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A NOVEL CONCEPT: THE PRODUCT OF TWO NEGATIVELY DIRECTED NUMBERS IS A NEGATIVELY DIRECTED NUMBER THOUGH THE NEGATIVE OF A NEGATIVE NUMBER IS A POSITIVE NUMBER

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Abstract

In this paper, the author proved that the product of two negatively directed numbers is a negatively directed number. This article is the outcome of previously published (2021-2024) ten (10) articles of this author. It is true that the negative of a negative number is a positive number. It has been done by applying the inversion process to a negative number. The process of inversion does not satisfy the basic concept of multiplication. Multiplication is defined as the adding of a number concerning another number repeatedly. So, the process of inversion does not comply with the fundamental concept of multiplication. According to the Theory of Dynamics of Numbers there exist three types of numbers: (1) Neutral or discrete numbers (2) Positively directed numbers (3) Negatively directed numbers. In general, we use four types of operators: addition (+), subtraction (-), multiplication (x), and division (÷) in mathematical calculations. Besides these, we use the negative sign (-) as an inversion operator. The positive sign (+) and negative sign (-) also represent the direction of neutral or discrete numbers. In this paper, the author introduced Fermat's Last Theorem: $x^n + y^n = z^n$ *for* $n = 2$ *in Bhattacharyya's Theorem to prove that the product of two negatively directed numbers is a negatively directed number using the concept of the Theory of Dynamics of Numbers. In this paper, the author framed new laws of multiplication and inversion. Also, the author has given a comparative study between the conventional method of multiplication and the present concept of multiplication citing some practical examples. The author has become successful in finding the root of a quadratic equation in real numbers even if the discriminant,* $b^2 - 4ac < 0$ *without using the concept of the imaginary number and also in determining the radius of a circle even if* $g^2 + f^2 < c$*, in real number without using the concept of complex numbers. With one example the author proved that this theorem is applicable in 'Calculus' also.*

Keywords: Bhattacharyya's Theorem, Concept of limit, Number Theory, Rectangular Bhattacharyya's coordinates, Role of multiplication and inversion, Theory of Dynamics of Numbers.

I. Introduction

The fundamental concept of multiplication of numbers is adding a number with respect to another number repeatedly. However, the question arises regarding the nature of numbers. According to the author, the inherent nature of numbers is three types: Neutral or discrete numbers, positively directed numbers, and negatively directed numbers. For example: 5, + 5, -5. Here 5 represents a neutral number or discrete number, + 5 represents a positively directed number and -5 represents a negatively directed number. According to the Theory of Dynamics of Numbers 0 (zero) is defined as the starting point of any number and the numbers can move in an infinite direction from 0 (zero). 0 (Zero) is defined as the neutral number, the numbers that are moving away from 0 (zero) are called positively directed numbers or countup numbers, and the numbers that are moving towards 0 (zero) are called negatively directed numbers or countdown numbers.

According to conventional mathematicians (+) sign and (-) sign represent addition and subtraction operators respectively in general. They also represent the negative of a negative number by the negative sign multiplied by the negative number. For example: $- \times - 5 = +5$.

But the author introduced a new symbol of inversion I_n, where n = number of inversions of a number. For example:

$$I_n(-1) = +1, \text{ for all } n = \text{odd number.}$$

$$= -1, \text{ for all } n = \text{even number.}$$

So, negative of a negative number 5 should be represented as

$$I_1(-5) = +5, \text{ but not as } - \times - 5 = +5.$$

Oscillatory numbers occur due to repeated inversion of a number but not due to repeated multiplication.

Also, the author states that

$$(-1)^n = -1 \quad \text{for all n, but not an oscillatory number.}$$

The inversion of a number means the number which is in the vertically opposite direction of that number.

Therefore, the inversion of a number can never be represented as multiplication because it does not satisfy the fundamental concept of multiplication. In major cases of multiplication, the conventional mathematicians ignored the (+) sign and (-) sign as the direction of numbers in mathematical calculations especially in the case of inversion. The conventional concept that $(-1)^n$ represents an oscillatory number is wrong.

The author framed some new laws of multiplication and inversion considering three types of numbers according to the Theory of Dynamics of Numbers.

In this article, the author proved that the product of two negatively directed numbers is a negatively directed number using Bhattacharyya's theorem introducing Fermat's Last Theorem for n = 2. This is a unique approach to finding the square root of any negative number in real numbers without using the concept of imaginary numbers.

The author has become successful in finding the root of a quadratic equation even if the discriminant, $b^2 - 4ac < 0$ and also in finding the radius of a circle even if $g^2 + f^2 < c$, in real number in both the cases without using the concept of imaginary numbers with the help of this theorem. The author proved $\lim_{x \to 0} \sqrt{x - 4}$ exists whereas according to the conventional concept of limit, it does not exist.

II. Literature Review

The date of creation and origin of the concept of multiplication is unknown. Most likely out of necessity for combining a wide range of numbers and object values the concept of multiplication emerged to solve every day's problems initially. Today, we find various applications of multiplications in science, engineering, and technology including business and finance.

Multiplication is a form of arithmetic. We know that the word arithmetic appeared from Proto-Indo-European origin, meaning to reason and count, Geometry, algebra, and topology are the other forms of arithmetic.

The first multiplication tables were found in ancient Babylonia and Egypt over 4000 years ago. The tables were developed on the groundwork of base 60, which is although similar to base 10 which is used today. In ancient China (around 305 BC), they used tables that were very close to our modern-day representation of multiplication tables. The method of multiplication was migrated to ancient Greece by the Egyptians. The Greeks developed the first formal multiplication algorithm. The ancient Greek. Mathematician Pythagoras (570-495BC) also presented a multiplication table, French, Italian and Russian refer to it as a table of Pythagoras.

Most probably, Indian mathematics had been developed independently of Babylonian, Egyptian, and Chinese mathematics. Mantras from the beginning stage of the Vedic period (before 1000 BCE) put forward powers of 10 and have given evidence of arithmetic operations such as addition, subtraction, multiplication, division, squares, cubes, and roots. The Indian mathematician used seven distinct modes of multiplications which are as old as 200 AD.

The popular methods of multiplications are Gomutrika, Kapat - Sandhi Khanda / bheda, Sthana-Kanda, and tastha ete.

Ancient Indian mathematics was extremely advanced even if the written materials are only available from 900 BC. Out of 20 different operations in calculation one of the basic operations was multiplication. Aryabhata (510 AD), Brahmagupta (628 AD), Sridhara (750 AD) BhaskaraII (1150 AD), and many others have given their own interpretation and modification regarding multiplication [VII, VIII, XIV – XVI, XIX, XXIII, XXXVII].

Most probably the method of multiplication was carried forward to the West by Persian mathematician, Muhammad Al-Khwarizmi. The symbol 0 (zero) played an important role in the Indian numeral system with its place value concept. Considering the power and efficiency of the Indian numeral system Al-Khwarizmi accepted it to revolutionize Islamic and Western mathematics. He named these " Hindu numerals 1-9 and 0" This concept was adopted by the complete Islamic world very soon and later, by the entire world. William Oughtred (1574-1660) was the first person who used the symbol "x" for multiplication. At present, we are using a Computer for long multiplication.

In the last four years (2021-2024) the author developed a new structure in mathematics based on the Theory of Dynamics of Numbers by publishing ten (10) research articles. According to the Theory of the Dynamics of Numbers, there are three types of numbers: (1) Neutral numbers or discrete numbers (2) Positively directed numbers or countup numbers (3) Negatively directed numbers or countdown numbers. A neutral or Discrete number means a number without any motion. The number 0 (zero) is defined as a neutral number. There are three laws of the Theory of Dynamics of Numbers [XXV]:

(I) 0 (Zero) is the starting point of any number. The numbers that are moving away from 0 (zero) are called positively directed numbers or countup numbers and the numbers that are moving towards 0 (zero) are called negatively directed numbers or countdown numbers.

(II) Positively directed numbers or countup numbers can move independently but the motion of negatively directed numbers is dependent on the positively directed numbers

(III) For every equation the number of positively directed numbers or countup numbers is always equal to the number of countdown numbers or negatively directed numbers.

With the help of the Theory of dynamics of numbers the author developed a new concept of "Rectangular Bhattacharyya's Coordinate System" [XXIV] where all axes and ordinates are positive instead of a Cartesian coordinate system where out of two axes one axis is positive and the other one is negative and also out of two ordinates one ordinate is positive and the other one is negative. Bhattacharyy’s Coordinate system not only finds the distance between two points but also indicates the direction of motion of the connecting straight line between two points.

Based on the Theory of Dynamics of Numbers the author solved all types of problems of quadratic equations even if the discriminant, $b^2 - 4ac < 0$ in real number without using the concept of imaginary number [XXVI]. Also, the author states that the quadratic equations, which are factorizable, are actually pseudo-quadratic equations [XVIII] having two roots.

Again, introducing this theory to the equation of a circle where $g^2 + f^2 < c$, the author determined the value of the radius in real numbers whereas the conventional mathematician finds the value of the radius in imaginary numbers [XXVII]

Further, by introducing this theory the author proved that the Cauchy-Riemann equation,

$$\frac{\partial \varphi}{\partial x} = \frac{\partial \Psi}{\partial y} \text{ and } \frac{\partial \varphi}{\partial y} = -\frac{\partial \Psi}{\partial x}$$

where Ψ and φ represent stream function and velocity potential respectively in two-dimensional fluid motion using real variables only whereas these relations had been established by Cauchy-Riemann with the help of imaginary numbers [XXIX]

Moreover, the author proved that, $\sqrt{-1} = -1$ in both geometric and algebraic methods without using the concept of imaginary numbers [XXXI] using the Extended form of Pythagoras Theorem [XXX].

Lastly, the author proved the Bhattacharyya's Theorem [XXXIII]: $\sqrt{-(x^2 + y^2)} = -\sqrt{(x^2 + y^2)}$ to find the square root of any negative number introducing Fermat's last theorem for n = 2 [XVII] in real numbers without using the concept of imaginary number. The present theorem: The product of two negatively directed numbers is a negatively directed number is the outcome of all previous works of the present author.

III. Formulation of the problem and its solution

Theorem: <u>The product of two negatively directed numbers is a negatively directed number</u>.

<u>Proof</u>:

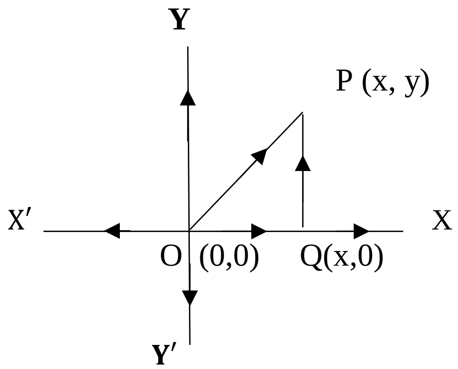

Fig. 1(a).

Let us consider a right-angled triangle OQP where base = OQ = $x > 0$, altitude = QP = y > 0, and hypotenuse = OP = z > 0, and x, y, z are all real numbers.

First of all, we shall prove that

$$\sqrt{-(x^2 + y^2)} = -\sqrt{(x^2 + y^2)}$$

These arrows in Fig. 1(a) and Fig. 1(b) represent the direction of the lines only.

According to Pythagoras Theorem, we have from Fig. 1(a)

$$OQ^2 + QP^2 = OP^2 \quad (1)$$

$$\text{or, } \sqrt{OQ^2 + QP^2} = \sqrt{OP^2}$$

$$\text{or, } \sqrt{OQ^2 + QP^2} = +OP$$

$$\text{or, } \sqrt{x^2 + y^2} = +z \quad (2)$$

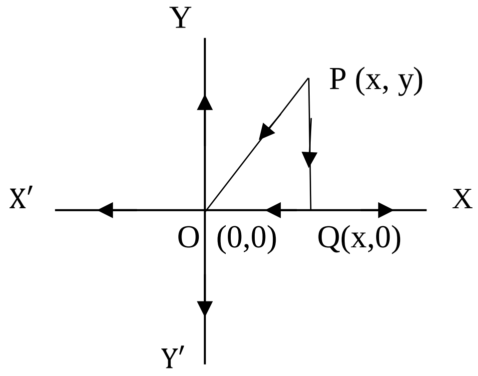

Fig. 1(b).

Similarly, from Fig. 1(b) we have,

$$PQ^2 + QO^2 = PO^2 \quad (3)$$

$$\text{or, } \sqrt{PQ^2 + QO^2} = \sqrt{PO^2}$$

$$\text{or, } \sqrt{QO^2 + PQ^2} = PO$$

$$\text{or, } \sqrt{-OQ^2 - QP^2} = -OP \quad [\because QO^2 = -OQ^2, PQ^2 = -QP^2 \text{and } PO = -OP]$$

$$\text{or, } \sqrt{-x^2 - y^2} = -z$$

$$\text{or, } \sqrt{-(x^2 + y^2)} = -z \quad (4)$$

Now from equation (2) and (4) we have,

$$\sqrt{-(x^2 + y^2)} + \sqrt{x^2 + y^2} = -z + z \quad (5)$$

$$\text{or, } \sqrt{-(x^2 + y^2)} + \sqrt{x^2 + y^2} = 0$$

$$\text{or, } \sqrt{-(x^2 + y^2)} = -\sqrt{x^2 + y^2} \quad (6)$$

The equation (6) is known as the Bhattacharyya's Theorem.

From Fermat's Last Theorem, we know.

$$x^n + y^n = z^n \quad (7)$$

This theorem has not been yet solvable for all n but it is solvable for n = 2. It has already been proved (G. N. Hardy and E. Wright: An introduction to the Theory of numbers, Sixth Edition, page: 245-247)

So,

$$x^n + y^n = z^n \quad (8)$$

is solvable for n = 2 where x > 0, y > 0 and z > 0 and x, y, z are real numbers.

Let us consider n = 2

$$x^2 + y^2 = z^2 = p \quad (9)$$

where p > 0, a real number

Now putting the value of $x^2 + y^2 = p$ in equation (6) we have

$$\sqrt{-p} = -\sqrt{p} \quad (10)$$

Again, let us consider

$$\sqrt{p} = q, \ q > 0, \text{ a real number}$$

putting the value of $\sqrt{p} = q$ in equation (10) we get

$$\sqrt{-p} = -q \quad (11)$$

Now squaring both sides of equation (11) we get

$$(-q)^2 = (\sqrt{-p})^2 \quad (12)$$

or, $(-q)^2 = -p$

or, $(-q) \text{ X } (-q) = -p$ (13)

The equation (13) proved that the product of two negatively directed numbers is a negatively directed number.

Note that the negative of a negative number is a positive number. For example:

$$-(-2) = +2$$

Here, we have obtained + 2 as an inversion of – 2 but that does not mean the product of a negative and a negative number. Inversion of a number cannot represent multiplication because the process of inversion does not satisfy the fundamental concept of multiplication which states that the process of multiplication of numbers is adding a number concerning another number repeatedly. So, the process of inversion cannot be claimed as the process of multiplication.

III.i. Important observations :

1. $\sqrt{-\mathrm{p}} = -\sqrt{\mathrm{p}}$ where p > 0, a real number. We can find the square root of any negative number in a real number without using the concept of an imaginary number.

2. (a) Multiplication between two numbers can occur if the two numbers are in the same direction only.

   (b) Multiplication between two numbers cannot occur if the two numbers are in different directions.

   (c) Multiplication between two numbers can occur where one number is a neutral number or discrete number and the other number is a directed number.

   (d) Inversion of a number can occur in the case of any directed number.

   (e) Inversion of a number does not represent as multiplication.

   (f) Negative of a negative number is a positive number. In that case, this inversion cannot be stated as multiplication.

3. (a) A positive sign has two significance.

(i) It may represent an addition operator between two numbers.

(ii) It may represent the direction of a neutral number or discrete number in the positive direction, ie, a positively direct number

(b) A negative sign has three significance:

(i) It may represent a subtraction operator in between two numbers.

(ii) It may represent the direction of a neutral number or discrete number in the negative direction, ie, a negatively directed number.

(iii) It may represent the inversion operator for neutral or discrete numbers, positive numbers, or negative numbers.

(c) Any neutral number or discrete number that undergoes repeated inversion represents an oscillatory number.

Examples:

(a) (+2) x (+3) = + 6

(b) (-2) x (-3) = – 6

(c) 2 x (+3) = + 6

(d) 2 x (-3) = – 6

(e) +2 x – 3 does not hold good if + 2 represents a positively directed number and – 3 represents a negatively directed number.

But it may hold good if we consider (+) sign of + 2 as addition operator and the number 2 as a discrete or neutral number.

For example:

8 + 2 x – 3 = 8 + (2 x – 3) = 8 + (- 6) = 8 – 6 = 2

Note that + (– 6) means addition of – 6 with 8 but + (– 6) does not mean + x (– 6)

Note that in that case, the positive sign represents as addition operator but not as direction of neutral or discrete number 2

Again, - 2 x +3 = – (2 x + 3) = – 6

Note that in that case, the negative sign represents as subtraction or inversion operator but not as direction of neutral or discrete number 2

III.ii. Laws of Multiplications

1) Neutral or Discrete number x positively directed number = Positively directed number.

2) Neutral or Discrete number x Negatively directed number = Negatively directed number

3) Positively directed number x Positively directed number = Positively directed number

4) Negatively directed number X Negatively directed number = Negatively directed number

5) Positively directed number x Negatively directed number or Negatively directed number x Positively directed number does not exist

6) $(-1)^n = -1$ for all n.

III.iii. Laws of Inversion

1) Inversion of Neutral or discrete number or positively directed number = Negatively directed number.

2) Inversion of negatively directed number = Positively directed number

3) Repeated inversion of Neutral or discrete number or positively directed number = Oscillatory number.

4) Repeated Inversion of negatively directed number = Oscillatory number

5) Repeated Inversion of negatively directed numbers and Repeated multiplication of negatively directed numbers cannot be represented as one and same.

III.iv. Symbol of Inversion

$I_n(N)$ where N is a negatively directed number and I_n represents n times of inversion of the number N.

For n = odd number, $I_n(N)$ will be a positive number.

and for n = even number, $I_n(N)$ will be a negative number.

For example:

$I_1(-1) = -(-1) = +1$, $I_2(-1) = -(+1) = -1$, $I_3(-1) = -(-1) = +1$, $I_4(-1) = -(+1) = -1$ etc.

IV. Applications :

New Symbols :

According to the Theory of Dynamics of Numbers the symbol '$\overline{}\!\!\uparrow$' represents Countup or positively directed number and the symbol '$\overline{}\!\!\downarrow$' represents Countdown or negatively directed number. At present, these new symbols are not available in any mathematics book.

For example: (a) $+3 = \overset{\uparrow}{\overline{3}}$, (b) $-3 = \overset{\downarrow}{\overline{3}}$

IV.i. Application to the quadratic equation :

Solve :

$$x^2 + 2x + 4 = 0$$

Solution :

$$x^2 + 2x + 4 = 0 \qquad (1)$$

Since 4 > 0, the inherent nature of unknown quantity x of the quadratic equation (1) will be countdown $x = \overset{\downarrow}{\overline{x}}$. In the quadratic equation the discriminant $= (2)^2 - 4.1.4 = 4 - 16 = -12 < 0$. According to the Novel Concept of the Theory of

Quadratic Equations [XXV], the equation (1) is a pure quadratic equation. According to the Theory of Dynamics of Numbers [XXVI], the equation takes the form

$$\overrightarrow{x^2 + 2x} + \overrightarrow{4} = 0 \tag{2}$$

According to the third law of the Theory of Dynamics of Numbers

$$x^2 + 2x = 4 \tag{3}$$

$$\Rightarrow x^2 + 2x + 1 = 4 + 1$$

$$\Rightarrow (x + 1)^2 = 5$$

$$\Rightarrow x + 1 = \sqrt{5}$$

$$\Rightarrow x = -1 + \sqrt{5} \tag{4}$$

Since, the inherent nature of x is countdown $x = \overrightarrow{x}$ and the numerical value of $\overrightarrow{x} = -x$, we have

$$\overrightarrow{x} = -(-1 + \sqrt{5}) = +1 - \sqrt{5} = 1 - 2.236 = -1.236 \text{ (approx.)} \tag{5}$$

Now, let $\overrightarrow{x} = \alpha$, be the root of equation (1), then we have

$$\alpha^2 + 2\alpha + 4 = 0 \quad \text{where } \alpha = -1.236 \text{ (approx.)} \tag{6}$$

Let us verify whether the value of α satisfies the equation (6) or not. Now, substituting the value of α in the expression.

$$\alpha^2 + 2\alpha + 4$$

we get,

$$(-1.236)^2 + 2(-1.236) + 4$$

$$= -1.527698 - 2.472 + 4 \quad [\therefore \text{ According to the Theorem : (Negatively directed number)}^2 = \text{A Negatively directed number}]$$

$$= -3.999696 \text{ (Approx.)} + 4$$

$$\cong -4 + 4$$

$$= 0$$

Therefore, α as the root of the equation (1) is satisfied.

So, $x = -1.236$, a real number.

Significance :

Using the concept of multiplication and with the help of Bhattacharyya's theorem the author has shown that the root of a quadratic equation can be determined in a real number even if the discriminant of the quadratic equation, $b^2 - 4ac < 0$ without using the concept of imaginary number.

Again, let us solve the equation by conventional method by using an imaginary number,

$$x^2 + 2x + 4 = 0$$

Then

$$x = \frac{-2 \pm \sqrt{2^2 - 4.1.4}}{2}$$

or, $x = \frac{-2 \pm i\,2\sqrt{3}}{2}$

or, $x = -1 \pm i\sqrt{3}$

Remarks :

We are unable to find the numerical value of the roots of the quadratic equation by using an imaginary concept since the numerical value of $\sqrt{-1} = i$ is unknown whereas we can find the numerical value of the root of a quadratic equation in real numbers by using the concept of Bhattacharyya's Theorem and the new concept of multiplication.

IV.ii. Application to the equation of a circle.

Problem : Find the location of the centre of the circle and its radius,

$$x^2 + y^2 - 3x + 2y + 20 = 0$$

Solution

$$x^2 + y^2 - 3x + 2y + 20 = 0 \qquad (1)$$

Here, $g = -\frac{3}{2}$, $f = 1$ and $c = 20$

$$g^2 + f^2 = \left(-\frac{3}{2}\right)^2 + (1)^2 = -\frac{9}{4} + 1 = -\frac{5}{4}$$

[(Negatively directed number)^2 = a negatively directed number). Here, $-\frac{3}{2}$ represents a negatively directed number]

So, $g^2 + f^2 = -\frac{5}{4}$

and $-\frac{5}{4} < 20$

Therefore, $g^2 + f^2 < c$

Since, $g^2 + f^2 < c$, the inherent nature of x and y are countdown x and countdown y in the equation (1). The equation takes the form:

$$\overrightarrow{x^2 + y^2 - 3x + 2y} + \overrightarrow{20} = 0 \qquad (2)$$

or, $\overrightarrow{\left(x - \frac{3}{2}\right)^2} + \overrightarrow{(y + 1)^2} + \overrightarrow{\frac{9}{4} + 1 + 20} = 0$ [in $\left(x - \frac{3}{2}\right)^2$, x and $\frac{3}{2}$ represents the neutral number and in between *x and* $\frac{3}{2}$ the negative sign represents as subtraction operator. So, $\frac{3}{2}$ cannot be represented as a negatively directed number] or, $$\overrightarrow{\left(x-\frac{3}{2}\right)^2} + \overrightarrow{(y+1)^2} = \overrightarrow{\frac{93}{4}} \quad (3)$$

or, $$\overrightarrow{\left(x-\frac{3}{2}\right)^2} + \overrightarrow{(y+1)^2} = \overrightarrow{\left(\frac{\sqrt{93}}{2}\right)}^2$$

According to Bhattacharyya's Coordinate System, the equation takes the form

$$\overrightarrow{\left(x-\frac{3}{2}\right)^2} + \overrightarrow{(y+1)^2} = \overrightarrow{\text{r}}^2 \quad (4)$$

Where, $\overrightarrow{\text{r}} = \overrightarrow{\left(\frac{\sqrt{93}}{2}\right)} = -\frac{\sqrt{93}}{2}$

The equation (4) represents the countdown circle. According to Bhattacharyya's Coordinate System, the point P(x, y) lies on the first quadrant, and the coordinates of the center, c $\left(\frac{3}{2},\ 1'\right)$ lies on the fourth quadrant of the real plane and its radius,

$\overrightarrow{\text{r}} = -\frac{\sqrt{93}}{2}$

Note that according to Bhattacharyya's Coordinate System, ordinate, $1'$ is equivalent to ordinate – 1 according to the Cartesian Coordinate System.

Now, let us solve the said problem by conventional method.

$$x^2 + y^2 - 3x + 2y + 20 = 0 \quad (1)$$

or,

$$\left(x-\frac{3}{2}\right)^2 + (y+1)^2 - \frac{9}{4} - 1 + 20 = 0$$

or,

$$\left(x-\frac{3}{2}\right)^2 + (y+1)^2 = -\frac{67}{4}$$

Let, $$\left(x-\frac{3}{2}\right)^2 + (y+1)^2 = r^2 \quad (2)$$

where, $r = \sqrt{\frac{-67}{4}} = i\,\frac{\sqrt{67}}{2}$

It is clear from equation (2) that the coordinates of the point P(x, y) lie in the first quadrant, and the center, c $\left(\frac{3}{2}, -1\right)$ lies in the fourth quadrant of the real plane but the radius $\text{r} = \text{i}\frac{\sqrt{67}}{2}$ an imaginary number.

Remarks:

(A) If we connect two points A (x_1, y_1) and B (x_2, y_2) on a real plane where (x_1, y_1) and (x_2, y_2) are real coordinates then the Straight line AB may be (1) $\overline{AB}$, a neutral straight line without any direction (2) $\overrightarrow{AB}$, a directed straight line from A to B, or countup straight line. (3) $\overrightarrow{BA}$, a directed straight line from. B to A means countdown straight line $\overrightarrow{AB}$ but in no way we can define the straight line as an imaginary line. So, the radius of the circle on a real plane having its center, c $\left(\frac{3}{2}, -1\right)$ are real coordinates and the point P (x, y) where (x, y) are real coordinates cannot be imaginary whereas we have found the radius, $r = i\,\frac{\sqrt{67}}{2}$, an imaginary number when we have applied the Conventional method to find the radius of the circle to the said problem. It is absurd.

(B) In case of the same problem if we apply Bhattacharyya's Theorem with the help of Bhattacharyya's coordinate system which is based on the Theory of Dynamics of Numbers, we can find the numerical value of the radius of the circle is $-\frac{\sqrt{93}}{2}$, a real number that means the radius of the circle directed towards the centre of the circle, and the circle represents a countdown circle.

IV.iii. Application to the Concept of Limit (Calculus)

Evaluate: $\lim_{x\to 0}\sqrt{x-4}$

Solution

$$\lim_{x\to 0}\sqrt{x-4}$$
$$= \sqrt{0-4}$$
$$= \sqrt{-4}$$
$$= -\sqrt{4} \quad \text{(According to Bhattacharyya's theorem)}$$
$$= -2$$

So, $\lim_{x\to 0}\sqrt{x-4}$ exists and its value is – 2

But, according to the conventional concept of limit :

$$\lim_{x\to 0}\sqrt{x-4}$$
$$= \sqrt{0-4}$$
$$= \sqrt{-4}$$

So, $\lim_{x\to 0}\sqrt{x-4}$ does not exist.

According to the author, the conventional concept of limit is wrong in that case.

V. Conclusion

The new concept that the product of two negatively directed numbers is a negatively directed number will enable the present and future mathematicians of the world to solve any problem in real numbers in mathematical calculation without using the concept of imaginary numbers. The author established this new concept is true by citing three simple examples from (1) Algebra (Quadratic Equation), (2) Coordinate Geometry (Equation of a circle), (3) Calculus (To evaluate the limit of a function) with a comparative study between the conventional method of calculation by using imaginary numbers and the new concept, which is based on the Theory of Dynamics of Numbers, in real numbers. The present article is the outcome of Bhattacharyya's Theorem where the square root of any negative number can be determined in real numbers without using the concept of imaginary numbers. This new concept may be used in Science, Engineering, and Technology and also in other fields of mathematical Calculations.

Conflicts of interest

The author declares that there are no conflicts of interest.

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