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CRACK RESISTANCE OF REINFORCED CONCRETE STRUCTURES OF RING SECTIONS

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Abstract

In this paper, the issues of developing a calculation method for the formation of cracks normal to the longitudinal axis of bent reinforced concrete structures of circular cross-section without prestressing the reinforcement based on a nonlinear deformation model using a bilinear diagram of the state of concrete are considered. The prerequisites based on which theoretical dependences are constructed to determine the complex internal forces of a round normal section before the formation of cracks is presented. Based on stereometry, dependencies are presented to determine the forces in the concrete and reinforcement of the compressed zone and the forces in the stretched zone, respectively, for concrete and reinforcement. Since it is difficult to analytically solve a system of equations with negative and positive exponents, it is recommended to carry out numerical iterative processes to determine the desired unknowns: heights of the compressed zone, maximum stresses in concrete and reinforcement of the compressed zone. Numerical studies have made it possible to determine the value of the elastic-plastic moment of the annular section and to identify its dependence on the strength of concrete.

Keywords: Circular cross-section, moment of crack formation, neutral axis, arc length, sector area, elastic-plastic moment of resistance.

I. Introduction

Reinforced concrete structures of ring cross-section are widely used in load-bearing structures of multi-storey public buildings, in high-voltage power transmission poles and street lighting, which are massively used in megacities and settlements [III, IV, V, VI, VII, X, XI]. Currently, the supports of wind power plants are also proposed to be made in the form of ring-reinforced concrete elements.

One of the conditions for ensuring the design durability of reinforced concrete structures operated in an open environment is to preserve the continuity of the sections and the integrity of the protective layer to prevent corrosion of the reinforcement. The possible cause of these defects is the insufficient crack resistance of the sections, including the lack of a calculation method for the formation of normal cracks in ring-reinforced concrete sections.

Adopted in SP 63.13330.2018 "Concrete and reinforced concrete structures. Basic provisions" the calculation method for the formation of cracks in structures without prestressing the reinforcement is applicable for rectangular and T-sections and is based on the determination of the elastic-plastic moment of resistance, which depends on the actual reduced geometric characteristics and the coefficient γ, which takes into account the inelastic deformations of the concrete of the stretched zone [XVI]. For these cross-section shapes, the coefficient γ was set to 1,3 [XVI, XVII]. In our opinion, this approach significantly underestimates the calculated values of the moment of crack formation, especially for structures without prestressing the reinforcement. In previous domestic standards for the design of reinforced concrete structures (SNiP 2.03.01-84* Concrete and reinforced concrete structures), the coefficient γ was accepted only depending on the shape of the section and varied within 1,25-2. For ring sections, it was the highest value equal to 2 – 0,4d/D [II].

Recent studies in the field of crack resistance of bent elements [XIII-XIV, VIII, IX] have shown that the elastic-plastic moment of resistance of reinforced concrete sections depends more on the strength of concrete. At the same time, the coefficient γ takes the highest value, for example, 1,8 for rectangular sections with concrete grade B15, and decreases to values of 1,4 for concrete grade B60. This fact leads to the need to study the dependence of the coefficient γ for other forms of sections. In this article, in continuation of the work [XV], the ring section is considered.

II. Materials and Methods

A nonlinear deformation model was adopted as the basis for the computational analysis of the stress-strain state of a normal section of a ring shape, which makes it possible, based on accepted assumptions, to model the stress-strain state before the formation of cracks of sections of any shape.

The main prerequisites adopted for the analytical determination of the moment of crack formation in reinforced concrete elements of a ring section are adopted by analogy with a circular section [I, XV]. Figure 1 shows a design diagram of the stress-strain state of a reinforced concrete ring section along the diameter line before the formation of cracks.

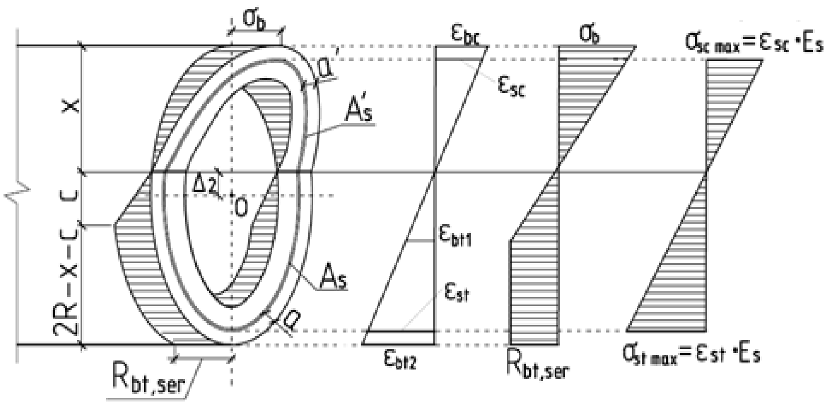

Fig. 1. Diagram of the stress state of a ring section with longitudinal reinforcement in the form of a continuous distributed layer before cracking

The compressed zone of concrete is a smaller part of a straight circular truncated cylinder with a cross-section of one base – a cylindrical "hoof". The accepted symbols and the design scheme are shown in Figure 2.

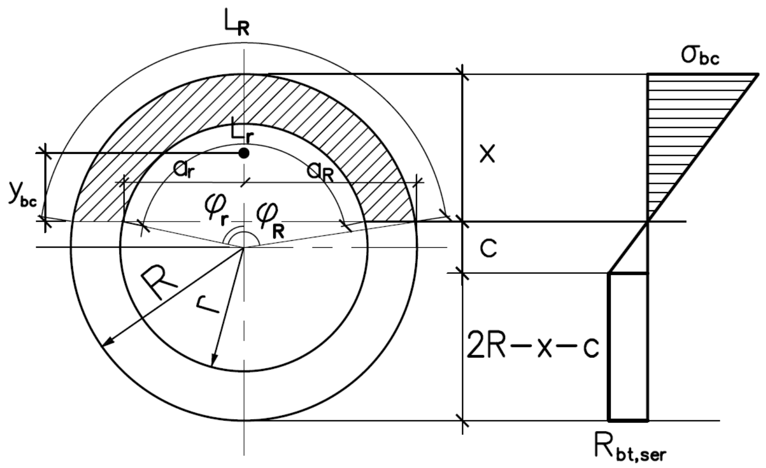

Fig. 2. Diagram for determining the longitudinal force and bending moment in concrete of the compressed zone

The maximum stresses in the stretched concrete are equal to $R_{bt\ ser}$, and the maximum stresses in the concrete of the compressed face, respectively, at the level of the external (R) and internal (r) radii, based on the hypothesis of flat sections, will be:

$$\sigma_{bc,R} = \varepsilon_{bR} \cdot E_{b,red} = \frac{x}{2R-x}\varepsilon_{bt2} \cdot E_{b,red}; \quad (1)$$

$$\sigma_{bc,r} = \varepsilon_{br} \cdot E_{b,red} = \frac{x-\Delta r}{2R-x}\varepsilon_{bt2} \cdot E_{b.red}. \quad (2)$$

Where x is the height of the compressed zone; R is the outer radius of the annular section; ε_{bt2} = 0,00015 is the value of the relative deformation of the stretched concrete.

The wall thickness of the ring Δr, here, is equal to:

$$\Delta r = R - r.$$

The reduced modulus of elasticity of concrete:

$$E_{b,red} = 0{,}85 \cdot E_b.$$

Where E_b is the initial modulus of elasticity of concrete.

The volume of the figure of the cylinder segment (hoof) in Fig.1 is the total force in the concrete of the compressed zone. Using the known dependences of stereometry [I], the expression for the force in the concrete of the compressed zone is equal to:

$$N_{bc} = \sigma_{bc,R}[a_R(3R^2 - a_R^2) + 3R^2(x - R)\varphi_R]\frac{1}{3x} -$$

$$-\sigma_{bc,r}[a_r(3r^2 - a_r^2) + 3r^2(x - \Delta r - r)\varphi_r]\frac{1}{3(x - \Delta r)}. \quad (3)$$

Where $a_R = \sqrt{2xR - x^2}$ and $a_r = \sqrt{2(x - \Delta r)r - (x - \Delta r)^2}$ – half of the chord cut off by the neutral axis; φ_R and φ_r are half of the central angle in radians.

Half of the central angle can be determined through the arc length l_R and l_r cut off by the neutral axis:

$$\varphi_R = \frac{l_R}{2R} \text{ and } \varphi_r = \frac{l_r}{2r}.$$

Where

$$l_R = \sqrt{4a_R^2 + \frac{16}{3}x^2} \text{ and } l_r = \sqrt{4a_r^2 + \frac{16}{3}(x - \Delta r)^2}$$

The center of gravity is located at a distance y_{bc} from the neutral axis:

$$y_{bc} = \frac{2}{3}X.$$

Accordingly, the moment in compressed concrete is defined as:

$$M_{bc} = N_{bc} \cdot y_{bc}$$

The stretched concrete zone has two sections (Fig. 3):

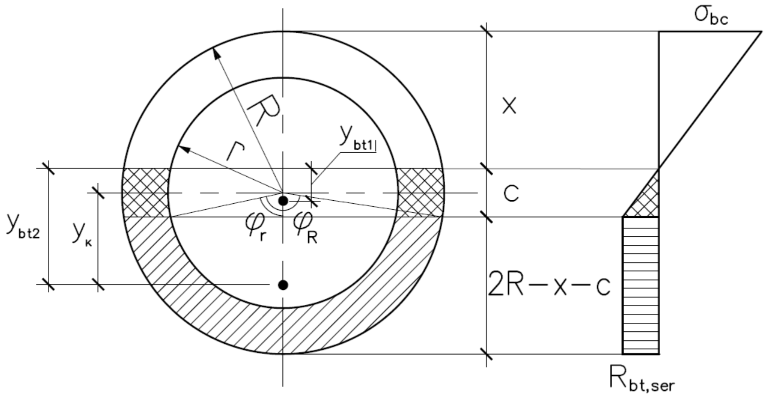

Fig. 3. A diagram for determining the longitudinal force and bending moment of a stretched concrete zone

The height of the truncated part of the stretched segment of the cylinder:

$$c = \frac{\varepsilon_{bt1}}{\varepsilon_{bt2}}(2R - x) \approx 0,533(2R - x).$$

The longitudinal force in this section is defined as:

$$N_{bt,1} = c(R - r)R_{bt,ser}.$$

Where $R_{bt,ser}$ are the calculated resistance of concrete to axial tension for the limiting states of the first and second groups, respectively.

Accordingly, the moment in the concrete of the stretched zone for the first section is:

$$M_{bt,1} = N_{bt,1} \cdot y_{bt,1}.$$

Where $y_{bt,1} = \frac{2}{3}x$ is the coordinate of the center of gravity of the section.

The area of the considered section $F_{bt,2}$ is equal to the difference between the area of the segment of the circle F_R with radius R and the area of the segment of the circle F_r with radius r.

$$F_{bt,2} = F_R - F_r .$$

where

$$F_R = \frac{R^2}{2}\left(2\arccos\left(\frac{R-h_R}{R}\right) - \sin\left(2\arccos\left(\frac{R-h_R}{R}\right)\right)\right);$$

$$F_r = \frac{r^2}{2}\left(2\arccos\left(\frac{r-h_r}{r}\right) - \sin\left(2\arccos\left(\frac{r-h_r}{r}\right)\right)\right).$$

Where h_R and h_r are the height of the segment with radii R and r respectively.

Therefore, the force in the stretched zone of the concrete of the second section can be calculated by the formula:

$$N_{bt,2} = F_{bt,2} \cdot R_{bt,ser}.$$

The value of the moment in the second section of the stretched zone of concrete relative to the neutral axis is determined by the formula:

$$M_{bt,2} = N_{bt,2} \cdot y_{bt,2}.$$

Where $y_{bt,2} = (R - x) + y_k$. Here y_k is the coordinate of the center of gravity of the second figure of the stretched zone relative to the center of the circle.

To determine the forces in compressed and stretched reinforcement, as in [XV], we distribute it evenly over the area in a circle with a radius of r_s and a thickness of d. Deformations and their corresponding stresses will be determined based on the hypothesis of plane sections and Hooke's law. The force in the compressed N_{sc} reinforcement is the product of the lateral surface of the steel figure in the form of a "hoof", the height of which is the stress σ_{sc} on the thickness of the distributed reinforcement layer δ:

$$N_{sc} = \frac{2\sigma_{sc}(R-a')}{x-a'}\left((x-a')\varphi_R + (a_R - 2a')\right)\delta.$$

Where the designations are accepted according to Fig. 2.

The bending moment along the compressed reinforcement relative to the neutral axis is equal to:

$$M_{sc} = N_{sc} \cdot y_{sc}.$$

Where $y_{sc} = \frac{2}{3}(x - a')$ is the distance from the center of gravity of the compressed reinforcement figure to the neutral axis.

The force in the valve along the stretched zone will be determined (see Figure 2 for the designations)

$$N_{st} = \frac{2\sigma_{st}(R-a)}{x-a}\left((x-a)\varphi_R + (a_R - 2a)\right)\delta.$$

The bending moment of a stretched reinforcement can be defined as:

$$M_{st} = N_{st} \cdot y_{st}.$$

Where $y_{sc} = \frac{2}{3}(2R - x - a)$ is the distance from the center of gravity of the adopted reinforcement figure of the stretched zone to the neutral axis.

The moment of cracking of the normal section of the annular shape will be equal to:

$$M_{crc} = M_{bc} + M_{bt,1} + M_{bt,2} + M_{sc} + M_{st}.$$

The final expression for the moment of cracking of ring sections is quite difficult to obtain since the accepted formulas include expressions with positive and negative exponents. Let's use numerical methods by analogy with [XV], which consists in sequentially changing the height of the compressed section zone until the equilibrium of longitudinal forces in the normal section is reached.

III. Results

In the numerical analysis, the determination of the moment of crack formation was varied by the following parameters: concrete strength –B30, B40, and B60; diameter values from 6 to 0,5 m and the ratio of the inner radius "r" to the outer "R". Figure 4 shows the results of calculating the coefficient of accounting for inelastic deformations of concrete in a stretched zone, depending on the strength of concrete and the r/R ratio.

The presented technique is associated with a large number of geometric parameters of the ring section and is based on an iterative calculation process. To develop an engineering approach, we will use the classical formula for determining the crack resistance of normal sections.

$$M_{crc} = R_{bt,ser} \cdot W_{pl}.$$

Where W_{pl} is the elastic-plastic moment of cross-section resistance equal.

$$W_{pl} = W_{red}\gamma.$$

Here γ is a coefficient taking into account inelastic deformations of stretched concrete; W_{red} is the reduced moment of resistance of the section.

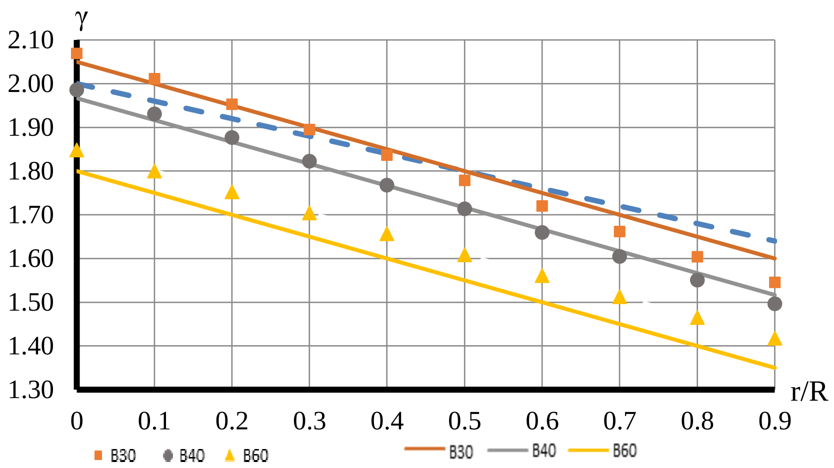

Fig. 4. Graph of the dependence of the coefficient of accounting for inelastic deformations of the stretched concrete zone on the ratio of the inner to outer radii of the ring sections for various classes of concrete strength

The values of γ determined based on a nonlinear deformation model are indicated by dot icons in the color corresponding to the concrete class. A separate dotted line shows the dependence γ=f(r/R) recommended by [II].

The obtained graphs (see Fig.4) show that the dependence adopted in [II] by the definition of γ satisfactorily coincides with the calculation using a nonlinear deformation model for concretes of class B30 and B40. For more durable concretes, the discrepancies increase.

To determine the coefficient γ, the following dependence is proposed

$$\gamma = 2{,}3 - \frac{B}{120} - 0{,}5\frac{r}{R}.$$

Where B is the concrete compressive strength class.

The results of calculations according to the proposed formula are shown on the graph in solid lines (see Fig.4) for the corresponding classes of concrete.

IV. Conclusion

Based on the conducted research, the following conclusions can be drawn:

1. based on a nonlinear deformation model, a method has been developed for determining the moment of formation of normal cracks when bending elements with a ring section without prestressing the longitudinal reinforcement.

2. Numerical studies have shown that for ring section elements without prestressing the reinforcement, the moment of crack formation mainly depends on the ratio of the inner and outer radii and the strength of concrete, without significant influence of longitudinal reinforcement.

3. A dependence is proposed for determining the coefficient that takes into account inelastic deformations of stretched concrete γ when determining the elastic-plastic moment of resistance. For ring sections, the coefficient γ varies almost linearly within the concrete classes B30 – B60 and the ratio of radii.

4. We consider it advisable, when making changes to the current SP 63.13330.2018, to supplement the methodology for determining the moment of cracking for ring sections.

Conflict of Interest:

There was no relevant conflict of interest regarding this paper.

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