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SOME CHARACTERIZATIONS OF SLIGHTLY $\Sigma\alpha$ - COMPACT FUZZY SETS

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Abstract

Within this paper, we create the ideas of slightly $\Sigma\alpha$-shelter, shortly $s\Sigma\alpha$- shelter, and slightly $\Sigma\alpha$-compact, shortly $s\Sigma\alpha$-compact fuzzy sets. In addition, we have established certain theorems concerning our vision in fuzzy topological spaces, fuzzy subspaces, fuzzy continuity mapping, fuzzy open mapping, fuzzy $\mathscr{T}_2$-spaces (Hausdorff spaces), and their different characterizations. Finally, we have discussed our vision about the "good extension" characteristics.

Keywords: fuzzy sets, fuzzy topological spaces, $s\Sigma\alpha$-shelter, and $s\Sigma\alpha$-compact.

I. Introduction

In 1965, for the first time, Zadeh [XXXVIII] addressed fuzzy sets and described fuzziness mathematically. Chang [V] first presented the concept of fuzzy compactness using open cover, and Wong [XXXV] further developed the fuzzy topological spaces theory introduced by Chang [V] and investigated a lot of covering characterizations that comprise these spaces. The following fuzzy compactness levels were developed by Lowen [XIX, XXI]: fuzzy, strong, and ultra fuzzy, while Chadwick [IV] expanded the compactness of Lowen's. Additionally, Lowen [XX, XXII] also studied compactness within the fuzzy real line and compact product fuzzy topological spaces. Ying [XXXVI, XXXVII] initially presented the concept of fuzzifying compactness in fuzzifying topological spaces, followed by Guojun [XI] with N-compactness, and then Dongsheng [VII] with a generalization of N-compactness. Shi [XXXI] proposed a new way to define fuzzy compactness topological spaces that are in L-fuzzy, and L-fuzzy topological spaces. Li and Shi [XVII] presented the concept of the degree of fuzzy compactness. Hong [XII] addressed the concept of RS-compactness, while Noiri [XXVII] discussed various properties related to RS-compactness. Additionally, Kudri and Warner [XVI] and EL-Shafei et al. [VIII] discussed the characteristics of RS-compactness. Georgiou and Papadopoulos [X] characterized well known fuzzy compactness using the notion of fuzzy upper limit and Warner [XXXIV] discussed the modeling of topology in general through fuzzy topology which is fuzzy compactness serves as an illustration. Warner and McLean [XXXIII] provided an effective definition of compactness within L-fuzzy topological space, while Kudri [XV] expanded this concept to an arbitrary L-fuzzy subset and discussed its characteristics. In reference [VI], Chen addressed the fuzzy compactness of induced I(L)-fuzzy topological spaces. An extension of the g-inverses of regular fuzzy matrices, the g-inverses of interval-valued fuzzy matrices were investigated by Meenakshi et al. [XXV]. Saha Apu Kumar and Debasish Bhattacharya [XXIX] discussed countable fuzzy topological spaces. Mahbub et al. [XXIII] presented two concepts of intuitionistic fuzzy compactness within intuitionistic fuzzy topological spaces. In their presentation of intuitionistic (compactness, almost compactness, near compactness), Abbas[I] used intuitionistic fuzzy topological spaces as a larger framework. Gantner et al.[IX] presented the idea of α-shading and α-compactness for L-fuzzy topological spaces, while Mashhour et al. [XXIV] broadened this idea. Kim [XIV] discussed the characteristics of α-compact fuzzy topological spaces along with Benchalli and Siddapur [III] and formed a connection between general topological spaces and α-compact fuzzy topological spaces. In spaces with L-topologies, Shi [XXX] presented a new type of α-compactness, while in L-fuzzy topological spaces, Aygün [II] defined an effective version of fuzzy α-compactness. Talukder and Ali [XXXII] proposed partially α-compact fuzzy sets and discussed their different characteristics in topological spaces that are fuzzy. We introduced and discussed the idea of $s\Sigma\alpha$-compact fuzzy sets with several characterizations. We investigate $\Sigma\alpha$ - compact fuzzy sets and observe that it has various outward results.

II. Preliminaries

This section will go over a few key definitions that are important for the next section. These are found in the documents that are cited and are essential to our discussion.

Definition 2.1. [XXXVIII**]:** Suppose $\mathscr{S}$ is a set that is not empty and a unit interval $I = [0,1]$ that is closed. A function $e : \mathscr{S} \to I$ which assigns to each ingredient $g \in \mathscr{S}$ is a fuzzy set. $I^{\mathscr{S}}$ stands for all of $\mathscr{S}$'s fuzzy sets. Any ingredient of $I^{\mathscr{S}}$ may likewise be thought of as a fuzzy subset of $\mathscr{S}$.

Definition 2.2. [XXVIII**]:** Presume a fuzzy set e in $\mathscr{S}$, after that the set $\{g \in \mathscr{S} : e(g) > 0\}$ is treated as the support of e which is identified by e_0.

Also, fuzzy sets that are (empty, whole, equal), fuzzy subset, a fuzzy set of complement, disjoint fuzzy sets; fuzzy sets (union and intersection), laws of (De-Morgan's distributive) in Zadeh [XXXVIII], quasi-coincident in Pao-Ming, P. & Ying-Ming [XXVIII], image and inverse image of a fuzzy set in Chang [V], topology and compact set in Lipschutz [XVIII] standard now.

Definition 2.3[V]**:** Presume $\mathscr{S}$ is a non-empty set and ρ is a gathering of fuzzy sets in $\mathscr{S}$. After that ρ is treated as fuzzy topology on $\mathscr{S}$

when

$(1)\ 0,\ 1 \in \rho$

$(2)\ e_i \in \rho$ for each $i \in \mathscr{J}$, after that $\bigcup_{i \in \mathscr{J}} e_i \in \rho$

$(3)\ e\ ,\ z \in \rho$, after that $e \cap z \in \rho$.

The pair $(\mathscr{S}, \rho)$ is treated as a fuzzy topological space and promptly, fts. Every component in ρ is treated as an open fuzzy set. If the complements to a fuzzy set are open, afterward the fuzzy set is considered closed.

Definition 2.4[V]**:** Presume $(\mathscr{S}, \rho)$ and $(\mathscr{B}, \ell)$ are fuzzy topological spaces. A mapping $p:\ (\mathscr{S}, \rho) \to (\mathscr{B}, \ell)$ is treated as fuzzy continuous iff inverse any open fuzzy set in ℓ is likewise an open fuzzy set in ρ or equivalently inverse owing to each closed fuzzy set in ℓ is also likewise a closed fuzzy set in ρ .

Definition 2.5[XXXV]: Presume $(\mathscr{S}, \rho)$ and $(\mathscr{B}, \ell)$ are fuzzy topological spaces. Consider $p:\ (\mathscr{S}, \rho) \to (\mathscr{B}, \ell)$ is a mapping from an fts $(\mathscr{S}, \rho)$ to another fts $(\mathscr{B}, \ell)$. After that p is treated as fuzzy open mapping iff $p(e) \in \ell$ for each $e \in \rho$.

Definition 2.6[XXVIII]**:** Presume $(\mathscr{S}, \rho)$ is an fts and $\mathscr{A} \subseteq \mathscr{S}$ (subset). After that, the gathering $\rho_{\mathscr{A}} = \{e \mid \mathscr{A} : e \in \rho\}$ is also fuzzy topology about $\mathscr{A}$, treated as subspace topology that are fuzzy about $\mathscr{A}$, and the pair $(\mathscr{A}, \rho_{\mathscr{A}})$ is treated as subspace of $(\mathscr{S}, \rho)$ that are fuzzy.

Definition 2.7[XIII]: Presume $(\mathscr{S}, \rho)$ is an fts. After that $(\mathscr{S}, \rho)$ is mentioned fuzzy $\mathscr{T}_2$ -space (Hausdorff space) iff for every $g\ ,\ d \in \mathscr{S}$, $g \neq d$, there exist $e\ ,\ z \in \rho$ such that $e(g) > 0$, $z(d) > 0$ and $e \cap d = 0$.

Definition 2.8[XIX]: Presume $(\mathscr{S}, \mathscr{V})$ is a topological space. After that, a mapping $p:\ \mathscr{S} \to \mathscr{R}$ (using the usual topology) is treated as lower semi-continuous (l. s. c.) when owing to every $a \in \mathscr{R}$ and $f^{-1}(a, \infty) \in \mathscr{V}$. Regarding a topology $\mathscr{V}$ on a set $\mathscr{S}$, presume $\omega(\mathscr{V})$ is the collection of every l. s. c. function from $(\mathscr{S}, \mathscr{V})$ to I (using the usual topology); hence $\omega(\mathscr{V}) = \{e \in I^{\mathscr{S}} : e^{-1}(a, 1] \in \mathscr{V},\ a \in I_1\}$. It will be proven that $\omega(\mathscr{V})$ is a fuzzy topology on $\mathscr{S}$.

Assume that FP is the fuzzy topology analogue of P, where P is a property of topological spaces. After that FP is treated as a ‘good extension’ of P “iff the announcement $(\mathscr{S}, \mathscr{V})$ has P iff $(\mathscr{S}, \omega(\mathscr{V}))$ has FP" remains valid for each topological space $(\mathscr{S}, \mathscr{V})$. In this manner characteristic functions are l. s. c.

III. Principal Findings

Within this section, we presented the thought of $s\Sigma\alpha$-compact fuzzy sets and set out some discernible characterizations of $s\Sigma\alpha$-compact fuzzy sets.

Definition 3.1: Presume $\mathscr{S}$ is not an empty set $0 < \alpha \le 1$. Consider $\{e_i : i \in \mathscr{J}\}$ is a gathering of fuzzy sets in $\mathscr{S}$. After that $\{e_i : i \in \mathscr{J}\}$ is treated as slightly $\Sigma\alpha$-shelter, shortly $s\Sigma\alpha$-shelter of a fuzzy set ξ in $\mathscr{S}$ iff $\sum_i e_i(g) \ge \alpha$ for whole $g \in \xi_0$ $(\xi_0 \ne \mathscr{S})$. A subgathering of $s\Sigma\alpha$-shelter of ξ which is also a $s\Sigma\alpha$-shelter of ξ is treated as $s\Sigma\alpha$-subshelter of ξ.

For instance, assume $\mathscr{S} = \{a, b, c\}$, $I = [0,1]$ and $0 < \alpha \le 1$. Consider e_1, $e_2 \in I^{\mathscr{S}}$ is defined with $e_1(a) = 0.02$, $e_1(b) = 0.01$, $e_1(c) = 0.03$ and $e_2(a) = 0.04$, $e_2(b) = 0.03$, $e_2(c) = 0.02$. Again, presume $\xi \in I^{\mathscr{S}}$ is defined with $\xi(a) = 0.01$, $\xi(b) = 0.00$, $\xi(c) = 0.07$, so $\xi_0 = \{a, c\}$. Choose $\alpha = 0.05$. It is clear that $u_1(a) + u_2(a) > \alpha$, $e_1(c) + e_2(c) \ge \alpha$, where $a, c \in \xi_0$. Therefore $\{e_1, e_2\}$ is a $s\Sigma\alpha$-shelter of ξ.

Definition 3.2: Presume $(\mathscr{S}, \rho)$ is an fts and $0 < \alpha \le 1$. A fuzzy set ξ in $\mathscr{S}$ is treated as slightly $\Sigma\alpha$-compact, shortly $s\Sigma\alpha$-compact iff each open $s\Sigma\alpha$-shelter of ξ has a finite $s\Sigma\alpha$-subshelter i.e. there exist e_{i_1}, e_{i_2}, ….., $e_{i_n} \in \{e_i\}$ such that $\sum_i e_{i_k}(g) \ge \alpha$ for whole $g \in \xi_0$ and $(k \in \mathscr{J}_n)$.

The concepts of some theorems are taken from Lipschutz[XVIII] and Murdeswar[XXVI].

Theorem 3.3: Presume $(\mathscr{S}, \rho)$ is an fts with $\mathscr{H} \subset \mathscr{S}$ (proper subset) and ξ is a fuzzy set in $\mathscr{S}$ where $\xi_0 \subset \mathscr{H}$. After that ξ is $s\Sigma\alpha$-compact in $(\mathscr{S}, \rho)$ iff ξ is $s\Sigma\alpha$-compact in $(\mathscr{H}, \rho_{\mathscr{H}})$.

Proof: Regard ξ is $s\Sigma\alpha$-compact in $(\mathscr{S}, \rho)$. Presume $\mathscr{E} = \{e_i : i \in \mathscr{J}\}$ is an open $s\Sigma\alpha$-shelter of ξ in $(\mathscr{H}, \rho_{\mathscr{H}})$. So, there exists $z_i \in \rho$ such that $e_i = z_i \mid \mathscr{H} \subseteq z_i$. Thus $\{z_i : i \in \mathscr{J}\}$ is an open $s\Sigma\alpha$-shelter of ξ in $(\mathscr{S}, \rho)$. As ξ is $s\Sigma\alpha$-compact in $(\mathscr{S}, \rho)$, so $\{z_i : i \in \mathscr{J}\}$ has a finite $s\Sigma\alpha$-subshelter, say

$\{z_{i_r} : r \in \mathscr{I}_n\}$ such that $\sum_i z_{i_r}(g) \geq \alpha$ for all $g \in \xi_0$ and $r \in \mathscr{I}_n$. For if, $g \in \xi_0$, then there exist $z_{i_{r_0}}$ such that $\sum_i z_{i_{r_0}}(g) \geq \alpha \quad (r \in \mathscr{I}_n)$ which implies that $\sum_i (z_{i_{r_0}} \mid \mathscr{X})(g) \geq \alpha \quad (r \in \mathscr{I}_n)$ and consequently $\sum_i e_{i_{r_0}}(g) \geq \alpha \quad (r \in \mathscr{I}_n)$, as $\xi_0 \subset \mathscr{X}$. Hence $e_{i_0} \in \mathscr{E}$ and thus $\{e_{i_r} : r \in \mathscr{I}_n\}$ is a finite $s\Sigma\alpha$-subshelter of $\mathscr{E}$. Thus ξ is $s\Sigma\alpha$-compact in $(\mathscr{X}, \rho_{\mathscr{X}})$.

Conversely, presume ξ is $s\Sigma\alpha$-compact in $(\mathscr{X}, \rho_{\mathscr{X}})$. Assume $\mathscr{W} = \{z_i : i \in \mathscr{I}\}$ is an open $s\Sigma\alpha$-shelter ξ in $(\mathscr{S}, \rho)$. Set $e_i = z_i \mid \mathscr{X}$. To show this, presume $g \in \mathscr{S}$. When $g \in \mathscr{X}$, after that, there exists $z_{i_0} \in \mathscr{W}$ such that $e_{i_0} = z_{i_0} \mid \mathscr{X}$. On the other hand, $e_{i_0} \in \rho_{\mathscr{X}}$, so $\sum_i e_{i_0}(g) \geq \alpha$ for whole $g \in \xi_0$. Thus $\{e_i : i \in \mathscr{I}\}$ is an open $s\Sigma\alpha$-shelter of ξ in $(\mathscr{X}, \rho_{\mathscr{X}})$. But ξ is $s\Sigma\alpha$-compact in $(\mathscr{X}, \rho_{\mathscr{X}})$, so $\{e_i : i \in \mathscr{I}\}$ has a finite $s\Sigma\alpha$-subshelter, say $\{e_{i_r} : r \in \mathscr{I}_n\}$ such that $\sum_i e_{i_r}(g) \geq \alpha$ for all $g \in \xi_0$. For if, $g \in \xi_0$, then there exists $e_{i_{r_0}}$ such that $\sum_i e_{i_{r_0}}(g) \geq \alpha$ implies that $\sum_i (z_{i_{r_0}} \mid \mathscr{X})(g) \geq \alpha$ implies that $\sum_i z_{i_{r_0}}(g) \geq \alpha$, as $\xi_0 \subset \mathscr{X}$. So, $\{z_{i_r} : r \in \mathscr{I}_n\}$ is a finite $s\Sigma\alpha$-subshelter of $\mathscr{W}$. Therefore ξ is $s\Sigma\alpha$-compact in $(\mathscr{S}, \rho)$.

Theorem 3.4: Presume $(\mathscr{S}, \rho)$ is an fts, $0 < \alpha \leq 1$ and ξ, Ω are $s\Sigma\alpha$-compact fuzzy sets in $\mathscr{S}$. After that $\xi \cup \Omega$ $((\xi \cup \Omega)_0 \neq \mathscr{S})$ is also $s\Sigma\alpha$-compact in $(\mathscr{S}, \rho)$.

Proof: Presume $\mathscr{E} = \{e_i : i \in \mathscr{I}\}$ is an open $s\Sigma\alpha$-shelter of $\xi \cup \Omega$. So, $\mathscr{E}$ is open $s\Sigma\alpha$-shelter of both ξ and Ω. But, ξ is $s\Sigma\alpha$-compact in $(\mathscr{S}, \rho)$, so every open $s\Sigma\alpha$-shelter of ξ has a finite $s\Sigma\alpha$-subshelter i.e. there is a, say $e_{i_r} \in \mathscr{E}$ for which $\sum_i e_{i_r}(g) \geq \alpha$ for all $g \in \xi_0 \, (r \in \mathscr{I}_n)$. Again, Ω is $s\Sigma\alpha$-compact in $(\mathscr{S}, \rho)$, so every open $s\Sigma\alpha$-shelter of Ω has a finite $s\Sigma\alpha$-subshelter i.e. there exists, say $e_{i_p} \in \mathscr{E}$ such that $\sum_i e_{i_p}(g) \geq \alpha$ for all $g \in \Omega_0 \, (p \in \mathscr{I}_n)$. Therefore, $\{e_{i_r}, e_{i_p}\}$ is a finite $s\Sigma\alpha$-subshelter of $\mathscr{E}$. So, $\xi \cup \Omega$ is $s\Sigma\alpha$-compact in $(\mathscr{S}, \rho)$.

Theorem 3.5: Presume an fts in $(\mathscr{O}, \rho)$ and ξ is a fuzzy set in $\mathscr{O}$ where $\xi_0 \subset \mathscr{O}$. If each collection of closed fuzzy sets in $(\mathscr{O}, \rho)$ whose intersection is empty and has a finite subcollection with the empty intersection, consequently ξ is $s\Sigma\alpha$-compact in $(\mathscr{O}, \rho)$. On the contrary, it is not right everywhere.

Proof: Presume $\{e_i : i \in \mathscr{J}\}$ is an open $s\Sigma\alpha$-shelter of ξ in $(\mathscr{O}, \rho)$ i.e. $\sum_i e_i(g) \geq \alpha$ for whole $g \in \xi_0$. For the first context of the theorem, we get $\bigcap_{i \in \mathscr{J}} e_i^c = 0_{\mathscr{O}}$, so we write $\bigcup_{i \in \mathscr{J}} e_i = 1_{\mathscr{O}}$ and hence clearly $\sum_i e_i(g) \geq \alpha$ for the whole $g \in \xi_0$. Again, for the second context of the theorem, we have $\bigcap_{k \in \mathscr{J}_n} e_{i_k}^c = 0_{\mathscr{O}}$, so we can write $\bigcup_{k \in \mathscr{J}_n} e_{i_k} = 1_{\mathscr{O}}$, and hence, it clearly shows that $\sum_i e_{i_k}(g) \geq \alpha$ for all $g \in \xi_0$ $(k \in \mathscr{J}_n)$, and hence, it is clear that $\{e_k : k \in \mathscr{J}_n\}$ is a finite $s\Sigma\alpha$-subshelter of $\{e_i : i \in \mathscr{J}\}$. Thus ξ is $s\Sigma\alpha$-compact in $(\mathscr{O}, \rho)$.

For the contrary, consider the following instance:

Presume $\mathscr{O} = \{a, b, c\}$, $I = [0, 1]$ and $0 < \alpha \leq 1$. Presume $e_1, e_2, e_3, e_4 \in I^{\mathscr{O}}$ is defined by $e_1(a) = 0.03$, $e_1(b) = 0.04$, $e_1(c) = 0.02$; $e_2(a) = 0.04$, $e_2(b) = 0.08$, $e_2(c) = 0.01$; $e_3(a) = 0.04$, $e_3(b) = 0.08$, $e_3(c) = 0.02$ and $e_4(a) = 0.03$, $e_4(b) = 0.04$, $e_4(c) = 0.01$. Choose $\rho = \{0, e_1, e_2, e_3, e_4, 1\}$, then $(\mathscr{O}, \rho)$ is an fts. Again, presume $\xi \in I^{\mathscr{O}}$ is defined by $\xi(a) = 0.06$, $\xi(b) = 0.05$, $\xi(c) = 0.00$, after that $\xi_0 = \{a, b\}$. Take $\alpha = 0.09$. After that, clearly ξ is $s\Sigma\alpha$-compact in $(\mathscr{O}, \rho)$. On the other hand, closed fuzzy sets are $e_1^c(a) = 0.97$, $e_1^c(b) = 0.96$, $e_1^c(c) = 0.98$; $e_2^c(a) = 0.96$, $e_2^c(b) = 0.92$, $e_2^c(c) = 0.99$; $e_3^c(a) = 0.96$, $e_3^c(b) = 0.92$, $e_3^c(c) = 0.98$ and $e_4^c(a) = 0.97$, $e_4^c(b) = 0.96$, $e_4^c(c) = 0.99$. Hence, we see that $e_1^c \cap e_2^c \cap e_3^c \cap e_4^c \neq 0_{\mathscr{O}}$. So, the contrary of the theorem is not true everywhere.

Theorem 3.6: Presume p: $(\mathscr{O}, \rho) \to (\mathscr{B}, \ell)$ is fuzzy continuous and onto mapping, where $(\mathscr{O}, \rho)$ and $(\mathscr{B}, \ell)$ are fuzzy topological spaces. If ξ is $s\Sigma\alpha$-compact in $(\mathscr{O}, \rho)$, after that $p(\xi)$ is $s\Sigma\alpha$-compact in $(\mathscr{B}, \ell)$.

Proof: Presume $\{e_i : e_i \in \mathcal{C}\}$ is an open $s\Sigma\alpha$-shelter of $p(\xi)$ in $(\mathscr{B}, \mathcal{C})$ for $i \in \mathscr{J}$ i.e. $\sum_i e_i(d) \ge \alpha$ for whole $d \in (p(\xi))_0$. As p is fuzzy continuous, so $p^{-1}(e_i) \in \rho$ and hence, $\{p^{-1}(e_i) : e_i \in \mathcal{C}\}$ is an open $s\Sigma\alpha$-shelter of ξ in $(\mathscr{S}, \rho)$. For if, $g \in \xi_0$, then $p(g) \in (p(\xi))_0$ and so, there exists $e_{i_o} \in \{e_i\}$ such that $\sum_i e_{i_o}(p(g)) \ge \alpha \Rightarrow \sum_i p^{-1}(e_{i_o})(g) \ge \alpha$. Since, ξ is $s\Sigma\alpha$-compact in $(\mathscr{S}, \rho)$, so $\{p^{-1}(e_i) : e_i \in \mathcal{C}\}$ has a finite $s\Sigma\alpha$-subshlter, say $\{p^{-1}(e_{i_k}) : k \in \mathscr{J}_n\}$. Now, if $d \in (p(\xi))_0$, then $d = p(g)$ for some $g \in \xi_0$. So, there exist $e_{i_k} \in \{e_i\}$ such that $\sum_i p^{-1}(e_{i_k})(g) \ge \alpha \Rightarrow \sum_i (e_{i_k})(p(g)) \ge \alpha \Rightarrow \sum_i (e_{i_k})(d) \ge \alpha \ (k \in \mathscr{J}_n)$ i.e. $\{e_{i_k}\}\ (k \in \mathscr{J}_n)$ form a finite $s\Sigma\alpha$-subshelter of $\{e_i\}$. So, $p(\xi)$ is $s\Sigma\alpha$-compact in $(\mathscr{B}, \mathcal{C})$.

Theorem 3.7: Presume $p: (\mathscr{S}, \rho) \rightarrow (\mathscr{B}, \mathcal{C})$ is fuzzy open and bijective mapping, where $(\mathscr{S}, \rho)$ and $(\mathscr{B}, \mathcal{C})$ are fuzzy topological spaces. If ξ is $s\Sigma\alpha$-compact in $(\mathscr{B}, \mathcal{C})$, after that $p^{-1}(\xi)$ is $s\Sigma\alpha$-compact in $(\mathscr{S}, \rho)$.

Proof: Presume $\{e_i : e_i \in \rho\}$ is an open $s\Sigma\alpha$-shelter of $p^{-1}(\xi)$ in $(\mathscr{S}, \rho)$ for $i \in \mathscr{J}$ i.e. $\sum_i e_i(g) \ge \alpha$ for all $g \in (p^{-1}(\xi))_0$. But p is fuzzy open, so $p(e_i) \in \mathcal{C}$ and hence, $\{p(e_i) : e_i \in \rho\}$ is an open $s\Sigma\alpha$-shelter of ξ in $(\mathscr{B}, \mathcal{C})$. For if, $d \in \xi_0$, then $p^{-1}(d) \in (p^{-1}(\xi))_0$ and so, there exists $e_{i_o} \in \{e_i\}$ such that $\sum_i e_{i_o}(p^{-1}(d)) \ge \alpha \Rightarrow \sum_i p(e_{i_o})(d) \ge \alpha$. Since, ξ is $s\Sigma\alpha$-compact in $(\mathscr{B}, \mathcal{C})$, so $\{p(e_i) : e_i \in \rho\}$ has a finite $s\Sigma\alpha$-subshlter, say $\{p(e_{i_k}) : k \in \mathscr{J}_n\}$. Now, if $g \in (p^{-1}(\xi))_0$, then $g = p^{-1}(d)$ for some $d \in \xi_0$. So, there exist $e_{i_k} \in \{e_i\}$ such that $\sum_i p(e_{i_k})(d) \ge \alpha \Rightarrow \sum_i e_{i_k}(p^{-1}(d)) \ge \alpha \Rightarrow \sum_i e_{i_k}(g) \ge \alpha \ (k \in \mathscr{J}_n)$ i.e. $\{e_{i_k}\}\ (k \in \mathscr{J}_n)$ is a finite $s\Sigma\alpha$-subshelter of $\{e_i\}$. Thus $p^{-1}(\xi)$ is $s\Sigma\alpha$-compact in $(\mathscr{S}, \rho)$.

Theorem 3.8: Presume a fuzzy $\mathscr{T}_2$-space (Hausdorff) is $(\mathscr{S}, \rho)$ and ξ is a fuzzy set in $\mathscr{S}$ when $\xi_0 \subset \mathscr{S}$. If ξ is $s\Sigma\alpha$-compact in $(\mathscr{S}, \rho)$ and $g \notin \xi_0$, after that there exist $e, z \in \rho$ such that $e(g) > 0$ and $\xi_0 \subseteq z^{-1}(0, 1]$ and $e \cap z = 0$. But, the reverse is not right in everywhere.

Proof: Suppose $d \in \xi_0$. Since, $g \notin \xi_0$, then it is clear that $g \neq d$. But, $(\mathscr{S}, \rho)$ is fuzzy Hausdorff, then there exist $e_d, z_d \in \rho$ such that $e_d(g) > 0$, $z_d(d) > 0$ and $e_d \cap z_d = 0$. Since, ξ is $s\Sigma\alpha$-compact in fuzzy Hausdorff space $(\mathscr{S}, \rho)$, then it clearly shows that $\sum_d z_d(d) \geq \alpha$ for all $d \in \xi_0$ i.e. $\{z_d : d \in \xi_0\}$ is an open $s\Sigma\alpha$-shelter of ξ in $(\mathscr{S}, \rho)$ and hence $\{z_d : d \in \xi_0\}$ has a finite $s\Sigma\alpha$-subshelter, say $\{z_{d_k} : d \in \xi_0\}$ $(k \in \mathscr{I}_n)$ such that $\sum_d z_{d_k}(d) \geq \alpha$ for all $d \in \xi_0$.

Now, let $e = e_{d_1} \cap e_{d_2} \cap \ldots\ldots \cap e_{d_n}$ and $z = z_{d_1} \cup z_{d_2} \cup \ldots\ldots \cup z_{d_n}$. Hence, we observe that $e, z \in \rho$, as e and z are the finite intersection and union of open fuzzy sets, respectively. Moreover, $e(g) > 0$, as $e_{d_k}(g) > 0$ for each k and $\xi_0 \subseteq z^{-1}(0, 1]$.

Finally, we have to show that $e \cap z = 0$. Since $e_{d_k} \cap z_{d_k} = 0 \Rightarrow e \cap z_{d_k} = 0$, by distributive law, we have $e \cap z = e \cap (z_{d_1} \cup z_{d_2} \cup \ldots\ldots \cup z_{d_n}) = 0$.

To observe the reverse, consider $\mathscr{S} = \{a, b\}$, $I = [0, 1]$ and $0 < \alpha \leq 1$. Presume $e_1, e_2, e_3 \in I^{\mathscr{S}}$ is defined with $e_1(a) = 0.01$, $e_1(b) = 0.00$; $e_2(a) = 0.00$, $e_2(b) = 0.02$ and $e_3(a) = 0.01$, $e_3(b) = 0.02$. Put $\rho = \{0, e_1, e_2, e_3, 1\}$, then $(\mathscr{S}, \rho)$ is a fuzzy Hausdorff space. Presume $\xi \in I^{\mathscr{S}}$ is defined by $\xi(a) = 0.06$, $\xi(b) = 0.00$, then $\xi_0 = \{a\}$ and $b \notin \xi_0$. Here $e_1, e_2 \in \rho$, where $e_2(b) > 0$ and $e_1^{-1}(0, 1] = \{a\}$. Therefore, $\xi_0 \subseteq e_1^{-1}(0, 1]$. Take $\alpha = 0.04$. Thus ξ is not $s\Sigma\alpha$-compact in $(\mathscr{S}, \rho)$, as $\sum_k e_k(a) < \alpha$, where $a \in \xi_0$, for $k = 1, 2, 3$. Thus, the theoretical discussion is not generally true everywhere.

Theorem 3.9: Presume $(\mathscr{S}, \rho)$ is a fuzzy Hausdorff space and ξ, Ω are fuzzy sets in $\mathscr{S}$ when ξ_0, $\Omega_0 \subset \mathscr{S}$. If ξ and Ω are disjoint $s\Sigma\alpha$-compacts in $(\mathscr{S}, \rho)$, after that there exist $e, z \in \rho$ such that $\xi_0 \subseteq e^{-1}(0, 1]$, $\Omega_0 \subseteq z^{-1}(0, 1]$ and $e \cap z = 0$. Conversely, it is not true everywhere.

Proof: Let $d \in \xi_0$. So, it is clear that $d \notin \Omega_0$, as ξ and Ω are disjoint. Since, Ω is $s\Sigma\alpha$-compact in fuzzy Hausdorff space $(\mathscr{S}, \rho)$, then by theorem (3.8), there exists e_d, $z_d \in \rho$ such that $e_d(d) > 0$ and $\Omega_0 \subseteq z_d^{-1}(0, 1]$ and $e_d \cap z_d = 0$. Since, $e_d(d) > 0$ and ξ is $s\Sigma\alpha$-compact in $(\mathscr{S}, \rho)$, then it is clear that

$\sum_{d} e_{d}(d) \geq \alpha$ for all $d \in \xi_0$ and hence, $\{e_d : d \in \xi_0\}$ is an open $s\Sigma\alpha$-shelter of ξ and so $\{e_d : d \in \xi_0\}$ has a finite $s\Sigma\alpha$-subshelter, say $\{e_{d_k} : d \in \xi_0\}\ (k \in \mathcal{I}_n)$ such that $\sum_{d} e_{d_k}(d) \geq \alpha\ (k \in \mathcal{I}_n)$ for all $d \in \xi_0$. Again, since Ω is $s\Sigma\alpha$-compact in $(\mathcal{S}, \rho)$, so $\{z_d : g \in \Omega_0\}$ has a finite $s\Sigma\alpha$-subshelter, say $\{z_{d_k} : g \in \Omega_0\}\ (k \in \mathcal{I}_n)$ such that $\sum_{d} z_{d_k}(g) \geq \alpha\ \ (k \in \mathcal{I}_n)$ for all $g \in \Omega_0$, as $\Omega_0 \subseteq z^{-1}_{d_k}(0, 1]$ because of each k. Now, suppose $e = e_{d_1} \cup e_{d_2} \cup \ldots\ldots \cup e_{d_n}$ and $z = z_{d_1} \cap z_{d_2} \cap \ldots\ldots \cap z_{d_n}$. Hence, we observe that $\xi_0 \subseteq e^{-1}(0, 1]$ and $\Omega_0 \subseteq z^{-1}(0, 1]$. Thus $e, z \in \rho$, as e and z are union and the finite intersection of open fuzzy sets serially.

Finally, we will demonstrate that $e \cap z = 0$. At the beginning, we notice that $e_{d_k} \cap z_{d_k} = 0$ owing to each $k \Rightarrow e_{d_k} \cap z = 0$, by distributive law, we have $e \cap z = (e_{d_1} \cup e_{d_2} \cup \ldots\ldots \cup e_{d_n}) \cap z = 0$.

To observe the reverse, presume fuzzy Hausdorff space in the context of the theorem (3.8). Presume ξ, $\Omega \in I^{\mathcal{S}}$ defined with $\xi(a) = 0.00$, $\xi(b) = 0.09$ and $\Omega(a) = 0.08$, $\Omega(b) = 0.00$, then $\xi_0 = \{b\}$, $\Omega_0 = \{a\}$. Now e_1, $e_2 \in \rho$, because of $e_1^{-1}(0, 1] = \{a\}$ and $e_2^{-1}(0, 1] = \{b\}$. Thus, we see that $\xi_0 \subseteq e_2^{-1}(0, 1]$, $\Omega_0 \subseteq e_1^{-1}(0, 1]$ and $e_1 \cap e_2 = 0$, where ξ and Ω are disjoint. Choose $\alpha = 0.07$. Hence, we notice that ξ and Ω are not $s\Sigma\alpha$-compacts in $(\mathcal{S}, \rho)$, since $\sum_{k} e_k(b) < \alpha$, where $b \in \xi_0$ and $\sum_{k} e_k(a) < \alpha$, where $a \in \Omega_0$ because of $k = 1, 2, 3$. Therefore, the theorem's opposite is not always true.

Theorem 3.10: Presume $(\mathcal{S}, \mathcal{V})$ is a topological space, $(\mathcal{S}, \omega(\mathcal{V}))$ is an fts, along with ξ is a fuzzy set in $\mathcal{S}$ where $\xi_0 \subset \mathcal{S}$. If ξ is $s\Sigma\alpha$-compact in $(\mathcal{S}, \omega(\mathcal{V}))$, after that ξ_0 is compact in $(\mathcal{S}, \mathcal{V})$. On the contrary, it is not right everywhere.

Proof: Assume ξ is $s\Sigma\alpha$-compact in $(\mathcal{S}, \omega(\mathcal{V}))$. Presume the open cover of ξ_0 in $(\mathcal{S}, \mathcal{V})$ is $\mathcal{E} = \{\mathcal{A}_i : i \in \mathcal{J}\}$. After that for each, $\mathcal{A}_i$ there is a $b_i \in \omega(\mathcal{V})$ such that $\mathcal{A}_i = b_i^{-1}(0, 1]$. Thus, $\mathcal{E} = \{b_i^{-1}(0, 1] : i \in \mathcal{J}\}$. But, the collection $\mathcal{F} = \{b_i : i \in \mathcal{J}\}$ is an open $s\Sigma\alpha$-shelter of ξ in $(\mathcal{S}, \omega(\mathcal{V}))$ i.e. $\sum_{i} b_i(g) \geq \alpha$

for all $g \in \xi_o$. Since, $\mathscr{E}$ is an open cover of ξ_o in $(\mathscr{S}, \mathscr{V})$, then there is $\mathcal{A}_{i_o} \in \mathscr{E}$ such that $g \in \mathcal{A}_{i_o}$. But $\mathcal{A}_{i_o} = b_{i_o}^{-1}(0, 1]$ for some $b_{i_o} \in \omega(\mathscr{V})$. Thus, $g \in b_{i_o}^{-1}(0, 1]$ which shows that $b_{i_o}(g) > 0$ and hence, $\sum_i b_{i_o}(g) \geq \alpha$. Since, ξ is $s\Sigma\alpha$-compact in $(\mathscr{S}, \omega(\mathscr{V}))$, therefore, $\mathscr{F}$ has a finite $s\Sigma\alpha$-subshelter, say $\{b_{i_k} : k \in \mathscr{I}_n\}$. Thus, $\{b_{i_k}^{-1}(0, 1] : k \in \mathscr{I}_n\}$ construct a finite subcover of $\mathscr{E}$ and so ξ_o is compact in $(\mathscr{S}, \mathscr{V})$.

We provide the following example to illustrate the opposite.

Presume $\mathscr{S} = \{a, b, c\}$, $I = [0, 1]$ and $0 < \alpha \leq 1$. Consider $\mathscr{V} = \{\{a\}, \{b\}, \{a, b\}, \phi, \mathscr{S}\}$. After that $(\mathscr{S}, \mathscr{V})$ is a topological space. Again, presume $e_1, e_2, e_3 \in I^{\mathscr{S}}$ is defined by $e_1(a) = 0.03$, $e_1(b) = 0.00$, $e_1(c) = 0.00$; $e_2(a) = 0.00$, $e_2(b) = 0.02$, $e_2(c) = 0.00$ and $e_3(a) = 0.03$, $e_3(b) = 0.02$, $e_3(c) = 0.00$. So, $\omega(\mathscr{V}) = \{e_1, e_2, e_3, 0, 1\}$ and $(\mathscr{S}, \omega(\mathscr{V}))$ is an fts. Furthermore, presume $\xi \in I^{\mathscr{S}}$ is defined with $\xi(a) = 0.08$, $\xi(b) = 0.09$, $\xi(c) = 0.00$, after that $\xi_o = \{a, b\}$. Clearly ξ_o is compact in $(\mathscr{S}, \mathscr{V})$. Now, choose $\alpha = 0.07$. So, ξ is not $s\Sigma\alpha$-compact in $(\mathscr{S}, \omega(\mathscr{V}))$, as $\sum_k e_k(a) < \alpha$ where $a \in \xi_o$, because of $k = 1, 2, 3$. So the contrary of the theorem is not right everywhere.

Conflict of Interest

With regard to this paper, no conflicts of interest exist at this time.

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