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SOME FEATURES OF PAIRWISE $\alpha - R_0$ SPACES IN SUPRA FUZZY BITOPOLOGY

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Abstract

This paper introduces and studies four concepts of R_0 supra fuzzy bitopological spaces. We have exhibited that all these four concepts are 'good extensions' of the corresponding concepts R_0 bitopological spaces and building relationships among them. It has been justified that all the definitions are hereditary, productive, and projective. Furthermore, additional properties of these concepts are studied.

Keywords: Fuzzy set, Fuzzy bitopological space, Good extension, Supra fuzzy bitopological space.

I. Introduction

American Mathematician Zadeh [XVIII] first time in 1965 introduced the concepts of fuzzy sets. Chang[V] and Lowen[X]] developed the theory of fuzzy topological space using fuzzy sets. Mashhour et al [XI] introduced supra-topological spaces and studied s-continuous functions and s^* -continuous functions. After that plenty of research has been conducted to extend the theory of fuzzy topological spaces in different areas. Wong, C. K. [XVI], Srivastava, A. K., and Ali, D. M. [XV] have introduced fuzzy topological spaces as well as fuzzy subspace topology. Ali, D. M.[III] has given notes on T_0 and R_0 fuzzy topological spaces. Hossain and Ali [VII] generalized R0 and R1 fuzzy topological spaces.

The research for fuzzy bitopological spaces began in the beginning 1990s. The fuzzy bitopological spaces with the property of separation axioms have become interesting as these spaces possesses many useful properties and can be found throughout various areas in fuzzy topologies. The concept of fuzzy pairwise T_0 bitopological space has been given by Kandil and El-Shafee [VIII, IX]. Abu Sufiya et al[II] and Nouh[XIV] have also introduced Fuzzy pairwise T_0 separation axioms. Hannan Miah and Ruhul Amin[[VI] also developed the features of pairwise $\alpha - T_0$ spaces in supra fuzzy bitopology.

In this paper, we study, some features of $\alpha - R_0$-spaces in supra fuzzy bitopological spaces and establish relationships among them.

As usual I=[0, 1] and $I_1 = [0, 1)$.

II. Preliminaries

We review various types of concepts in this section which will be needed in the sequel. Through paper X and Y are always represented as nonempty sets.

Definition 2.1 [XVIII]: For a set X, a function $u: X \to [0, 1]$ is called a fuzzy set in X. For every $x \in X, u(x)$ represents the grade of membership of x in the fuzzy set u. A fuzzy subset of X is denoted by u by some author. Therefore a usual subset of X is a special type of a fuzzy set in which the function values in {0, 1}.

Definition 2.2 [XVIII]: Let X be a nonempty set and A be a subset of X. The function $I_A: X \to [0, 1]\{0, 1\}$ defined by $I_A(x) = \begin{cases} 1 \ if \ x \in A \\ 0 \ if \ x \notin A \end{cases}$ is called the characteristic function of A. Now a day's author also writes the characteristic function of {x} as 1_x . The characteristic functions of subsets of a set X are referred to as the crisp sets in X.

Definition 2.3 [V]: Let X be a nonempty set and t be the family of fuzzy sets in I^X. Then t is called a fuzzy topology on X if it satisfies the following conditions:

(i) $1, \ 0 \in t$

(ii) If $u_i \in t$ for each $i \in A$, then $\cup_{i \in A} u_i \in t$.

(iii) If $u_1, \ u_2 \in t$ then $u_1 \cap u_2 \in t$.

If t is a fuzzy topology on X, then the pair (X, t) is called a fuzzy topological space(fts, in short), and members of t are called t-open(or simply open) fuzzy sets. If u is an open fuzzy set, then the fuzzy sets of the form 1-u are said to be t-closed(or simply closed) fuzzy sets.

Definition 2.4 [X]: Let X be a nonempty set and t be the collection of fuzzy sets in I^X such that

(i) $1, \ 0 \in t$

(ii) If $u_i \in t$ for each $i \in A$, then $\cup_{i \in A} u_i \in t$.

(iii) If $u_1, \ u_2 \in t$ then $u_1 \cap u_2 \in t$.

(iv) All constants fuzzy sets in X belong to t.

Then t is called a fuzzy topology on X.

Definition 2.5 [XI]: Let X be a set which is non-empty. A subfamily t^* of I^X is called a supra topology on X if and only if

(i) $1, \ 0 \in t^*$

(ii) If $u_i \in t^*$ for all $i \in A$, then $\cup_{i \in A} u_i \in t^*$.

Then the pair (X, t^*) is said to be a supra fuzzy topological spaces. The elements of t^* are said to be supra open sets in (X, t^*) and complement of a supra open set is called supra closed set.

.**Definition 2.6 [XI]**: Let (X, s) and (Y, t) be two topological spaces. Let s^* and t^* are associated supra topologies with s and t respectively and $f:(X,s^*)\to(Y,t^*)$ is a function. Then the function f is called a supra fuzzy continuous if the inverse image of each i.e. if for any $v\in t^*, f^{-1}(v)\in t^*$. The function f is called supra fuzzy homeomorphic if and only if f is supra one -one and onto and both f and f^{-1} are supra fuzzy continuous..

Definition 2.7 [IV]: Let (X,s^*) and (X,t^*) are two supra-topological spaces. If u_1 and u_2 be supra fuzzy subsets of X and Y respectively, then the Cartesian product $u_1\times u_2$ is also a supra fuzzy subset of $X\times Y$ defined by $(u_1\times u_2)(x,y)=\min[u_1(x),u_2(y)]$, for every pair $(x,y)\in X\times Y$.

Definition 2.8 [XVII]: Consider$\{X_i, i\in A\}$, be any family of sets and X denotes the Cartesian product of these sets, i.e.,$X=\prod_{i\in A}X_i$. Here X consists of all points $p=<a_i, i\in A>$, where $a_i\in X_i$. For each $j_0\in A$, the authors defined the projection π_{j_0} by $\pi_{j_0}(a_i: i\in A)=a_{j_0}$. The product supra topology is defined by using these projections.

Definition 2.9 [XVII]: Let $\{X_\alpha\}_{\alpha\in A}$ be a family of nonempty sets. Let $X=\prod_{\alpha\in A}X_\alpha$ be the usual products of X_α's and let $\pi_\alpha: X\to X_\alpha$ be the projection. Further, consider that every X_α be a supra fuzzy topological space with supra fuzzy topology t_α^*. Now the supra fuzzy topology is called the product supra fuzzy topology on X which is generated by a basis $\{\pi_\alpha^{-1}(b): b_\alpha\in t_\alpha^*, \alpha\in A\}$. Thus if w is a basis element in the product, then there exists $\alpha_1,\alpha_2,\dots\dots\alpha_n\in A$ such that $w(x)=\min\{b_\alpha(x_\alpha): \alpha=1,2,3,\dots.,n\}$, where $x=(x_\alpha)_{\alpha\in A}\in X$.

Definition 2.10 [I]: Let (X, T) be a topological space and T^* be associated supra topology with T. Then a function $f:X\to R$ is lower semi-continuous if and only if $\{x\in X: f(x)>\alpha\}$ is open for all $\in R$.

Definition 2.11 [XII]: Let (X, T) be a topological space and T^* be associated supra topology with T. Then the lower semi-continuous topology on X is associated with T^* defined by $\omega(T^*)=\{\mu: X\to[0,1], \mu \textit{ is supra lsc}\}$. If $\omega(T^*)=(X,T^*)\to[0,1]$ be the collection of all lower semi-continuous (lsc) functions. We can easily prove that $\omega(T^*)$ is a supra fuzzy topology on X.

Definition 2.12 [XIII]: Let (X,s_1^*,t_1^*) and (Y,s_2^*,t_2^*) are two bitopological spaces and $f:(X,s_1^*,t_1^*)\to(Y,s_2^*,t_2^*)$ be a function. Then the function f is said to be a supra fuzzy pairwise continuous if the two functions $f:(X,s_1^*)\to(Y,s_2^*)$ and $f:(X,t_1^*)\to(Y, t_2^*)$ are supra fuzzy continuous.

Definition 2.13 [XIII]: Let (X,s_1^*,t_1^*) and (Y,s_2^*,t_2^*) be two bitopological spaces and $f:(X,s_1^*,t_1^*)\to(Y,s_2^*,t_2^*)$ be a function. Then the function f is said to be a supra fuzzy pairwise open if both the function $f:(X,s_1^*)\to(Y,s_2^*)$ and $f:(X,t_1^*)\to(Y, t_2^*)$ are supra fuzzy open. i.e. for each open set $u\in s_1^*$, $f(u)\in s_2^*$ and for each $v\in t_1^*, f(v)\in t_2^*$.

Definition 2.14 [XVII]: Assume that $\{(X_i,s_i,t_i): i\in A\}$ is a family of fuzzy bitopological spaces. Then the space $(\prod X_i,\prod s_i,\prod t_i)$ is called the product fuzzy bitopological space of the family $\{(X_i,s_i,t_i): i\in A\}$, where $\prod s_i \textit{ and } \prod t_i$ denote the usual product fuzzy topologies of the families $\{\prod s_i : i \in A\}$ and $\{\prod t_i : i \in A\}$ of the fuzzy topologies respectively on X.

Let S^* and T^* are two supra topologies associated with two topologies S and T respectively. Let P is the property of a supra bitopological space (X, S^*, T^*) and FP is its supra fuzzy topological analogue. Then FP is said to be a 'good extension' of P 'if and only if the statement (X, S^*, T^*) has P if and only if $(X, \omega(S^*), \omega(T^*))$ has FP" holds good for every supra topological space (X, S^*, T^*).

III. Results and Discussions

$\alpha - R_0(I), \alpha - R_0(II), \alpha -$ R_0(III) and $R_0(IV$ SPACES IN SUPRA FUZZY BITOPOLOGICAL SPACE

Definition 3.1 : Let (X, s^*, t^*) be a fuzzy bitopological space and s^* and t^* be two supra topologies and $\alpha \in I_1$, then

(a) (X, s^*, t^*) be a pairwise $\alpha - R_0(i)$ space if and only if for all different elements $x, y \in X$, whenever there exists $u \in s^*$ with the property u(x)=1 and $u(y) \leq \alpha$, then there exists $v \in t^*$ with $v(x) \leq \alpha,$ $and\ v(y) = 1.$

(b) (X, s^*, t^*) be a pairwise $\alpha - R_0(ii)$ space if and only if for all different elements $x, y \in X$, whenever there exists $u \in s^*$ with the property u(x)=0 and $u(y) > \alpha$, then there exists $v \in t^*$ with $v(x) > \alpha,$ $and\ v(y) = 0.$

(c) (X, s^*, t^*) be a pairwise $\alpha - R_0(iii)$ space if and only if for all different elements $x, y \in X$, whenever there exists $u \in s^*$ with the property $0 \leq u(x) \leq \alpha < u(y) \leq 1,$ then there exists $v \in t^*$ $with$ $0 \leq v(y) \leq \alpha < v(x) \leq 1.$

(d) (X, s^*, t^*) be a pairwise $R_0(iv)$ space if and only if for all different elements $x, y \in X$, whenever there exists $u \in s^*$ with the property $u(y) < u(y)$, then there exists $v \in t^*$ with $v(x) > v(y)$.

The following examples show that $\alpha - R_0(i),$ $\alpha - R_0(ii),$ $\alpha - R_0(iii)$ and $R_0(iv)$ are all independent.

Example 3.1: Let X={x, y} and $u, v \in I^X$ are defined by u(x)=0, u(y)=1 and v(x)=0.42, v(y)=1. Consider the supra fuzzy topologies s^* and t^* on X are generated by {0, u, 1, $constants$} and {0, v, 1, $constants$} respectively. Then by definition, for $\alpha = 0.52, (X, s^*, t^*)$ is $\alpha - R_0(i)$, but (X, s^*, t^*) is not $\alpha - R_0(ii)$.

Example 3.2: Let X={x, y} and $u, v \in I^X$ are defined by u(x)=1, u(y)=0 and v(x)=0.65, v(y)=1. Consider the supra fuzzy topologies s^* and t^* on X are generated by {0, u, 1, $constants$} and {0, v, 1, $constants$} respectively. Then by definition, for $\alpha = 0.56, (X, s^*, t^*)$ is $\alpha - R_0(ii)$, but (X, s^*, t^*) is not $\alpha - R_0(i)$.

Example 3.3: Let X={x, y} and $u, v \in I^X$ are defined by u(x)=1, u(y)=0 and v(x)=0.42, v(y)=0.79. Consider the supra fuzzy topologies s^* and t^* on X are generated by {0, u, 1, constants} and {0, v, 1, $constants$} respectively. Then by definition , for $\alpha = 0.52, (X, s^*, t^*)$ is $\alpha - R_0(iii)$, but (X, s^*, t^*) is not $\alpha - R_0(ii)$.

Example 3.4: Let X={x, y} and $u, v, \in I^X$ are defined by u(x)=1, u(y)=0 and v(x)=0.34, v(y)=0.58. Consider the supra fuzzy topologies s^* and t^* on X are generated by {0, u, 1, constants} and {0, v, 1, constants} respectively. Then by definition, for $\alpha = 0.74, (X, s^*, t^*)$ is $\alpha - R_0(iv)$, but (X, s^*, t^*) is not $\alpha - R_0(i)$, (X, s^*, t^*) is not $\alpha - R_0(ii)$, and (X, s^*, t^*) is not $\alpha - R_0(iii)$,

Example: 3.5: Let X={x, y, z} and $u, v, w \in I^X$ are defined by u(x)=1, u(y)=1, u(z)=0 and v(x)=0, v(y)=0, v(z)=1 and w(x)=0.94, w(y)=0.54, w(z)=0. Let the supra fuzzy topologies s^* and t^* on X be generated by {0, u, w, 1, constants} and {0, v, w, 1, constants} respectively. Then by definition, for $\alpha = 0.66, (X, s^*, t^*)$ is $\alpha - R_0(i)$ and (X, s^*, t^*) is $\alpha - R_0(ii)$. But we observe (X, s^*, t^*) is not $\alpha - R_0(iii)$, and (X, s^*, t^*) is not $\alpha - R_0(iv)$. Since $w(x) > \alpha \geq w(y)$ but there does not exist $q \in t^*$ such that $q(x) \leq \alpha < q(y)$.

Example 3.6: Let X={x, y, z} and $u, v \in I^X$ are defined by u(x)=0.86, u(y)=0.45 , u(z)=0.38 and v(x)=0.38, v(y)=0.86, v(z)=0.26. Consider the supra fuzzy topologies s^* and t^* on X are generated by {0, u, 1, constants} and {0, v, 1, constants} respectively. Then by definition, for $\alpha = 0.52, (X, s^*, t^*)$ is $\alpha - R_0(iii)$, but (X, s^*, t^*) is not $\alpha - R_0(iv)$, since there exists $u \in s^*$ with $u(y) > u(z)$ but we have no $q \in t^*$ such that $q(y) < q(z)$.

Theorem 3.1: Prove that (X, s^*, t^*) **is 0-**$R_0(ii)$ if and only if (X, s^*, t^*) **is 0-**$R_0(iii)$.

Proof: The proof is trivial.

Theorem 3.2: Let (X, s^*, t^*) be a supra fuzzy bitopological space, $\alpha \in I_1$ and let $I_\alpha(s^*) = \{u^{-1}(\alpha, 1): u \in s^*\}$ and $I_\alpha(t^*) = \{v^{-1}(\alpha, 1): v \in t^*\}$, then the following statement is true:

(a) (X, s^*, t^*) is a pairwise $\alpha - R_0(iii)$ space if and only if $(X, I_\alpha(s^*), I_\alpha(t^*))$ is a pairwise R_0.
(b) If (X, s^*, t^*) is a pairwise $\alpha - R_0(i)$ space then $(X, I_\alpha(s^*), I_\alpha(t^*))$ is not pairwise R_0 space and conversely.
(c) If (X, s^*, t^*) is a pairwise $\alpha - R_0(ii)$ space then $(X, I_\alpha(s^*), I_\alpha(t^*))$ is not pairwise R_0 space and conversely.
(d) If (X, s^*, t^*) is a pairwise $R_0(iv)$ space then $(X, I_\alpha(s^*), I_\alpha(t^*))$ is not pairwise R_0 space and conversely.

Proof: Let (X, s^*, t^*) is a pairwise $\alpha - R_0(iii)$ space. We have to prove that $(X, I_\alpha(s^*), I_\alpha(t^*))$ is a pairwise R_0. Let $x, y \in X$ with $x \neq y$ and $M \in I_\alpha(s^*)$

With $x \in M, y \notin M$ or $x \notin M, y \in M$. Suppose that $x \in M, y \notin M$. We can write, $M = u^{-1}(\alpha, 1]$, for some $u \in s^*$. Then we have $u(x) > \alpha, u(y) \leq \alpha, i.e., 0 \leq u(y) \leq \alpha < u(x) \leq 1$. As (X, s^*, t^*) be a pairwise $\alpha - R_0(iii)$, $\alpha \in I_{1,}$ then there exists $v \in t^*$ such that $0 \leq v(x) \leq \alpha < v(y) \leq 1$, i.e., $v(x) \leq \alpha, v(y) > \alpha$. It follows that $x \notin v^{-1}(\alpha, 1], y \in v^{-1}(\alpha, 1]$ and also $v^{-1}(\alpha, 1] \in I_\alpha(t^*)$. Thus $(X, I_\alpha(s^*), I_\alpha(t^*))$ is a pairwise R_0.

Conversely, suppose that $(X, I_\alpha(s^*), I_\alpha(t^*))$ is a pairwise R_0. We have to prove that (X, s^*, t^*) is a pairwise $\alpha - R_0(iii)$ space. Let $x, y \in X$ with $x \neq y$ and $u \in s^*$ with $0 \leq u(x) \leq \alpha < u(y) \leq 1$ i.e., $u(x) \leq \alpha, u(y) > \alpha$, it follows that $x \notin u^{-1}(\alpha, 1], y \in u^{-1}(\alpha, 1]$ and $u^{-1}(\alpha, 1] \in I_\alpha(s^*)$, for every $u \in s^*$. As $\left(X, I_\alpha(s^*), I_\alpha(t^*)\right)$ be a pairwise R_0, then there exists $M \in I_\alpha(t^*)$ such that $\in M, y \notin M$. We can write $M = v^{-1}(\alpha, 1]$, where $v \in t^*$, it follows that $v(x) > \alpha, v(y) \leq \alpha, i.e., 0 \leq v(y) \leq \alpha < v(x) \leq 1$. Thus (X, s^*, t^*) is a pairwise $\alpha - R_0(iii)$ space.

Example 3.7: Let $X = \{x, y, z\}$ and $u, v \in I^X$ are defined by $u(x) = 1, u(y) = 0, u(z) = 0.7$ and $v(x) = 0, v(y) = 1, v(z) = 0.6$. Consider the supra fuzzy topology s^* and t^* on X generated by $\{0, u, 1, constants\}$ and $\{0, v, 1, constants\}$ respectively. Then for $\alpha = 0.5$, we have (X, s^*, t^*) is $\alpha - R_0(i)$. Now$I_\alpha(s^*) = \{X, \varphi, \{x, z\}, \{z\}\} and\ I_\alpha(t^*) = \{X, \varphi, \{y, z\}, \{z\}\}$. It observed that (X, s^*, t^*) is not R_0 space, since $y, z \in X, y \neq z$ and $\{x, z\} \in I_\alpha(s^*)$, with $z \in \{x, z\}, y \notin \{x, z\}$, but no such $U \in I_\alpha(t^*)$ with $x \notin U, y \in U$.

Example 3.8: Let $X = \{x, y, z\}$ and $u, v \in I^X$ are defined by $u(x) = 0.4, u(y) = 0, u(z) = 0.85$ and $v(x) = 0.84, v(y) = 1, v(z) = 0$. Consider the supra-fuzzy topology s^* and t^* on X generated by $\{0, u, 1, constants\}$ and $\{0, v, 1, constants\}$ respectively. Then for $\alpha = 0.52$, we have (X, s^*, t^*) is $\alpha - R_0(ii)$ and also (X, s^*, t^*) is $R_0(iv)$. Now $I_\alpha(s^*) = \{X, \varphi, \{z\}, \{x\}, \{x, z\}\} and\ I_\alpha(t^*) = \{X, \varphi, \{z\}, \{y\}, \{y, z\}\}$. It observed that $(X, I_\alpha(s^*), I_\alpha(t^*))$ is not R_0 space, since $x, y \in X, x \neq y$ and $\{x\} \in I_\alpha(s^*)\ with\ y \notin \{x\}, x \in \{x\}$ but no such $U \in I_\alpha(t^*)$ with $x \in U, y \notin U$.

Example 3.9 : Let $X = \{x, y\}$ and $u, v, w \in I^X$ are defined by $u(x) = 1, u(y) = 0$ and $v(x) = 0.45, v(y) = 0.94, w(x) = 0.74, w(y) = 0.34$. Consider the supra fuzzy topology s^* and t^* on X generated by $\{0, u, w, 1, constants\}$ and $\{0, v, w, 1, constants\}$ respectively. Then for $\alpha = 0.64$, we have (X, s^*, t^*) is not $\alpha - R_0(i)$ and also (X, s^*, t^*) is not $\alpha - R_0(ii)$. Now $I_\alpha(s^*) = \{X, \varphi, \{x\}\} and\ I_\alpha(t^*) = \{X, \varphi, \{y\}\}$. Then we observed that $I_\alpha(s^*)\ and\ I_\alpha(t^*)$ are topology on X and $(X, I_\alpha(s^*), I_\alpha(t^*))$ is R_0 space.

Example 3.10: Let $X = \{x, y\}$ and $u, v \in I^X$ are defined by $u(x) = 0.45, u(y) = 0.55$ and $v(x) = 0.34, v(y) = 0.43$. Consider the supra-fuzzy topology s^* and t^* on X generated by $\{0, u, 1, constants\}$ and $\{0, v, 1, constants\}$ respectively. Then for $\alpha = 0.6$, we have (X, s^*, t^*) is not $\alpha - R_0(iv)$.
Now $I_\alpha(s^*) = \{X, \varphi\} and\ I_\alpha(t^*) = \{X, \varphi\}$. Then $(X, I_\alpha(s^*), I_\alpha(t^*))$ is R_0 space.

Theorem 3.3: Let (X, S^*, T^*) be supra fuzzy bitopological space. Then (X, S^*, T^*) is a pairwise R_0 if and only if $(X, w(S^*), w(T^*))$ is a pairwise $\alpha - R_0(p)$, where p=i, ii, iii, iv.

Proof: Let $(X, w(S^*), w(T^*))$ be a pairwise $\alpha - R_0(i)$. Let $x, y \in X$ with $x \neq y$ and $U \in S^*$ with $x \in U, y \notin U$. But $I_U \in w(S^*)$ and $I_U(x) = 1, I_U(y) = 0$. Now, we have $I_U \in w(S^*)$ with $I_U(x) = 1, I_U(y) \leq \alpha$. Since $(X, w(S^*), w(T^*))$ is a pairwise $\alpha -$

$R_0(i)$, there exists $v \in w(T^*)$ such that $v(x) \le \alpha, v(y) = 1$. Then $x \notin v^{-1}(\alpha, 1), y \in v^{-1}(\alpha, 1)$ as $v(x) \le \alpha, v(y) = 1$ and also there exists $v^{-1}(\alpha, 1) \in T^*$. Thus (X, S^*, T^*) is a pairwise R_0 −space.

Conversely, let (X, S^*, T^*) be a pairwise R_0 −space. We have to show that $(X, w(S^*), w(T^*))$ be a pairwise $\alpha - R_0(i)$. Let $x, y \in X$ with $x \neq y$ and there exists $u \in S^*$ such that $u(x) = 1, u(y) \le \alpha$. Then $x \in u^{-1}(\alpha, 1), y \notin u^{-1}(\alpha, 1)$ as $u(x) = 1, u(y) \le \alpha$. Hence $u^{-1}(\alpha, 1) \in S^*$. As (X, S^*, T^*) be a pairwise R_0 −space, then there exists $V \in T^*$ such that $x \notin V, y \in V$ but $I_V \in w(T^*)$ and $I_V(x) = 0, I_V(y) = 1$ i.e., there exists $I_V \in w(T^*)$ such that $I_V(x) \le \alpha, I_V(y) = 1$. Thus $(X, w(S^*), w(T^*))$ be a pairwise $\alpha - R_0(i)$.

Hence (X, S^*, T^*) is a pairwise R_0 if and only if $(X, w(S^*), w(T^*))$ be a pairwise$\alpha - R_0(i)$.

In the same way, we can prove that

(a) (X, S^*, T^*) is a pairwise R_0 if and only if $(X, w(S^*), w(T^*))$ be a pairwise $\alpha - R_0(ii)$.
(b) (X, S^*, T^*) is a pairwise R_0 if and only if $(X, w(S^*), w(T^*))$ be a pairwise $\alpha - R_0(iii)$.
(c) (X, S^*, T^*) is a pairwise R_0 if and only if $(X, w(S^*), w(T^*))$ be a pairwise $\alpha - R_0(iv)$.

Therefore it is justified that $\alpha - R_0(p)$ is a good extension of its bitopolgical counterpart (p=I, ii, iii, iv).

Theorem 3.4: Let (X, s^*, t^*) be a supra fuzzy bitopological space. $A \subseteq X$ and

$s_A^* = \{u/A : u \in s^*\}$ and $t_A^* = \{v/A : v \in t^*\}$

Then (a) (X, s^*, t^*) is a pairwise $\alpha - R_0(i)$if and only if (A, s_A^*, t_A^*) is a pairwise $\alpha - R_0(i)$.

(b) (X, s^*, t^*) is a pairwise $\alpha - R_0(ii)$if and only if (A, s_A^*, t_A^*) is a pairwise $\alpha - R_0(ii)$

(c) (X, s^*, t^*) is a pairwise $\alpha - R_0(iii)$if and only if (A, s_A^*, t_A^*) is a pairwise $\alpha - R_0(iii)$.

(d) (X, s^*, t^*) is a pairwise $\alpha - R_0(iv)$ implies (A, s_A^*, t_A^*) is a pairwise $\alpha - R_0(iv)$

Proof: Suppose (X, s^*, t^*) is a pairwise $\alpha - R_0(i)$. Then for $x, y \in A$, with $x \neq y$ and $u \in s_A^*$ such that $u(x) = 1, u(y) \le \alpha$, then also $x, y \in X, x \neq y$. But we can write $u = w/A$, where $w \in s^*$ and hence $w(x) = 1, w(y) \le \alpha$. Since (X, s^*, t^*) is a pairwise $\alpha - R_0(i)$, then there exists $m \in t^*$ such that $m(x) \le \alpha, m(y) = 1$. But from the definition $m/A \in t_A^*$, for every $m \in t^*$ and $m/A\,(x) \le \alpha, m/A\,(y) = 1$. Thus (A, s_A^*, t_A^*) is a pairwise $\alpha - R_0(i)$. i.e., (a) proved.

Similarly (b), (c), and (d) can be proved.

Theorem 3.5: Let $\{(X_i, s_i^*, t_i^*), i \in A\}$ be a collection of supra fuzzy bitopological spaces and $(\prod X_i, \prod s_i^*, \prod t_i^*) = (X, s^*, t^*)$ be the product topological space on X, then

(a) $\forall\, i \in A$, (X_i, s_i^*, t_i^*) is a pairwise $\alpha - R_0(i)$ if and only if (X, s^*, t^*) is a pairwise $\alpha - R_0(i)$.
(b) $\forall\, i \in A$, (X_i, s_i^*, t_i^*) is a pairwise $\alpha - R_0(ii)$ if and only if (X, s^*, t^*) is a pairwise $\alpha - R_0(ii)$.
(c) $\forall\, i \in A$, (X_i, s_i^*, t_i^*) is a pairwise $\alpha - R_0(iii)$ if and only if (X, s^*, t^*) is a pairwise $\alpha - R_0(iii)$.
(d) $\forall\, i \in A$, (X_i, s_i^*, t_i^*) is a pairwise $\alpha - R_0(iv)$ if and only if (X, s^*, t^*) is a pairwise $\alpha - R_0(iv)$.

Proof: Suppose $\forall\, i \in A$, (X_i, s_i^*, t_i^*) is a pairwise $\alpha - R_0(i)$. We have to show that (X, s^*, t^*) is a pairwise $\alpha - R_0(i)$ .Let $x, y \in X$ with $x \neq y$ and $u \in s^*$ such that $u(x) = 1, u(y) \leq \alpha$. But we have $u(x) = \min\{u_i(x_i): i \in \Lambda\}$ $and\, u(y) = \min\{u_i(y_i): i \in \Lambda\}$ and hence we can find an $u_i \in s_i^*$ and $x_i \neq y_i$ such that $u_i(x_i) = 1$ and$u_i(y_i) \leq \alpha$. Since $(X_i, s_i^*, t_i^*), i \in \Lambda$ is α pairwise $-T_0(i)$, $\alpha \in I_1$, then there exists $v_i \in t_i^*$, $i \in$ such that $v_i(x_i) \leq \alpha, v_i(y_i) = 1$. But we have $\pi_i(x) = x_i$ and $\pi_i(y) = y_i$. Thus $v_i(\pi_i(x)) \leq \alpha$ and $v_i(\pi_i(y)) = 1$. It follows that there exists $v_i \circ \pi_i \in t^*$ such that$(v_i \circ \pi_i)(x) \leq \alpha, (v_i \circ \pi_i)(y) = 1$. Hence by definition (X, s^*, t^*) is a pairwise $\alpha - R_0(i)$.

Conversely, let (X, s^*, t^*) be a pair wise $\alpha - R_0(i)$. We have to prove that (X_i, s_i^*, t_i^*) , $i \in A$ is a pairwise $\alpha - R_0(i)$. Let a_i be a fixed point in X_i and $A_i = \{x \in X = \prod_{i \in \Lambda} X_i : x_j = a_{i\,,}\ for\ some\ i \neq j\}$. Thus A_i is a subset of X and hence $(A_i, s_{A_i}^*, t_{A_i}^*)$ is also a subspace of (X, s^*, t^*). Since (X, s^*, t^*) is a pairwise $\alpha - R_0(i)$, $(A_i, s_{A_i}^*, t_{A_i}^*)$ is also a pairwise $\alpha - R_0(i)$. Now we have A_i is homeomorphic image of X_i. Thus (X_i, s_i^*, t_i^*) , $i \in \Lambda$ is a pairwise $\alpha - R_0(i)$. i.e., (a) is proved.

Similarly (b), (c), and (d) can be proved.

IV. Conclusion

One of the important results of this paper is defining some new concepts of supra fuzzy pairwise $\alpha - R_0$ bitopological spaces. We represent their good extension, heridetity, productive and projective properties. These concepts would play a vital role in future research work in supra-fuzzy bitopological spaces.

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Conflicts of interest

All authors declare that they have no conflicts of interest.

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