinations. Consistency in arriving at convergence from a fairly short sequence of iterations, namely from 4 to 9, testifies to the method's steadiness over complex-valued realms.

For example, starting with $w_0 = 0.5 + 0.5i$, the function$w^6 - w^2 + 3$ converged to α=1.0429+0.5019i in just 5 steps, while the more intricate expression$\tan w + \tanh w - (1 + 1i)$ required 9 iterations to find α=1.9564+1.9564i. These findings evidence the flexibility of the method with respect to the type of function, as well as pinpointing its validity for solving the complex equations in scientific and genetic engineering applications.

Table 3: First Derivative at the Initial Guess is zero, i.e. $h'(w_0) = 0$

f(w)	w_0	α	Number of Iterations
$2w^3 - 3w^2 - 1$	1	1.6777	4
$w^4 - 4w^3 + 4w^2$	1	1.9995	11
$\sin w - \frac{w}{2}$	$\frac{\pi}{3}$	1.0472	4
$\tan w - 2w$	$\frac{\pi}{4}$	1.1656	7
$w^5 - 5w^4 + 5w^3$	1	1.382	5
$\cos w - 2$	π	1.317i	10
$\ln w - w + 2$	1	3.1462	7
$\sin w + \cos w - 2$	$\frac{\pi}{4}$	0.7854+0.8814i	9

Table 3 presents the results obtained using the considered method in some selected nonlinear equations. One of the main features in all entries is that the first derivative of the function at the initial guess 'w_0' is zero, a situation in which most traditional methods like Newton-Raphson will fail or become unstable.

Nonetheless, the method converged to real or complex roots α successfully with an acceptable number of iterations. The functions were started from both real numerical values and symbolic expressions (e.g. $\pi, \frac{\pi}{3}$), and still converged.

For example, the function $\cos w - 2$ with $w_0 = \pi$ led to a purely imaginary root α=1.317iin 10 iterations and $\sin w + \cos w - 2$ with $w_0 = \frac{\pi}{4}$ produced the complex root 0.7854+0.8814i in 9 iterations.

These outcomes demonstrate the robustness of the method in scenarios where standard derivative-based techniques are typically unreliable, highlighting its applicability in solving nonlinear equations with flat initial slopes.

The authors compared the proposed method with the existing methods like Newton-Raphson Method and Helly's Method (Here not shown to avoid the length of the paper).

V. Graphical Illustrations

The visualizations of convergence behaviour and root trajectories for the above numerical examples are shown in this section. The plots demonstrate how the method develops from an initial guess to the final root, and how complex solutions emerge from real starting points. Additionally, the graphs depict the function's landscape, fixed points, and behaviour near critical points where the first derivative vanishes. These visual insights further support the robustness and accuracy of the proposed method.

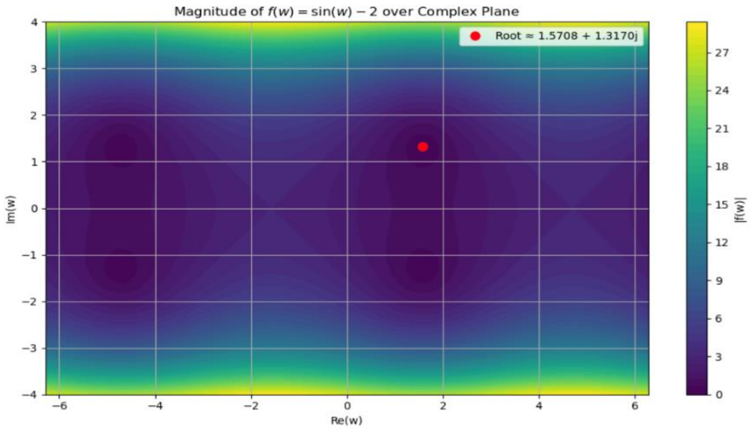

Fig. 1. Graphical Analysis of $f(w) = \sin w - 2$ over the Complex Plane

Figure 1 presents a complex domain visualization of the function $f(w) = \sin w - 2$, showing the magnitude $|f(w)|$over the complex plane. The horizontal axis represents the real part of w (Re(w)), while the vertical axis represents the imaginary part (Im(w)). A colour gradient, ranging from dark purple (low values of $|f(w)|$to bright yellow (high values), is used to represent the magnitude $|f(w)|$ at each complex coordinate.

A key feature of this graph is the red dot, which marks the approximate complex root w ≈ 1.5708 + 1.3170i. This root was identified using the proposed iterative method and lies in a region where the magnitude of f(w) approaches zero, corresponding to a deep trough (dark region) in the surface plot.

The colormap clearly illustrates how the function behaves around this root. The surrounding area transitions smoothly from dark to lighter colours, indicating a localized minimum in magnitude. This is consistent with the nature of a root in the complex domain, where$|f(w)| = 0$.

Furthermore, this plot visually confirms that although the root is complex, it is surrounded by a well-behaved landscape, allowing the iterative method to converge reliably. It also provides insight into the nonlinear structure of the sine function extended into the complex plane, showing symmetry and periodicity consistent with $\sin w$ in both real and imaginary directions.

This graphical representation not only reinforces the numerical result but also offers an intuitive understanding of why and where the root occurs, validating the correctness of the computed solution.

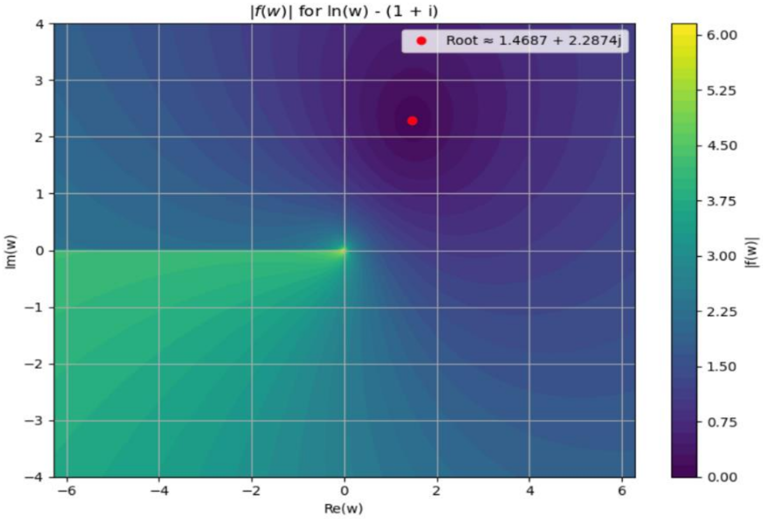

Fig. 2. Graphical Analysis of $f(w) = \ln w - (1 + i)$ over the Complex Plane

Figure 2 highlights the location of the complex root of the function $f(w) = \ln w - (1 + i)$ on the complex plane, where the iterative method successfully identified the solution w ≈1.4687 + 2.2874i. The root lies in a well-defined low-magnitude region, as indicated by the dark shading near the red marker.

Unlike the previous examples involving trigonometric functions, this plot reveals the more subtle and gradual variation in $|f(w)|$that is characteristic of the complex logarithm. The central bright spot near the origin reflects the singularity of the logarithmic function, but the root lies away from this region, allowing smooth convergence.

This illustration underscores the method's ability to manage multivalued functions and converge even in domains with analytical complexities such as branch cuts and singularities.

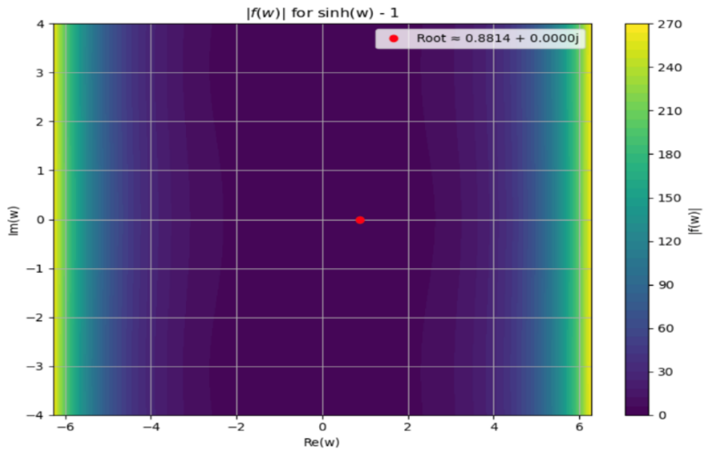

Fig. 3. Graphical Analysis of $f(w) = \sinh w - 1$ over the Complex Plane

This contour plot visualizes the magnitude of the function $f(w) = \sinh w - 1$ across the complex plane, with the computed root marked as a red dot at approximately w ≈ 0.8814. Unlike earlier examples where complex roots were involved, this function yields a purely real root, situated along the real axis.

The symmetric structure observed along the imaginary axis is characteristic of the hyperbolic sine function. The vertical gradient bands on either side indicate exponential growth in magnitude for large negative and positive real values of w, which aligns with the behaviour of $\sinh w$ as it increases rapidly with$|w|$. The darkest region near the center, where the red dot is located, confirms that the root lies in a flat, well-conditioned zone, promoting rapid convergence.

This plot is a clear example of a function where the solution space is dominated by real-valued behaviour, and the method precisely identifies the root with minimal complexity in the imaginary direction.

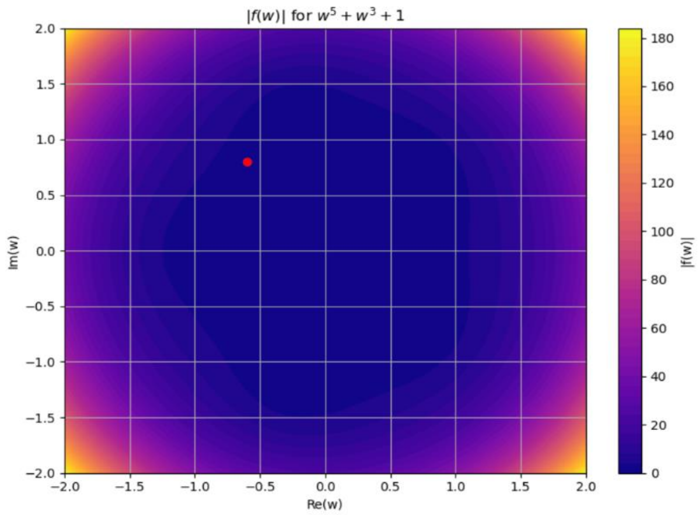

Fig. 4. Graphical Analysis of $f(w) = w^5 + w^3 + 1$ over the Complex Plane

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The contour plot displays the magnitude $|f(w)|$of the function $f(w) = w^5 + w^3 + 1$ over a bounded region in the complex plane. The red dot indicates the numerically found root, which is located in the upper-left quadrant of the complex plane at a position where the magnitude of the function is close to zero.

In contrast to some previous plots involving exponential or trigonometric functions, this plot consists of a polynomial with no discontinuities or singularities, and therefore the landscape is smooth and radially symmetrical around the root.

The colour gradient in the vicinity is gradual in its increase in|f(w)|from the root, and this indicates good conditions for convergence in an iterative process. This plot effectively visualizes the behaviour of a complex polynomial and confirms that the iterative method can efficiently locate roots in multi-dimensional, smooth solution spaces.

VI. Conclusion

This paper introduces a robust, convergent iterative method for nonlinear equationsolutionswithaspecialabilityto find real and complex roots even when the first derivative is zero. Using a cubic expression, it eliminates the use of higher-order derivatives while maintaining second-order convergence, finding an optimal balance between accuracy and work.

Through detailed convergence proofs and a wide array of numerical tests, we've shown that this method remains stable and reliable across many kinds of nonlinear functions like polynomials, trigonometric, logarithmic, and transcendental. It succeeds where classic approaches like Newton-Raphson often falter or diverge, particularly when real initial guesses lead to complex solutions.

The plots and tables always show fewer iterations to convergence, irrespective of function complexity or root type. The fact that this algorithm can solve multi-valued and complex-valued problems without sacrificing speed or accuracy makes it a useful addition to the root-finding arsenal.

In short, this derivative light approach provides an applicable alternative for computational mathematics, physics, engineering, and other related disciplines, where conventional methods would fail. It also opens doors for future developments, mentions more general and adaptive root-finding paradigms.

Conflict of Interest:

The author declares that there was no conflict of interest regarding this paper.

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