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CHARACTERISTICS OF INTEGRATION BETWEEN STATISTICAL MODELS AND MATHEMATICAL MODELS

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Abstract

This study focuses on integrating mathematical and statistical modeling, where a statistical model estimates the parameters of a mathematical model, or a mathematical model generates data to train a statistical model. This integration benefits both approaches: mathematical models improve the accuracy of statistical models, while statistical models help reduce bias in mathematical ones. The findings demonstrate that this combination is a valuable tool for understanding and predicting dynamic systems, offering more accurate and flexible models. Research consistently shows that integrating these models is an ideal approach for solving complex problems and understanding various systems.

Keywords: Complex problem, Mathematical Modeling, Statistical Modeling Sustainability, Ultimately Indicated

I. Introduction

Mathematical modeling and statistical modeling are two distinct approaches to representing dynamic, even non-dynamic, systems [XXIV, VI, IX]. Mathematical modeling focuses on building mathematical models based on the laws of physics or mathematics, while statistical modeling focuses on building statistical models based on experimental data. Mathematical modeling and statistical modeling can be used separately, or jointly [XIX]. When used separately, they each offer advantages and disadvantages. Therefore, if they are used together and we are able to obtain the advantages of each and avoid the disadvantages of both methods, we will have obtained an integration between both mathematical modeling and statistical modeling [XXVII].

One of the advantages of mathematical modeling is accuracy, as mathematical models can be very accurate, especially if they are based on the laws of physics or precise mathematics [XXI]. Darwinian approaches for cancer treatment: benefits of mathematical modeling. Also, understanding, as mathematical models can help in understanding how dynamic systems work. Like other branches of science, modeling also has some disadvantages such as that it may sometimes be [XXVI], e.g., as the mathematical models can be very complex, making it difficult to use or interpret. Another disadvantage is that it may sometimes depend on several assumptions, which may not be correct in all cases. Another disadvantage is that it takes a long time [XXV].

Statistical modeling is characterized by realism: statistical models can be very realistic, especially if they are based on experimental data. It is also characterized by flexibility as it can be easily modified to suit new data or changes in the system [II]. One of the disadvantages of statistical modeling is statistical modeling. The most important disadvantage of statistical modeling is bias, as statistical models can be biased, which means that they may not provide an accurate picture of the system. Another disadvantage of statistical modeling is uncertainty, as statistical models are always associated with a degree of uncertainty, which means that the accuracy of predictions cannot be completely certain.

Mathematical modeling is the process of representing a real or imaginary system using mathematical concepts and tools. Mathematical models can be very useful in a variety of fields, including Science: Mathematical models can be used to understand how the natural world works [XVIII, X, VIII, I]. For example, mathematical models can be used to describe the motion of planets or the behavior of waves [III]. Engineering and mathematical models can be used to design and build structures and systems. For example, mathematical models can be used to design bridges or airplanes. Economics: Mathematical models can be used to understand how the economy works. For example, mathematical models can be used to forecast economic growth or the impact of economic policies. Business: Mathematical models can be used to make better business decisions. For example, mathematical models can be used to determine the best location for a factory or forecast demand for a product Mathematical modeling has many tools and applications that can be used in the field of mathematical modeling, and these tools vary according to the needs and requirements of the users. Among these tools and applications: MATLAB, which is a powerful program for numerical computing and programming, widely used in various fields of engineering and science. Simulink: It is an add-on for MATLAB used to model and simulate dynamic and control systems. There are also many simulation and modeling programs that can be used in many applications, especially mechanical ones, and in light of the tremendous progress in computer science and programming, there are now many programs such as Python, where Python (with libraries such as NumPy and SciPy): Python is considered a powerful programming language. And with libraries like NumPy and SciPy. Mathematical modeling is used to solve many problems [XII].

Statistical modeling is the process of using data to create a model that describes the probability distribution of a phenomenon. Statistical models can be very useful in a variety of fields, including Science: Statistical models can be used to understand how data is distributed in nature. For example, statistical models can be used to describe the distribution of people's heights or daily temperatures [XXIII]. For Business: Statistical models can be used to make better business decisions. For example, statistical models can be used to estimate the probability of success of a new product or forecast demand for a product. Medicine: Statistical models can be used to understand how diseases are distributed in a population. For example, statistical models can be used to estimate the prevalence of a disease or predict the spread of a disease. There are many tools and applications used in the field of statistical modeling to analyze data and use statistics to make decisions. Here are some popular tools and applications in this context: R is a programming language and environment for statistics, widely used to analyze data and perform statistical tests. SPSS (Statistical Package for the Social Sciences): SPSS is a program intended for statistical data analysis and is mainly used in social research and behavioral sciences. STATA: used to perform statistical analysis and economic and social modeling [XXII].

The integration of mathematical modeling and statistical modeling can provide advantages to both approaches. Mathematical models can help develop more accurate statistical models, and statistical models can help reduce bias in mathematical models. There are several different ways to integrate mathematical modeling and statistical modeling (Andersen, P. K. et al 2012). Statistical models based on counting processes. Springer Science & Business Media. One way is to use a statistical model to estimate the parameters of a mathematical model. Another way is to use a mathematical model to generate experimental data to train a statistical model [XXVII].

The problem of studying the integration of mathematical modeling and statistical modeling is how to effectively integrate different approaches. There are several different ways to integrate mathematical modeling and statistical modeling, each of which has advantages and disadvantages [VII, XIII, XV, V]. One of the most important challenges is choosing the appropriate method for integrating mathematical modeling and statistical modeling. The appropriate method depends on the data set and the system being modeled. Another challenge is handling uncertainty in embedded models. Combined models are often more complex than separate mathematical or statistical models, making it difficult to quantify the degree of uncertainty in predictions.

The importance of studying the integration of mathematical modeling and statistical modeling is that it can provide advantages for both approaches. Mathematical models can help develop more accurate statistical models, and statistical models can help reduce bias in mathematical models. The importance of studying the integration of mathematical modeling and statistical modeling. The study of the integration of mathematical modeling and statistical modeling aims to develop effective methods for integrating different approaches. This study seeks to achieve the following objectives:

- Develop methods for selecting the appropriate method for integrating mathematical modeling and statistical modeling.
- Develop methods to address uncertainty in embedded models.
- Develop methods for dealing with assumptions in mathematical models.
- Develop methods to deal with bias in statistical models.

By achieving these goals, studying the integration of mathematical modeling and statistical modeling can contribute to the development of more accurate and flexible models, which can be used to understand and predict dynamical systems.

II. Literature Review and Analysis

The history of mathematical modeling dates back to ancient times, when Greek astronomers used mathematical models to describe the motion of the planets. In the 17th century, Isaac Newton developed his laws of motion, which provided a solid mathematical basis for mathematical models [XX]. Statistical modeling relies on using data to create a model that describes the probability distribution of a phenomenon. Statistical models can be simple or complex and may be based on probability theory or both. The integration of mathematical modeling and statistical modeling began in the twentieth century, with the development of complex models requiring a combination of mathematical and statistical methods. There are several different ways to integrate mathematical modeling and statistical modeling, each with advantages and disadvantages in the book “The Nature of Mathematical Modeling” by A. Gerstenfeld [XVI]. A comprehensive introduction to the subject of mathematical modeling. The book covers a wide range of topics, including different levels of mathematical models. Mathematical models can be divided into three main levels: Representation: This level of modeling aims to provide a description of the system or phenomenon to be modeled. For example, a mathematical model can be used to describe the movement of the planets or the behavior of financial markets. To be modeled works. For example, a mathematical model can be used to understand how diseases spread or how organisms interact with their environment. Prediction: This level of modeling aims to predict the behavior of the system or phenomenon to be modeled. For example, a mathematical model can be used to predict weather changes or product sales. He also discussed the different methods of building mathematical models, including analysis, as this approach relies on physical or mathematical laws to describe the system or phenomenon to be modeled. For example, Newton’s laws of motion can be used to describe the motion of planets. Understanding, as this level of modeling aims to understand how the system or phenomenon. This approach relies on experimental data to describe the system or phenomenon to be modeled. For example, a statistical model can be used to describe the distribution of temperatures throughout the year. Experimental method: This approach relies on experimentation to describe the system or phenomenon to be modeled. For example, a thermometer can be used to measure the temperature in the air.

As for statistical modeling, it presents “Applied Linear Statistical Models” [XIV], a comprehensive introduction to the topic of linear statistical models. The book covers a wide range of topics, including statistical techniques used to analyze experimental data that involve a relationship between two or more variables. Statistical techniques are used to test a hypothesis about the variance between sets of data. Analysis of variance is a statistical technique used to test a hypothesis about the variance between sets of data. Analysis of variance can be used to determine if there are statistically significant differences between data sets. Also how to design scientific experiments to collect accurate and meaningful data. The experimental design aims to reduce bias in the data and increase the power of the test.

In a book titled “Handbook of Multivariate Statistics and Applied Mathematical Modeling” [IV], the explained the importance of both multivariate statistics and mathematical modeling in analyzing complex phenomena that involve many interrelated variables and highlighted how both approaches can be combined to improve the robustness and accuracy of the results, as he explained the importance of preparing the data well before starting statistical analysis or mathematical modeling. He recommended a set of preliminary steps to better interpret the data before applying formal statistical techniques. He stressed the importance of choosing the appropriate statistical technique or mathematical model based on the research question and the characteristics of the data. He also discussed some of the factors that Things to consider when choosing the appropriate technique, such as the number of variables, the type of data, and the presence of the necessary assumptions. The authors pointed out the necessity of interpreting statistical results or mathematical models in the context of the research problem and avoiding incorrect conclusions.

In a study [XI] on how to achieve integration between mathematical modeling and statistical modeling by integrating approaches and methods from both fields. The authors pointed out that this can be done in several ways, including using mathematical models to create statistical models: Mathematical models can be used to create more accurate and applicable statistical models. Using statistical models to estimate the parameters of mathematical models. Statistical models can be used to estimate the parameters of mathematical models. For example, a regression statistical model can be used to estimate the parameters of a mathematical model that describes the relationship between two variables. Using Mathematical Models and Statistical Models Together to Understand Complex Systems Mathematical models and statistical models can be used together to understand complex systems that involve many interrelated variables. This has been confirmed by many studies, such as a study [XVII] that focused on the relationships between statistics, mathematical modeling, and statistical modeling. The study begins with a short historical review of mathematical and statistical models and then discusses the fundamental differences between these two fields. The study then focuses on how to combine statistical and mathematical methods to create more accurate and applicable models. The study identified three main relationships between statistics, mathematical modeling, and statistical modeling: The overlap relationship, which indicates that statistical and mathematical methods often overlap with each other. For example, statistical methods can be used to estimate parameters of mathematical models, and mathematical methods can be used to create more accurate statistical models. Dependency relationship This indicates that statistical methods are often based on mathematical models. For example, many statistical techniques rely on assumptions about the nature of the system to be modeled. Integration relationship, which indicates that statistical and mathematical methods can be combined to create more robust and efficient models. The study also discussed some challenges that must be considered when integrating statistical and mathematical methods. These challenges include the following items:

- ❖ Differences in approach: Statistical methods rely on the use of data, while mathematical methods rely on the use of mathematical concepts and tools. It can be difficult to integrate these two disparate approaches.
- ❖ Assumptions: Mathematical models may require assumptions about the nature of the system to be modeled. These assumptions may be unrealistic or unverifiable in some cases.

The alkaline activator solution (AAS) used in geopolymer concrete mixes was a combination of sodium silicate solution ($SiO_2/Na_2O=2.5$), sodium hydroxide pellets, and distilled water. The role of AAS is to dissolve the reactive portion of source materials Si and Al present in GGBS and Metakaolin providing a highly alkaline liquid medium for condensation polymerization reaction. The sodium hydroxide was taken in the form of flakes of approximately 2.5 mm in size. The sodium hydroxide (NaOH) solution with the required concentration was prepared by dissolving the computed amount of sodium hydroxide flakes in distilled water.

In general, most studies indicate that the integration of statistical and mathematical methods is an emerging field with great potential. Integrating methods from both fields can create more accurate and applicable models, which can help understand and solve complex problems in a wide range of fields. This is consistent with the aims and results of our current study.

III. Methodology

The study used descriptive analytical methodology. It included a short historical review of mathematical and statistical models to provide a historical background to the topic. Discussing the basic differences between statistics, mathematical modeling, and statistical modeling: This discussion aims to clarify the differences between these two fields. Defining the relationships between statistics, mathematical modeling, and statistical modeling: This outline aims to provide a conceptual framework for the relationships between these two fields. As well as discussing the challenges that must be considered when integrating methods. Statistics and Mathematics: This discussion aims to provide an understanding of the challenges that researchers may face when integrating statistical and mathematical methods.

We will use scientific, analytical, and simulation methodology to obtain the results of the effect of Integration of Mathematical Modeling and Statistical Modeling The methodology and method were implemented on a set of basic steps according to the practical framework described, see Figure 1.

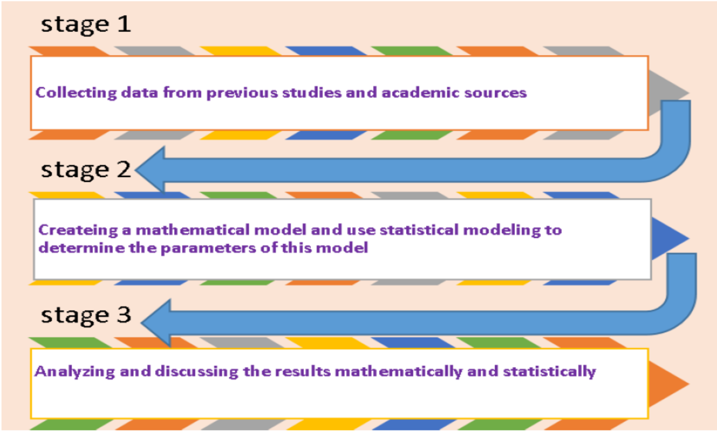

Fig. 1. The Practical Framework.

Table 1: The Actual Values of Displacement and Velocity of a Moving Body

Displacement (m)	Speed (m/s)
100	120
120	144
110	132
150	180
200	240
250	300
300	360
310	372
280	336
290	348
320	384

Procedure

- Creating a reliable database and creating reliable standards from previous research and studies. The goal is to obtain parameters of the relationship between speed and displacement for the movement of a particle during a certain period. The differential equation for the relationship between speed and displacement is an equation that links the speed of a particle and its position. Let us assume the movement of a particle and the values of velocity and displacement at certain points in time are shown in Table 1.
- Mathematical Modeling Formulation: Formulating the differential equation for the relationship between speed and displacement is an equation that links the speed of a particle and its position.

$$m\frac{d^2x}{dt^2} = F(x),$$

where m is the mass of the particle, $\frac{d^2x}{dt^2}$ is the acceleration of the particle, and $F(x)$ is the force acting on a particle. If the force $F(x)$ is linear in displacement, then the differential equation is as follows:

$$m\frac{d^2x}{dt^2} = kx,$$

where k is the constant of proportionality. Also, if the force $F(x)$ is nonlinear in displacement, then the differential equation is as follows:

$$\frac{d^2x}{dt^2} = f(x).$$

- Choose an appropriate statistical model to describe the relationship between velocity and displacement. This model can be linear or non-linear. If you think that the relationship between velocity and displacement is nonlinear, you can choose a model like the following:

$$v(t) = ax^2(t) + bx(t) + c,$$

where a, b, and c are the model parameters.

- Using the least squares method, you will obtain the following estimates of the model parameters The least squares method is a statistical method to estimate the parameters of a mathematical model by minimizing the sum of the squares of the differences between observed values and expected values.

In the case of a linear regression model, the least squares method can be used to estimate the regression coefficients a and b by solving the following system of equations:

$$(y - ax - b)^2 = min,$$

where y is the observed value, x is the value of the independent variable, a is the regression coefficient, and b is the cut point. We can use the Newton-Ralphson method to estimate the parameters of the following model:

$$v(t) = ax^2(t) + bx(t) + c,$$

where the initial guess of the parameters is (a_0, b_0, c_0). Herein, we calculate the derivative of the equation

$$\frac{dv}{dx} = 2ax + b.$$

We also calculate the new approximation of the parameters

$$a_1 = a_0 - \frac{v(t) - a_0\, x^2(t) - b_0x(t) - c_0}{2ax(t) + b_0},$$

$$b_1 = b_0 - \frac{2ax(t)v(t) - b_0v(t) - c_0x(t)}{(2ax(t) + b_0)^2},$$

$$c_1 = c_0 - \frac{v^2(t) - a_0 x^3(t) - b_0 x^2(t) - c_0 x(t)}{(2ax(t) + b_0)^3}.$$

In the same regard, the Newton-Ralphson method for estimating multiple linear model parameters consists of the following steps:

- Starting from an initial guess of the model parameters (a_0, b_0, c_0) to calculate the derivative of the equation concerning the parameters.
- Use the derivative of the equation to create a new approximation for the parameters (a_1, b_1, c_1).
- Repeat steps 1 and 2 until the approximation of the parameters reaches the desired accuracy.

Suppose we have data on the displacement and velocity of a body moving under the action of a nonlinear force. Then we can use the Newton-Ralphson method to estimate the parameters of the following model:

$$v(t) = ax^2(t) + bx(t) + c.$$

with the initial guess of the parameters (a_0, b_0, c_0), where x is the displacement (m), and v is the velocity (m/s). Herein, we should calculate the derivative of the equation

$$\frac{dv}{dx} = 2ax + b.$$

In addition, we should also calculate the new approximation of the parameter, that is

$$a_1 = a_0 - \frac{v(t) - a_0\, x^2(t) - b_0 x(t) - c_0}{2ax(t) + b_0},$$

$$b_1 = b_0 - \frac{2ax(t)v(t) - b_0 v(t) - c_0 x(t)}{(2ax(t) + b_0)^2},$$

$$c_1 = c_0 - \frac{v^2(t) - a_0 x^3(t) - b_0 x^2(t) - c_0 x(t)}{(2ax(t) + b_0)^3}.$$

Now, we repeat steps 1 and 2 until the approximation of the parameters reaches the desired accuracy. This yields

$$a_2 = a_1 - \frac{v(2) - {a_1}^2(2) - b_2(2) - c_2}{2a_1(2) + b_1},$$

$$b_2 = b_1 - \frac{2a_1(2)v(2) - b_1 v(2) - c_1(2)}{(2a_1(2) + b_1)^2},$$

$$c_2 = c_1 - \frac{v^2(2) - {a_1}^3(2) - {b_1}^2(2) - c_1(2)}{(2a_1(2) + b_1)^3},$$

$$a_3 = a_2 - \frac{v(3) - {a_1}^2(3) - b_2(3) - c_2}{2a_1(3) + b_1},$$

$$b_3 = b_2 - \frac{2a_1(3)v(3) - b_1v(3) - c_1(3)}{(2a_1(3) + b_1)^2},$$

$$c_3 = c_2 - \frac{v^2(3) - {a_1}^3(3) - {b_1}^2(3) - c_1(3)}{(2a_1(3) + b_1)^3}.$$

We notice that the more steps we repeat, the more accurate the results become.

IV. Results and Discussion

In this section, we will present the results of using statistical modeling to determine the parameters of the relationship between velocity and displacement and discuss those results. However, we gain the following results:

$$x = 100, 120, 110, 150, 200, 250, 300, 310, 280, 290, 320,$$

and

$$v = 120, 144, 132, 180, 240, 300, 360, 372, 336, 348, 384,$$

for which $data = np.array(x, v)$. See Figures 2, 3, 4, 5, and Tables 2, 3. The study reached positive results, as it showed that integrating mathematical modeling with statistical modeling can achieve many benefits, including the subsequent details.

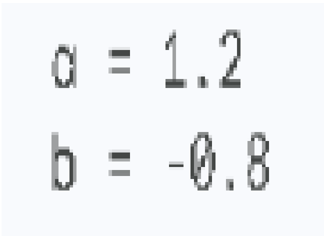

Fig. 2. Results of the Relationship Parameters Between Velocity and Displacement Using Statistical Modeling.

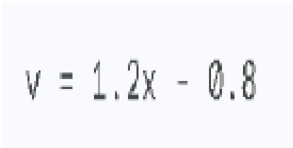

Fig. 3. Mathematical Relationship Between Speed and Displacement Using Statistical Modeling.

Looking at Figure 2 and Figure 3, it is clear that it is easy to obtain the parameters of the relation-ship or mathematical model using statistical modeling, as these parameters were obtained using the linear regression method. Table 2 shows the speed values obtained through the equation:

$$v = 1.2x - 0.8,$$

which was obtained by using the statistical model

$$v(t) = ax^2(t) + bx(t) + c,$$

where a and b are the relationship parameters, which were estimated using statistical modeling a linear regression.

Table 2: Displacement's and Velocity's Values of a Moving Body Using Statistical Modeling to Determine the Parameters of the Relationship Between Velocity.

x	v
100	119.2
120	143.2
110	131.2
150	179.2
200	239.2
250	299.2
300	359.2
310	371.2
280	335.2
290	347.2
320	383.2

Table 3: Error Percentage.

Displacement (m)	v(actual)	v	Error	Error percentage
100	120	119.2	0.8	0.006666667
120	144	143.2	0.8	0.005555556
110	132	131.2	0.8	0.006060606
150	180	179.2	0.8	0.004444444
200	240	239.2	0.8	0.003333333
250	300	299.2	0.8	0.002666667
300	360	359.2	0.8	0.002222222
310	372	371.2	0.8	0.002150538
280	336	335.2	0.8	0.002380952
290	348	347.2	0.8	0.002298851
320	384	383.2	0.8	0.002083333

Figure 4 and Figure 5 depict the actual relationship and the relationship between speed and displacement through the use of mathematical modeling and statistical modeling. Looking at the two figures, it is clear that they express a linear relationship between speed and displacement, and that the slope of the two straight lines is almost the same. This, however, indicates the effectiveness of statistical modeling to determine the modeling parameters. Mathematical relationship between velocity and displacement. In the same regard, looking at Table 3, it is clear that the actual values of the relationship between speed and displacement, through the use of mathematical and statistical modeling, and the values produced by using statistical modeling to determine the parameters of the mathematical model between speed and displacement, are very close, and that the percentage of error is very small, as the percentage of error ranged between 0.002083333 and 0.006666667. If there is an indication of this, it indicates the effectiveness of statistical modeling in determining the mathematical modeling parameters of the relationship between velocity and displacement.

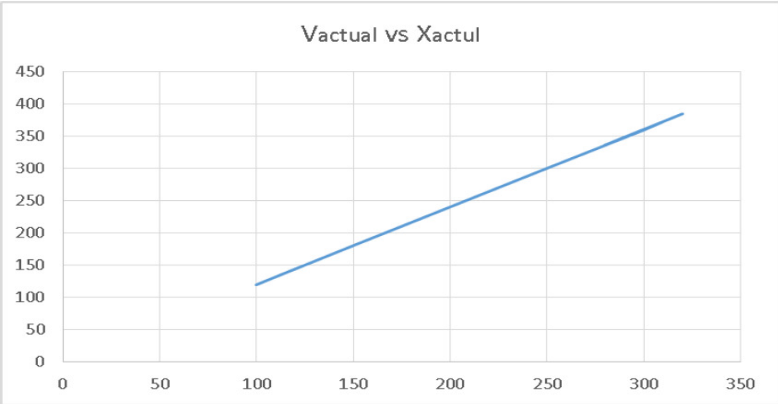

Fig. 4. The Actual Relationship Between Speed and Displacement.

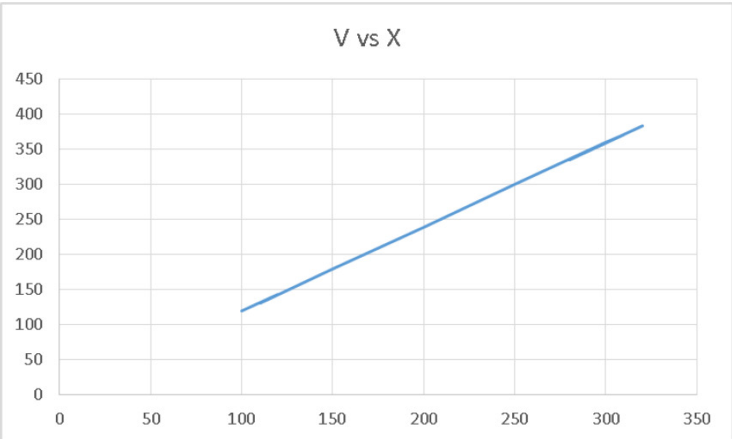

Fig. 5. The Actual Relationship Between Speed and Displacement After Modeling it Statistically and Mathematically.

V. Conclusion

The most important conclusions of the study can be explained as follows:

I. Integrating mathematical modeling with statistical modeling can bring many benefits, including improving the quality of models, increasing the accuracy of predictions, and expanding the range of applications.

II. Diversity of integration methods between mathematical modeling and statistical modeling.

III. There are some challenges that must be overcome when combining mathematical modeling with statistical modeling, including heterogeneity and difficulty of interpretation.

IV. The quality of results can be improved using multiple statistical methods. Integrating mathematical modeling with statistical modeling can help improve the quality of models by combining the strengths of each. Mathematical modeling can help understand the relationship between variables more accurately, while statistical modeling can help predict the values of variables more accurately under conditions of uncertainty.

V. Increasing the accuracy of predictions through which integration of mathematical modeling with statistical modeling can help increase the accuracy of predictions by using statistical data to improve the accuracy of mathematical models. For example, statistical data can be used to estimate unknown parameters in mathematical models or to evaluate how well models fit the data.

VI. Expanding the range of applications Integrating mathematical modeling with statistical modeling can help expand the range of possible applications of mathematical models. Statistical modeling can help handle complex, non-linear data, which may not be suitable for traditional mathematical modeling.

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VII. Conflicts of Interest

The authors declare that they have no conflicts of interest.

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