JOURNAL OF MECHANICS OF CONTINUA AND MATHEMATICAL SCIENCES
www.journalimcms.org

ISSN (Online) : 2454 -7190 Vol.-20, No.-8, August (2025) pp 1 - 10 ISSN (Print) 0973-8975

PROPERTIES OF A CLASS OF ANALYTIC FUNCTIONS ASSOCIATED WITH EXPONENTIALLY CONVEX FUNCTIONS

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https://doi.org/10.26782/jmcms.2025.08.00001

(Received: April 27, 2025; Revised: July 22, 2025; Accepted: August 06, 2025)

Abstract

Studies in univalent function theory comprising the exponential of differential characterizations are rarely considered. The prominent study in this direction is the study of so-called α-exponentially convex functions. Here we study a class of analytic functions which satisfy an analytic characterization influenced by the definition of the multiplicative derivative and α-exponentially convex functions. Integral representation and coefficient inequalities of the defined function class are the main results of the paper.

Keywords: Analytic function, exponentially convex functions, multiplicative derivative, starlike functions.

I. Introduction

Arango et al. in [I] introduced the family $\varepsilon(\alpha)$, $\alpha \in \mathbb{C}$ of α-exponentially convex functions $\chi \in \mathcal{A}$ which is normalized by $\chi(0) = \chi'(0) - 1 = 0$ in the unit disc $\mathbb{E} = \{\nu : \nu \in \mathbb{C} \text{ and } |\nu| < 1\}$ such that $e^{\alpha \chi(\nu)}$ is convex univalent in $\mathbb{E}$. Precisely for $\alpha \in \mathbb{C} \setminus \{0\}$, a function $\chi \in \mathcal{A}$ is in $\varepsilon(\alpha)$ if and only if

$$Re\left(1+\frac{\nu\chi''(\nu)}{\chi'(\nu)}+\alpha\,\nu\chi'(\nu)\right)>0, \qquad (1)$$

where $\chi \in \mathcal{A}$is of the form

$$\chi(\nu)=\nu+\sum_{n=2}^{\infty}a_n\nu^n, \;\; (\nu\in\mathbb{E};\; a_n\in\mathbb{C}). \qquad (2)$$

The region of variability of α-exponentially convex functions was illustrated by Ponnusamy et al. in [XII]. The so-called class of α-exponentially bi-convex functions was introduced by Sharma et al. [XV]. Let Ω and $\mathcal{S}$ denote the respective families of univalent functions and Caratheòdory's functions (see [III, XIV]). The subfamily of $\mathcal{A}$ popularly known as starlike ($\mathcal{S}^*$) and convex functions ($\mathcal{C}$) satisfy the differential inclusions

$$\frac{\nu\chi'(\nu)}{\chi(\nu)}\in\Omega \quad \text{and} \quad \frac{\left(\nu\,\chi'(\nu)\right)'}{\chi'(\nu)}\in\Omega,$$

respectively. Ma-Minda [IX] defined a family of functions $\Psi\in\Omega$ of the form

$$\Psi(\nu)=1+\psi_1\nu+\psi_2\,\nu^2+\psi_3\nu^3+\cdots, \quad (\psi_1>0;\;\nu\in\mathbb{E}) \qquad (3)$$

They obtained the coefficient estimates for the functions of the form Ψ, which are starlike and symmetric with respect to the real axis. Motivated by classes $\mathcal{S}^*$ and $\mathcal{C}$, Ma, and Minda defined the following

$$\mathcal{S}^*(\Psi):=\left\{\chi\in\mathcal{A};\frac{\nu\chi'(\nu)}{\chi(\nu)}\prec\Psi(\nu)\right\}$$

and

$$\mathcal{C}(\Psi):=\left\{\chi\in\mathcal{A};\,\frac{\left(\nu\chi'(\nu)\right)'}{\chi'(\nu)}\prec\Psi(\nu)\right\}$$

Studies replacing the Ma-Minda function Ψ in $\mathcal{S}^*(\Psi)$ and $\mathcal{C}(\Psi)$ are enormous in literature; here we spotlight only a few.

Table 1: Studies Impacted by Conic Domains

Conic Region	$\Psi(\nu)$	Reference
Balloon-shaped region	$\frac{2\sqrt{1+\nu}}{1+e^{-\nu}}$	Khan et al. [VIII]
Three leaf-type region	$1+\frac{4}{5}\nu+\frac{1}{5}\nu^4$	Murugusundaramoorthy et al. [X]
Cardioid	$1+\frac{4}{3}\nu+\frac{2}{3}\nu^2$	Sharma et al. [XV]
Crescent or Lune shape	$\nu+\sqrt{1+\nu^2}$	Raina and Sokół [XIII]
Limacon	$1+\sqrt{2}\nu+\frac{\nu^2}{2}$	Cho et al. [IV]

Sunil Verma et al. [XVI] obtained the radius of exponential convexity as 0.36 and $\frac{1}{3}$ for the classes of functions $\chi \in \mathcal{A}$ satisfying the respective characterizations

$$Re\left(\frac{\nu\chi'(\nu)e^{\alpha\chi(\nu)}}{g(\nu)}\right) > 0, \qquad (\nu \in \mathbb{E}; \alpha \in \mathbb{C} \setminus \{0\}),$$

and

$$\left|\frac{\nu\chi'(\nu)e^{\alpha\chi(\nu)}}{g(\nu)} - 1\right| < 1, \qquad (\nu \in \mathbb{E}; \alpha \in \mathbb{C} \setminus \{0\}),$$

where $g(\nu)$ satisfies $Re\left(\frac{\chi(\nu)}{\nu}\right) > 0$. The other motivation to make this study stems from the definition of the so-called *multiplicative calculus.* Although its applications are limited when compared to classical calculus, it is very interesting and has been useful in the field of economics and finance. For a positive real-valued function $\chi: \mathcal{R} \to \mathcal{R}$, the multiplicative derivative is defined by

$$\chi(x) = \lim_{h\to 0}\left(\frac{\chi(x+h)}{\chi(x)}\right)^{\frac{1}{h}} = e^{\frac{\chi'(x)}{\chi(x)}} = e^{[\ln x]'}$$

where $\chi'(x)$ is the classical derivative. For the complex case of multiplicative derivative, for ν defined in a small neighborhood where the function χ does not vanish, the multiplicative derivative is defined by

$$\chi^*(\nu) = e^{\frac{\chi'(\nu)}{\chi(\nu)}}$$

Motivated by the definition of the multiplicative derivative, Karthikeyan and Murugusundaramoorthy in [VI] studied a class of functions $\chi \in \mathcal{A}$ that satisfy

$$\frac{\nu e^{\frac{\nu^2\chi'(\nu)}{\chi(\nu)}}}{\chi(\nu)} \prec \Psi(\nu), \tag{4}$$

where $\Psi \in \Omega$ is defined as in (3). Functions $\chi \in \mathcal{A}$ that satisfy (4) will be denoted by $\mathcal{R}(\Psi)$. The class is non-empty and possesses good geometrical implications, but it does not reduce to well-known subclasses of $\mathcal{S}$. For the detailed analysis and closure properties of the class $\mathcal{R}(\Psi)$, refer to [VI, VII]. The meromorphic analogue of R(Ψ) was studied by Breaz et al. [II].

Motivated by the class studied by Arango et al. in [1], we now define the following.

Definition 1. A function $\chi \in \mathcal{A}$ belongs to the class $\mathcal{E}(\Psi)$ if it satisfies

$$1 + \left[\frac{\nu^2\chi''(\nu)}{\chi(\nu)} + \frac{\nu\chi'(\nu)}{\chi(\nu)} - \left(\frac{\nu\chi'(\nu)}{\chi(\nu)}\right)^2\right] e^{\frac{\nu\chi'(\nu)}{\chi(\nu)}-1} \prec \Psi(\nu), \tag{5}$$

where $\Psi(\nu) \in \Omega$ has a power series representation of the form (3)

Remark 1.:The defined class $\mathcal{E}(\Psi)$ does not unify or extend the existing studies. But it is closely related to the well-known function classes like starlike and convex functions.

Example 1. The function class $\mathcal{E}(\Psi)$ is non-empty. Let $\chi(\nu) = \frac{5\nu}{5-\nu}$, then it can easily be seen that χ satisfies the normalization required for the class $\mathcal{A}$. On computation, the left side of (5) will reduce to

$$\mathcal{L}(\nu) = 1 + \left(\frac{5\nu}{5-\nu} + \frac{5\nu^2}{(5-\nu)^2}\right) e^{\frac{\nu}{5-\nu}}$$

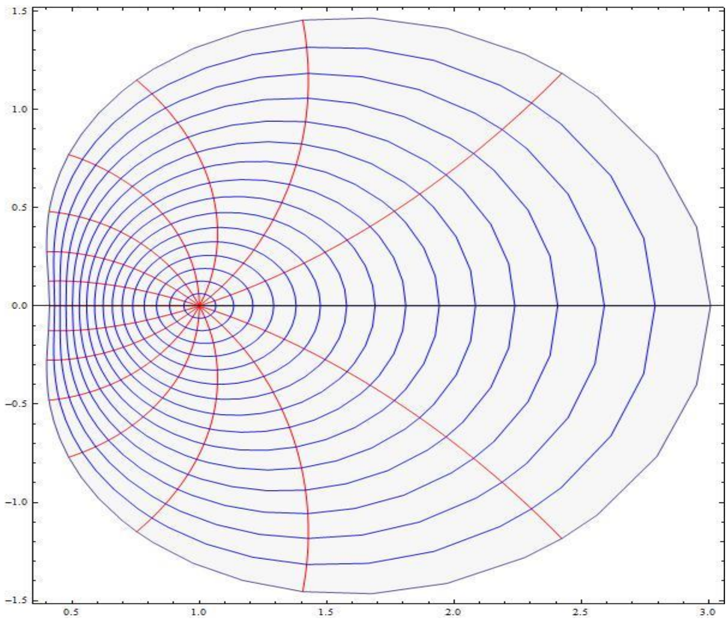

Fig. 1. The image of $|\nu| < 1$ under $\mathcal{L}(\nu) = 1 + \left(\frac{5\nu}{5-\nu} + \frac{5\nu^2}{(5-\nu)^2}\right) e^{\frac{\nu}{5-\nu}}$

The function $\mathcal{L}(\nu)$ maps the unit disc onto a convex domain in the right half plane (see Figure 1). Hence $\chi(\nu) = \frac{5\nu}{5-\nu} \in \mathcal{E}(\Psi)$.

Remark 2. If $\chi \in \mathcal{E}(\Psi)$, then χ is not convex univalent. On the other hand, the function $\chi(\nu) = \frac{\nu}{1-\nu}$ is well-known for being an extremal function in the family of convex functions and its power series representation of the form $\chi(\nu) = \nu + \sum_{n=2}^{\infty} \nu^n$. Now the left-hand side of (5) is equivalent to

$$\Lambda(\nu) = 1 + \left(\frac{\nu}{1-\nu} + \frac{\nu^2}{(1-\nu)^2}\right) e^{\frac{\nu}{1-\nu}}.$$

The function $\Lambda(\nu)$ maps the disc of radius $\frac{1}{3}$ onto a cardioid in the right half plane (see Figure 2). That is, the function $\chi(\nu) = \frac{\nu}{1-\nu}$ belongs to the class $\mathcal{E}(\Psi)$ if $|\nu| < \frac{1}{3}$ (see Figure 2). Hence, all functions that are convex do not belong to $\mathcal{E}(\Psi)$. Similarly, we can show that the Koebe function $\chi(\nu) = \frac{\nu}{(1-\nu)^2}$ which is an extremal function in a class of starlike functions that do not belong to $\mathcal{E}(\Psi)$.

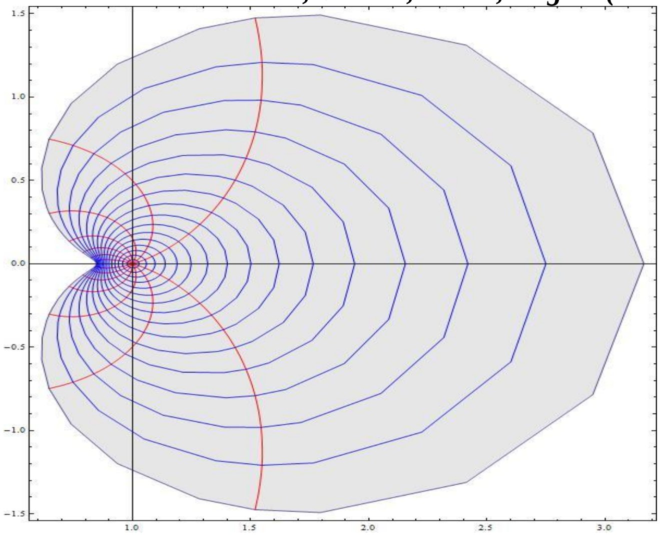

Fig. 2. The image of $|\nu| < \frac{1}{3}$ under $\Lambda(\nu) = 1 + \left(\frac{\nu}{1-\nu} + \frac{\nu^2}{(1-\nu)^2}\right) e^{\frac{\nu}{1-\nu}}$

II. Integral Representation and Coefficient Inequalities

We will begin with finding the integral representation of functions belonging to class $\mathcal{E}(\Psi)$.

Theorem 1. A function $\chi \in \mathcal{E}(\Psi)$ if and only if there exists a function $p \in \Omega$ such that $p \prec \Psi$ and

$$\chi(\nu) = \nu \ \exp\left(\int_0^\nu \frac{1}{\eta} \log\left\{\int_0^\eta \frac{p(t)-1}{t} dt\right\}\right), (\nu \in \mathbb{E}; \log 1 = 0). \qquad (6)$$

Proof. Let us suppose that a function $p(\nu) \in \Omega$ satisfies $p(\nu) \prec \Psi(\nu)$ and also suppose that $\chi \in \mathcal{E}(\Psi)$. Now (5) can be rewritten as

$$\left[\frac{\nu^2\chi''(\nu)}{\chi(\nu)} + \frac{\nu\chi'(\nu)}{\chi(\nu)} - \left(\frac{\nu\chi'(\nu)}{\chi(\nu)}\right)^2\right] e^{\frac{\nu\chi'(\nu)}{\chi(\nu)}-1} = \frac{p(\nu)-1}{\nu}. \qquad (7)$$

Equivalently, (7) can be rewritten as

$$\frac{d}{d\nu}\left[e^{\frac{\nu\chi'(\nu)}{\chi(\nu)}-1}\right] = \frac{p(\nu)-1}{\nu}, \qquad (8)$$

which, upon integrating, would lead to

$$\frac{d}{d\nu}\log\frac{\chi(\nu)}{\nu} = \frac{1}{\nu}\log\left\{\int_0^\nu \frac{p(t)-1}{t} dt\right\}, \qquad (\nu \in \mathbb{E}; \log 1 = 0). \qquad (9)$$

Again, integrating the expression (9), we have

$$\chi(\nu) = \nu \ exp\left(\int_0^\nu \frac{1}{\eta}\log\left\{\int_0^\eta \frac{p(t)-1}{t} dt\right\} d\eta\right), \qquad (\nu \in \mathbb{E}; \log 1 = 0).$$

Hence, the proof of Theorem 1.

Ma-Minda [IX] obtained the bounds of $\left|\ell_2 - \rho\, \ell_1^2\right|$ for $L(\nu) = 1 + \sum_{r=1}^{\infty} \ell_r \nu^r \in \Omega$ when ρ is real. Generalizing the inequality of Livingston, Efraimidis [V] obtained the following result.

Lemma 1. [V] If $L(\nu) = 1 + \sum_{r=1}^{\infty} \ell_r \nu^r \in \Omega$, and $\rho \in \mathbb{C}$, then

$$|\ell_\varepsilon - \rho\, \ell_r \ell_{\varepsilon - r}| \le 2 \max\{1, |2\rho - 1|\},$$

for all $1 \le r \le \varepsilon - 1$. In particular, for $\varepsilon = 2$ the above inequality will reduce to

$$\left|\ell_2 - \rho\, \ell_1^2\right| \le 2 \max\{1, |2\rho - 1|\}$$

and the result is sharp for the functions:

$$L_1(\nu) = \frac{1+\nu}{1-\nu} \quad \text{and} \quad L_2(\nu) = \frac{1+\nu^2}{1-\nu^2}.$$

We will now proceed to find the upper bound of the coefficient estimates in $\chi \in \mathcal{E}(\Psi)$.

Theorem 1. Let $\chi(\nu) \in \mathcal{E}(\Psi)$, then we have

$$|a_2| \le \psi_1 \text{ and } |a_3| \le \frac{\psi_1}{4} \max\left\{1, \left|\frac{\psi_2}{\psi_1} + \psi_1\right|\right\} \qquad (10)$$

and for all $\rho \in \mathbb{C}$

$$\left|a_3 - \rho\, a_2^2\right| \le \frac{\psi_1}{4} \max\left\{1, \left|\frac{\psi_2}{\psi_1} + (1 - 4\rho)\psi_1\right|\right\}. \qquad (11)$$

The inequalities are sharp.

Proof. As $\chi(\nu) \in \mathcal{E}(\Psi)$, by (5), we have

$$1 + \left[\frac{\nu^2 \chi''(\nu)}{\chi(\nu)} + \frac{\nu \chi'(\nu)}{\chi(\nu)} - \left(\frac{\nu \chi'(\nu)}{\chi(\nu)}\right)^2\right] e^{\frac{\nu\chi'(\nu)}{\chi(\nu)} - 1} = \Psi[w(\nu)]. \qquad (12)$$

To expand the right-hand side of (12), we let $h(\nu) \in \Omega$ be of the form $h(\nu) = 1 + \sum_{n=1}^{\infty} \vartheta_n z^n$, we consider

$$h(\nu) = \frac{1 + w(\nu)}{1 - w(\nu)},$$

where $w(\nu)$ is such that $w(0) = 0$ and $|w(z)| < 1$. On simple computation, we have

$$w(\nu) = \frac{h(\nu) - 1}{h(\nu) + 1} = \frac{\vartheta_1 \nu + \vartheta_2 \nu^2 + \vartheta_3 \nu^3 + \cdots}{2 + \vartheta_1 \nu + \vartheta_2 \nu^2 + \vartheta_3 \nu^3 + \cdots}$$
$$= \frac{1}{2}\vartheta_1 \nu + \frac{1}{2}\left(\vartheta_2 - \frac{1}{2}\vartheta_1^2\right)\nu^2 + \frac{1}{2}\left(\vartheta_3 - \vartheta_1\vartheta_2 + \frac{1}{4}\vartheta_1^3\right)\nu^3 + \cdots.$$

Using the above equation in (3), we have

$$\Psi[w(\nu)] = 1 + \psi_1 w(\nu) + \psi_2[w(\nu)]^2 + \psi_3[w(\nu)]^3 + \cdots$$

$$= 1 + \frac{\psi_1\vartheta_1}{2}\nu + \frac{\psi_1}{2}\left[\vartheta_2 - \frac{1}{2}\left(1 - \frac{\psi_2}{\psi_1}\right)\vartheta_1^2\right]\nu^2 + \cdots. \quad (13)$$

The left-hand side of (12) will be of the form

$$1 + \left[\frac{\nu^2\chi''(\nu)}{\chi(\nu)} + \frac{\nu\chi'(\nu)}{\chi(\nu)} - \left(\frac{\nu\chi'(\nu)}{\chi(\nu)}\right)^2\right] e^{\frac{\nu\chi'(\nu)}{\chi(\nu)}-1} = 1 + \nu\frac{d}{d\nu}\left[e^{\frac{\nu\chi'(\nu)}{\chi(\nu)}-1}\right]$$

$$= 1 + \nu\frac{d}{d\nu}\left[e^{\frac{\nu+\sum_{n=2}^{\infty} n\, a_n\nu^n}{\nu+\sum_{n=2}^{\infty} a_n\nu^n}-1}\right] = 1 + \nu\frac{d}{d\nu}\left[e^{a_2\nu+(2a_3-a_2^2)\nu^2+(3a_4-3a_2a_3+a_2^2)\nu^3} + \cdots\right]$$

$$= 1 + \nu\frac{d}{d\nu}\left[1 + \frac{\{a_2\nu + (2a_3 - a_2^2)\nu^2 + (3a_4 - 3a_2a_3 + a_2^2)\nu^3 + \cdots\}}{1!} + \frac{\{a_2\nu + (2a_3 - a_2^2)\nu^2 + (3a_4 - 3a_2a_3 + a_2^2)\nu^3 + \cdots\}^2}{2!} + \cdots\right]$$

$$= 1 + a_2\nu + (-a_2^2 + 4\,a_3)\,\nu^2 + \left(\frac{a_2^3}{2} - 3\,a_2\,a_3 + 9\,a_4\right)\nu^3 + \cdots. \quad (14)$$

From (13) and (14), we obtain

$$a_2 = \frac{\vartheta_1\psi_1}{2} \quad and \quad a_3 = \frac{\psi_1}{8}\left[\vartheta_2 - \frac{\vartheta_1^2}{2}\left(1 - \frac{\psi_2}{\psi_1} - \psi_1\right)\right]. \quad (15)$$

On using $|\vartheta_1| \leq 2$ (see[p. 41][XI) in the above equation, we get the bound for

$$|a_2| = \left|\frac{\vartheta_1\psi_1}{2}\right| = \frac{\psi_1|\vartheta_1|}{2} \leq \frac{2\psi_1}{2} = \psi_1.$$

Using Lemma 1 in the equality of a_3, we get the bounds (10)

$$|a_3| = \left|\frac{\psi_1}{8}\left[\vartheta_2 - \frac{\vartheta_1^2}{2}\left(1 - \frac{\psi_2}{\psi_1} - \psi_1\right)\right]\right|$$

$$\leq \frac{\psi_1}{8} \times 2max\left\{1, \quad \left|2 \times \frac{1}{2}\left(1 - \frac{\psi_2}{\psi_1} - \psi_1\right) - 1\right|\right\}$$

$$\leq \frac{\psi_1}{4} max\left\{1, \quad \left|\frac{\psi_2}{\psi_1} + \psi_1\right|\right\}.$$

Now to prove (11), we consider

$$|a_3 - \rho\, a_2^2| = \left|\frac{\psi_1}{8}\left[\vartheta_2 - \frac{\vartheta_1^2}{2}\left(1 - \frac{\psi_2}{\psi_1} - \psi_1\right)\right] - \frac{\rho\vartheta_1^2\psi_1^2}{4}\right|$$

$$= \left|\frac{\psi_1}{8}\left[\vartheta_2 - \frac{\vartheta_1^2}{2}\left(1 - \frac{\psi_2}{\psi_1} - (1 - 4\rho)\psi_1\right)\right]\right|$$

$$\leq \frac{\psi_1}{8}\left[2 + \frac{|\vartheta_1|^2}{2}\left(\left|\frac{\psi_2}{\psi_1} + (1 - 4\rho)\psi_1\right| - 1\right)\right]. \quad (16)$$

Denoting

$$\mathfrak{B} := \left|\frac{\psi_2}{\psi_1} + (1-4\rho)\psi_1\right|,$$

If $\mathfrak{B} \le 1$, from (16) we obtain

$$|a_3 - \rho\, a_2^2| \le \frac{\psi_1}{4}. \tag{17}$$

Further, if $\mathfrak{B} \ge 1$ from (16), we deduce

$$|a_3 - \rho\, a_2^2| \le \frac{\psi_1}{4}\left|\frac{\psi_2}{\psi_1} + (1-4\rho)\psi_1\right| \tag{18}$$

An examination of the proof shows that equality for (17) holds if $\vartheta_1 = 0,\ \vartheta_2 = 2$. Equivalently, by Lemma 1, we have

$$\Psi(w(\nu)) = \Psi\left(\frac{\frac{1+\nu^2}{1-\nu^2} - 1}{\frac{1+\nu^2}{1-\nu^2} + 1}\right)\Psi\left(\frac{2\nu^2 + 2\nu^4 + \cdots}{2 + 2\nu^2 + 2\,\nu^4 + \cdots}\right) = \Psi_2\,(\nu).$$

Therefore, the extremal function of the class $\mathcal{E}(\Psi)$ is given by

$$1 + \left[\frac{\nu^2\chi''(\nu)}{\chi(\nu)} + \frac{\nu\chi'(\nu)}{\chi(\nu)} - \left(\frac{\nu\chi'(\nu)}{\chi(\nu)}\right)^2\right] e^{\frac{\nu\chi'(\nu)}{\chi(\nu)} - 1} = \Psi_2(\nu)$$

Similarly, the equality for (18) holds if $\vartheta_1 = \vartheta_2 = 2$. Equivalently, by Lemma 1, we have

$$\Psi(w(\nu)) = \Psi\left(\frac{\frac{1+\nu}{1-\nu} - 1}{\frac{1+\nu}{1-\nu} + 1}\right) = \Psi\left(\frac{2\nu + 2\nu^2 + \cdots}{2 + 2\nu + 2\nu^2 + \cdots}\right) = \Psi_1\,(\nu).$$

Therefore, the extremal function in $\mathcal{E}(\Psi)$ is given by

$$1 + \left[\frac{\nu^2\chi''(\nu)}{\chi(\nu)} + \frac{\nu\chi'(\nu)}{\chi(\nu)} - \left(\frac{\nu\chi'(\nu)}{\chi(\nu)}\right)^2\right] e^{\frac{\nu\chi'(\nu)}{\chi(\nu)} - 1} = \Psi_1(\nu),$$

and the proof of the theorem is complete.

Letting $\Psi(\nu) = \frac{1+\nu}{1-\nu}$ in Theorem 2, we get the following result.

Corollary Let $\chi(\nu) \in \mathcal{E}\left(\frac{1+\nu}{1-\nu}\right)$, then we have

$$|a_2| \le 2, \quad |a_3| \le \frac{3}{2}$$

and for all $\rho \in \mathbb{C}$

$$\left|a_3 - \rho\, a_2^2\right| \leq \frac{1}{2}\max\{1, |3 - 8\rho|\}$$

The inequalities are sharp.

III. Conclusions

The defined function class does not reduce to well-known function classes of the geometric function theory. That's the primary reason; we could not provide applications for our main results. However, in examples, we have illustrated the relationship between the defined function class and the classes that exist in literature. There are very few studies that involve differential characterizations associated with special functions. This study intends to fill the gap in the existing literature.

Since our class relied on higher-order derivatives, this study can be extended by replacing f with a differential operator defined using Hadamard product or convolution. Furthermore, since the right-hand side of the superordinate function involved a more general function, the defined function can be restudied by replacing Ψ with a conic domain, as illustrated in Table 1

Conflict of Interest:

There was no relevant conflict of interest regarding this paper.

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