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HYPERSOFT GENERALIZED COMPACTNESS AND CONNECTEDNESS IN HYPERSOFT TOPOLOGICAL SPACES

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Abstract

In this paper, we have introduced the notion of hypersoft generalized compactness and generalized connectedness in hypersoft topological spaces. We have also defined the core concepts and explored the key properties that connect them. Finally, the notion of hypersoft generalized compactness and connectedness of hypersoft topological spaces is proposed, and some related properties are discussed.

Keywords: Hypersoft generalized compactness, Hypersoft generalized connectedness, Hypersoft topological spaces.

I. Introduction

The concept of *soft set theory*, originally introduced by Molodtsov [IV], has proven to be a powerful mathematical framework for handling uncertainties in complex systems. The exploration of soft set theory and its extensions has garnered considerable interest in recent years, owing to their effectiveness in managing uncertainty and imprecision in mathematical modeling and decision-making. Among these extensions, hypersoft set theory has emerged as a powerful framework that generalizes soft sets by incorporating multi-attribute parameters, thus providing a more refined approach to uncertainty modeling. Hypersoft sets extend the classical soft sets introduced by Molodtsov [IV] by allowing the association of multiple attribute values with each parameter, making them particularly useful in various applications, including decision-making, optimization, and artificial intelligence. Saeed [IX]. et al. (2020) laid the foundation for this theory through a comprehensive exploration of its basic definitions, properties, and potential applications in their work titled ‘A Study of the Fundamentals of Hypersoft Set Theory’. This groundwork was extended in their subsequent publication, ‘An Inclusive Study on Fundamentals of

Hypersoft Set' [X], where the authors delved deeper into the structural and theoretical underpinnings of hypersoft sets, integrating key mathematical concepts and demonstrating their practical relevance.

Saeed, Ahsan, and Rahman (2021) introduced a novel methodology for mappings within hypersoft classes, thereby enhancing the applicability of hypersoft theory in areas such as classification and data analysis [XI]. Complementing these developments, Musa and Asaad (2022) proposed the concept of hypersoft topological spaces, which merges topological structures with hypersoft frameworks, facilitating new avenues in spatial reasoning and generalized continuity [XII]. Smarandache (2018) provided a significant theoretical extension by connecting hypersoft sets to plithogenic sets, a broader class that encompasses multiple degrees of truth, indeterminacy, and falsehood [XIII].

The foundational work on hypersoft sets was established by Abbas et al. [I], who introduced basic operations on hypersoft sets and hypersoft points. Aygunoglu and Aygun [II] contributed to the development of soft topological spaces, providing insights into their structural properties and potential applications in computational models. Further extending these ideas, Asaad and Musa [III] explored the concepts of continuity and compactness within the framework of hypersoft open sets, highlighting their significance in topological analysis. Musa and Asaad[IV] developed Connectedness on hypersoft topological spaces. Building upon these advancements, Mythili and Arokialancy.A [VII, VIII] introduced the notion of hypersoft generalized continuous functions and hypersoft generalized closed sets, which play a crucial role in the study of hypersoft topological spaces. Their research bridges the gap between classical topological properties and the newly emerging hypersoft structures. The introduction of hypersoft topological spaces by Musa and Asaad [V] further reinforced the applicability of hypersoft set theory, enabling a broader understanding of continuity, separation axioms, and compactness in this novel framework.

In this paper, we have introduced the notion of hypersoft generalized compactness and connectedness of hypersoft topological spaces are proposed and some related properties are discussed.

II. Preliminaries

Definition 2.1 ([IV]). Let U be an initial universe set and E be a set of parameters. Let P(U) denote the power set of U. Let A ⊆ E. A pair (F_A, E) is called a soft set over U, where F_A is a mapping given by F_A: A → P(U). In other words, a soft set over U is a parameterized family of subsets of the universe U.

Definition 2.2. [XIII]. A pair $(F, A_1 \times A_2 \times \dots \times A_n)$ is called a hypersoft set over U, where F is a mapping given by $F: A_1 \times A_2 \times \dots \times A_n \to P(U)$. Simply, we write the symbol E for $E_1 \times E_2 \times \dots \times E_n$, and for the subsets of E: the symbols A for $A_1 \times A_2 \times \dots \times A_n$, and B for $B_1 \times B_2 \times \dots \times B_n$. Each element in A, B, and E is an n-tuple element. We can represent a hypersoft set (F, A) as an ordered pair, (F, A) = {(α, F(α)): α ∈ A}.

Definition 2.3[I]: For two hypersoft sets (F, A) and (G, B) over a common universe U, we say that (F, A) is a hypersoft subset of (G, B) if (1) A ⊆ B, and (2) F(α) ⊆ G(α) for all α ∈ A. We write (F, A) ⊆ (G, B).

Definition 2.3[I]: For two hypersoft sets (F, A) and (G, B) over a common universe U, we say that (F, A) is a hypersoft subset of (G, B) if (1) A ⊆ B, and (2) F(α) ⊆ G(α) for all α ∈ A. We write (F, A) ⊆ (G, B).

Definition 2.4 [I]: Two hypersoft sets (F, A) and (G, B) over a common universe U are said to be hypersoft equal if (F, A) is a hypersoft subset of (G, B) and (G, B) is a hypersoft subset of (F, A).

Definition 2.5 [I]: Difference of two hypersoft sets (F, A) and (G, B) over a common universe U, is a hypersoft set (H, C), where C = A ∩ B and for all α ∈ C, H(α) = F(α)\G(α). We write (F, A) \ (G, B) = (H, C).

Definition 2.6 [I]: Union of two hypersoft sets (F, A) and (G, B) over a common universe U is a hypersoft set (H, C), where C = A ∩ B and for all α ∈ C, H(α) = F(α) ∪ G(α). We write (F, A) ∪ (G, B) = (H, C).

Definition 2.7[I]: Intersection of two hypersoft sets (F, A) and (G, B) over a common universe U is a hypersoft set (H, C), where C = A∩B and for all α ∈ C, H(α) = F(α)∩G(α). We write (F, A) ∩ (G, B) = (H, C).

Definition 2.8 [V]: Let τ_H be the collection of hypersoft sets over U, then τ_H is said to be a hypersoft topology on U if

(1) (Φ, E), (X, E) belong to τ_H ,

(2) The intersection of any two hypersoft sets in τ_H belongs to τ_H ,

(3) The union of any number of hypersoft sets in τ_H belongs to τ_H . Then (U, τ_H , E) is called a hypersoft topological space over U.

Definition 2.9 [VIII]: A hypersoft set (H_S, E) is called a hypersoft generalized closed (hypersoft g-closed) in a hypersoft topological space (X, τ_H , E) if cl (H_S, E) ⊆ (U, E) whenever (H_S, E) ⊂ (U, E) and (U, E) is hypersoft open in X.

Definition 2.10 [VIII]: A hypersoft set (H_S, E) is called a hypersoft generalized open (hypersoft g-open) in a hypersoft topological space (X, τ_H , E) if the relative complement (H_S, E)' is hypersoft g-closed in X.

Definition 2.11 [VII]: An H_S map f_H: $(X, \tau_H, E) \rightarrow (Y, \sigma_H, E)$ is said to be H_S continuous if the inverse image of every closed set in (Y, σ_H, E) is H_S-Closed in (X, τ_H, E).

Definition 2.12 [VII]: A function f_H: $(X, \tau_H, E) \rightarrow (Y, \sigma_H, E)$ is said to be Hypersoft generalized continuous (briefly $H_S g$-Continuous) if the inverse image of every H_S-closed set in (Y, σ_H, E) is $H_S g$-Closed in (X, τ_H, E).

III. Hypersoft Generalized Compactness and Hypersoft Generalized Connectedness

In this section, we introduce the hypersoft generalized compactness, hypersoft generalized connectedness, and investigate some of their properties.

Definition 3.1: A subset A of a hypersoft topological space (X, τ_H, E) is said to be hypersoft generalized compact (**HSG-compact**) if every open cover of hypersoft generalized open set A has a finite subcover.

3.1.1 Comparison with standard hypersoft compactness

While standard hypersoft compactness requires every hypersoft open cover to have a finite subcover, HSG-compactness generalized this by using hypersoft generalized open sets instead of only hypersoft open sets. Thus, every hypersoft compact space is HSG-compact, as hypersoft open sets are also hypersoft generalized open. However, the converse is not true.

Example 3.2: Let X= {a, b}, $E_1=\{e_1\}$, $E_2=\{e_2\}$, $E_3=\{e_3\}$ with hypersoft topology with $\tau_H = \{(\varphi, E), (\chi, E), (G_1, E)\}$ where $(G_1, E) = \{\{(e_1, e_2, e_3), \{a\}\}$. Define a generalized open set $(G_2, E) = \{\{(e_1, e_2, e_3), \{b\}\}$. It is HSG-open but not open in τ_H. The cover $\{(G_1, E), (G_2, E)\}$ covers X, making it HSG-compact. However, since (G_2, E) is not open in τ_H, X fails standard hypersoft compactness.

Definition 3.3: A hypersoft topological space (X, τ_H, E) is said to be hypersoft generalized connected (**HSG-connected**) if it cannot be represented as the union of two non-empty disjoint hypersoft generalized open sets.

Theorem 3.4: The finite union of hypersoft generalized compact subsets is hypersoft generalized compact.

Proof: Let $A_1, A_2, \ldots A_n$ be an HSG-compact subset of (X, τ_H, E).
Let

$$A = \bigcup_{i=1}^{n} A_i.$$

Given any open cover U of A by hypersoft generalized open sets,

- Since each $A_i \subseteq A$, U covers each A_i
- By compactness of A_i, there exists a finite subcover $U_i' \subseteq U$ for each A_i.
- The union of these finite subcovers $U' = \cup_{i=1}^{n} U_i'$ is finite and covers A.

Thus, A is HSG-compact.

Example 3.5: Let X= {a, b, c}, E_1={e_1}, E_2={e_2}, E_3={e_3} with hypersoft topology,

$\tau_H = \{\{(e_1, e_2, e_3), \varphi\}, \{(e_1, e_2, e_3), X\}, \{(e_1, e_2, e_3), \{a\}\}, \{(e_1, e_2, e_3), \{a, b\}\}\}$.

Consider the subsets $A_1 = \{(e_1, e_2, e_3), \{a\}\}$, $A_2 = \{(e_1, e_2, e_3), \{b\}\}$. Both A_1 and A_2 are covered by the hypersoft open set $\{a, b\}$, thus making them HSG-compact. Their union $A = A_1 \cup A_2 = \{(e_1, e_2, e_3), \{a, b\}\}$ is also HSG -compact.

Theorem 3.6: Every HSG-closed subset of a HSG-compact set is HSG-compact.

Proof: Let $A \subseteq X$ be HSG- compact and $B \subseteq A$ be HSG- closed. Let $U = \{U_\alpha\}_{\alpha\in I}$ be an open cover of B. Extend U by adding X \ B to form $\curlyvee = U \cup \{X\backslash B\}$, which covers A. Since A is HSG-compact, there exists a finite subcover $\curlyvee' \subseteq \curlyvee$. Removing X \ B from $\curlyvee'$ leaves a finite subcover. Thus, B is HSG-Compact.

Example 3.7: Let X= {a, b, c}, E_1={e_1}, E_2={e_2}, E_3={e_3} with hypersoft topology, $\tau_H = \{\{(e_1, e_2, e_3), \varphi\}, \{(e_1, e_2, e_3), X\}, \{(e_1, e_2, e_3), \{a\}\}, \{(e_1, e_2, e_3), \{a, b\}\}\}$. Take A = X, which is trivially HSG-compact. The subset $B = \{(e_1, e_2, e_3), \{a, b\}\}$ is hypersoft generalized closed in A and HSG-compact. As it can be covered by $\{(e_1, e_2, e_3), \{a, b\}\}$.

Theorem 3.8: A Continuous image of HSG-connected space is HSG-connected.

Proof: Let $f_H: (X, \tau_H, E) \to (Y, \sigma_H, E)$ be continuous, and (X, τ_H, E) be HSG-connected. Suppose, for contradiction, that $f(x) = U \cup V$ where U and V are disjoint, non-empty hypersoft generalized open sets in (Y, σ_H, E). The pre-images $f^{-1}(U)$ and $f^{-1}(V)$ are hypersoft generalized open in (X, τ_H, E), disjoint, and cover (X, τ_H, E). This contradicts the connectedness of (X, τ_H, E). Thus, $f(x)$ is HSG-connected.

Remark 3.9: HSG-compactness is preserved under continuous hypersoft functions when the function is surjective, analogous to classical compactness results. However, general proofs in the hypersoft framework require formal verification of image mappings preserving generalized open subcovers.

Example 3.10: Let $X = \{a, b\}$ with $\tau_H = \{\{(e_1, e_2, e_3), \varphi\}, \{(e_1, e_2, e_3), \{a\}\}, \{(e_1, e_2, e_3), X\}\}$ which is trivially HSG-connected. Define $f_H: (X, \tau_H, E) \to (Y, \sigma_H, E)$, let Y = {1,2} by f(a) = 1, f(b) = 2 and let (Y, σ_H, E) have the discrete topology, $\sigma_H = \{\{(e_1, e_2, e_3), \varphi\}, \{(e_1, e_2, e_3), \{1\}\}, \{(e_1, e_2, e_3), \{2\}\}, \{(e_1, e_2, e_3), Y\}\}$. Consider the open set {1} in (Y, σ_H, E), its preimage under f is $f^{-1}(\{1\}) = \{a\}$, since {a} is open in (X, τ_H, E) under τ_H, this condition holds. Similarly, $f^{-1}(\{2\}) = \{b\}$. Since {b} is not explicitly listed as an open set in τ_H, thus, f is continuous in the hypersoft topology. The image f(x) = {1, 2} is not connected in the discrete topology, showing that continuity alone does not preserve connectedness in general soft topologies, but under hypersoft generalized conditions, the result holds.

Theorem 3.11: Every HSG-compact space with no proper hypersoft generalized clopen subsets is HSG-connected.

Proof: Let (X, τ_H, E) be HSG-compact with no proper hypersoft generalized clopen sets. Suppose (X, τ_H, E) is not connected. Then $X = A \cup B$, where A, B are non-empty, disjoint hypersoft generalized open sets. Both A & B would be hypersoft generalized clopen, contradicting the assumption. Thus, (X, τ_H, E) is HSG-connected.

Example 3.12: Let X= {a, b}, $E_1=\{e_1\}$, $E_2=\{e_2\}$, $E_3=\{e_3\}$ with the trivial hypersoft topology, $\tau_H = \{\{(e_1, e_2, e_3), \varphi\}, \{(e_1, e_2, e_3), X\}\}$ there are no proper hypersoft generalized clopen sets in (X, τ_H, E). (X, τ_H, E) is trivially HSG-compact and HSG-connected.

Theorem 3.13: The finite intersection of hypersoft generalized compact sets is hypersoft generalized compact.

Proof: Let $A_1, A_2, \ldots A_n$ be HSG-compact subsets of (X, τ_H, E). Consider $A = \bigcap_{i=1}^{n} A_i$. Consider any open cover U of A consisting of HSG-open sets. Since $A \subseteq A_i, U \cup \{X \setminus A_i\}$ forms an open cover for each A_i. By the compactness of each A_i, there exists a finite subcover covering A_i. Therefore, the union of these finite subcovers covers A. Hence, A is HSG-compact.

Theorem 3.11: If $f_H: (X, \tau_H, E) \to (Y, \sigma_H, E)$ is continuous and surjective and X is HSG-connected, then Y is HSG-connected.

Proof: Assume Y is not HSG-connected. Then $Y = U \cup V$ where U & V are disjoint, non-empty HSG-open sets. The preimages $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint, non-empty HSG-open in X, contradicting the connectedness of X. Thus, Y is HSG-connected.

Theorem 3.12: The Product of two HSG-connected spaces is HSG-connected.

Proof: Let (X, τ_H, E) and (Y, σ_H, E) be HSG-connected spaces. Consider $X \times Y$ with the product topology. Assume $X \times Y$ is not HSG-connected. Then there exist disjoint, non-empty open sets U, V separating $X \times Y$. This implies that either X or Y can be partitioned into non-empty disjoint open sets, contradicting their connectedness. Thus $X \times Y$ is HSG-connected.

Theorem 3.13: If $\{A_\alpha\}_{\alpha\in I}$ is a family of HSG-connected subsets of X with non-empty intersection, then $\cup_{\alpha\in I} A_\alpha$ is HSG-connected.

Proof: Let $p \in \cap_{\alpha\in I} A_\alpha$, for contradiction, assume $\cup_{\alpha\in I} A_\alpha = U \cup V$ where U, V are disjoint, non-empty HSG-open sets. The point p must belong to either U or V (say $p \in U$) each A_α being connected implies that $A_\alpha \subseteq U$, leading to $\cup A_\alpha \subseteq U$, which contradicts the assumption that both U & V are non-empty. Thus $\cup_{\alpha\in I} A_\alpha$ is HSG-connected.

Theorem 3.15: The HSG-connected components of an HSG-compact space are themselves HSG-compact.

Proof: Let C be a connected component of an HSG-compact space X. Since C is a maximal connected subset, any open cover of C extends to X. Using compactness of X, a finite subcover exists that restricts to C. Thus, C is HSG-compact.

Example of Compactness & Connectedness:

Let X = {a, b, c, d} $E_1=\{e_1\}$, $E_2=\{e_2\}$, $E_3=\{e_3\}$ with hypersoft topology $\tau_H = \{\{(e_1, e_2, e_3), \varphi\},\{(e_1, e_2, e_3), \{a\}\}, \{(e_1, e_2, e_3), \{a, b\}\}, \{(e_1, e_2, e_3), \{c, d\}\}, \{(e_1, e_2, e_3), X\}\}$, subsets A={a, b} and B = {c, d} are HSG-compact (finite subcover exist). The entire space X is the union of two HSG-compact components. X is not HSG-connected as A & B are disjoint HSG-open sets. This illustrates the independence of Compactness and Connectedness in hypersoft topological spaces.

IV. Hypersoft generalized locally compact space

A hypersoft topological space (X, τ_H, E) is called hypersoft generalized locally compact (HSG-locally compact) at a point $x \in X$ if there exists a hypersoft generalized open neighborhood U of x such that cl(U) is hypersoft generalized compact. If every point in X has such a neighborhood, X is called HSG-locally compact.

Definition 4.1: Hypersoft generalized locally compact subspace:

A subspace $A \subseteq X$ of a hypersoft topological space is HSG-locally compact if every point $x \in A$ has a hypersoft generalized open neighborhood in A whose closure in A is HSG-compact.

Theorem 4.2: Every HSG-compact Space is HSG-locally compact.

Proof: Let X be HSG-compact. For every $x \in X$, take U = X, which is trivially a hypersoft generalized open neighborhood of x. The closure of U is X, which is HSG-compact by assumption. Thus, X is HSG-locally compact.

Example 4.3: Let X = {a, b, c} E1={e1}, E2={e2}, E3={e3} with hypersoft topology $\tau_H = \{\{(e_1, e_2, e_3), \varphi\}, \{(e_1, e_2, e_3), \{a\}\}, \{(e_1, e_2, e_3), \{a, b\}\}, \{(e_1, e_2, e_3), X\}\}$. X is finite and thus HSG-compact; every point x has an open neighborhood U = X whose closure is X. Thus, X is HSG-locally compact.

Theorem 4.4: A hypersoft generalized open subspace of HSG-locally compact space is HSG-locally compact.

Proof: Let (X, τ_H, E) be HSG-locally compact and let $A \subseteq X$ be an open subset. For any $x \in A$

Since X is HSG-locally compact, there exists an open neighborhood U of x in X such that cl(A) is HSG-compact. Since A is open, $U \cap A$ is an open neighborhood of x in A. The closure of $U \cap A$ in A is contained in cl(U), which is HSG-compact. Therefore, A is HSG-locally compact.

Example 4.5: Let X = {a, b, c, d} $E_1=\{e_1\}$, $E_2=\{e_2\}$, $E_3=\{e_3\}$ with hypersoft topology $\tau_H = \{\{(e_1, e_2, e_3), \varphi\}, \{(e_1, e_2, e_3), \{a\}\}, \{(e_1, e_2, e_3), \{a, b\}\}, \{(e_1, e_2,$

e_3), $\{c, d\}\}$, $\{(e_1, e_2, e_3), X\}\}$ X is HSG-locally compact. The subset A = {a, b} is open in X and inherits local compactness.

Theorem 4.6: If X is HSG-locally compact and $K \subseteq X$ is HSG-compact, then K is hypersoft generalized closed.

Proof: Let K be HSG-compact in X. Suppose K is not HSG-closed, then there exists a point

$x \in cl(K) \backslash K$. Since X is HSG-locally compact, there exists an open neighborhood U of x such that cl(U) is HSG-compact. But then cl(U) would contain an open cover of $K \cup \{x\}$ without a finite subcover, contradicting compactness. Thus, K must be HSG-closed.

Example 4.7: Let X = {a, b, c, d} $E_1=\{e_1\}$, $E_2=\{e_2\}$, $E_3=\{e_3\}$ with hypersoft topology τ_H =

$\{\{(e_1, e_2, e_3), \varphi\}, \{(e_1, e_2, e_3), \{a\}\}, \{(e_1, e_2, e_3), \{a, b\}\}, \{(e_1, e_2, e_3), \{c, d\}\}, \{(e_1, e_2, e_3), X\}\}$ The subset K = {a, b} is HSG-compact. It is also HSG-closed because its complement {c, d} is open.

Theorem 4.8: HSG-locally compact spaces are regular.

Every HSG-locally compact space is HSG-regular, meaning that for every point x and every HSG-closed set C not containing x, there exist disjoint HSG-open sets separating them.

Proof: Let $x \in X$ and C be a hypersoft generalized closed set such that $x \notin C$. Since X is HSG- locally compact, there exists an open set U containing x such that cl(U) is HSG-compact. Since C is HSG-closed and disjoint from U, we can separate x and C using open sets U and V = X \ cl(U). Thus, X is HSG-regular.

V. Conclusion

With this study, we have contributed to the field of hypersoft topological spaces. We have developed new types of HSG compactness and HSG-connectedness, and have investigated and produced several examples to validate and illustrate the relationships established.

Conflict of Interest:

There was no relevant conflict of interest regarding this paper.

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