JOURNAL OF MECHANICS OF CONTINUA AND MATHEMATICAL SCIENCES
www.journalimcms.org

ISSN (Online) : 2454 -7190 Vol.-19, No.-11, November (2024) pp 207 - 216 ISSN (Print) 0973-8975

# REGULAR PARTIAL DOMATIC NUMBER ON ANTI FUZZY GRAPHS

**Rengasamy Muthuraj[1], Palanisamy Vijayalakshmi[2]**
**Anandaraman Sasireka[3]**

[1,2] PG & Research Department of Mathematics, H.H. The Rajah's College, Pudukkottai – 622 001, Tamilnadu, India.
(Affiliated to Bharathidasan University, Tiruchirappalli, Tamil Nadu, India).

[2,3] Department of Mathematics, PSNA College of Engineering and Technology, Dindigul – 624 622, Tamilnadu, India.

Corresponding Author: **Rengasamy Muthuraj**

Email: [1]rmr1973@gmail.com, [2]vijibharathi2020@gmail.com
[3]sasireka.psna@gmail.com

https://doi.org/10.26782/jmcms.2024.11.00014

(Received: September 03, 2024; Revised: October 27, 2024; Accepted: November 06, 2024)

## Abstract

$A_G$ = (N, A, σ, μ) be a anti fuzzy graph. A partition of $N(A_G)$ Π = {$D_1$, $D_2$, ...., $D_k$} is a regular anti fuzzy partial domatic partition of $A_G$ if (i) for each $D_i$, < $D_i$ > is an anti fuzzy regular and (ii) $D_i$ is an anti fuzzy dominating set of $G_A$. The maximum fuzzy cardinality of a regular anti fuzzy partial domatic partition of $A_G$ is called the regular anti fuzzy partial domatic number [RAPDN]of $A_G$ and it is denoted by $d_r^{af}(A_G)$. Also these numbers are determined for various anti fuzzy graph. In this work, random r-regular anti fuzzy graph, regular partial domatic number in anti fuzzy graphs, regular partial anti domatic number in anti fuzzy graphs are introduced. Some bounds for anti fuzzy domatic numbers are discussed.

**Keywords**: Anti fuzzy graph, Dominating set, Domatic number, Vertex degree.

## I. Introduction

The idea of an anti fuzzy structure on a graph from the fuzzy relation started by Zadeh [XIII] is well known to Muhaamad Akram [I]. The idea of a graph's domatic number was first presented by S.T. Hedetniemi and E.J. Cockayne [III]. Bohdan Zelinka [XVI] first proposed the idea of a graph's anti domatic number. A. Sasireka and R. Muthuraj [VIII, XI, IX] determined the dominance parameters for anti-fuzzy graphs and demonstrated the generality of specific anti fuzzy graph types. The novel ideas of regular domatic partition and regular anti-domatic partition in fuzzy graphs were first presented by K.M. Dharmalingam and K. Valli [V]. The novel ideas of regular partial domatic number, random r-regular anti fuzzy graph, and regular partial anti fuzzy number in an anti fuzzy.

**Definition 1.1 [IX]**

An anti fuzzy graph [AFG] $G_A = (\sigma, \mu)$ is a pair of functions $\sigma$: V→ [0,1] and $\mu$: V×V→ [0,1], with $\mu$ (u, v) ≥ $\sigma$ (u) ∨ $\mu$ (v) for all (u, v) ϵ V.

## II. Random r- Regular Anti Fuzzy Graph

$A_G$ is an AFG with n nodes. $A_G$ is represented as a r-regular AFG on n nodes and it is denoted by $A_G$ (n, r). To build a random r-regular AFG at random on the node set {$v_1$, $v_2$, ……, $v_n$}. To collapse each set {$v_{i,1}$, $v_{i, 2}$, …$v_{i, r}$} into a single node vi by taking a random matching on the node set {$v_{1,1}$, $v_{1,2}$, ……$v_{1, r}$, $v_{2,1}$, $v_{2,2}$…...$v_{n, r}$}. Throw away the generated AFG if it has any loops or more than one arc.

**Note**

1. $\bar{d}(A_G)$ = d( $\overline{A_G}$) is the domatic partition of the complement of AFG.
2. $A_G = C_n ∪ C_M$ is a 3- regular graph which forms a cycle that has length even.
3. If $G_A = C_n ∪ C_M$ is a 3- regular graph then $\overline{A_G}$ is $(n - \delta - 1)$ regular graph where $\delta$ is a minimum degree of an AFG.

## III. Lower Bound for Anti Fuzzy Domatic Number

Under this part, r ≥ 3 is fixed for finding the anti fuzzy domatic number of a random r-regular AFG. Consider that the domatic number of a random r-regular AFG is atleast 3.

**Definition 3.1**

Let $A_G$ be a 3- regular AFG that is created by adding a perfect matching M to an anti fuzzy cycle C = $v_1$, $v_2$, ……, $v_n$, $v_1$. If $v_i$ and $v_{i+1}$ have matching partners $v_j$ and $v_k$ respectively, so that the anti fuzzy cycle segments [$v_j$, $v_i$] and [$v_{i+1}$, $v_k$] are disjoint and have cardinality 0 (mod 3), then an arc $v_j$, $v_{j+1}$ of C (indices mod n) is a 3-edge.

**Note**

To the following theorem consider an $A_G$ = C∪M where C is anti fuzzy cycle with

n ≥ 9 and n ≡ 0 mod 3.

**Theorem 3.2**

Let $A_G$ be a random 3-regular AFG. Then

(i) $d_f(A_G) \leq d_f(C_n) \leq d_f(C_M)$
(ii) $d(C_M) \leq d(C_n) \leq d(A_G)$
(iii) $d(A_G) \geq 3$

**Proof**

Let's think of $A_G$ as an AFG that is created by adding a random perfect matching M to a cycle.

That is, $A_G = C_n ∪ C_M$

If $C_n$ is a cycle with length n > 9 and n ≡ 0 mod 3. $C_n$ has a 3- edge. Consider $v_i$ $v_{i+1}$ is a 3- arc of $C_n$. Let $v_j$, $v_k$ be their corresponding matching partners respectively. Let $D_i$ be a dominating set of $A_G$ and $D_l$ = {$v_n$ / n ≡ $l$ mod 3} when $l$ = 1, 2, 3.

The cycle $c' = v_1, v_j, v_{j+1}, \ldots, v_n, v_1$ has length 0 mod 3 if n ≡ 1 mod 3. As $v_1v_2$ has 3-nodes. In case the length of the cycle $c'' = v_1, v_j, v_{j-1}, v_{j-2}, \ldots v_1$ has length $n-|c'|+2 \equiv 0$ mod 3. Then n ≡ 1 mod 3.

Since $A_G$ has at least three disjoint dominating set by choosing each node individually from each cycle $c'$ and $c''$. Let $D_i$ be the dominating set for $l = 1, 2, 3, \ldots$

$D_1 = \{v_1, v_4, v_7, \ldots, v_{j-2}\} \cup \{v_{n-2}, v_{n-5}, \ldots, v_{j+1}\}$

$D_2 = \{v_2, v_5, v_8, \ldots, v_{j-1}\} \cup \{v_{n-1}, v_{n-4}, v_{n-7}, \ldots, v_{j+2}\}$

$D_3 = \{v_3, v_6, v_9, \ldots, v_j\} \cup \{v_n, v_{n-3}, v_{n-6}, \ldots, v_j\}$

Any one of $D_i$'s are minimal dominating set of $A_G$.

If n ≡ 2 mod 3, then k ≡ j ≡ 1 (mod 3). We select a three disjoint dominating set of AFG $A_G$ by choosing, for the same collection, all three nodes of the anti fuzzy cycle except $v_1$ and $v_2$.

$D_1 = \{v_4, v_7, \ldots, v_{n-5}, v_{n-2}\} \cup \{v_1\}$

$D_2 = \{v_5, v_8, \ldots, v_{n-4}, v_{n-1}\} \cup \{v_2\}$

$D_3 = \{v_3, v_6, v_9, \ldots, v_{n-3}, v_n\}$

Each of the dominating sets $D_1$, $D_2$, and $D_3$ are disjoint minimal dominating sets of $A_G$. which yields a domatic partition of $A_G$. DP = $\{D_1, D_2, D_3\}$. $D(A_G) = 3$.

If $A_G = C_n \cup M$ with $C_n$ has length < 9. Then $A_G$ has atleast 3 dominating sets which forms a domatic partition of $A_G$.

$$d(A_G) \geq 3 \text{ and } d(C_n) \leq d(A_G) \qquad (1)$$

By the results in [14] $d(C_n) \geq 3$.

Let $C_M$ be a graph which consider the node set attained by adding a random matching M to an even – order cycle $C = v_1, v_2, \ldots v_n, v_1$. At least two components and one dominating set larger than the cycle $C_n$ are present in $C_m$.

$$d(C_M) \leq d(C_n) \qquad (2)$$

From (1) and (2) we get, $d(C_M) \leq d(C_n) \leq d(A_G)$.

Similarly, the anti fuzzy value of these graphs has the inequality as,

$d_{af}(A_G) \leq d_{af}(C_n) \leq d_{af}(C_m)$.

## IV. Regular Partial Domatic Number on Anti Fuzzy Graphs

In this section, the concept of a regular partial domatic number on an anti fuzzy graph [AFG] is defined in this section. We explore the associated outcomes and theorems.

**Definition 4.1**

Let $A_G = (N, A, \sigma, \mu)$ be an AFG. A partition $\Pi = \{D_1, D_2, \ldots, D_n\}$ of $N(A_G)$ is called regular anti fuzzy partial domatic partition [RAPDP] of $A_G$ if (i) for each $D_i$, $< D_i >$ is an anti fuzzy regular and (ii) $D_i$ is an anti fuzzy dominating set of $G_A$. The maximum fuzzy cardinality of a regular anti fuzzy partial domatic partition of $A_G$ is called the regular anti fuzzy partial domatic number [RAPDN]of $A_G$ and it is denoted by $d_r^{af}(A_G)$.

**Example 4.2**

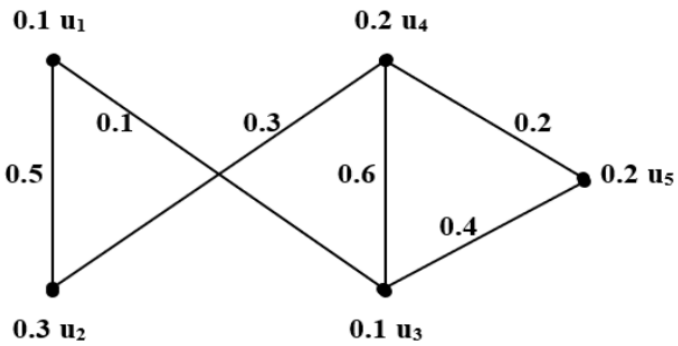


**Figure 1 Anti Fuzzy Graph $A_G$**

From Figure1.

$d_{A_G}(u_1)$ = 0.1 + 0.5 = 0.6
$d_{A_G}(u_2)$ = 0.5 + 0.3 = 0.8
$d_{A_G}(u_3)$ = 0.1 + 0.6 + 0.4 = 1.1
$d_{A_G}(u_4)$ = 0.3 + 0.6 +0.2 = 1.1
$d_{A_G}(u_5)$ = 0.2 + 0.4 = 0.6
$d_{A_G}(u_1) = d_{A_G}(u_5)$ = 0.6
$d_{A_G}(u_3) = d_{A_G}(u_4) = 1.1$

$D_1 = \{u_1, u_5\}$ is a dominating and anti fuzzy regular.
$D_2 = \{u_3, u_4\}$ is a dominating and anti fuzzy regular.
$\Pi = \{D_1, D_2\}$ is a regular anti fuzzy partial domatic partition of $A_G$. Therefore $d_r^{af}(A_G) = 2$.

**Remark**

All anti fuzzy graphs have no regular anti fuzzy partial domatic partition.

**Example 4.3**

(i) $K_1 \cup K_2$ has no partial regular anti fuzzy domatic partition
(ii) Any anti fuzzy graph $A_G \neq \overline{K_n}$ of order ≥ 3 with an isolated node has no partial regular anti fuzzy domatic partition.

**Note**

i) If $A_G$ is a regular AFG with n nodes, then $d_r^{af}(A_G) \geq 1$.
ii) $d_r^{af}(A_G) \leq d_{af}(A_G)$.

**Theorem 4.4**

Let $A_G$ be an AFG with a pendent node. Then $\delta_{af}(A_G) \leq d_r^{af}(A_G) + 1$.

**Proof**

Suppose $\Pi = \{D_1, D_2, \ldots, D_{d_r^{af}(A_G)}\}$ is a regular anti fuzzy domatic partition of $A_G$. Assume that u is a pendent node with support v. Without loss of generality, let $u \in D_1$.

If $v \in D_1$, then the node in $D_2$ cannot dominate u. If $v \in D_2$ then any node in $V_i$, $i \geq 3$ cannot dominate u. Therefore, $|DP| > 2$.

**Case(i)**

If $|\Pi| = 1$, then $A_G$ is anti fuzzy regular. If $A_G$ has a pendent node, then all the pendent nodes should be contained in any one $D_i$. Therefore, $\delta_{af}(A_G) \leq d_r^{af}(A_G) + 1$.

**Case(ii)**

If $|\Pi| > 1$ then the minimum fuzzy degree values of $\delta_{af}(G_A)$ is at least 0.1. In such case $d_r^{af}(A_G) = 1$. Therefore, $\delta_{af}(A_G) \leq d_r^{af}(A_G) + 1$.

**Note**

If $A_G$ has no pendent node, then the result is not true.

**Result**

If $A_{G_1}$ and $A_{G_2}$ are two AFG's for which $d_r^{af}(A_{G_1})$ and $d_r^{af}(A_{G_2})$ exist. Then, $d_r^{af}(A_{G_1} \cup A_{G_2})$ exists iff there exists partition $\Pi_1 = \{D_1, D_2, \ldots, Dd_r^{af}(A_{G_1})\}$ and $\Pi_2 = \{U_1, U_2, \ldots, Ud_r^{af}(A_{G_1})\}$ such that (i) if the anti fuzzy regular domatic number $d_r^{af}(A_{G_1}) = d_r^{af}(A_{G_2})$ then for each $D_i$ there exists $U_i$ such that $D_i$ and $U_i$ are having the same anti fuzzy regularity for each i, $1 \leq i \leq d_r^{af}(A_{G_1}) = d_r^{af}(A_{G_2})$. (ii) if $d_r^{af}(A_{G_1}) < d_r^{af}(A_{G_2})$ then $D_i$ and $U_i$ are having the same fuzzy regularity for each i, $1 \leq i \leq d_r^{af}(A_{G_1})$ and $Dd_r^{af}(A_{G_2}) + 1, \ldots\ldots, Dd_r^{af}(A_{G_2})$ must have the same anti fuzzy regularity with any one $D_1, D_2, \ldots, Dd_r^{af}(A_{G_1})$.

## V. Regular Partial Anti Domatic Number in Anti Fuzzy Graphs

The idea of a regular partial anti domatic number in an AFG is defined in this section. Results and theorems in correspondence are given.

**Definition 5.1**

Let $A_G = (N, A, \sigma, \mu)$ be an AFG. A partition $\Pi = \{D_1, D_2, \ldots, D_K\}$ of $N(A_G)$ is called regular partial anti fuzzy anti domatic partition of $A_G$ if (i) for each $V_i$, $\langle V_i \rangle$ is anti fuzzy regular and (ii) $V_i$ is a non anti fuzzy dominating set of $A_G$. The minimum cardinality of regular anti fuzzy anti domatic partition is called the regular anti fuzzy anti domatic number of $A_G$ and is denoted by $\overline{d_r^{af}(A_G)}$ .

**Example 5.2**

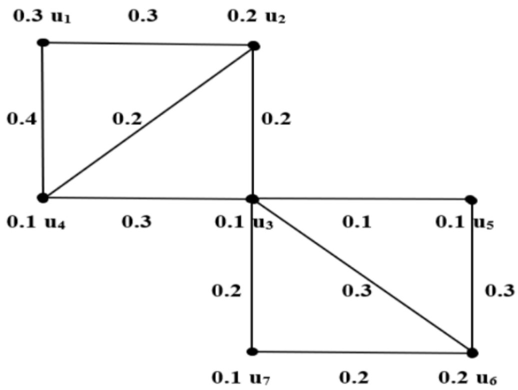


**Figure 2 Anti Fuzzy Graph $A_G$**

From Figure2.

$d_{A_G}(u_1)$ = 0.3 + 0.4 = 0.7
$d_{A_G}(u_2)$ = 0.3 + 0.2 + 0.2 = 0.7
$d_{A_G}(u_3)$ = 0.3 + 0.2 + 0.2 + 0.3 + 0.1 = 1.1
$d_{A_G}(u_4)$ = 0.4 + 0.2 + 0.3 = 0.9
$d_{A_G}(u_5)$ = 0.1 + 0.3 = 0.4
$d_{A_G}(u_6)$ = 0.2 + 0.3 + 0.3 = 0.8
$d_{A_G}(u_7)$ = 0.2 + 0.2 = 0.4
$d_{A_G}(u_1) = d_{A_G}(u_2)$ = 0.7

$D_1 = \{u_1, u_2\}$ is a non-dominating and anti fuzzy regular.

$d_{A_G}(u_5) = d_{A_G}(u_7)$ = 0.4

$D_2 = \{u_5, u_7\}$ is a non-dominating and anti fuzzy regular.

Therefore, $\Pi = \{D_1, D_2\}$ is a regular anti fuzzy anti domatic partition of $A_G$.

Therefore, $|\Pi| = \overline{d_r^{af}(A_G)} = 2$.

**Definition 5.3**

An anti domatic partition of an anti fuzzy graph $A_G$ is a partition of $N(A_G)$ into non dominating sets. The minimum cardinality of an anti domatic partition of an anti fuzzy graph $A_G$ is called anti fuzzy anti domatic number of $A_G$ and it is denoted by $\bar{d}_r^{af}$ ($A_G$).

**Remark**

Let $A_G = (N, A, \sigma, \mu)$ be an AFG.

(i) If $A_G$ has a full degree node, then $A_G$ has no regular partial anti fuzzy anti domatic partition. Since $A_G$ has non dominating sets. Therefore, $A_G$ has no full degree node. Suppose $A_G$ has full degree node, all nodes dominate each other nodes of $A_G$ and anti fuzzy regular. Therefore, $A_G$ has no regular partial anti fuzzy anti domatic partition.

**(ii)** Let N($A_G$) = {$N_1$, $N_2$, ……, $N_n$}. Then Π={{$N_1$}, {$N_2$}, ……, {$N_n$}} is a regular anti fuzzy anti domatic partition of $A_G$.

**Theorem 5.4**

Let $A_G$ be any AFG without full degree node, then $\bar{d}_r^{af}$ ($A_G$) ≥ 2.

**Proof**

Suppose $\overline{d_r^{af}(G_A)}$=1. Then Π={D}is an anti fuzzy regular and not dominating. N($A_G$) is a regular anti fuzzy anti domatic partition which is contradiction to our assumption. Therefore, $\bar{d}_r^{af}$ ($A_G$) ≥ 2.

**Definition 5.5 [V]**

The distance d (u, v) between two vertices u and v in an anti fuzzy graph $A_G$ is the length of a shortest u-v path in $A_G$. The diameter of a connected anti fuzzy graph $A_G$ is the length of any shortest u-v path and it is denoted by $diam^{af}(A_G)$.

**Theorem 5.6**

If $diam^{af}(A_G) \geq 3$ and x, y ϵ N($A_G$) such that d (x, y) = $diam^{af}(A_G)$with N[x] and (N-N[x]) are anti fuzzy regular. Then $\bar{d}_r^{af}$ ($A_G$) = 2.

**Proof**

Let $diam^{af}(A_G)$ ≥ 3. Let x, y ϵ N($A_G$) such that d (x, y) = $diam^{af}(A_G)$and N[x] and (N - N[x]) are anti fuzzy regular. Clearly, N[x] and (N - N[x]) are anti fuzzy dominating sets. Therefore, $\bar{d}_r^{af}$ ($A_G$) = 2.

**Theorem 5.7**

If $A_G$ is a disconnected AFG with k components $A_{G_1}, A_{G_2}$, …., $A_{G_k}$. Then $\bar{d}_r^{af}$ ($A_G$) = 2 if and only if for some i, 1 ≤ i ≤ k – 1, $A_{G_1}, A_{G_2}$, …., $A_{G_i}$are anti fuzzy $r_1$ - regular and $A_{G_{i+1}}, A_{G_{i+2}}, \dots\dots, A_{G_{i+k}}$are anti fuzzy $r_2$ -regular.

**Proof**

Let $A_G$ be disconnected anti fuzzy graph with components $A_{G_1}, A_{G_2}$, …., $A_{G_k}$. Suppose $A_{G_1}, A_{G_2}$, …., $A_{G_i}$ are anti fuzzy$r_1$-regular and $A_{G_{i+1}}, A_{G_{i+2}}, \dots\dots, A_{G_{i+k}}$ are anti fuzzy $r_2$-regular, for some i, 1 ≤ i ≤ k – 1. Let Π = {N( $A_{G_1}$)∪N( $A_{G_2}$) ∪……∪$A_{G_i}$, ∪ N($A_{G_{i+1}}$)∪ N($A_{G_{i+2}}$) ∪……..∪N($A_{G_{i+k}}$)}.Then Π is a regular partial anti fuzzy anti domatic partition of $A_G$. Therefore, $\bar{d}_r^{af}$ ($A_G$) = 2. The converse is obvious.

**Theorem 5.8**

Let $A_G$ be an AFG without full degree nodes. If u is a node of degree $\delta^{af}(A_G)$ such that N - N[u] is an anti fuzzy regular. Then $\bar{d}_r^{af}$ ($A_G$) ≤ $\delta^{af}(A_G)$ + 2.

**Proof**

Let u be a node of degree $\delta^{af}(A_G)$and N - N[u] is an anti fuzzy regular. Let N[u]= {u, $v_1$, $v_2$, …., $v_{\delta f}$}. Let Π = {V - N[u], {u}, {$v_1$}, ….,} {$v_{\delta f}$}. Then Π is a regular anti fuzzy anti domatic partition of $A_G$.

Therefore, $\bar{d}_r^{af}$ ($A_G$) ≤ |Π| $= \delta^{af}(A_G)$ + 2. Hence $\bar{d}_r^{af}$ ($A_G$) ≤ $\delta^{af}(A_G)$ + 2.

**Theorem 5.9**

If $A_G$ is an uninodal AFG without full degree nodes. Then $\bar{d}_r^{af}$ ($A_G$) = n if and only if n is even and $A_G$ = $\overline{k_2} \oplus \overline{k_2} \ldots\ldots \left(\frac{n}{2}\right)$ times.

**Proof**

Suppose n is even and $A_G$ = $\overline{k_2} \oplus \overline{k_2} \ldots\ldots \left(\frac{n}{2}\right)$ times. Then $\bar{d}_r^{af}$ ($A_G$) = n. Conversely, suppose $\bar{d}_r^{af}$ ($G_A$) = n, every node in $G_A$ is an anti fuzzy regular and which form a non dominating set. Since $A_G$ is an anti fuzzy regular. Then $A_G$ is uninodal anti fuzzy graph.

**Result**

The sum of the regular partial anti fuzzy domatic number and regular anti fuzzy anti – domatic number is greater than equal to two.

That is, $d_r^{af}(G_A)$ + $\bar{d}_r^{af}$ ($G_A$) ≥ 2.

**Theorem 5.10**

Let $A_G$ be a disconnected anti fuzzy graph with no isolated nodes. Then d $(A_G)$ ≤ d $(\overline{A_G})$ if and only if $A_G$ has exactly two components as anti fuzzy cycle each with same domatic number.

**Theorem 5.11**

If $A_G$ is a disconnected graph of two components of cycle with n nodes, then

$$d(A_G) = \begin{cases} 3 & if\ n \equiv 0\ mod\ 3 \\ 2 & otherwise \end{cases}$$

where $n_1$ and $n_2$ are number of nodes in two components ($n_1$= $n_2$). d $(\overline{A_G})$ = $n_1$.

**Theorem 5.12**

Let L [$A_G$] be an anti fuzzy line graph of n ≥ 3. Then d[L($A_G$)] = 2.

**Proof**

Let L [$A_G$] be an anti fuzzy line graph. $D_1$ is a dominating set of $A_G$.

**Case (i):** Suppose n is the odd number of nodes. Consider u ∊ $D_1$is adjacent to at least two vertices in L [$A_G$]. Then L [$A_G$] - $D_1$ ∊ N (L [$A_G$ ]) does not dominate with L[$A_G$]. That is, the number of nodes of an anti fuzzy line graph required to dominate all the nodes. Suppose there exists another dominating set, which cover all nodes in L [$A_G$] - $D_1$. The anti fuzzy line graph $G_A$ has atleast one minimal dominating set and has two domatic partition. Therefore d [L ($G_A$)] = 2.

**Case (ii):** Suppose n is the even number of vertices in anti fuzzy line graph. Let $D_1$and $D_2$ be two dominating sets with equal number of vertices. That is both dominating sets are minimal. Therefore, the domatic partitions becomes $D_1 = \{v_1, v_3, \ldots\ldots v_{n-1}\}$ and $D_2 = \{v_2, v_4, \ldots\ldots v_{n-2}\}$. Hence $d\,[L\,(G_A)] = 2$.

## VII. Results and Discussion

In path anti fuzzy graph, $\gamma\,(A_G) + d\,(A_G) \le \left\lceil \frac{n}{2} \right\rceil + 2$.

1) In cycle anti fuzzy graph, $\gamma\,(A_G) + d\,(A_G) \le \left\lceil \frac{n}{2} \right\rceil + 2$.
2) In complete anti fuzzy graph, $\gamma\,(A_G) + d\,(A_G) = n + 1$.
3) In any anti fuzzy graph, $\gamma\,(A_G) + d\,(A_G) \le \left\lceil \frac{n}{2} \right\rceil + 2$.
4) If $A_G$ is an anti fuzzy wheel graph of order n, then $d\,(A_G) = \begin{cases} 4 \;; & if\ n \equiv 1\ mod\ 4 \\ 3 \;; & otherwise \end{cases}$

## VII. Conclusion

The definitions of regular partial domatic number and regular partial anti domatic number in anti fuzzy graphs are given in this paper. Lower bound for anti fuzzy domatic number is discussed. A vast array of issues may be solved with the help of the regular partial domatic and regular partial anti domatic numbers. This will result in an anti fuzzy graph with a new notation. For graphical research, the regular anti fuzzy domatic number and regular anti fuzzy anti domatic number are very useful for solving very wide range problems. We can impose additional restriction. This will lead us to a new notation for anti fuzzy graph. Also, the regular fuzzy domatic number and the regular anti fuzzy anti domatic numbers are useful to solve Transportation problems.

## Conflict of interest:

There is no conflict of interest regarding this paper.

## References


I. Akram, Muhammad. "Anti fuzzy structures on graphs." *Middle East Journal of Scientific Research* 11.12 (2012): 1641-1648. 10.5829/idosi.mejsr.2012.11.12.131012

II. Allan, Robert B., and Renu Laskar. "On domination and independent domination numbers of a graph." *Discrete mathematics* 23.2 (1978): 73-76. 10.1016/0012-365X(78)90105-X

III. Chang, Gerald J. "The domatic number problem." *Discrete Mathematics* 125.1-3 (1994): 115-122. 10.1016/0012-365X(94)90151-1

IV. Cockayne, Ernest J., and Stephen T. Hedetniemi. "Towards a theory of domination in graphs." *Networks* 7.3 (1977): 247-261. 10.1002/net.3230070305

V. Dharmalingam, K.M., and Valli, K., "Regular Domatic Partition in Fuzzy Graph." *World Journal of Engineering Research and Technology*, 5.5 (2019), 100-107.b http://wjert.org/admin/assets/article_issue/34082019/1567157413.pdf

VI. Gani, A. Nagoor, and K. Prasanna Devi. "2-domination in fuzzy graphs." *International Journal of Fuzzy Mathematical Archive* 9.1 (2015): 119-124. http://www.researchmathsci.org/IJFMAart/IJFMA-V9n1-14.pdf

VII. Haynes, Teresa W., Stephen Hedetniemi, and Peter Slater. *Fundamentals of domination in graphs*. CRC press, 2013. 10.1201/9781482246582

VIII. Muthuraj, R., and A. Sasireka. "Domination on anti fuzzy graph." *International Journal of Mathematical Archive* 9.5 (2018): 82-92. https://sadakath.ac.in/naac/criterion_iii/research/maths_supportingdocuments.pdf

IX. Muthuraj, R., and A. Sasireka. "Total domination on anti fuzzy graph." *New Trends in Mathematical Sciences* 6.4 (2018): 28-39. 10.20852/ntmsci.2018.312

X. Muthuraj, R.,Vijayalakshmi, P., and Sasireka, A., "Domatic Number On Anti Fuzzy Graph. " *AIPCP*. [ Accepted]

XI. Muthuraj, R., and A. Sasireka. "On anti fuzzy graph." *Advances in Fuzzy Mathematics* 12.5 (2017): 1123-1135. https://www.ripublication.com/afm17/afmv12n5_06.pdf

XII. Somasundaram, A., and S. Somasundaram. "Domination in fuzzy graphs–I." *Pattern recognition letters* 19.9 (1998): 787-791. 10.1016/S0167-8655(98)00064-6

XIII. Zadeh, Lotfi Asker, George J. Klir, and Bo Yuan. *Fuzzy sets, fuzzy logic, and fuzzy systems: selected papers*. Vol. 6. World scientific, 1996. https://books.google.co.in/books/about/Fuzzy_Sets_Fuzzy_Logic_and_Fuzzy_Systems.html?id=wu0dMiIHwJkC

XIV. Zelinka, Bohdan. "Antidomatic number of a graph." *Archivum Mathematicum* 33.2 (1997): 191-195. https://dml.cz/bitstream/handle/10338.dmlcz/107610/ArchMathRetro_033-1997-2_2.pdf