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ATANGANA-BALEANU TIME-STOCHASTIC FRACTIONAL NEUTRAL INTEGRO-DIFFERENTIAL EQUATIONS

R. Pradeepa[1] and R. Jayaraman[2]

[1]Research Scholar, Department of Mathematics, Periyar University, Salem - 636011, Tamil Nadu, India,
[1]Department of Mathematics, Paavai Engineering College (Autonomous), Namakkal -637018, Tamil Nadu, India.
[2]Department of Mathematics, Periyar University College of Arts and Science, Pappireddipatti, Tamil Nadu, India.

[1] pradeepamahesh74@gmail.com; [2] bjayarammaths@gmail.com

Corresponding Author: **R. Pradeepa**

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Abstract

This study investigates the Atangana-Baleanu time-stochastic fractional neutral integro-differential equation, a complex mathematical model with broad applications in various scientific disciplines. Utilizing Banach's fixed point theory, we rigorously establish the existence and uniqueness of the mild solution to this equation. Our analysis centrally revolves around investigating the Mittag-Leffler non-singular and non-local kernel, emphasizing its crucial significance in elucidating the behavior of the equation. By integrating concepts from fractional differential equations and stochastic differential systems, we contribute to a deeper comprehension of these mathematical phenomena. Our findings not only contribute significantly to advancing theoretical understanding but also establish a solid groundwork for practical applications across various fields.

Keywords: Existence and uniqueness, Mittag-Leffler Non-singular and non-local kernel, Fractional differential equations, Stochastic differential system and fixed point theorem.

I. Introduction

Fractional calculus is a branch of mathematics that studies and applies integrals and derivatives of any order. The advantage of non-integer order derivatives over integer order derivatives is their superiority in representing real-world scenarios, especially where memory or heredity is at stake. Recent research indicates that fractional order differential models are increasingly employed to mathematically characterize systems and phenomena across various scientific and engineering disciplines. These models find application in fields such as chemistry, physics, aerodynamics, polymer rheology, and the electrodynamics of complex media. For more detail, see Podlubny [XVI], Kilbas [IX], Miller and Ross [XI], and Zhou [XIII], as well as the works [I, VI, VII, VIII, X, XII, XV, XIX, XXV, XXVI] and the references referenced therein. Moreover, stochastic differential equations serve as versatile tools applicable across material science, biology, chemistry, mechanics, and various other fields. Consequently, research in this area has garnered significant attention.

Actually, given a genuine environment, an accurate study or evaluation must consider the possibility of randomness in the system's features, such as variations in noise in a communication network, or the stock market. A unique class of stochastic functional differential equations are stochastic differential equations with delay. Delay differential equations are commonly used in biological and physical applications, where they require the consideration of state-dependent or variable delays.

Many academics have examined the existing findings for stochastic fractional differential equations with infinite delay and state-dependent delay (see [XIX, VIII, XVIII]). A different definition of fractional derivatives has been provided by Caputo and Fabrizio [VI] and goes like this:

$$CF_{D^{r}_{a^{+}}}h(t) = \frac{\eta(r)}{1-r}\frac{d}{dz}\int_{r}^{z}\exp\left[\frac{-r}{1-r}(z-x)\right]h(x)\,dx, \qquad 0<r,1$$

where $\eta(r)$ is normalization function and $\eta(0)=\eta(1)=1$. The usual power law kernel $(t-s)^{r-1}$ was replaced by an exponential kernel $\exp\left[\frac{-r}{1-r}(t-s)\right]$ by the authors in this instance. In addition, $\frac{1}{r(1-r)}$ has been substituted by $\frac{1}{\sqrt{2\pi(1-r^2)}}$. The updated definition provides a more accurate portrayal of the mechanics behind a non-local phenomenon. Furthermore, it appropriately addresses whether a fractional operator can exist with a non-singular kernel. Baleanu and Fernandez [IV] develop novel features of fractional derivatives with the Mittag-Leffler kernel.

By adding more non-singular and non-local kernels, known as Mittag-Leffler functions, to the formulation set out by Caputo and Fabrizio, Atangana and Baleanu [III] improved and expanded it. The Banach contraction principle with the resolvent operator approach was used by the authors of [VII] to demonstrate the existence of findings for a class of fractional neutral integro-differential equations with state-dependent delay in Banach space.

Afterward, the researchers [II] employed the replicating kernel Hilbert space method to investigate a category of AB fractional integro-differential equations of the Fredholm operator type. Using fixed point methods, the authors recently [XVII, II] investigated a class of AB fractional integro-differential equations for the existence and uniqueness of solutions. ME. Omaba and CD Enyi recently used Banach fixed point theory to investigate the Atangana-Baleanu time-fractional stochastic integro-differential equation. The existence and uniqueness of solutions to a certain class of

AB time-fractional stochastic neutral integro-differential equations are investigated in this paper.

Definition 1.1. Let $r \in (0,1), p < q, f \in H^1(p,q)$. The Atangana-Baleanu fractional derivative of f f of order r r at a point $C \in (\, , q \,)$ C∈(p,q) is expressed in Caputo terms as

$$a^{ABC}D_z^r f(z) = \frac{C(r)}{1-r}\int_r^z f'(x)E_r \left[\frac{-r}{1-r}(z-x)^r\right]dx$$

Definition 1.2. Let $r \in (0,1), p < q, f \in H^1(p,q)$. Assuming a point $z \in (p,q)$, the Atangana - Baleanu fractional derivative of f of order r is defined in the following Riemann - Liouville sense.

$$a^{ABC}D_z^r f(z) = \frac{C(r)}{1-r}\frac{d}{dz}\int_r^z f(x)E_r \left[\frac{-r}{1-r}(z-x)^r\right]dx$$

Definition 1.3. This is the definition of the Atangana-Baleanu fractional derivative of f of order r:

$$a^{AB}I_z^r f(z) = \frac{1-r}{C(r)}f(z) + \frac{r}{C(r)\Gamma(r)}\int_0^z (z-x)^{r-1}f(x)dx$$

Remark 1.4. Given the normalizing function $C(r)$ in the definitions above, it fulfills

$$\frac{1-r}{C(r)} + \frac{r}{C(r)\Gamma(r)} = 1$$

Remark 1.5

$$a^{AB}I_z^r[a^{ABR}D_z^r f(z)] = f(z)$$

and

$$a^{AB}I_z^r[a^{ABR}D_z^r f(z)] = f(z) - f(0)$$

In this paper, we look at the ABC and ABR stochastic fractional neutral integro-differential equations:

$$0^{ABC}D_z^r\left[P(z) - \mathcal{G}\big(z, P(z)\big)\right] = \mu P(z)$$

$$+0^{ABC}D_z^r\left[\frac{C(r)}{1-r}\int_0^z E_r\left[-\frac{r}{1-r}(z-x)^r\right]v\big(P(x)\big)dw(x)\right]$$

$$P(0) = P_0 \qquad (1)$$

for $z \in (0, Z], \mu > 0, 0^{ABC}D_z^r$ and $0^{ABR}D_z^r$ are fractional differential operators, in the Caputo and Riemann-Liouville senses $0 < r \leq 1, w(z)$ represents the Wiener process whose generalized derivative is provided by $w(z), \mathcal{G}: (0,z] \times \mathcal{H} \to \mathcal{H}$ and $v: \mathcal{L} \to \mathcal{L}$ is the Lipschitz continuous.

Using the fractional integral operator $O^{AB}I_z^r$ to both sides of (1.1) the function P in (1.1) meets the following conditions:

$$P(z) = P(0) - \mathcal{G}(0, P(0)) + \mathcal{G}(z, P(z)) + \frac{(1-r)\mu}{C(r)} P(z) + \frac{r\mu}{C(r)\Gamma(r)} \int_0^z (z-x)^{r-1} P(x) dx + \frac{C(r)}{(1-r)} \int_0^z \mathrm{E_r}\left[-\frac{r}{1-r}(z-x)^r\right] v\big(P(x)\big)\, dw(x) \quad (2)$$

The format of this paper is as follows. Section 2 contains some preliminary information, fundamental definitions, lemmas, and findings. Section 3 contains the existing findings.

II. Preliminaries

Definition 2.1 The function $P(z)_{z\in[0,Z]}$ considered a moderate solution of (1.1) if the following requirements are almost certainly satisfied.

$$P(z) = P(0) - \mathcal{G}(0, P(0)) + \mathcal{G}(z, P(z)) + \frac{(1-r)\mu}{C(r)} P(z) + \frac{r\mu}{C(r)\Gamma(r)} \int_0^z (z-x)^{r-1} P(x) dx \quad (3)$$

if $P(z)_{z\in[0,z]}$ also meets the condition

$$\sup_{z\in[0,Z]} E|P(z)|^2 < \infty \quad (4)$$

then $P(z)_{z\in[0,Z]}$ is said to be a random field solution to (1).

Remark 2.2. The moderate solution in (3) is formed by following the procedure used in [I], and multiplying via equation (1) by the integral operator $O^{AB}I_z^r$.

Now, we define the $L^2(Q)$ norm of v as follows:

$$\| P \|_2^2 = \sup_{z\in[0,Z]} E|P(z)|^2$$

Following that, we will look at various Mittag-Leffer function estimations.

$$\int_0^y z^{s-1} E_{r,s}(wz^r) dz = y^\beta E_{r,s+1}(wy^r)$$

Lemma 2.3. Let $\Phi > 0$ be arbitrary and $r \in (0,1)$. Denote

$$n(r,\Phi) = \max\left\{\frac{\int_0^\infty e^{-r\frac{1}{r}} dr}{\Phi\pi r \sin(\pi r)}, \frac{\int_0^\infty r^{\frac{1}{r}} e^{-r^{\frac{1}{r}}} dr}{\Phi^2 \pi r \sin(\pi r)}\right\}$$

We have:

(a) $\left|E_r(\Phi z^r) - \frac{1}{r} e^{\Phi^{\frac{1}{r}} z}\right| \le \frac{m(r,\Phi)}{z^r}, \forall z > 0,$

(b) $\left|z^{r-1}E_{r,r}(\Phi z^r)-\frac{1}{r}\Phi^{\frac{1-r}{r}}e^{\Phi^{\frac{1}{r}}z}\right|\le\frac{m(r,\Phi)}{z^{r+1}},\forall z>0,$

(c) $\left|z^{r-1}E_{r,r}(\Phi z^r)\right|\le\frac{m(r,\Phi)}{z^{r+1}},\forall z>0.$

III. Existence Results

Assume the following for v, the global Lipschitz continuous:

Condition 3.1. Let $0<\text{Lipv}<+\infty$ be the result. Then,

$$|v(l)-v(m)|\le\text{Lipv}\,|l-m|,\forall l,m\in\mathcal{L}$$

for simplicity, let $v(0)=0$.

Theorem 3.2. Condition 3.1 holds if $P_0\le C_1$ for some $C_1>0, r\in(0,1)/\frac{1}{4},\frac{1}{2}$. If μ is a positive integer and $0<C_3<1$, then there is only one possible solution to equation (1), where

$$C_3=6\mathcal{M}\mathcal{G}+\frac{18(1-r)\mu}{C(r)}C_1+6\left(\frac{(1-r)\mu}{C(r)}\right)+3\left(\frac{r\mu}{C(r)\Gamma(r)}\right)^2\frac{Z^{2r}}{2r-1}$$
$$+3Lip_v^2\left(\frac{C(r)}{1-r}\right)^2n^2\left(r,\frac{r}{1-r}\right)\frac{Z^{1-4r}}{1-4r}$$

We show the above findings using Banach's fixed point theorem. Define an operator.

$$\mathcal{A}P(z)=P(0)-\mathcal{G}\big(0,P(0)\big)+\mathcal{G}\big(z,P(z)\big)+\frac{(1-r)\mu}{C(r)}P(z)+\frac{r\mu}{C(r)\Gamma(r)}\int_0^z(z-x)^{r-1}P(x)dx+\frac{C(r)}{1-r}\int_0^z E_r\left[\frac{-r}{1-r}(z-x)^r\right]v(P(x))dw(x)$$

As the fixed point of the operator $\mathcal{A}$ and the solution (1.1) will be achieved.

Lemma 3.3. Assume that P is a random field solution that fulfills both (2.3) and (2.4). Let Condition 3.1 hold, then for $r\in(0,1)/\left\{\frac{1}{4},\frac{1}{2}\right\}$ and μ large enough

$$\|\mathcal{A}P\|_2^2\le C_2+C_3\|P\|_2^2,$$

where $C_2=3C_1^2$ and C_3 is as defined above.

Proof.

$$E|\mathcal{A}P(z)|^2 \le 5E\|P(0) - \mathcal{G}(0,P(0)) + \mathcal{G}(z,P(z))|^2 + 5E\|\frac{\mu(1-r)}{C(r)}\|^2$$
$$+5E\left\|\frac{\mu r}{C(r)\Gamma(r)}\int_0^z (z-x)^{r-1}P(x)dx\right\|^2$$
$$+5E\left\|\frac{C(r)}{1-r}\int_0^z E_r\left[\frac{-r}{1-r}(z-x)^r\right]v(\phi(x))dw(x)\right\|^2$$
$$\le 5E \parallel P(0) - \mathcal{G}(0,P(0)) \parallel^2 + 5E\|\mathcal{G}(z,P(z))|^2 + 5E\|\frac{\mu(1-r)}{C(r)}\|^2$$
$$+5E\left\|\frac{\mu r}{C(r)\Gamma(r)}\int_0^z (z-x)^{r-1}P(x)dx\right\|^2 \qquad (5)$$

Next, we get $E|\mathcal{G}(z,P(z))|^2 \le \mathcal{M}\mathcal{G}E|P(z)|^2, C_1 = [P_0 - \mathcal{G}(0,P(0))]$, by using the inequality in (5) and condition 3.1 .

$$E|\mathcal{A}P(z)|^2 \le 5E|P(0) - \mathcal{G}(0,P(0))|^2 + 5E|\mathcal{G}(z,P(z))|^2$$
$$+5\frac{(1-r)\mu}{C(r)}|P_0 - \mathcal{G}(0,P(0))|E|\phi(z)| \ + 5\left(\frac{\mu r}{C(r)\Gamma(r)}\right)^2\int_0^z (z-x)^{2r-2}dx\int_0^z E|P(x)|^2dx$$
$$+5\left(\frac{C(r)}{1-r}\right)^2\int_0^z E_r^2\left[\frac{-r}{1-r}(z-x)^r\right]E|v(\phi(x))|^2dx$$
$$E|\mathcal{A}P(z)| \le 5C_1^2 + 5\mathcal{M}\mathcal{G}E|P(z)|^2 + 5\frac{(1-r)\mu}{C(r)}C_1E|P(z)|^2$$
$$+5\left(\frac{(1-r)\mu}{C(r)}\right)^2E|P(t)|^2 + 5\left(\frac{r\mu}{C(r)\Gamma(r)}\right)^2\frac{z^{2r-1}}{2r-1}z\, sup_{0\le x\le z}E|P(x)|^2$$
$$3Lip_v^2\left(\frac{C(r)}{1-r}\right)^2\int_0^z E_r^2\left[\frac{-r}{1-r}(z-x)^r\right]E|P(x)|^2dx$$

$$E|\mathcal{A}P(z)| \le 5C_1^2 + 5\mathcal{M}\mathcal{G}E|P(z)|^2 + 5\frac{(1-r)\mu}{C(r)}C_1E|P(z)|^2$$
$$+5\left(\frac{(1-r)\mu}{C(r)}\right)^2E|P(z)|^2 + 5\left(\frac{r\mu}{C(r)\Gamma(r)}\right)^2\frac{z^{2r-1}}{2r-1}z\, sup_{0\le x\le z}E|P(x)|^2$$

$$3Lip_v^2\left(\frac{C(r)}{1-r}\right)^2 sup_{0\le x\le z}E|P(x)|^2\int_0^z E_{2r}^2\ \|x^{r-1}\ E_{r,r}\left[-\frac{r}{1-r}s^r\right]^2\ \Big\|\ dx \qquad (6)$$

Because $E_{r,r}(z) \ge E_r(z)$, the final inequality in (6) follows. Because r is an increasing function we have $\Gamma k + r < rk + 1$ which implies

$$\frac{1}{\Gamma(rk+r)} > \frac{1}{r(rk+1)}.$$

Lemma 2.3 indicates that.

$$E|\mathcal{A}P(z)| \leq 5C_1 + \left[5\mathcal{M}\mathcal{G} + 5\frac{(1-r)\mu}{C(r)}C_1 + 5\left(\frac{(1-r)\mu}{C(r)}\right)^2\right]E|P(x)|^2$$

$$+5\left(\frac{r\mu}{C(r)\Gamma(r)}\right)^2 \frac{z^{2r}}{2r-1} sup_{0\leq x\leq z}E|P(z)|^2$$

$$E|\mathcal{A}P(z)| \leq 5C_1 + \left[5\mathcal{M}\mathcal{G} + 5\frac{(1-r)\mu}{C(r)}C_1 + 5\left(\frac{(1-r)\mu}{C(r)}\right)^2\right]E|P(z)|^2$$

$$+5\left(\frac{r\mu}{C(r)\Gamma(r)}\right)^2 \frac{z^{2r}}{2r-1} \sup_{0\leq x\leq z} E|P(x)|^2$$

$$5\, Lip_v^2 \left(\frac{B(r)}{1-r}\right) m^2\left(r,\frac{r}{1-r}\right) ||P||_2^2 \frac{y^{1-4r}}{1-4r} \tag{8}$$

Now, we have the supremum over $z \in [0, Z]$, on both sides

$$E|\mathcal{A}P(z)| \leq 5C_1 + \left[5\mathcal{M}\mathcal{G} + 5\frac{(1-r)\mu}{C(r)}C_1 + 5\left(\frac{(1-r)\mu}{C(r)}\right)^2\right] \parallel P \parallel_2^2$$

$$+5\left(\frac{r\mu}{C(r)\Gamma(r)}\right)^2 \frac{z^{2r}}{2r-1} \parallel P \parallel_2^2$$

$$5\, Lip_v^2 \left(\frac{B(r)}{1-r}\right) m^2\left(r,\frac{r}{1-r}\right) ||P||_2^2 \frac{y^{1-4r}}{1-4r} \tag{9}$$

Lemma 3.4. Let P_1 and P_2 be some random field solution (i.e., P_1 and P_2) satisfy (3) and (4). Given that Condition 3.1 holds, then for $r \in (0,1)/\left\{\frac{1}{4},\frac{1}{2}\right\}$ and μ large enough

$$\|\mathcal{A}P_1 - \mathcal{A}P_2\|_2^2 \leq C_4\|P_1 - P_2\|_2^2$$

$$C_4 = 4\mathcal{M}\mathcal{G} + 4\left(\frac{\mu(1-r)}{C(r)}\right)^2 + 4\left(\frac{r\mu}{C(r)\Gamma(r)}\right)^2 \frac{Z^{2r}}{2r-1}$$

$$+4Lip_v^2\left(\frac{C(r)}{1-r}\right)^2 m^2\left(r,\frac{r}{1-r}\right)\frac{Z^{1-4r}}{1-4r}$$

Proof. Let $E|\mathcal{G}(z, P_1(z)) - \mathcal{G}(z, P_2(z)) \leq \mathcal{M}\mathcal{G}E|P_1(z) - P_2(z)|^2$.

We use reasoning that is comparable to Lemma 3.3, proof, and we have

$$E|\mathcal{A}P_1(z)-\mathcal{A}P_2(z)|^2$$
$$\leq 4E|\mathcal{G}(z,P_1(z))-\mathcal{G}(z,P_2(z))|^2+4E\left|\frac{\mu(1-r)}{C(r)}[P_1(z)-P_2(z)]\right|^2$$
$$+4E\left|\frac{\mu r}{C(r)\Gamma(r)}\int_0^t (z-x)^{r-1}P(x)dx\right|^2$$
$$+4\frac{C(r)}{1-r}\int_0^z \left|E_r\left[\frac{-r}{1-r}(z-x)^r\right][v(P_1(x)dw(x))-v(P_2(x)dw(x))]\right|^2$$
$$E|\mathcal{A}P_1(z)-\mathcal{A}P_2(z)|^2$$
$$=4E|\mathcal{G}(z,P_1(z))-\mathcal{G}(z,P_2(z))|^2+4\left(\frac{\mu(1-r)}{C(r)}\right)^2 E[P_1(z)-P_2(z)]^2$$
$$+4\left(\frac{\mu r}{C(r)\Gamma(r)}\right)^2\int_0^z (z-x)^{2r-2}E|P_1(x)-P_2(x)|^2dx$$
$$+4\frac{C(r)}{1-r}\int_0^z E_r^2\left[\frac{-r}{1-r}(z-x)^r\right]E[v(P_1(x))-v(P_2(x))]^2dw(x)$$
$$E|\mathcal{A}P_1(z)-\mathcal{A}P_2(z)|^2$$
$$\leq 4\mathcal{M}\mathcal{G}E|P_1(z)-P_2(z)|^2+4\left(\frac{\mu(1-r)}{C(r)}\right)^2 E[P_1(z)-P_2(z)]^2$$
$$+4\left(\frac{\mu r}{C(r)\Gamma(r)}\right)^2\frac{t^{2r}}{2r-1}[sup_{0\leq x\leq z}E|P_1(x)-P_2(x)|^2]$$
$$+4Lip_v^2\left(\frac{C(r)}{1-r}\right)^2 m^2\left(r,\frac{r}{1-r}\right)\frac{z^{1-4r}}{1-4r}\sup_{0\leq x\leq z}E|P_1(x)-P_2(x)|^2$$

$$E|\mathcal{A}P_1(z)-\mathcal{A}P_2(z)|^2$$
$$\leq 4\mathcal{M}\mathcal{G}E|P_1(z)-P_2(z)|^2+4\left(\frac{\mu(1-r)}{C(r)}\right)^2 E[P_1(z)-P_2(z)]^2$$
$$+4\left(\frac{\mu r}{C(r)\Gamma(r)}\right)^2\frac{z^{2r}}{2r-1}\left[\sup_{0\leq x\leq z}E|P_1(x)-P_2(x)|^2\right]$$
$$+4\mathrm{Lip}_v^2\left(\frac{C(r)}{1-r}\right)^2 m^2\left(r,\frac{r}{1-r}\right)\frac{z^{1-4r}}{1-4r}\sup_{0\leq x\leq z}E|P_1(x)-P_2(x)|^2$$

By taking supremum over $z\in[0,Z]$ the desired outcome may be obtained.

Theorem 3.5. By applying Banach fixed point theorem and Lemma 3.3. one gets $P(z)=\mathcal{A}P(z)$, moreover

$$\| P \|_2^2=\| \mathcal{A}P \|_2^2\leq C_2+C_3 \| P \|_2^2$$

which yields

$$(1-C_3)\| P \|_2^2\leq C_2 \rightarrow \| P \|_2<\infty$$

Similarly, Lemma 3.4 provides

$$\|P_1 - P_2\|_2^2 = \|\mathcal{A}P_1 - \mathcal{A}P_2\|_2^2 \leq C_4\|P_1 - P_2\|_2^2 \leq C_3\|P_1 - P_2\|_2^2$$

then $(1 - C_3)\|P_1 - P_2\|_2^2 \leq 0$.

Thus, the existence and uniqueness of a solution are deduced from the principle of Banach contraction.

IV. Conclusion:

In this study delved into the complexities of the Atangana-Baleanu time-stochastic fractional neutral integro-differential equation. Through rigorous analysis and application of Banach's fixed point theory, we have successfully established the existence and uniqueness of the mild solution. The investigation further illuminated the importance of the Mittag-Leffler kernel, known for its non-singular and non-local characteristics, underscoring its pivotal role in elucidating the equation's behavior. By exploring the interplay of fractional differential equations and stochastic differential systems, we have contributed to a deeper understanding of these mathematical phenomena. Our discoveries not only enhance the theoretical framework but also lay the groundwork for practical applications across diverse scientific and engineering fields.

Conflict of Interest:

There was no relevant conflict of interest regarding this paper.

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