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# COMPARATIVE STUDY OF B&B WITH HEURISTICS NEH AND CDS FOR BI STAGE FLOW SHOP SCHEDULING PROBLEM UNDER FUZZY ENVIRONMENT

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**Abstract**

*A flow shop is a workspace where machines, which could be humans or machines perform a range of tasks. It involves figuring out how to arrange several jobs in the most effective way possible. In the manufacturing sector, production scheduling is essential for several reasons, including lower product costs, higher productivity, customer happiness, and competitiveness. To adequately satisfy consumer needs and meet product demand, proper scheduling offers and promotes the proper usage of criteria such as available commodities, labour, and machines. This study illustrates the general algorithm and methodology comparison using fuzzy numbers, which is beneficial in figuring out the order of tasks. The aim is to provide the best way to minimize the makespan required to distribute shared resources over time to finish competing tasks. Furthermore, the machine processing times are not fixed; rather, they are interpreted as trapezoidal fuzzy numbers (TrFN). First, during the initial step, three parallel equipotential machines are taken and one machine is in the subsequent phase. Three parallel equipotential machines are taken initially and a single machine in the subsequent phase. Then a comparative study between branch and bound and heuristic methods like CDS (Campbell, Dudek, and Smith) and NEH (Nawaz, Enscore, and Ham) is done.*

**Keywords:** Makespan, Trapezoidal Fuzzy Number (TrFN), Flow shop scheduling (FSS), Yager's Ranking Formula, Transportation technique

## I. Introduction

A conscious investigation of decision-making issues is scheduling. The selection-making philosophy employed in engineering and industrial manufacturing services today is called FSS. The FSS model includes a variety of tasks as well as some processes that must be completed on distinct equipment. Railway networks, ophthalmologists, automobile production machinery and many more types of machineries are some examples of this model. The aim of scheduling is to accomplish a number of objectives by allocating different tasks to available machines.

Although deterministic processing times are the focus of most studies, there are many challenges with unknown scenarios in the actual world. Techniques that focus on precise processing times are unable to address problems with uncertainty. Zadeh was the first to use fuzzy sets as a mathematical representation of imprecision or ambiguity in daily life. This prompted the creation of fuzzy set theory, which is now a commonly used theory in intelligent control. Due to its ease of use and resemblance to human thought processes, fuzzy set theory has many applications in several fields, including manufacturing, engineering, and medicine.

Due to the various contemporary and financial uses of flow shop scheduling difficulties, several scientists and researchers periodically study these applications. The first person to solve two or three-stage machine difficulties to reduce the overall elapsed time was Johnson [XIX]. Palmer et al. [IX] investigated how jobs are sequenced across a multi-step process and contrasted several approaches. For the case of three machines, Lomnicki et al. [XXII] developed the algorithm to efficiently solve the scheduling challenge. Ignall et al. [X] developed a noteworthy approach called Branch and Bound for n-job, three machines problems. Zadeh et al. [XIV] establish numerous features of concepts like convexity, complement, etc. in the context of fuzzy sets. McMahon et al. [XI] used the B&B approach to work on the flow shop scheduling issue. The technique was expanded by Smith et al. [XVI], who provided an algorithm on n jobs, m machines. Zadeh et al. [XV] defined possibility distribution as a fuzzy limitation to explain how the theory of possibility relates to the theory of fuzzy sets. Then algebra of fuzzy sets was studied by Dubois [IV]. Yee Chung et al. [III] extended the classical multiprocessor scheduling problems with all machines available at time zero by working on n-independent jobs on m distinct identical computers. The development of the fuzzy simplex method was researched by Ganesan et al. [XII]. Chen et al. [XVIII] introduced a novel approach to fuzzy risk analysis in their work. A thorough assessment on scheduling issues was provided by Allahverdi et al. [I], who also classified the literature according to flow shop, job shop, open shop, and other categories. Abbasbandy et al. [XVII] presented a novel method for ranking TrFNs. The suggested method's computation was much more straightforward. In their endeavour to investigate the issues where task probabilities and processing durations are related, S Sharma et al. [XX] also included the idea of interval breakdown and job block criteria. Gupta et al. [II] [V] worked on a heuristic algorithm to minimize makespan.

By taking into account various concepts like left, and right fuzziness of TrFNs, Ponnialagan et al. [VIII] proposed a new ranking mechanism on the class of TrFNs. Two and three-stage FSSP with equipotential machines using branch and bound method was studied by Sonia Goel et al. [VI] [VII] [XIII]. In their endeavour to investigate the Hybrid FSS in parallel sequential mode, Feng et al. [XXI] included the idea of handling time and setup time.

## II. Preliminaries

**Definition 2.1**

let F be a crisp set and $\mu_F(x)$ is a function from F to [0, 1]. A fuzzy set F is defined with the membership function $\mu_F(x)$ is

$$F = \{(x, \mu_F(x)) ; x \in F \text{ and } \mu_F(x) \in [0,1]\}.$$

**Definition 2.2**

Fuzzy set F is a fuzzy number if the following conditions are satisfied:

(1) F is convex fuzzy set ($\mu_F(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_F(x_1), \mu_F(x_2))$).

(2) F is normalized fuzzy set ($\exists\, x/\mu_F(x) = 1$).

(3) Its membership function is piecewise continuous.

**Definition 2.3**

A Trapezoidal fuzzy number denoted by $A$ is defined as ($l$, $m$, $n$, $u$) where the membership function is given by

$$\mu_A(x) = \begin{cases} 0, & x < l \\ \frac{x-l}{a2-a1}, & l \leq x \leq m \\ \frac{a3-x}{a3-a2}, & a2 \leq x \leq a3 \\ 0, & x > a3 \end{cases}$$

## III. Practical Situation

A manufacturing company produces two types of products and the production process consists of two stages: machining and assembly. The trapezoidal fuzzy processing times represent the uncertainty in the processing times due to various factors like machine wear, operator skill, and material variability. The goal is to determine the optimal schedule for these jobs on the devices to optimize the total production times.

Hospitals require sterilization of medical instruments and equipment to prevent the spread of infections. A hospital has a sterilization department that processes surgical instruments. The department has 3 identical autoclaves (M1, M2, M3) at the first stage, which sterilize instruments. The sterilized instruments are then transferred to a single packaging machine (M4) at the second stage, which packages them for use in surgeries. Sterilizer machines are used to sterilize these items, and the process involves two stages: washing /sterilization and packaging. Every day, the hospital has a lot of equipment and tools that need to be sterilized. The objective is to plan the sterilizing procedure to satisfy demand while reducing the overall processing time. In the initial phase, machines wash and sterilize the instruments and equipment in parallel. Each machine has the same capacity. After washing and sterilization, the instruments and equipment are transferred to the packaging machines. This is shown in Figures 1(a) and 1(b).

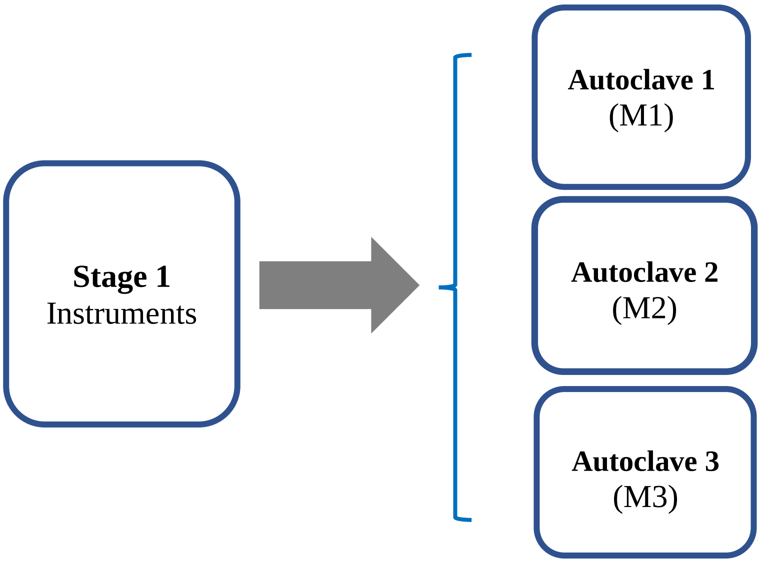


**Fig.1(a).** Three parallel autoclaves at 1st stage.

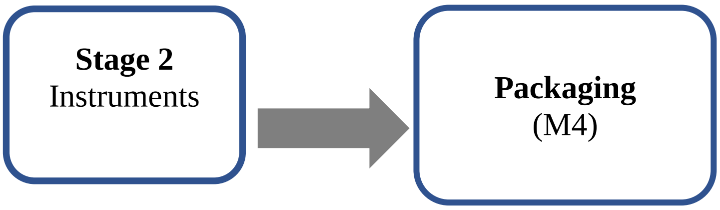


**Fig. 1(b).** Single packaging machines at 2nd stage.

By optimizing the washing, sterilization, and packaging processes as shown in Fig. 1(a) and 1(b) respectively, the hospital can minimize the total processing time, meet the demand, and improve resource utilization, ultimately leading to enhanced patient care.

## IV. Mathematical model

Assume that three parallel equipotential machines will be used for the first stage of n tasks (i=1,2,3..., n) and one machine will be used for the subsequent stage. The makespan of every job is considered a fuzzy trapezoidal number. The available time of every identical machine and its cost of operation per unit are also provided. The problem's description is provided here in Table 1.

**Table 1: Tabular presentation of the model**

| **Job** | **Machine K** | | | **Processing time (machine K)** | **Processing time (machine P)** |
|---|---|---|---|---|---|
| i | $K_1$ | $K_2$ | $K_3$ | Trapezoidal Fuzzy time | Trapezoidal Fuzzy time |
| 1<br>2<br>3<br>.<br>.<br>.<br>.<br>n<br>$t_{pj}$ | $k_{11}$<br>$k_{21}$<br>$k_{31}$<br>.<br>.<br>.<br>.<br>$k_{n1}$<br>$t_{11}$ | $k_{12}$<br>$k_{22}$<br>$k_{32}$<br>.<br>.<br>.<br>.<br>$k_{n2}$<br>$t_{12}$ | $k_{13}$<br>$k_{23}$<br>$k_{33}$<br>.<br>.<br>.<br>.<br>$k_{n3}$<br>$t_{13}$ | (å$_{1,}$ß$_{1,}c_1$, ý$_1$)<br>(å$_{2,}$ß$_{2,}c_2$, ý$_2$)<br>(å$_{3,}$ß$_{3,}c_3$, ý$_3$)<br>.<br>.<br>.<br>.<br>(å$_{n,}$ß$_{n,}c_n$, ý$_n$) | ($d_{1,}$é$_1$ƒ$_1$, $J_1$)<br>($d_{2,}$é$_2$ƒ$_2$$J_{2,}$)<br>($d_{3,}$é$_3$ƒ$_3$, $J_3$)<br>.<br>.<br>.<br>.<br>($d_{n,}$é$_n$ƒ$_n$, $J_n$) |

Where

i : Index of the job

(å$_{i,}$ß$_i$, $c_{i,}$ý$_i$) : Fuzzy utilization time of $i^{th}$ task on $K_j$

($d_{i,}$é$_{i,}$ƒ$_i$, $J_i$) : Fuzzy utilization time of $i^{th}$ task on processor E

$K_j$ : Parallel equipotential machines for Machine K

$t_{pj}$ : Total available time of parallel equipotential machines of type k for p=1,2,3

### IV.ii. Assumptions

- The tasks to be operated are autonomous.
- Another task cannot be operated until the already started job is completed.
- Every job and machine is accessible at the start of the processing.
- Machines are available during the whole process and never malfunction.
- Every task is run via every machine just once.
- Machines are identical and continuously available.
- No pre-emption or interruption is allowed.
- Processing times are not certain and are taken as trapezoidal fuzzy numbers.

## V. Algorithm

The aforesaid issue can be resolved by taking the following course of action.

**Step 1:** Use Yager's ranking rule to find the crisp value

$$k_i = \frac{1}{2}[(ß_i+c_i)-\frac{4}{5}(ß_i-å_{i,}) + \frac{2}{3}(ý_i - c_i)]$$

Where $(å_{i,}ß_i, c_i, ý_i)$ is the trapezoidal fuzzy processing time and

$$p_i = \frac{1}{2}[(é_i + f_i) - \frac{4}{5}(é_i - d_{i,}) + \frac{2}{3}(J_i - f_i)]$$

Where $(d_{i,}é_{i,}f_i, J_i)$ is the trapezoidal fuzzy processing time.

**Step 2**: Use a transit approach such as VAM. Before using any transportation method, we look at the restrictions.

$$\sum_{j=1}^{3} t_{1j} = \sum_{i=1}^{n} k_i$$

To find out if the issue is properly balanced or not.

- If the problem is balanced then apply the transportation technique and find the most ideal allocation of unit cost of all tasks by MODI method.
- If the problem is not balanced then add a dummy entry with zero operational cost.

**Step 3:** Apply the branch and bound technique by using the rule.

$$g' = \max\left(\sum_{i=1}^{n} k_{ij}\right) + \min_{i\in J_k'}(p_i)$$

$$g'' = \max_{i\in J_k'} k_{ij} + \sum_{i}^{n} p_i$$

Where $J_k$ denote the tasks not taken into consideration under branching.

Then evaluate $G = \max(g', g'')$ for all the tasks.

Determine the lowest value of G among all values, then begin the optimal subsequence's first task at that vertex. Continue doing the preceding steps until we reach the branch termination point, which will provide us with the best possible order for our jobs.

### VI.i. Numerical problem:

Consider the FSSP with 3 equipotential machines working in parallel at the first stage and a single machine at the second stage. The processing times of the machines are considered as trapezoidal fuzzy numbers. Total available time and operating cost (in hours) are also given in Table 2.

**Table 2: Mathematical problem**

| Jobs | Machine K | | | | Machine P |
|---|---|---|---|---|---|
| I | $K_1$ | $K_2$ | $K_3$ | Processing time $(å_{i,}ß_i, c_i, ý_i)$ | Processing time $(d_{i,}é_{i,}f_i, J_i)$ |
| 1 | 8 | 5 | 4 | (5,7,8,11) | (9,11,12,14) |
| 2 | 2 | 8 | 6 | (5,6,7,8) | (12,15,16,19) |
| 3 | 4 | 7 | 8 | (5,8,9,12) | (5,10,12,17) |
| 4 | 7 | 5 | 6 | (5,7,8,10) | (7,9,10,12) |
| Available time | 8 | 9 | 12.7 | | |

Step 1: Using Yager's ranking formula to transform the fuzzy utilization time into crisp value,

given as

$$k_i = \frac{1}{2}[(b+c)-\frac{4}{5}(b-a) + \frac{2}{3}(d-c)]$$

the problem in simplified form is represented in Table 3 and the distribution after VAM is shown in Table 4.

**Table 3: Mathematical issue after applying Yager's-ranking formula**

| Tasks | Machine K | | | | Machine P |
|---|---|---|---|---|---|
| I | $K_1$ | $K_2$ | $K_3$ | Processing time $k_i$ | Processing time $p_i$ |
| 1 | 8 | 5 | 4 | 7.7 | 11.3 |
| 2 | 2 | 8 | 6 | 6.4 | 15.3 |
| 3 | 4 | 7 | 8 | 8.3 | 10.6 |
| 4 | 7 | 5 | 6 | 7.3 | 9.3 |
| Available time | 8 | 9 | 12.7 | | |

**Table 4: Distribution of processing time among parallel equipotential machines after VAM**

| Jobs | $K_1$ | $K_2$ | $K_3$ |
|---|---|---|---|
| 1 | 0 | 0 | 7.7 |
| 2 | 6.4 | 0 | 0 |
| 3 | 1.6 | 6.7 | 0 |
| 4 | 0 | 2.3 | 5 |

Now to optimize the solution obtained above, applying Modified Distribution Method (MODI). The reduced problem is given in table 5.

**Table 5: Reduced problem's tabular representation**

| Jobs | $K_1$ | $K_2$ | $K_3$ | P ( $p_i$) |
|---|---|---|---|---|
| 1 | 0 | 0 | 7.7 | 11.3 |
| 2 | 6.4 | 0 | 0 | 15.3 |
| 3 | 1.6 | 6.7 | 0 | 10.6 |
| 4 | 0 | 2.3 | 5 | 9.3 |

Now we apply the B&B Algorithm and calculate the lower bound of jobs to find their order for the optimal solution shown in Table 6.

**Table 6: Depicting lower bound of jobs**

| **(i)** | $g' = \max\ (\sum_{i=1}^{n} K_{ij}) + \min_{i\in J_k'}(p_i)$ | $g'' = \max_{i\in J_k'} K_{ij} + \sum_i^n p_i$ | G=max $\{g', g''\}$ |
|---|---|---|---|
| **1** | $\max\ (\sum_{i=1}^{n} K_{ij}) + \min_{i\in\{2,3,4\}}(p_i)$ =12.7+9.3=22 | $\max_{i\in\{2,3,4\}} K_{ij} + \sum_i^n p_i$ =7.7+46.5=54.2 | 54.2 |
| **2** | $\max\ (\sum_{i=1}^{n} K_{ij}) + \min_{i\in\{1,3,4\}}(p_i)$ =12.7+9.3=22 | $\max_{i\in\{1,3,4\}} K_{ij} + \sum_i^n p_i$ =6.4+46.5=52.9 | 52.9 |
| **3** | $\max\ (\sum_{i=1}^{n} K_{ij}) + \min_{i\in\{1,2,4\}}(p_i)$ =12.7+9.3=22 | $\max_{i\in\{1,2,4\}} K_{ij} + \sum_i^n p_i$ =6.7+46.5=53.2 | 53.2 |
| **4** | $\max\ (\sum_{i=1}^{n} K_{ij}) + \min_{i\in\{1,2,3\}}(p_i)$ =12.7+10.6=23.3 | $\max_{i\in\{1,2,3\}} K_{ij} + \sum_i^n p_i$ =5+46.5=51.5 | 51.5 |

Here lower bound is 51.5 which concerns job 4. Therefore, job 4 is placed at the very first place in the required ideal sequence. Now we continue with the same process and determine the nodes for the second branch in Table 7.

**Table 7: Depicting the lower bound of jobs when job 4 is fixed in the first place**

| (i) | $g'$ | $g''$ | G=max $\{g', g''\}$ |
|---|---|---|---|
| **41** | $\max\ (\sum_{i=1}^{n} K_{ij}) + \min_{i\in\{2,3\}}(p_i)$ =12.7+10.6=23.3 | $\max_{i\in\{2,3\}} K_{ij} + \sum_i^n p_i$ =12.7+46.5=59.2 | 59.2 |
| **42** | $\max\ (\sum_{i=1}^{n} K_{ij}) + \min_{i\in\{1,3\}}(p_i)$ =12.7+10.6=23.3 | $\max_{i\in\{1,3\}} K_{ij} + \sum_i^n p_i$ =6.4+46.5=52.9 | 52.9 |
| **43** | $\max\ (\sum_{i=1}^{n} K_{ij}) + \min_{i\in\{1,2\}}(p_i)$ =12.7+11.3=24 | $\max_{i\in\{1,2\}} K_{ij} + \sum_i^n p_i$ =9+46.5=55.5 | 55.5 |

So the lower bound is again 52.9 which is for subsequence {4,2}. Therefore job 2 is fixed at second place. We continue searching for the next task in the ideal order shown in Table 8.

**Table 8: Depicting lower bound of jobs when job 4 and 2 is fixed at first and second place respectively**

| (i) | $g'$ | $g''$ | G=max{$g', g''$} |
|---|---|---|---|
| 421 | max $(\sum_{i=1}^{n} K_{ij})$ + $\min_{i=3}(p_i)$ =12.7+10.6=23.3 | $\max_{i=3} K_{ij}+\sum_{i}^{n} p_i$ =12.7+46.5=59.2 | 59.2 |
| 423 | max $(\sum_{i=1}^{n} K_{ij})$ + $\min_{i=1}(p_i)$ =12.7+11.3=24 | $\max_{i=1} K_{ij}+\sum_{i}^{n} p_i$ =9+46.5=55.5 | 55.5 |

Since min.{G}=55.5 which is related to the subsequence {4,2,3}. So, the initial three places in the required optimal sequence are acquired by jobs 4,2,3 respectively. Also, by default job 1 will take the last i.e. fourth place. Therefore {4,2,3,1} is the required optimal sequence.

Table 9 represents the in-out table for the best feasible sequence.

**Table 9: In-out table**

| Jobs | $K_1$ | $K_2$ | $K_3$ | P |
|---|---|---|---|---|
| 4 | - | 0-2.3 | 2.3-7.3 | 7.3-16.6 |
| 2 | 0-6.4 | - | - | 16.6-31.9 |
| 3 | 6.4-18 | 18-14.7 | - | 31.9-42.5 |
| 1 | - | - | 7.3-15 | 42.5-53.8 |

Therefore, the elapsed time is 53.8 hrs.

## Vii. Solution by the NEH method

The NEH (Nawaz, Enscore, and Ham) method is a heuristic algorithm accustomed to optimizing the processing time in a flow shop scheduling problem. It's an easy and efficient technique to determine the order of jobs to minimize total processing time.

**1**. Sort jobs by processing time: By their processing timeframes, sort the jobs.

**2**. Select the longest job: Assign the job that takes the longest to process as the first one in the order.

**3**. Select the next job: Choose a job that has the least sum of processing times with the already selected jobs.

**4**. Repeat step 3: Until every work has been scheduled, keep choosing jobs based on the minimum total of processing times.

**5**. Output the sequence: The final sequence of jobs is the optimal order to minimize the total processing time.

**Solution:** Let's take the tabular form of the reduced problem from Table 5 following the time allocation to parallel equipotential machines.

**Step 1:** As mentioned in table 10, determine the total processing time for each job.

Table 10: Processing time of all jobs

| Jobs | K | P | Total processing time |
|---|---|---|---|
| 1 | 7.7 | 11.3 | 19 |
| 2 | 6.4 | 15.3 | 21.7 |
| 3 | 8.3 | 10.6 | 18.9 |
| 4 | 7.3 | 9.3 | 16.6 |

**Step 2:** Now take the processing time of all the tasks in descending order, which are $J_2 < J_1 < J_3 < J_4$ .

**Step 3:** Choose the first two jobs from the selected schedule which are $J_2$ ,$J_1$ and its other possibility which can be $J_2$ ,$J_1$ .Compute the makespan of these two possibilities from Tables **11(a), 11(b).**

Table 11(a): makespan of {2,1}

| Jobs | K | P |
|---|---|---|
| 2 | 0-6.4 | 6.4-21.7 |
| 1 | 6.4-14.1 | 21.7-33 |

From this table, the makespan comes out to be 33.

**Table 11(b):** makespan of {1,2}

| Jobs | K | P |
|---|---|---|
| 1 | 0-7.7 | 7.7-19 |
| 2 | 7.7-14.1 | 19-34.3 |

And the makespan from the second possibility comes out to be 34.3.Therefore, the minimum makespan is 33 which is corresponding to possibility $J_2$ ,$J_1$.

**Step4:** Now, after placing job 3 in the already selected sequence we get the three possibilities {$J_2$ ,$J_1$,$J_3$ },{$J_2$ ,$J_3$,$J_1$ } and {$J_3$ ,$J_2$,$J_1$ }.

Again, Calculate the makespan for each of the above three scenarios from table **12(a), 12(b), 12(c).**

**Table 12(a): makespan of {2,1,3}**

| Jobs | K | P |
|---|---|---|
| 2 | 0-6.4 | 6.4-21.7 |
| 1 | 6.4-14.1 | 21.7-33 |
| 3 | 14.1-22.4 | 32.3-**43.6** |

**Table 12(b): makespan of {2,3,1}**

| Jobs | K | P |
|---|---|---|
| 2 | 0-6.4 | 6.4-21.7 |
| 3 | 6.4-14.7 | 21.7-32.3 |
| 1 | 14.7-22.4 | 32.3-**43.6** |

**Table 12(c): makespan of {3,2,1}**

| Jobs | K | P |
|---|---|---|
| 3 | 0-8.3 | 8.3-18.9 |
| 2 | 8.3-14.7 | 18.9-34.2 |
| 1 | 14.7-22.4 | 34.2-**45.5** |

Therefore, the minimum makespan is 43.6 from the makespan 43.6,43.6 and 45.5 corresponding to possibilities $\{J_2, J_1, J_3\}$,$\{J_2, J_3, J_1\}$ and $\{J_3, J_2, J_1\}$.

**Step5:** Repeat the above process, place the fourth job i.e. $J_4$ in the sequence $\{J_2, J_1, J_3\}$ It gives birth to four possibilities $\{J_4, J_2, J_1, J_3\}$,$\{J_2, J_4, J_1, J_3\}$,$\{J_2, J_1, J_4, J_3\}$ and $\{J_2, J_1, J_3, J_4\}$.. Placing $J_4$ in the sequence$\{J_2, J_3, J_1\}$ gives the following possibility $\{J_4, J_2, J_3, J_1\}$$\{J_2, J_4, J_3, J_1\}$,$\{J_2, J_3, J_4, J_1\}$ and $\{J_2, J_3, J_1, J_4\}$ and likewise again inserting $J_4$ in the sequence $\{J_3, J_2, J_1\}$, we get four more possibilities $\{J_4, J_3, J_2, J_1\}$,$\{J_3, J_4, J_2, J_1\}$,$\{J_3, J_2, J_4, J_1\}$ and $\{J_3, J_2, J_1, J_4\}$

Now, compute the makespan of each possibility using table **13(a), 13(b) and 13(c).**

**Table 13(a): Possibility 1**

| Sequence | Makespan |
|---|---|
| $\{J_4, J_2, J_1, J_3\}$ | 53.4 |
| $\{J_2, J_4, J_1, J_3\}$ | 53.3 |
| $\{J_2, J_1, J_4, J_3\}$ | 54.1 |
| $\{J_2, J_1, J_3, J_4\}$ | 54.2 |

**Table 13(a): Possibility 2**

| Sequence | Makespan |
|---|---|
| $\{J_4, J_2, J_3, J_1\}$ | 53.2 |
| $\{J_2, J_4, J_3, J_1\}$ | 53.1 |
| $\{J_2, J_3, J_4, J_1\}$ | 53.5 |
| $\{J_2, J_3, J_1, J_4\}$ | 53.6 |

**Table 13(a):** Possibility 3

| Sequence | Makespan |
|---|---|
| $\{J_4, J_3, J_2, J_1\}$ | 53.2 |
| $\{J_3, J_4, J_2, J_1\}$ | 53.2 |
| $\{J_3, J_2, J_4, J_1\}$ | 54.2 |
| $\{J_3, J_2, J_1, J_4\}$ | 54.3 |

The optimal sequence is 2-4-3-1, with a minimum makespan of **53.1**
Table 14 represents the in-out table for the optimal sequence.

**Table 14: In-out table**

| Jobs | K | P |
|---|---|---|
| | In-out | In-out |
| 2 | 0-6.4 | 6.4-21.7 |
| 4 | 6.4-13.7 | 21.7-31.3 |
| 3 | 13.7-22 | 31.3-41.9 |
| 1 | 22-29.7 | 41.9-53.1 |

## VII. CDS method

The CDS (Campbell, Dudek, and Smith) method is another popular heuristic algorithm for solving FSSP. It is similar to the NEH method but with some modifications to improve the solution quality.

The CDS method is an improvement over the NEH method, as it considers the slack time and priority index to minimize idle time and reduce the makespan.

**Step 1:** Determine the total processing time (TPT)

- Calculate TPT for each machine, which is the sum of processing times for all jobs on that machine.

**Step 2:** Calculate the slack time (SLK)

- Compute SLK for each task on each machine, which refers to the discrepancy between the task's execution time on the device and its TPT.

**Step 3:** Calculate the priority index(PI)

- Calculate the PI for each job, which is the maximum slack time across all machines for that job.

**Step 4:** Sort jobs by priority index

- In accordance with their priority indices, put the tasks in descending order.

**Step 5:** Schedule jobs

- Schedule the jobs in the sorted order, one by one, on each machine.

- For each job, assign it to the machine with the earliest available time slot.

**Step 6:** Update the schedule

- After scheduling a job, update the schedule by shifting the start times of the remaining jobs on the same machine to minimize idle time.

**Step 7:** Output the schedule
- The final schedule is the optimal order to minimize the total processing time.

**Problem Solution by CDS Method:**

**Table 15: Jobs with** Processing Time

| Jobs | K | P |
|---|---|---|
| 1 | 7.7 | 11.3 |
| 2 | 6.4 | 15.3 |
| 3 | 8.3 | 10.6 |
| 4 | 7.3 | 9.3 |

**Step 1:** Calculate Total Processing Time (TPT) using table 15
TPT(K) = 7.7 + 6.4 + 8.3 + 7.3 = 29.7
TPT(P) = 11.3 + 15.3 + 10.6 + 9.3 = 46.5

**Step 2:** Calculate Slack Time (SLK)

SLK(1, K) = 29.7 - 7.7 = 22 SLK(1, P) = 46.5 - 11.3 = 35.2

SLK(2, K) = 29.7 - 6.4 = 23.3 SLK(2, P) = 46.5 - 15.3 = 31.2

SLK(3, K) = 29.7 - 8.3 = 21.4 SLK(3, P) = 46.5 - 10.6 = 35.9

SLK(4, K) = 29.7 - 7.3 = 22.4 SLK(4, P) = 46.5 - 9.3 = 37.2

**Step 3:** Calculate Priority Index (PI)

PI(1) = max{22, 35.2} = 35.2

PI(2) = max{23.3, 31.2} = 31.2

PI(3) = max{21.4, 35.9} = 35.9

PI(4) = max{22.4, 37.2} = 37.2

**Step 4:** Sort Jobs by Priority Index
Sorted order: 4, 3, 1, 2

**Step 5-7:** Schedule Jobs and Update the Schedule

**Table 16:** Scheduled **Jobs**

| Jobs | K | P |
|---|---|---|
| | In-out | In-out |
| 4 | 0-7.3 | 7.3-16.6 |
| 3 | 7.3-15.6 | 16.6-27.2 |
| 1 | 15.6-23.3 | 27.2-38.6 |
| 2 | 23.3-29.7 | 38.6-54 |

So from table 16,Minimum Makespan is 54.
Using the CDS method, For the above two-stage FSSP, the optimum makespan is 54.

## VIII. Comparison between B & B, NEH and CDS

| No of jobs | B&B | NEH | CDS |
|---|---|---|---|
| 2 | 14 | 15 | 16 |
| 4 | 52.4 | 53.1 | 54 |
| 5 | 30 | 31.2 | 32 |

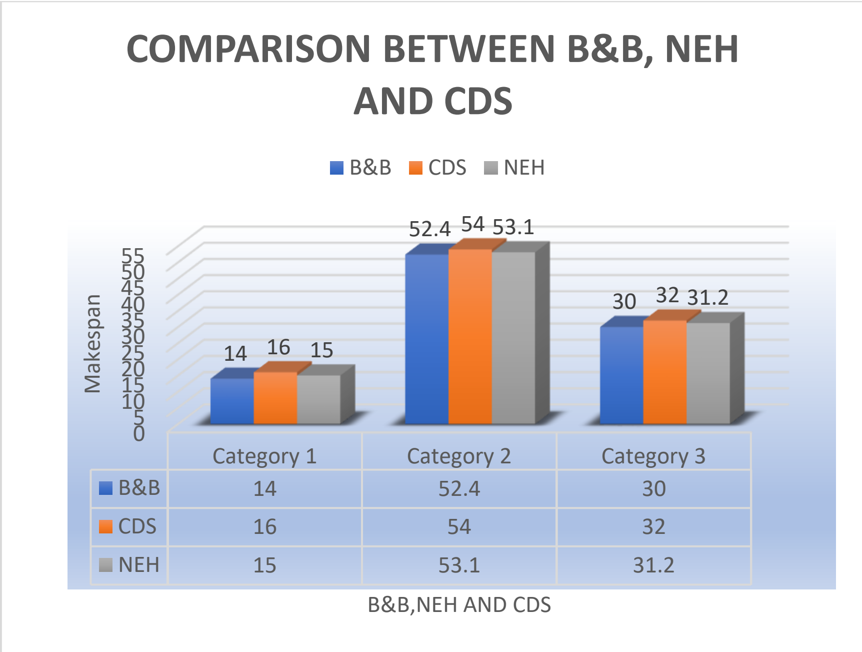


**Fig.2.** Comparison chart

## IX. Result

After finding the solution by B&B, GA and Palmar, from fig 2 we observed that the minimum make-span obtained by the B&B method is less as compared to NEH and CDS.

## X. Conclusion

While NEH and CDS might not always find the best answer, B&B, which employs a tree-based search algorithm, is an exact method for doing so. The idea of adding jobs in a particular order is the cornerstone of NEH, while CDS divides the issue into more manageable subproblems. The simplest to implement is NEH, which is followed by CDS and B&B. Massive issues can be handled by B&B and CDS, whereas huge problems may be difficult for NEH to manage. As a result, we can conclude that while heuristic approaches are simple to use and may result in solutions, they do not ensure the best answer that comes from accurate approaches.

## XI. Future scope

Studies can be done to create hybrid methods that combine the advantages of heuristic and accurate approaches to produce better results and faster calculation times. To identify the benefits and drawbacks of exact and heuristic approaches, as well as to provide new theoretical bounds and guarantees, more thorough evaluations can be conducted. It is also possible to compare these techniques in various settings.

## Conflict of Interest

No potential conflict of interest relevant to this article was reported.

## References

I. Allahverdi, Ali, et al. "A survey of scheduling problems with setup times or costs." *European journal of operational research* 187.3 (2008): 985-1032. 10.1016/j.ejor.2006.06.060

II. Gupta, Arun, and S. Chauhan. "A heuristic algorithm for scheduling in a flow shop environment to minimize makespan." *International Journal of Industrial Engineering Computations* 6.2 (2015): 173-184. 10.5267/j.ijiec.2014.12.002

III. Lee, Chung-Yee. "Parallel machines scheduling with no simultaneous machine available time." *Discrete Applied Mathematics* 30.1 (1991): 53-61. 10.1016/0166-218X(91)90013-M

IV. Dubois, Didier, and Henri Prade. "Fuzzy real algebra: some results." *Fuzzy sets and systems* 2.4 (1979): 327-348. 10.1016/0165-0114(79)90005-8

V. Gupta, Deepak, and Sonia Goel. "Three stage flow shop scheduling model with m-equipotential machines." *International Journal on Future Revolution in Computer Science & Communication Engineering* 4.3 (2018): 269-274. https://www.ijrar.org/papers/IJRAR19J1166.pdf

VI. Gupta, Deepak, Sonia Goel, and Neeraj Mangla. "Optimization of production scheduling in two stage flow shop scheduling problem with m equipotential machines at first stage." *International Journal of System Assurance Engineering and Management* 13.3 (2022): 1162-1169. https://link.springer.com/article/10.1007/s13198-021-01411-5

VII. Gupta, Deepak, and Sonia Goel. "Branch and bound technique for two stage flow shop scheduling model with equipotential machines at every stage." *International Journal of Operational Research* 44.4 (2022): 462-472. 10.1504/IJOR.2022.125132

VIII. Ponnialagan, Dhanasekaran, Jeevaraj Selvaraj, and Lakshmana Gomathi Nayagam Velu. "A complete ranking of trapezoidal fuzzy numbers and its applications to multi-criteria decision making." *Neural Computing and Applications* 30 (2018): 3303-3315. 10.1007/s00521-017-2898-7

IX. Palmer, Douglas S. "Sequencing jobs through a multi-stage process in the minimum total time—a quick method of obtaining a near optimum." *Journal of the Operational Research Society* 16.1 (1965): 101-107. 10.1057/jors.1965.8

X. Ignall, Edward, and Linus Schrage. "Application of the branch and bound technique to some flow-shop scheduling problems." *Operations research* 13.3 (1965): 400-412. 10.1287/opre.13.3.400

XI. McMahon, G. B., and P. G. Burton. "Flow-shop scheduling with the branch-and-bound method." *Operations Research* 15.3 (1967): 473-481. 10.1287/opre.15.3.473

XII. Ganesan, K., and P. Veeramani. "Fuzzy linear programs with trapezoidal fuzzy numbers." *Annals of operations research*143 (2006): 305-315.https://doi.org/10.1007/s10479-006-7390-1

XIII. Malhotra, K., et al. "Bi-objective flow shop scheduling with equipotential parallel machines." *Malays J Math Sci* 16.3 (2022): 451-470. 10.47836/mjms.16.3.04

XIV. Zadeh, Lotfi A. "Fuzzy sets." *Information and Control* (1965). http://liphy-annuaire.univ-grenoble-alpes.fr/pages_personnelles/bahram_houchmandzadeh/biblio/Zadeh_FuzzySetTheory _1965.pdf

XV. Zadeh, Lotfi Asker. "Fuzzy sets as a basis for a theory of possibility." *Fuzzy sets and systems* 1.1 (1978): 3-28. 10.1016/0165-0114(78)90029-5

XVI. Smith, Richard D., and Richard A. Dudek. "A general algorithm for solution of the n-job, M-machine sequencing problem of the flow shop." *Operations Research* 15.1 (1967): 71-82. 10.1287/opre.15.1.71]

XVII. Abbasbandy, Saeid, and T. Hajjari. "A new approach for ranking of trapezoidal fuzzy numbers." *Computers & mathematics with applications* 57.3 (2009): 413-419. 10.1016/j.camwa.2008.10.090

XVIII. Chen, Shi-Jay, and Shyi-Ming Chen. "Fuzzy risk analysis based on the ranking of generalized trapezoidal fuzzy numbers." *Applied intelligence* 26 (2007): 1-11. 10.1007/s10489-006-0003-5

XIX. Johnson, Selmer Martin. "Optimal two-and three-stage production schedules with setup times included." *Naval research logistics quarterly* 1.1 (1954): 61-68. 10.1002/nav.3800010110

XX. Sharma, Sameer, and Deepak Gupta. "Minimizing rental cost under specified rental policy in two stage flow shop, the processing time associated with probabilities including break-down interval and job block criteria." *European Journal of Business and Management* 3.2 (2011): 85-103.https://citeseerx.ist.psu.edu/document?repid=rep1&type=pdf&doi=00ebcb8318cb8481a13af7f6096b8da5eccc6820

XXI. Lomnicki, Z. A. "A “branch-and-bound” algorithm for the exact solution of the three-machine scheduling problem." *Journal of the operational research society* 16.1 (1965): 89-100. 10.1057/jors.1965.7