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APPROXIMATE APPROACH FOR THE MATHEMATICAL MODEL OF DISPLACEMENT-TRACTION WITH DEAD LOADS

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Abstract

This paper is concerned with the mathematical modeling of the displacement-traction with elastic bases, under specially selected boundary conditions for place and traction. The approximate approach is based on the theoretical results of the iterative factorization of operators given sufficiently smooth data, and smooth solutions. The problem resulting from the discretization of the original problem using the approximate approach occurs twice at each step of the proposed iterative process. The efficiency of the numerical method of iterative factorization explains that it is suitable for practical implementation in the computer.

Keywords: Iterative factorization and extension, Mathematical model of displacement-traction, Place and traction boundary conditions.

I. Introduction

Different phenomena in engineering and physics are described by various models concluded from higher-order differential equations with conditions. A nonlinear elasticity theory poses many important problems. A nonlinear character is inherent to a problem with three dimensions of elasticity. A problem with place and traction boundary conditions is called a displacement-traction problem [V, VI].

The equations that describe the balance and movement of elastic phenomena have a nonlinearity form which means that the necessity excludes certain noncompressible components, it is impossible to get theoretical solutions for such equations. In such cases, there is a necessity to recourse for some steps to reach convergent evolution to solutions that are so beneficial in applications. A rising demand for the precision of modeling phenomena under consideration causes a rise in the size of equations and rising in the size of arithmetic operations for their solutions [I, II].

A perturbence approach which used to reach for solutions to problems of bounded elasticity in the existence of loads that depend on the distortion. Later, Stoppelli showed a locality of existence and uniqueness solution to the traction model [VII, VIII]. A structure which applied for construction numerical solution to the staticbalance model of elastic materials under loads and study the significance of Fredholm condition for the stability of structure [III]. When considering various boundary conditions at the edges, diverse models appear, sometimes differing considerably in the approaches of solution and the difficulty of the model. These problems arise in mathematical problems of hydrodynamics, and nonlinear models of elasticity theory [IX, IV].

II. Analytical analysis of the displacement-traction model

Consider a mathematical model of displacement-traction

$$-div\hat{T} = \hat{f}\,, \text{ in } \Omega \tag{1}$$

$$\hat{T} = \hat{T}_0, \text{ on } \Gamma_1 \tag{2}$$

$$\hat{T}\,n = \hat{g}\,, \text{ on } \Gamma_2 \tag{3}$$

where $\hat{T}$ is a tensor function defined by $\hat{T}:\overline{\Omega}\times\mathbb{R}^3_+ \longrightarrow \mathbb{R}^3$, $\Omega = (0, a_1, a_2)\times(0, b_1, b_2)$ is an open subset of $\mathbb{R}^3$ whose boundary Γ, disjoint relatively open subsets Γ_1 and Γ_2 of Ω ($\Gamma = \Gamma_1 \cup \Gamma_2$), $\hat{f}:\overline{\Omega}\times\mathbb{R}^3_+ \longrightarrow \mathbb{R}^3$ is a vector-valued function that measures a density per volume, $\hat{g}\,;\ \Gamma_2\ \times\mathbb{R}^3_+ \longrightarrow \mathbb{R}^3$ is a vector-valued function that measures a density per area and n is a normal to $\overline{\Omega}$.

From the Eulerian formulation of equilibrium and linearity theory of bending of plates based on an elastic, there are homogeneous conditions of hinge support and symmetry, which can be written as:

$$\hat{\mathrm{E}}(\hat{T}) = \frac{1}{2}\hat{D}\int_\Omega (\Delta\hat{T})^2 + 2(1-\mu)\left(\hat{T}^2 - \hat{T}_{xt} - \hat{T}_{xx}\hat{T}_{tt}\right)d\Omega - \hat{D}\int_\Omega \hat{\mathrm{C}}(\Delta\hat{T})^2 d\Omega \tag{4}$$

such that $\hat{\mathrm{E}}$ is the tensile module, $\hat{\mathrm{C}}$ is the rigidity coefficient of elastic foundations, $\mu \in (0,1)$, and $\hat{D}$ is the cylindrical stiffness of the plate. If we equate the energy variation to zero, then we get

$$\delta\hat{\mathrm{E}}(\hat{T}) = \hat{D}\int_\Omega \left(\Delta\hat{T}\Delta\hat{v} + (1-\mu)\left(2\hat{T}\hat{v} - \hat{T}_{xt}\hat{v}_{xt} - \hat{T}_{xx}\,\hat{v}_{tt} - \hat{T}_{tt}\hat{v}_{xx}\right)\right)d\Omega - \int_\Omega \hat{\mathrm{C}}\hat{v}d\Omega = 0 \tag{5}$$

where $\hat{v} = \delta\hat{T}$, put $\hat{f}\ = \frac{\hat{\mathrm{C}}}{\hat{D}}$, implies to

$$\int_\Omega (\Delta^2\hat{T} + \hat{T})\hat{v}\,d\Omega + \int_\Omega \hat{T}\frac{\partial\hat{v}}{\partial n}d\Omega = \int_\Omega \hat{f}\,\hat{v}d\Omega \tag{6}$$

Thus, we obtain the problem under mixed boundary conditions

$$\Delta^2\hat{T} + \alpha\hat{T} = \hat{f} \tag{7}$$

$$\hat{T}\big|_{\Gamma_1} = 0 \tag{8}$$

$$\hat{T}n\big|_{\Gamma_2} = \hat{g} \tag{9}$$

such that $\alpha = \frac{\hat{k}}{\hat{D}}$, $\hat{k}$ is the pressure.

If we put $\hat{T} \equiv \hat{\omega}$, then the problem (7) – (9) becomes

$$\Delta^2\widehat{\omega} + \alpha\widehat{\omega} = \widehat{f} \tag{10}$$

$$\widehat{\omega}|_{\Gamma_1} = 0 \tag{11}$$

$$\widehat{\omega}n|_{\Gamma_2} = \widehat{g} \tag{12}$$

A model of displacement (10) – (12) is considered in variational shape to apply continuation. A general expression of the displacement-traction model of elastic basis with mixed conditions, it is determined by

$$\widehat{\omega} \in \hat{V}: L_1(\widehat{\omega}, I\hat{v}) + L_2(\widehat{\omega}, \hat{v}) = h_1(I\hat{v}) + h_2(\hat{v}) \tag{13}$$

$$h_3(\hat{v}) \tag{14}$$

where $\hat{V} = \{\hat{v} \in W_2^2: \hat{v}|_{\Gamma_1} = 0, \hat{v}n|_{\Gamma_2} = \widehat{g}\}$.

Assume that

$$L(\widehat{\omega}, \hat{v}) = L_1(\widehat{\omega}, \hat{v}) + L_2(\widehat{\omega}, \hat{v}), \text{ for all } \widehat{\omega}, \hat{v} \in \hat{V} \tag{15}$$

and

there exist

$$\lambda_1, \lambda_2 \in \mathbb{R}_+ \text{ such that } \lambda_1\|\hat{v}\|^2_{W_2^2} \le L(\hat{v}, \hat{v}) \le \lambda_2\|\hat{v}\|^2_{W_2^2} \tag{16}$$

Proposition 1. The equality

$$L_{1,2}(\widehat{\omega}_0, \hat{v}_1) = L_{1,2}(\hat{v}_1, \widehat{\omega}_0) = 0 \tag{17}$$

holds true for all $\widehat{\omega}_0, \hat{v}_1 \in \hat{V}$.

Proposition 2. The following estimate

$$\left\|\widehat{\omega}^k - \widehat{\omega}\right\|_{\hat{V}} \le \lambda\|\widehat{\omega}^0 - \widehat{\omega}\|_{\hat{V}}, k \in \mathbb{N}, \lambda > 0 \tag{18}$$

exists for the iterative processes of the problem (13) – (14).

Theorem 1. There is $\widehat{\omega} \in \hat{V}$ a solution of the model (13) – (14) in Ω.

Proof. Assume that $\widehat{\omega}_1, \widehat{\omega}_2 \in \hat{V}$ to be solutions of the model (13) – (14), then for $\widehat{\omega}_0 = \widehat{\omega}_1 + \widehat{\omega}_2$, we have

$$L_1(\widehat{\omega}_0, I\hat{v}_0) + L_2(\widehat{\omega}_0, \hat{v}) = 0, \text{ for all } \hat{v} \in \hat{V} \tag{19}$$

put $\hat{v} = \hat{v}_0$, we obtain

$L_1(\widehat{\omega}_0, I\hat{v}_0) + L_2(\widehat{\omega}_0, \hat{v}) = L_1(\widehat{\omega}_0, \hat{v}_0) + L_2(\widehat{\omega}_0, \hat{v}_0) = L(\widehat{\omega}_0, \hat{v}_0) = 0$, for all $\hat{v} \in \hat{V}$.

Let $\hat{v} = \widehat{\omega}_1$, implies that

$$L_1(\widehat{\omega}_1, I\widehat{\omega}_1) + L_2(\widehat{\omega}_1, \widehat{\omega}_1) = 0$$

hence

$$L_2(\widehat{\omega}_1, \widehat{\omega}_1) = 0$$

then

$$0 \leq \lambda_1 L(\widehat{\omega}_1, \widehat{\omega}_1) \leq L_2(\widehat{\omega}_1, \widehat{\omega}_1) = 0$$

therefore

$$L(\widehat{\omega}_1, \widehat{\omega}_1) = 0.$$

III. Approximation approach and algorithm for displacement-traction model

Theoretical analysis which modified an algorithm for approximate solution of the model (10) – (12), algorithms for calculation programs based on the iterative factorization of operators, and the Eulerian formulation of the equilibrium.

The steps of the proposed calculation algorithm:

1. Determined the size of the domain Ω.
2. Input the problem's parameters and the initial approximation is selected and computed of the first approximation.
3. Calculated the first discrepancy.
4. Finding the square of the error rate.
5. Checking the iteration-stopping condition.
6. Calculated additional vector.
7. Computed the iteration parameter.
8. Calculated a new approximation.
9. Finding the square of the error rate.
10. Checking the iteration-stopping condition.
11. The ending of a cycle.
12. Finding a solution to the model.
13. Displayed a solution of the model as a graph.

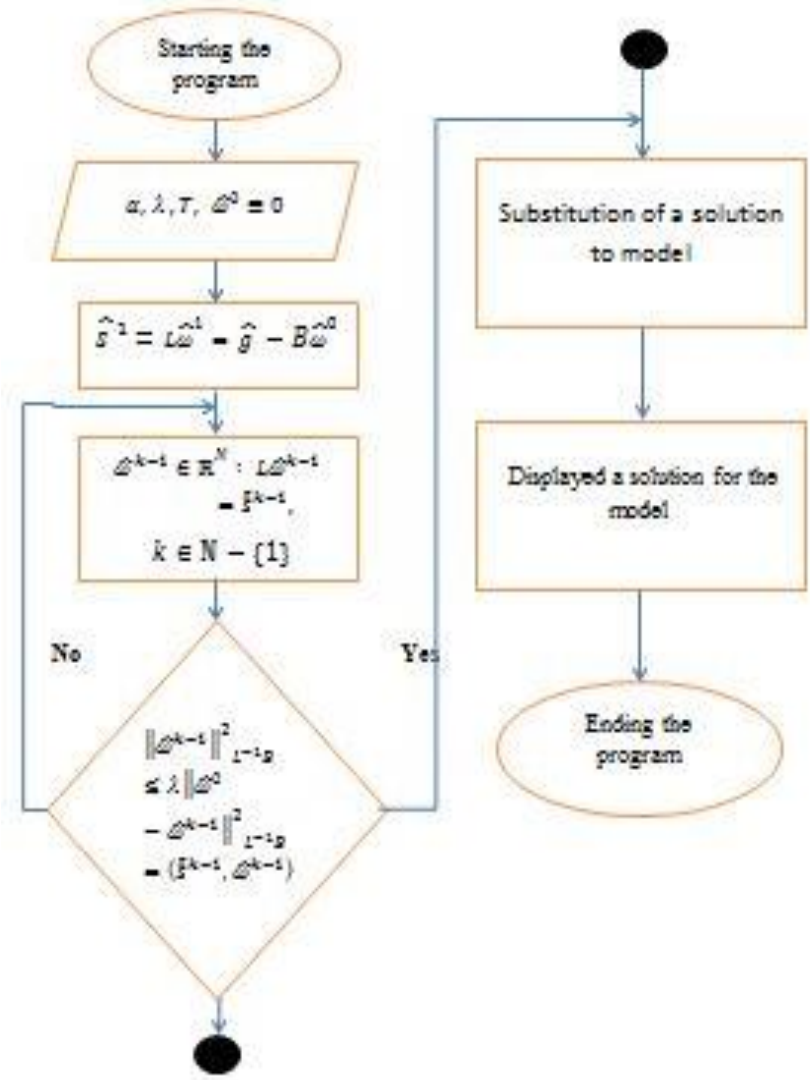

Fig. 1. Flowchart of the proposed algorithm

IV. Results and Discussions

The following examples illustrate applying the proposed algorithm

Example 1. To find an approximate solution to the problem (13) – (14), we choose $\alpha = 1$, $\lambda = 0.01$, $T = 10$, $\widehat{\omega}^0 \equiv 0$, and $\widehat{g} = 2 \sin t \sin x$ in the domain $[0,\pi] \times [0,\pi]$, applying the proposed algorithm with zero initial approximation is selected $\widehat{\omega}^0$and an amendment in progress returns the square of the error rat, the iteration parameter is calculated, a new approximation is calculated, and verify the stopping condition in each cycle with accuracy $\lambda = 0.01$, figure 2 shows the approximate solution to the problem (13) – (14).

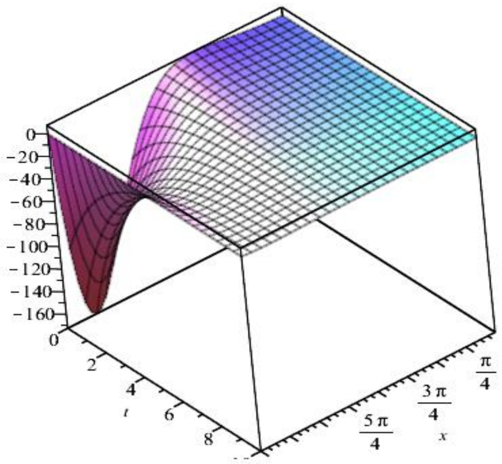

Fig.2. Approximate solution of example 1.

Example 2. To get an approximate solution to the problem (13) – (14), we suppose that $\alpha = 1$, $\lambda = 0.001$, $T = 20$, $\widehat{\omega}^0 \equiv 0$, and $\widehat{g} = -5 \cos t \cos t$ in the domain $[0, \pi] \times [0, \pi]$, by using the proposed algorithm with zero initial approximation is selected ω^0and an amendment in progress returns the square of the error rat, the iteration parameter is calculated, a new approximation is calculated, and verifying the stopping condition in each cycle with accuracy $\lambda = 0.001$, figure 3 shows the approximate solution to the problem (13) – (14).

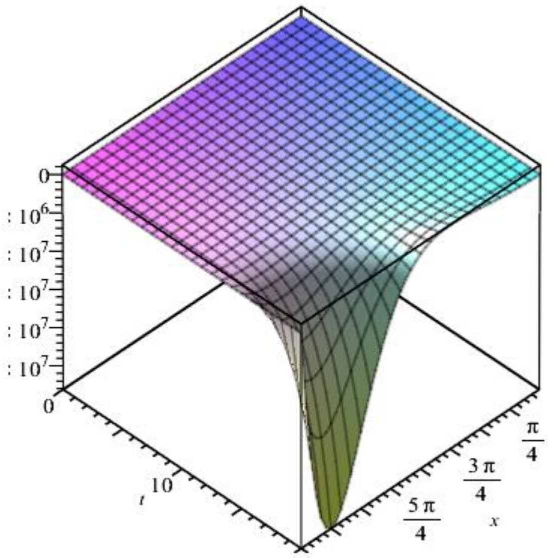

Fig. 3. Approximate solution of example 2.

V. Conclusion

An approximate analytical approach has been enhanced to compute displacements for the displacement-traction model under symmetry conditions. Estimates of convergence in this method are obtained. An approximating method has been developed for calculating displacements under symmetry conditions, which are asymptotically optimal regarding some steps on a computer. Computer programs have been written on which all calculations in the algorithms are based on the proposed approaches for computing the movements.

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Conflict of Intrest

The author declares that there is no conflict of interest regarding this article.

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