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DOUBLE ELZAKI TRANSFORM AND ADOMIAN POLYNOMIALS FOR SOLVING SOME PDEs

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Abstract

In this research, we provide novel methods for analyzing some models of PDEs that occur in a wide range of scientific and engineering applications. Adomian polynomials have been utilized for this purpose. The simplicity and accuracy of the suggested Integrated technique are confirmed by combining the Adomian decomposition method with the conventional Double Elzaki transform. An experimental study has been conducted. To illustrate the efficiency of the proposed scheme, the Rangaig transform-based Homotopy analysis method is used for the comparison study of the solutions.

Keywords: Adomian decomposition method; Double Elzaki transform; Benjamin-Bona-Mahony equations; KdV equations; Linear Schrodinger equations; Test examples.

I. Introduction

When the change in input is not proportionate to the change in output, the system is said to be nonlinear. Many scientists, including mathematicians, physicists, biologists, and engineers, are interested in nonlinear problems. In this research, we will discuss three classical models of partial differential equations.

The double Elzaki transform has been used for solving wave-like equations, and the outcomes are compared with the double Laplace transform approach in [I]. In [II], to find the solutions of differential equations, a novel integral transform known as the Elzaki transform has been devised. ODEs with variable coefficients have been solved using the Elzaki transform in [III]. The relationship between Laplace transforms and Elzaki has been described in [IV]. In [V], to solve both linear and nonlinear PDEs, a combination of the Elzaki and differential transforms has been used. To solve differential equations, two precise techniques based on the Elzaki and Sumudu transforms have been established and put into practice in [VI]. In [VII], the analytical solution of the telegraph equations is examined using the double Laplace transform. Singular systems of hyperbolic equations have been solved using the double integral transform approach in [VIII]. Nonlinear partial differential equations have been studied using the Double Elzaki decomposition approach in [IX]. The convergence of the double Elzaki transform scheme for solving PDEs has been discussed in [X]. A modification in the double Sumudu transform method has been carried out.

The composition of the research paper is as follows: The complete details on the double Elzaki transform and its characteristics are provided in section 2. Section 3 presents a suggested method for resolving partial differential equation mathematical models. In section 4, the suggested method for resolving such equations has been used in some computational work. The conclusion of the study is presented in Section 5.

II. Characteristics of the Double Elzaki Transform

The Double Elzaki transform, its inverse Elzaki transform, and a number of its important characteristics are examined in this section.

II.i. Double Elzaki Transform: An Introduction

Assume $f(\mu,\eta)$ be a function with $\mu\,,\eta\ > 0$. Because an infinite series can be used to represent the function. The Double Elzaki Transform can be expressed as follows:

$$DE\,\{f(\mu,\eta);\tau,\upsilon\} = T(\tau,\upsilon) = \tau\upsilon\int_0^\infty\int_0^\infty f(\mu,\eta)\,e^{-\left(\frac{\mu}{\tau}+\frac{\eta}{\upsilon}\right)}\,d\mu d\eta,$$

whenever an integral exists.

II.ii. Double Elzaki Inverse Transform

The following is the Double Elzaki transform's inverse:

$$DE^{-1}\{T(\tau,\upsilon)\} = f(\mu,\eta), \qquad \mu,\eta > 0$$

If $a > 0, b > 0$ in the area correspond to the interval $0 \le \mu < \infty, 0 \le \eta < \infty$,then $\exists$ a positive constant k for which the function $f(\mu,\eta)$ is said to have exponential order:

$$|f(\mu,\eta)| \le k\,e^{\left(\frac{\mu}{a}+\frac{\eta}{b}\right)}$$

II.iii. Double Elzaki Transform Standard Characteristics

This section will discuss a few characteristics of the double Elzaki transform:

LINEARITY PROPERTY: If $f(\mu,\eta)$ and $g(\mu,\eta)$ be two functions of $\mu,\eta > 0$ such that $DE[f(\mu,\eta)] = T_1(\tau,\upsilon)$ and $DE[g(\mu,\eta)] = T_2(\tau,\upsilon)$, then

$$\begin{aligned} DE\{\,af(\mu,\eta) + b\,g(\mu,\eta)\} &= a\,DE\{f(\mu,\eta)\} + b\,DE\,\{g(\mu,\eta)\} \\ &= a\,T_1(\tau,\upsilon) + bT_2(\tau,\upsilon) \end{aligned}$$

CHANGE SHIFTING PROPERTY: If $DE\{f(\mu,\eta)\} = T(\mu,\eta)$, then

$$DE\{f(a\mu,b\eta)\} = \frac{1}{ab}\,T(a\mu,b\eta)$$

FIRST SHIFTING PROPERTY:

(a) If $DE\{f(\mu,\eta)\} = T(\tau,\upsilon)$, then

$$DE\{e^{a\mu+b\eta}\ f(\mu,\eta\ ) = T\ \left[\frac{\tau}{1-a\tau},\frac{\upsilon}{1-b\upsilon}\right]$$

(b) If $DE\{f(\mu,\eta)\} = T(\tau,\upsilon)$, then

$$DE\{\ e^{-a\mu-b\eta}\ f(\mu,\eta\ ) =\ T\ \left[\frac{\tau}{1-a\tau},\frac{\upsilon}{1-b\upsilon}\right]$$

II.iv. Partial Derivatives' Double Elzaki Transform :

In this section, we introduce the double Elzaki transform of a few partial derivatives**:**

a) $DE\left\{\frac{\partial}{\partial\mu}\ f(\mu,\eta)\right\} = \frac{1}{\tau}\ T(\tau,\upsilon) - \tau T(0,\upsilon)$

b) $DE\left\{\frac{\partial}{\partial\eta}\ f(\mu,\eta)\right\} = \frac{1}{\upsilon}\ T(\tau,\upsilon) - \upsilon\ T(\tau,0)$

c) $DE\left\{\frac{\partial^2}{\partial\mu^2}\right)f(\mu,\eta)\ \} = \frac{1}{\tau^2}T(\tau,\upsilon) - T(0,\upsilon) - \tau\frac{\partial}{\partial\mu}\ T(0,\upsilon)$

d) $DE\{\frac{\partial^2}{\partial\eta^2}f(\mu,\eta)\ \} = \frac{1}{\upsilon^2}T(\mu,\upsilon) - T(\tau,0) - \upsilon\frac{\partial}{\partial q_3}\ T(\tau,0)$

e) $DE\{\frac{\partial^2}{\partial\mu\partial\eta}\quad f(\mu,\eta)\quad\} \ = \ \frac{1}{\tau\upsilon}\ \ T(\tau,\upsilon) - \frac{\tau}{\upsilon}T(\tau,0) - \frac{\tau}{\upsilon\upsilon}\ T(0{,}0) + \tau\upsilon\ T(0{,}0)$

III. Proposed Technique for Solving Models of PDEs

Consider the universal partial differential equation that is nonlinear and has the form:

$$L\ u(\mu,\eta) + N\ u(\ \mu,\eta) = g(\mu,\eta) \tag{2}$$

Under the initial condition

$$u(\mu,0) = h(\mu), \tag{3}$$

In this case $g(\mu,\eta)$ is the source term, a linear differential operator is denoted by L, and a nonlinear differential operator by N, where $L = \frac{\partial}{\partial\eta}$.

Equations (2) and (3) can be solved by applying the double Elzaki transform and the single Elzaki transform, respectively. We arrive at

$$DE\big(L\ u(\mu,\eta)\big) + DE\big(N\ u(\mu,\eta)\big) = DE\ \big(g(\mu,\eta)\big) \tag{4}$$

and

$$E_1\big(u(\mu,0)\big) =\ E_1\big(h(\mu)\big) = T(\tau,0) \tag{5}$$

Using Equation (4), we get

$$\frac{1}{\upsilon}\ T(\tau,\upsilon) - \upsilon\, T(\tau,0) = DE\ (g(\mu,\eta)) - DE\ (N\,u\,(\mu,\eta)$$

This implies

$$T(\tau,\upsilon) = \upsilon^2 T(\tau,0) + \upsilon DE\left(g(\mu,\eta)\right) - \upsilon DE\left(N\,u(\mu,\eta)\right)$$

Or

$$DE\big(u(\mu,\eta)\big) \quad = \upsilon\, E_1\big(h(\mu)\big) + \upsilon\, DE\left(g(\mu,\eta)\right) \ - \left\{\upsilon DE\big(N\,u(\mu,\eta)\big)\right\} \tag{6}$$

Applying the inverse double Elzaki transform to equation (6) yields,

$$u(\mu,\eta) = G(\mu,\eta) - DE^{-1}\left\{\upsilon DE\big(N\,u(\mu,\eta)\big)\right\} \tag{7}$$

where

$$G(\mu,\eta) = DE^{-1}\left\{\upsilon^2\, E_1\big(h(\mu)\big) + \upsilon\, DE\left(g(\mu,\eta)\right)\right\}$$

Assume that the following is the form of the solution:

$$u(\mu,\eta) = \sum_{n=0}^{\infty} u_n(\mu,\eta) \tag{8}$$

Write the nonlinear term as follows:

$$Nu(\mu,\eta) = \sum_{n=0}^{\infty} A_n(u), \tag{9}$$

Here $A_n(u)$ stands for the Adomian polynomials, which can be computed as follows:

$$A_n = \frac{1}{n!}\frac{d^n}{d\epsilon^n}\left\{N\left(\sum_{j=0}^{\infty}\epsilon^j u_j\right)\right\}_{\epsilon=0}, n = 0,1,2,3,\ldots$$

putting the values from (8) and (9) into (7), we get

$$\sum_{n=0}^{\infty} u_n(\mu,\eta) = S(\mu,\eta) - DE^{-1}\left\{\upsilon DE\left(\sum_{n=0}^{\infty} A_n(u)\right)\right\} \tag{10}$$

From (10), we obtain

$$u_0(\mu,\eta) = S(\mu,\eta),$$
$$u_1(\mu,\eta) = -DE^{-1}\{\upsilon\, DE(A_0)\},$$
$$u_2(\mu,\eta) = -DE^{-1}\{\upsilon\, DE(A_1)\},$$
$$\vdots$$

The following is the problem's approximate solution:

$$u(\mu,\eta) = \lim_{n\to\infty}\sum_{n=0}^{\infty} u_n(\mu,\eta).$$

IV. Computational Work

In this section, a few examples are presented to obtain the solution of nonlinear PDEs that develop during liquid drop formation.

Example 1: Take the nonlinear Benjamin-Bona-Mahony (BBM) equation

$$u_{\eta} + u_{\mu} + uu_{\mu} - u_{\mu\mu\eta} = 0 \tag{11}$$

with $u(\mu,0) = \mu$ as the initial condition. The precise solution is as follows:

$$u(\mu,\eta) = \frac{\mu-\eta}{1+\eta}$$

The double Elzaki transform applied to equation (11) yields,

$$DE(u_{\eta}) = -DE(u_{\mu} + uu_{\mu} - u_{\mu\mu\eta})$$

This implies

$$\frac{1}{\upsilon}T(\tau,\upsilon) - \upsilon.T(\tau,0) = -DE(u_{\mu} + uu_{\mu} - u_{\mu\mu\eta}) \tag{12}$$

When we apply a single Elzaki transform to the starting conditions, we get

$$E_1(u(\mu,0)) = T(\tau,0) = E(\mu) = u^3$$

From (12), we obtain

$$\frac{1}{\upsilon}T(\tau,\upsilon) = \upsilon(u^3) - DE(u_{\mu} + uu_{\mu} - u_{\mu\mu\eta})$$

This implies

$$T(\tau,\upsilon) = \upsilon^2(u^3) - \upsilon.DE(u_{\mu} + uu_{\mu} - u_{\mu\mu\eta})$$

When the inverse double Elzaki transforms is used, we get

$$DE^{-1}(T(\tau,\upsilon)) = DE^{-1}(\upsilon^2(u^3)) - DE^{-1}\{\upsilon.DE(u_{\mu} + uu_{\mu} - u_{\mu\mu\eta})\}$$

This implies

$$u(\mu,\eta) = \mu - DE^{-1}\{\upsilon.DE(u_{\mu} + uu_{\mu} - u_{\mu\mu\eta})\}$$

Applying Adomian decomposition approach, we get

$$\sum_{n=0}^{\infty} u_n(\mu,\eta) = \mu - DE^{-1}\left(\upsilon.DE\left\{\sum_{n=0}^{\infty} A_n(u)\right\}\right)$$

According to the Equation above, we get

$$u_0(\mu,\eta) = \mu,$$

$$u_1(\mu,\eta) = -DE^{-1}(\upsilon.DE\{A_0\}),$$
$$u_2(\mu,\eta) = -DE^{-1}(\upsilon.DE\{A_1\}),$$
$$\vdots$$

Some of the Adomian polynomials are:

$$A_0 = (1+\mu),$$
$$A_1 = -2(1+\mu)\eta,$$
$$A_2 = (1+\mu)\eta^2,$$
$$\vdots$$

Determine the value of $u_0, u_1, u_2, \ldots$ are given by

$$u_0(\mu,\eta) = \mu,$$
$$u_1(\mu,\eta) = -(1+\mu)\eta,$$
$$u_2(\mu,\eta) = (1+\mu)\eta^2,$$
$$\vdots$$

The solution is:

$$u(\mu,\eta) = u_0(\mu,\eta) + u_1(\mu,\eta) + u_2(\mu,\eta) + \cdots$$

Or

$$u(\mu,\eta) = (\mu - (1+\mu)\eta + (1+\mu)\eta^2 - \cdots)$$

The solution in closed form is:

$$u(\mu,\eta) = \frac{\mu-\eta}{1+\eta}.$$

Rangaig Transform Homotopy Analysis Method (XII):

Rewrite the given equation as:

$$u_\eta + u_\mu + uu_\mu - u_{\mu\mu\eta} = 0$$

Applying the Rangaig transform to both sides, we obtain

$$R\{u_\eta\} = -R\{(u_\mu + uu_\mu - u_{\mu\mu\eta})\}$$

This implies

$$R(u) = \frac{1}{\omega^2}u(\mu,0) - \frac{1}{\omega}R\{u_\mu + uu_\mu - u_{\mu\mu\eta}\} \tag{1}$$

Using the initial condition, we obtain

$$R(u) = \frac{1}{\omega^2}\mu - \frac{1}{\omega}R\{u_\mu + uu_\mu - u_{\mu\mu\eta}\}$$

The nonlinear operator is :

$$N = R(u) - \frac{1}{\omega^2}\mu + \frac{1}{\omega}R\{u_\mu + uu_\mu - u_{\mu\mu\eta}\}$$

Using the Homotopy Analysis Method, we obtain

$$\mathcal{R}_1\left(\overrightarrow{u_0}(\mu,\eta)\right) = -\frac{1}{\omega^3}(1+\mu),$$

$$\mathcal{R}_2\left(\overrightarrow{u_1}(\mu,\eta)\right) = \left(\frac{1}{\omega^3} - \frac{2}{\omega^4}\right)(1+\mu),$$

$$\vdots$$

Some components of solutions are:

$$u_0 = \mu,$$
$$u_1 = -(1+\mu)\eta,$$
$$u_2 = (1+\mu)\eta^2,$$
$$\vdots$$

The solution is:

$$u = u_0 + u_1 + u_2 + \cdots = \frac{\mu - \eta}{1+\eta}$$

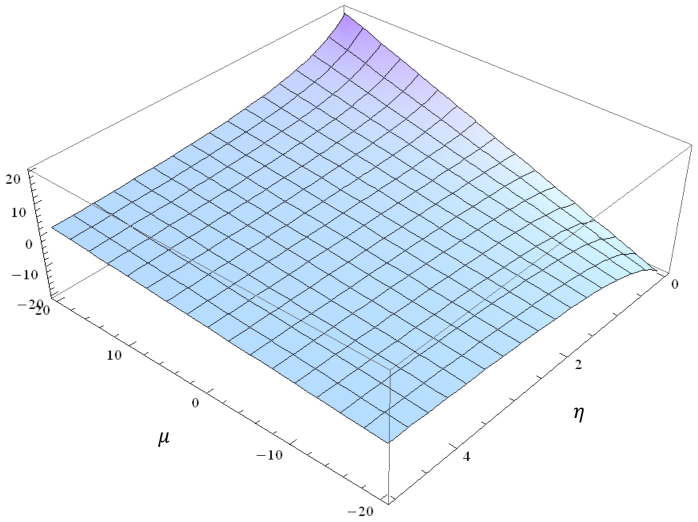
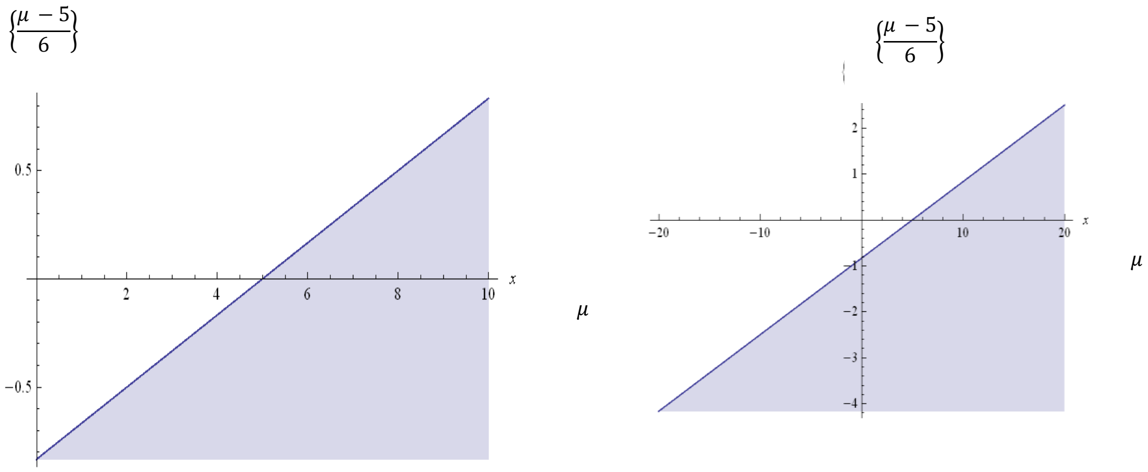


Fig. 1. Physical Interpretation of solutions of Example 1.



Fi. 2. Shows the representations of solutions at different values of parameters.

The dynamical and physical characteristics of analytical solutions produced by the Adomian decomposition approach based on double Elzaki transform at various ranges of μ and η. of Example 1, are shown in Figure 1. Figure 2 shows the representations of solutions at different ranges of parameters.

Example 2: Take the nonlinear KdV equation

$$u_\eta - auu_\mu + u_{\mu\mu\mu} = 0 \tag{13}$$

with initial condition

$$u(\mu, 0) = \frac{1}{a}(\mu - 1)$$

An exact solution is expressed as:

$$u(\mu, \eta) = \frac{1}{a}\left(\frac{\mu - 1}{1 - \eta}\right)$$

Using equation (13) and the double Elzaki transform, we obtain

$$DE(u_\eta) = DE(auu_\mu - u_{\mu\mu\mu})$$

This implies

$$\frac{1}{v}T(\tau, v) - v.T(\tau, 0) = DE(auu_\mu - u_{\mu\mu\mu}) \tag{14}$$

When we apply a single Elzaki transform on the starting condition, we obtain

$$E_1(u(\mu, 0)) = T(\tau, 0) = E\left(\frac{1}{a}(\mu - 1)\right) = \frac{1}{a}\{u^3 - u^2\}$$

From (14), we get

$$\frac{1}{v}T(\tau,v) = v\left\{\frac{1}{a}\{u^3 - u^2\}\right\} + DE\left(auu_\mu - u_{\mu\mu\mu}\right)$$

This implies

$$T(\tau,v) = v^2\left\{\frac{1}{a}\{u^3 - u^2\}\right\} + v.DE\left(auu_\mu - u_{\mu\mu\mu}\right)$$

When the inverse double Elzaki transforms is used, we get

$$DE^{-1}\left(T(\tau,\eta)\right) = DE^{-1}\left(v^2\left\{\frac{1}{a}\{u^3 - u^2\}\right\}\right) + DE^{-1}\left(v.DE\left(auu_\mu - u_{\mu\mu\mu}\right)\right)$$

This implies

$$u(\mu,\eta) = \frac{1}{a}(\mu - 1) + DE^{-1}\left(v.DE\left(auu_\mu - u_{\mu\mu\mu}\right)\right)$$

Applying Adomian decomposition approach, we get

$$\sum_{n=0}^{\infty} u_n(\mu,\eta) = \frac{1}{a}(\mu - 1) + DE^{-1}\left(v.DE\left\{\sum_{n=0}^{\infty} A_n(u)\right\}\right)$$

According to the Equation above, we get

$$u_0(\mu,\eta) = \frac{1}{a}(\mu - 1),$$
$$u_1(\mu,\eta) = DE^{-1}(v.DE\{A_0\}),$$
$$u_2(\mu,\eta) = DE^{-1}(v.DE\{A_1\}),$$
$$\vdots$$

Some of the Adomian polynomials are:

$$A_0 = \frac{1}{a}(\mu - 1),$$
$$A_1 = \frac{2}{a}(\mu - 1)\eta$$
$$A_2 = \frac{3}{a}(\mu - 1)\eta^2,$$
$$\vdots$$

The values of $u_0, u_1, u_2, \ldots$ are given by

$$u_0(\mu,\eta) = \frac{1}{a}(\mu - 1),$$
$$u_1(\mu,\eta) = \frac{1}{a}(\mu - 1)\eta$$
$$u_2(\mu,\eta) = \frac{1}{a}(\mu - 1)\eta^2,$$
$$\vdots$$

The solution is:

$$u(\mu,\eta) = u_0(\mu,\eta) + u_1(\mu,\eta) + u_2(\mu,\eta) + \cdots$$

Or

$$u(\mu,\eta) = \frac{1}{a}(\mu - 1) + \frac{1}{a}(\mu - 1)\eta + \frac{1}{a}(\mu - 1)\eta^2 + \cdots$$

Or

$$u(\mu,\eta) = \frac{1}{a}(\mu - 1)(1 + \eta + \eta^2 + \eta^3 + \cdots) = \frac{1}{a}\left(\frac{\mu - 1}{1 - \eta}\right)$$

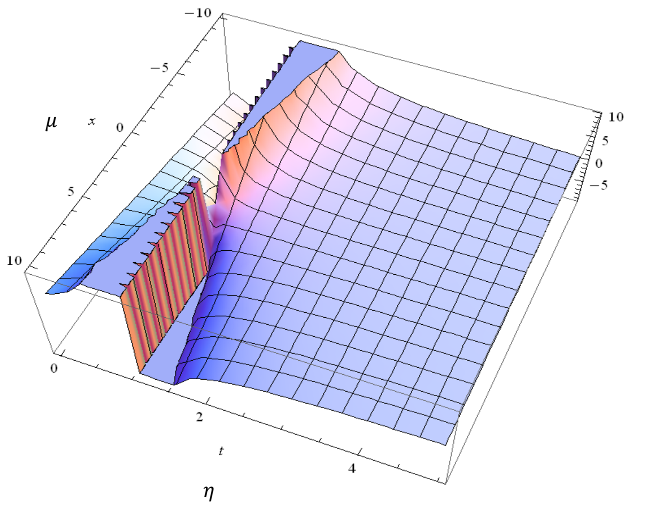


Fig. 3. Physical Interpretation of solutions of Example 2 for a = 2.

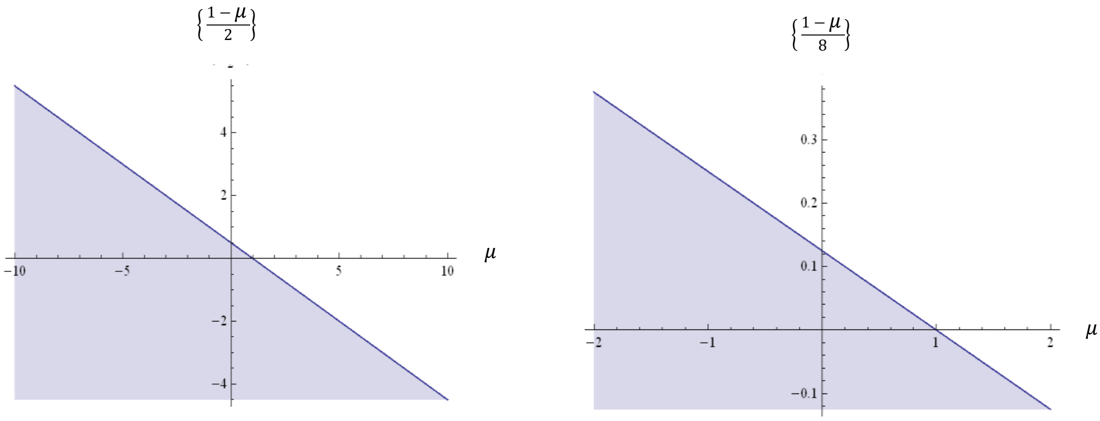


Fig. 4. Representations of the solutions of Example 1 at different values of the parameters

Figure 3 displays the physical and dynamical characteristics of analytical solutions produced by the Adomian decomposition approach based on the double Elzaki transform at various ranges of μ and η of Example 2. Figure 2 shows the representations of solutions at different ranges of parameters.

Example 3: Take the linear Schrodinger equation of the form

$$-iu_\eta = u_{\mu\mu} + (\pi^2 + 1)u, \tag{12}$$

with initial condition

$$u(\mu, 0) = \sin \pi\mu$$

An exact solution expressed as:

$$u(\mu, \eta) = e^{i\eta} \sin \pi\mu$$

Rewrite the Equation (12),

$$u_\eta = i\{u_{\mu\mu} + (\pi^2 + 1)u\}$$

Applying the double Elzaki transform to both sides, we obtain

$$DE(u_\eta) = DE\left(i\{u_{\mu\mu} + (\pi^2 + 1)u\}\right)$$

This implies

$$\frac{1}{v}T(\tau, v) - v.T(\tau, 0) = DE\left(i\{u_{\mu\mu} + (\pi^2 + 1)u\}\right)$$

After simplifications, we get

$$T(\tau, v) = v^2.T(\tau, 0) + v.DE\left[i\{u_{\mu\mu} + (\pi^2 + 1)u\}\right]$$

Applying initial conditions, we obtain

$$T(\tau, v) = v^2.\frac{\pi u^3}{1 + \pi^2 u^2} + v.DE\left[i\{u_{\mu\mu} + (\pi^2 + 1)u\}\right]$$

Taking the inverse double Elzaki transform used, we get

$$u(\mu, \eta) = \sin \pi\mu + DE^{-1}\{v.DE\left[i\{u_{\mu\mu} + (\pi^2 + 1)u\}\right]\}$$

Applying the Adomian decomposition approach, we get

$$\sum_{n=0}^{\infty} u_n(\mu, \eta) = \sin \pi\mu + DE^{-1}\left(v.DE\left\{\sum_{n=0}^{\infty} A_n(u)\right\}\right)$$

Comparing the different powers, we get

$$u_0(\mu, \eta) = \sin \pi\mu,$$
$$u_1(\mu, \eta) = DE^{-1}(v.DE\{A_0(u)\}),$$
$$u_2(\mu, \eta) = DE^{-1}(v.DE\{A_1(u)\}),$$
$$u_3(\mu, \eta) = DE^{-1}(v.DE\{A_2(u)\}),$$
$$\vdots$$

and so on. Some of the Adomian components are:

$$A_0(u) = \sin \pi\mu,$$
$$A_1(u) = i\eta \sin \pi\mu,$$
$$A_2(u) = \frac{i^2\eta^2}{2} \sin \pi\mu,$$
$$\vdots$$

and so on. Components of solutions are:

$$u_0(\mu, \eta) = \sin \pi\mu,$$
$$u_1(\mu, \eta) = i\eta . \sin \pi\mu,$$
$$u_2(\mu, \eta) = \frac{i^2\eta^2}{2!} \sin \pi\mu,$$
$$u_3(\mu, \eta) = \frac{i^3\eta^3}{3!} \sin \pi\mu,$$
$$\vdots$$

The solution is:

$$u(\mu, \eta) = u_0 + u_1 + u_2 + u_3 + u_4 + \cdots$$

Or

$$u(\mu, \eta) = \sin \pi\mu + i\eta . \sin \pi\mu + \frac{i^2\eta^2}{2!} \sin \pi\mu + \cdots$$

Or

$$u(\mu, \eta) = e^{i\eta} \sin \pi\mu$$

Rangaig Transform Homotopy Analysis Method (XII):

Rewrite the equation

$$u_\eta = i\big(u_{\mu\mu} + (\pi^2 + 1)u\big)$$

Applying the Rangaig transform to both sides, we obtain

$$R\{u_\eta\} = R\{i\big(u_{\mu\mu} + (\pi^2 + 1)u\big)\}$$

This implies

$$R(u) = \frac{1}{\omega^2} u(\mu, 0) - \frac{i}{\omega} R\{i\big(u_{\mu\mu} + (\pi^2 + 1)u\big)\} \quad (1)$$

Using the initial condition, we obtain

$$R(u) = \frac{1}{\omega^2} sin\pi\mu - \frac{i}{\omega} R\{i\big(u_{\mu\mu} + (\pi^2 + 1)u\big)\}$$

The nonlinear operator is :

$$N = R(u) - \frac{1}{\omega^2} sin\pi\mu + \frac{i}{\omega} R\{i(u_{\mu\mu} + (\pi^2 + 1)u)\}$$

Using the Homotopy Analysis Method, we obtain

$$\mathcal{R}_1(\overrightarrow{u_0}(\mu, \eta)) = \frac{i}{\omega^3} sin\pi\mu,$$

$$\mathcal{R}_2(\overrightarrow{u_1}(\mu, \eta)) = \left(-\frac{i}{\omega^3} - \frac{i^2}{\omega^4}\right) sin\pi\mu,$$

$$\vdots$$

Some components of solutions are:

$$u_0 = sin\pi\mu,$$

$$u_1 = it\, sin\pi\mu,$$

$$u_2 = \frac{(it)^2}{2!}\, sin\pi\mu,$$

$$\vdots$$

The solution is:

$$u = u_0 + u_1 + u_2 + \cdots = e^{it} sin\pi\mu$$

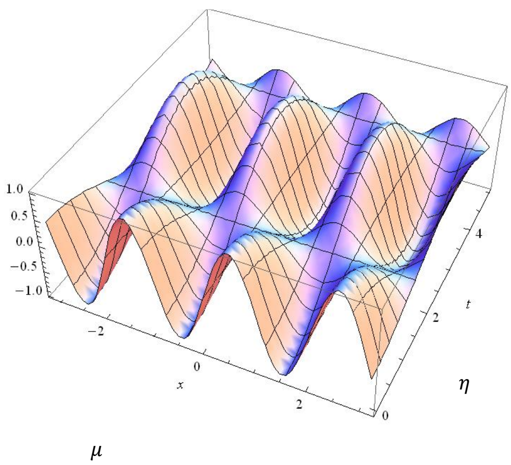


Fig. 5. Demonstrate the dynamical and physical behavior of the Real part of solutions at various ranges of μ and η.

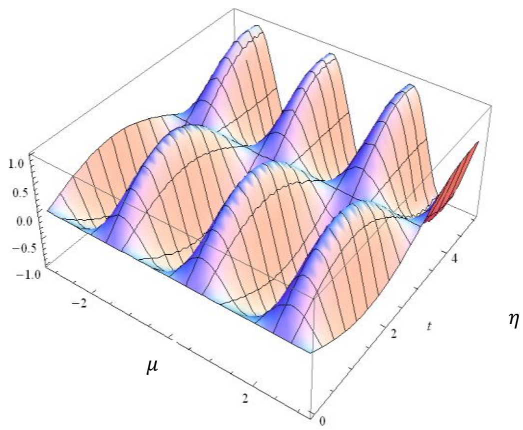


Fig. 6. Display the dynamical and physical behavior of the Imaginary part of solutions at different ranges of μ and η.

The physical and dynamical behavior of the imaginary and real parts of the solutions is shown in Figures 5 and 6 of Example 3 at various ranges of μ and η, respectively.

V. Conclusion

To get the solution of the BBM problem, when combined with the Adomian decomposition strategy, the double Elzaki transform is an effective mathematical tool., Kdv equations and Schrodinger equations, according to the computational data above. Even though the terms of infinite series can change, the solutions are still closer to the actual answer. The solutions obtained by the proposed scheme are similar and much closer to the solutions obtained by the Rangaig transform-based Homotopy analysis method. In the future, this technique will be used to solve nonlinear PDEs with semi-analytical solutions that appear in a range of engineering and scientific applications.

Conflict of Interest

The authors declare that there is no conflict of interest regarding this article.

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