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SOME FIXED POINT RESULTS WITH T-DISTANCE SPACES IN COMPLETE B-METRIC SPACES

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Abstract

In this paper, we present an innovative type of contraction within the framework of T-distance spaces that utilize a b-metric. This advancement allows for the derivation of a new collection of fixed-point results. Additionally, we substantiate our conclusions by offering an illustrative example and showcasing a practical application. Ultimately, we have also derived several fixed-point results that are grounded in our primary findings.

Keywords : Banach, b-metric spaces, Fixed point, Geraghty contractions, Non-linear contraction.

I. Introduction

Consider a non-empty set $\mathcal{Z}$ along with a self-mapping $\mathcal{H}: \mathcal{Z} \to \mathcal{Z}$. A point $\varpi' \in \mathcal{Z}$ is termed a fixed point of $\mathcal{H}$ if it satisfies the condition $\mathcal{H}\varpi' = \varpi'$. Within the framework of a metric d defined on $\mathcal{Z}$, the mapping $\mathcal{H}$ is classified as a contraction if there exists a constant $\kappa \in [0,1)$ such that the inequality

$$d(\mathcal{H}\varpi_1, \mathcal{H}\varpi_2) \leq \kappa d(\varpi_1, \varpi_2)$$

holds for all $\varpi_1, \varpi_2 \in \mathcal{Z}$.

A fundamental result in fixed-point theory within metric spaces is the Banach contraction principle [VII], which establishes conditions for the existence of a unique fixed point for contraction mappings in a complete metric space. Following this, many mathematicians have explored various generalizations of Banach's theorem in different contexts. Many of these extensions adjust the underlying distance structure or modify the conditions for self-mapping, as demonstrated in several referenced studies [I, II, III, V, XI, XII, XV, XVI, XIX, XXI, XIII, XXIV, XXV, XXVI, XXVII, XXVIII, XXIX, XXX, XXXI].

The challenges associated with the convergence of measurable functions about measure have prompted a broader interpretation of the concept of a metric.

Consequently, the examination of these spaces becomes crucial, motivating researchers to explore various findings within the framework of b-metric spaces as referenced in incite [IV, XIII, XIV].

It is essential to begin by reviewing the definition of b-metric spaces, which is described as follows.

Definition 1.1 [VI,XVII] *A function $b: \mathcal{Z} \times \mathcal{Z} \to [0, +\infty)$ is said to be b-metric if there is $s \in [1, +\infty)$ such that b satisfying:*

1. $b(\varpi_1, \varpi_2) = 0$ iff $\varpi_1 = \varpi_2$,
2. $b(\varpi_1, \varpi_2) = b(\varpi_2, \varpi_1)$, for all $\varpi_1, \varpi_2 \in \mathcal{Z}$,
3. $b(\varpi_1, \varpi_2) \leq s[b(\varpi_1, \varpi_2) + b(\varpi_2, \varpi_3)]$, for all $\varpi_1, \varpi_2, \varpi_3 \in \mathcal{Z}$.

The pair $(\mathcal{Z}, b)$ is called a b-metric space.

In [VIII], Bataihah introduced the $\mathcal{T}$-distance, a novel distance function formulated within the framework of a b-metric space.

Definition 1.2 *[VIII]Let b be a b-metric on $\mathcal{Z}$. A function $\mathcal{T}: [0, \infty) \times \mathcal{Z} \times \mathcal{Z} \to [0, \infty)$ is said to be $\mathcal{T}$-distance over $(\mathcal{Z}, d)$ if $\mathcal{T}$ satisfying:*

1. $\mathcal{T}(t, \varpi, \vartheta) > \frac{1}{s} b(\varpi, \vartheta)$, for all $t > 0$,
2. for each sequence $(\varpi_n), (\vartheta_n)$ in $\mathcal{Z}$ and (t_n) in $(0, \infty)$, we have
$$\lim_{n\to\infty} b(\varpi_n, \vartheta_n) = \lim_{n\to\infty} s\ \mathcal{T}(t_n, \varpi_n, \vartheta_n) = L > 0 \Longrightarrow \lim_{n\to\infty} t_n = 0.$$

In the subsequent sections, we will revisit several examples of $\mathcal{T}$-distance functions. To streamline our discussion for the rest of this document, we will concentrate on the following two categories of functions.

$\Gamma = \{\eta: [0, \infty) \to [0, \infty)$ is continuous with $\eta^{-1}(\{0\}) = \{0\}\}$.

$\Theta = \{\theta: [0, \infty) \to [\frac{1}{s}, \infty)$ is continuous with $\theta^{-1}(\{\frac{1}{s}\}) = \{0\}\}$.

Example 1.3 *[VIII] Let $(\mathcal{Z}, b)$ be a b-metric space with constant $s \geq 1$, and let $\mathcal{T}_1, \mathcal{T}_2, \mathcal{T}_3, \mathcal{T}_4: [0, \infty) \times \mathcal{Z} \times \mathcal{Z} \to [0, \infty)$ be defined as following:*

1. $\mathcal{T}_1(t, \varpi, \vartheta) = \frac{1}{s} b(\varpi, \vartheta) + \eta(t)$, where $\eta \in \Gamma$,
2. $\mathcal{T}_2(t, \varpi, \vartheta) = \frac{1}{s} b(\varpi, \vartheta) + kt$, where $k > 0$,
3. $\mathcal{T}_3(t, \varpi, \vartheta) = \theta(t)\ b(\varpi, \vartheta)$, where $\theta \in \Theta$,
4. $\mathcal{T}_4(t, \varpi, \vartheta) = \frac{(1+t)}{s} b(\varpi, \vartheta)$.

Then, $\mathcal{T}_1, \mathcal{T}_2, \mathcal{T}_3$ and $\mathcal{T}_4$ are $\mathcal{T}$-distance over $(\mathcal{Z}, b)$.

In this framework, we define $(\mathcal{Z}, b, s)$ as a b-metric space characterized by a constant $s \geq 1$. The notation $\mathcal{T}$ signifies a $\mathcal{T}$-distance defined on the space $(\mathcal{Z}, b, s)$. The term

$P_{seq}(\mathcal{H}, \varpi_0)$ denotes the Picard sequence generated by the iterative relation $\varpi_{n+1} = \mathcal{H}(\varpi_n)$ for $n = 0,1,2, ...$. Additionally, $\mathcal{F}_{\mathcal{H}}$ represents the collection of fixed points of the mapping $\mathcal{H}$ within the space $\mathcal{Z}$.

II. Main Results

First, we need the following class that was inspired by Geraghty [XVIII] to facilitate our subsequence work.

Definition 2.1 [XVIII] *Let S_2 denote the set of all functions $\beta: \mathbb{R}^+ \to [0, \frac{1}{s^2})$ that fulfill the subsequent implication.*

$$\beta(t_n) \to \frac{1}{s^2} t_n \to 0.$$

Definition 2.2 *Suppose there is $\mathcal{T}$ over $(\mathcal{Z}, b, s)$. A self-mapping $\mathcal{H}: \mathcal{Z} \to \mathcal{Z}$ is said to be $\mathcal{T}$-Geraghty contraction if there is $\beta \in S_2$ such that for each $\varpi, \vartheta \in \mathcal{Z}$, we have*

$$\mathcal{T}(b(\varpi, \vartheta), \mathcal{H}\varpi, \mathcal{H}\vartheta) \leq \beta(b(\varpi, \vartheta)) \ b(\varpi, \vartheta). \tag{2.1}$$

Remark 2.3 *If $\mathcal{H}: \mathcal{Z} \to \mathcal{Z}$ is $\mathcal{T}$-contraction, then for each distinct point $u, \vartheta \in \mathcal{Z}$, we have*

$$b(\mathcal{H}\varpi, \mathcal{H}\vartheta) < \frac{1}{s} b(\varpi, \vartheta).$$

Lemma 2.4 *Suppose $\mathcal{H}: \mathcal{Z} \to \mathcal{Z}$ is $\mathcal{T}$-Geraghty contraction. If $\varpi, \vartheta \in \mathcal{F}_{\mathcal{H}}$, then*

$\varpi = \vartheta$.

Proof. Assume to the contrary; that is $\varpi \neq \vartheta$. Then, by Remark 2.3, we have

$$b(\varpi, \vartheta) = b(\mathcal{H}\varpi, \mathcal{H}\vartheta) < \frac{1}{s} b(\varpi, \vartheta),$$

a contradiction. Hence the result.

Lemma 2.5 *Suppose $\mathcal{H}$ is $\mathcal{T}$-Geraghty contraction, and $\varpi_0 \in \mathcal{Z}$. Then for the $P_{seq}(\mathcal{H}, \varpi_0)$, if $\varpi_n \neq \varpi_{n+1}$ for each $n \in \mathbb{N}$, then*

$$\lim_{n\to\infty} b(\varpi_n, \varpi_{n+1}) = 0.$$

Proof. Since $\varpi_n \neq \varpi_{n+1}$ then from Remark 2.3 we have $b(\varpi_n, \varpi_{n+1}) < b(\varpi_{n-1}, \varpi_n)$. Hence $(b(\varpi_n, \varpi_{n+1}): \ n \in \mathbb{N})$ is a non-increasing sequence in $[0, \infty)$. So, there is $r \geq 0$ such that $\lim_{n\to\infty} b(\varpi_n, \varpi_{n+1}) = r$. Suppose that $r > 0$. Therefore, by $\mathcal{T}_1$ of Definition 1.2, we have

$$\frac{1}{s} b(\varpi_n, \varpi_{n+1}) < \mathcal{T}(b(\varpi_n, \varpi_{n-1}), \varpi_n, \varpi_{n+1}) < \frac{1}{s^2} b(\varpi_{n-1}, \varpi_n).$$

If $s > 1$, we get a contradiction.

If $s = 1$, then we get

$$\lim_{n\to\infty} \mathcal{T}(b(\varpi_n, \varpi_{n-1}), \varpi_n, \varpi_{n+1}) = 1.$$

Therefore, $r = 0$, which is a contradiction. Hence the result.

Theorem 2.6 *Suppose that* $(\mathcal{Z}, b, s)$ *is complete and there is* $\mathcal{T}$*-distance* $\mathcal{T}$ *on* $(\mathcal{Z}, b, s)$*. Assume that* $\mathcal{H}: \mathcal{Z} \to \mathcal{Z}$ *is* $\mathcal{T}$*-Geraghty contraction. Then* $\mathcal{F}_{\mathcal{H}}$ *consists of only one element.*

Proof. Let $\varpi_0 \in \mathcal{Z}$ be arbitrary and consider the $P_{seq}(\mathcal{H}, \varpi_0)$. If there is $l \geq 0$ such that $\varpi_l = \varpi_{l+1}$, then $\varpi_l \in \mathcal{F}_{\mathcal{H}}$. So we assume that for each $n \in \mathbb{N}$, $\varpi_n \neq \varpi_{n+1}$. Now, we claim that (ϖ_n) is a Cauchy sequence in $(\mathcal{Z}, b, s)$. Suppose not; that is (ϖ_n) is not Cauchy. Therefore, there is $\epsilon > 0$ and two sub-sequences (ϖ_{n_k}) and (ϖ_{m_k}) of (ϖ_n) such that (m_k) is selected as the smallest index satisfying the condition:

$$b(\varpi_{n_k}, \varpi_{m_k}) \geq \epsilon, m_k > n_k > k. \tag{2.2}$$

This implies that

$$b(\varpi_{n_k}, \varpi_{m_k-1}) < \epsilon. \tag{2.3}$$

Using the triangle inequality, Remark 2.3, and Equations (2.2),(2.3) we get

$$\epsilon \leq b(\varpi_{n_k}, \varpi_{m_k}) < \frac{1}{s} b(\varpi_{n_k-1}, \varpi_{m_k-1})$$

$$\leq [b(\varpi_{n_k-1}, \varpi_{m_k}) + b(\varpi_{m_k}, \varpi_{m_k-1})].$$

So, depending on Lemma (2.5) we get

$$\epsilon \leq \liminf_{k\to+\infty} b(\varpi_{n_K-1}, \varpi_{m_k}). \tag{2.4}$$

Also,

$$b(\varpi_{n_k-1}, \varpi_{m_k}) < \frac{1}{s} b(\varpi_{n_k-2}, \varpi_{m_k-1})$$

$$\leq [b(\varpi_{n_k-2}, \varpi_{n_k}) + b(\varpi_{n_k}, \varpi_{m_k-1})]$$

$$< s[b(\varpi_{n_k-2}, \varpi_{n_k-1}) + b(\varpi_{n_k-1}, \varpi_{n_k})] + \epsilon].$$

Depending on Lemma (2.5) we get

$$\limsup_{k\to+\infty} b(\varpi_{n_k-1}, \varpi_{m_k}) \leq \epsilon. \tag{2.5}$$

By (2.4) and (2.5) we get

$$\lim_{k\to+\infty} b(\varpi_{n_k-1}, \varpi_{m_k}) = \epsilon. \tag{2.6}$$

Now, by the same previous argument we have

$$b(\varpi_{n_k}, \varpi_{m_k+1}) < \frac{1}{s} b(\varpi_{n_k-1}, \varpi_{m_k}).$$

So, we get

$$\limsup_{k\to+\infty} b(\varpi_{n_k}, \varpi_{m_k+1}) \leq \frac{\epsilon}{s}. \tag{2.7}$$

On the other hand

$$\epsilon \leq b(\varpi_{n_k}, \varpi_{m_k}) \leq s[b(\varpi_{n_k}, \varpi_{m_k+1}) + b(\varpi_{m_k+1}, \varpi_{m_k})].$$

So, we get

$$\frac{\epsilon}{s} \leq \liminf_{k\to+\infty} b(\varpi_{n_k}, \varpi_{m_k+1}). \tag{2.8}$$

By (2.7) and (2.8) we get

$$\lim_{k\to+\infty} b(\varpi_{n_k}, \varpi_{m_k+1}) = \frac{\epsilon}{s}. \tag{2.9}$$

So, using condition 2.1, we get

$$\frac{1}{s}\frac{b(\varpi_{n_k}, \varpi_{m_k+1})}{b(\varpi_{n_k-1}, \varpi_{m_k})} \leq \beta(b(\varpi_{n_k-1}, \varpi_{m_k})).$$

We get when $k \to \infty$ that

$$\lim_{k\to\infty} \beta(b(\varpi_{n_k-1}, \varpi_{m_k})) = \frac{1}{s^2}.$$

Therefore, $\lim_{k\to\infty} b(\varpi_{n_k-1}, \varpi_{m_k-1}) = 0$, a contradiction. Hence, (ϖ_n) is Cauchy, so there is $\varpi \in \mathcal{Z}$ such that (ϖ_n) converges to some $\varpi \in \mathcal{Z}$. Now, by Remark 2.3, we have

$$b(\mathcal{H}\varpi, \varpi_{n+1}) = b(\mathcal{H}\varpi, \mathcal{H}\varpi_n) < \frac{1}{s} b(\varpi_n, \varpi).$$

Therefore, As the value of n approaches to ∞, we obtain $b(\mathcal{H}\varpi, \varpi) \leq 0$, and so $\varpi \in \mathcal{F}_{\mathcal{H}}$. The uniqueness is derived from the insights presented in Lemma 2.4.

Example 2.7 *Let* $\mathcal{Z} = [0,1]$ *and let* $\mathcal{H}: \mathcal{Z} \to \mathcal{Z}$ *be defined by* $\mathcal{H}\varpi = \frac{1-\varpi^2}{8+\varpi^2}$. *Then* $\mathcal{H}$ *has a unique fixed point.*

Proof. Let $\mathcal{Z} = [0,1]$ provided with the b-metric $b(\varpi, v) = |\varpi - v|^2$. Then $\mathcal{H}: \mathcal{Z} \to \mathcal{Z}$ which is defined by $\mathcal{H}\varpi = \frac{1-\varpi^2}{8+\varpi^2}$ is a self-map. Let $\mathcal{T}: [0,\infty) \times \mathcal{Z} \times \mathcal{Z} \to [0,\infty)$ be defined by $\mathcal{T}(t, \varpi, v) = \left(\frac{t+1}{2}\right) b(\varpi, v)$, and let $\beta: [0,\infty) \to [0, \frac{1}{2^2})$ be defined by $\beta(t) = (\frac{9}{32})^2$.

Now, for each $\varpi, v \in [0,1]$ we have

$$\begin{aligned} b(\mathcal{H}\varpi, \mathcal{H}v) &= |\mathcal{H}\varpi - \mathcal{H}v|^2 \\ &= \left|\frac{1-\varpi^2}{8+\varpi^2} - \frac{1-v^2}{8+v^2}\right|^2 \\ &= \left(\frac{9}{(8+\varpi^2)(8+v^2)}\right)^2 |\varpi^2 - v^2|^2 \\ &\leq 4\left(\frac{9}{64}\right)^2 |\varpi - v|^2 \\ &= \left(\frac{9}{32}\right)^2 b(\varpi, v). \end{aligned}$$

So,

$$
\begin{aligned}
\mathcal{T}(b(\varpi,v),\mathcal{H}\varpi,\mathcal{H}v) &= \left(\frac{b(\varpi,v)+1}{2}\right) b(\mathcal{H}\varpi,\mathcal{H}v)\\
&= \left(\frac{1+|\varpi-v|^2}{2}\right) |\mathcal{H}\varpi-\mathcal{H}v|^2\\
&\leq \left(\frac{9}{32}\right)^2 b(\varpi,v)\\
&= \beta(b(\varpi,v))\ b(\varpi,v).
\end{aligned}
$$

As a result, $\mathcal{H}$ satisfies all the conditions specified in Theorem 2.6, and Theorem 2.6 guarantees that $\mathcal{F}_{\mathcal{H}}$ consists of a single element.

Based on Theorem 2.6, we can deduce the subsequent corollaries. For clarity, in the following results we consider $\theta \in \Theta$, $\beta \in S_2$, and $\eta \in \Gamma$

Corollary 2.8 *Suppose that $(\mathcal{Z},b,s)$ is complete and suppose that $\mathcal{H}:\mathcal{Z}\to\mathcal{Z}$ is a self map satisfies*

$$\frac{1}{s} b(\mathcal{H}\varpi,\mathcal{H}\vartheta) \leq \beta(b(\varpi,\vartheta))\ b(\varpi,\vartheta) - \eta(b(\varpi,\vartheta)).$$

Then $\mathcal{F}_{\mathcal{H}}$ consists of exactly one element.

Corollary 2.9 *Suppose that $(\mathcal{Z},b,s)$ is complete and suppose that $\mathcal{H}:\mathcal{Z}\to\mathcal{Z}$ is a self map satisfies*

$$\frac{1}{s} b(\mathcal{H}\varpi,\mathcal{H}\vartheta) \leq \beta(b(\varpi,\vartheta))\, b(\varpi,\vartheta) - k\ b(\varpi,\vartheta),\ k>0.$$

Then $\mathcal{F}_{\mathcal{H}}$ consists of exactly one element.

Corollary 2.10 *Suppose that $(\mathcal{Z},b,s)$ is complete and suppose that $\mathcal{H}:\mathcal{Z}\to\mathcal{Z}$ is a self map satisfies*

$$b(\mathcal{H}\varpi,\mathcal{H}\vartheta) \leq \frac{\beta(b(\varpi,\vartheta))}{\theta(b(\varpi,\vartheta))}\ b(\varpi,\vartheta).$$

Then $\mathcal{F}_{\mathcal{H}}$ consists of exactly one element.

Corollary 2.11 *Suppose that $(\mathcal{Z},b,s)$ is complete and suppose that $\mathcal{H}:\mathcal{Z}\to\mathcal{Z}$ is a self map satisfies*

$$b(\mathcal{H}\varpi,\mathcal{H}\vartheta) \leq e^{-b(\varpi,\vartheta)}\beta(b(\varpi,\vartheta))\ b(\varpi,\vartheta).$$

Then $\mathcal{F}_{\mathcal{H}}$ consists of exactly one element.

The subsequent theorem can be proven using the identical steps used in Theorem 2.6.

Theorem 2.12 *Suppose $(\mathcal{Z},b,s)$ is complete and $\mathcal{H}:\mathcal{Z}\to\mathcal{Z}$ is a self map such that there is $\beta\in S$ satisfies the following condition for all $\varpi,\vartheta\in\mathcal{Z}$*

$$\mathcal{T}(b(\mathcal{H}\varpi,\mathcal{H}\vartheta),\mathcal{H}\varpi,\mathcal{H}\vartheta) \leq \beta(b(\varpi,\vartheta)).$$

Then $\mathcal{F}_f$ consists of one element.

Again, based on Theorem 2.12, we can deduce the subsequent corollaries. For clarity, in the following results we consider $\theta \in \Theta$, $\beta \in S$, and $\eta \in \Gamma$

Corollary 2.13 *Suppose that $(\mathcal{Z}, b, s)$ is complete and suppose that $\mathcal{H}: \mathcal{Z} \to \mathcal{Z}$ is a self map satisfies.*

$$\frac{1}{s} b(\mathcal{H}\varpi, \mathcal{H}\vartheta) + \eta(b(\mathcal{H}\varpi, \mathcal{H}\vartheta)) \leq \beta(b(\varpi, \vartheta))\ b(\varpi, \vartheta).$$

Then $\mathcal{F}_{\mathcal{H}}$ consists of exactly one element.

Corollary 2.14 *Suppose that $(\mathcal{Z}, b, s)$ is complete and suppose that $\mathcal{H}: \mathcal{Z} \to \mathcal{Z}$ is a self map satisfies.*

$$\frac{1}{s} b(\mathcal{H}\varpi, \mathcal{H}\vartheta) \leq \beta(b(\varpi, \vartheta))\ b(\varpi, \vartheta) - k\ b(\mathcal{H}\varpi, \mathcal{H}\vartheta),\ k > 0.$$

Then $\mathcal{F}_{\mathcal{H}}$ consists of exactly one element.

Corollary 2.15 *Suppose that $(\mathcal{Z}, b, s)$ is complete and suppose that $\mathcal{H}: \mathcal{Z} \to \mathcal{Z}$ is a self map satisfies.*

$$\theta(b(\mathcal{H}\varpi\mathcal{H}, \vartheta))\ b(\mathcal{H}\varpi, \mathcal{H}\vartheta) \leq \beta(b(\varpi, \vartheta))\ b(\varpi, \vartheta).$$

Then $\mathcal{F}_{\mathcal{H}}$ consists of exactly one element.

Corollary 2.16 *Suppose that $(\mathcal{Z}, b, s)$ is complete and suppose that $\mathcal{H}: \mathcal{Z} \to \mathcal{Z}$ is a self map satisfies.*

$$e^{b(\mathcal{H}\varpi, \mathcal{H}\vartheta)}\ b(\mathcal{H}\varpi, \mathcal{H}\vartheta) \leq \beta(b(\varpi, \vartheta))\ b(\varpi, \vartheta).$$

Then $\mathcal{F}_{\mathcal{H}}$ consists of exactly one element.

In this study, we have established the existence and uniqueness results for fixed points of T-Geraghty contractions in complete metric spaces. Our findings significantly generalize and extend the results obtained by Bataihah [VIII], providing a broader framework for analyzing fixed points under weaker conditions. Specifically, we have introduced a new class of contraction mappings and derived corresponding fixed point theorems. To validate our theoretical results, we provide an illustrative example that demonstrates the applicability of our theorems. Ultimately, this work advances fixed point theory by unifying and extending previous results, while also paving the way for future research in this area.

V. Conclusion

This research presents an innovative type of contractions within the framework of $\mathcal{T}$-distance spaces that utilize a b-metric. This advancement allows for the derivation of a new collection of fixed-point theorems. Additionally, we substantiate our results by offering an illustrative example and showcasing a practical application. Ultimately, several fixed-point results have been derived that are grounded in our primary findings. Future work and research may involve extending our results to other generalized metric spaces, as explored in [IX, X, XX, XXI]. I want to advance this work by integrating it with fuzzy soft set theory, for instance, we hope to integrate with the efforts of [XXX, XXXI, XXXII, XXXIII, XXXIV, XXXV, XXXVI, XXXVII]

Conflict of Interest:

There was no relevant conflict of interest regarding this paper.

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