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# SOME FEATURES OF FUZZY SOFT $\alpha - T_1$ TOPOLOGICAL SPACES USING FUZZY SOFT POINTS

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**Abstract**

*In this paper, we give new four notions of fuzzy soft α -T_1 topological space using the fuzzy soft points concept. We have discussed some implications and theorems, along with their proofs. Moreover, we have presented the hereditary, productive, and projective properties with proof. Finally, we have shown that our given notions are preserved under bijective, fuzzy soft open, and continuous mappings.*

**Keywords:** Soft set, Fuzzy soft set, Fuzzy soft topological space, Quasi-coincidence, Fuzzy Soft $T_1$ topological spaces.

## I. Introduction

Fuzzy sets are currently the most researched topic in mathematics. This fuzzy set was first defined by the American mathematician L. A. Zadeh [XVII]. After that many other researchers in different branches of mathematics worked on his definitions and theorems and got excellent results. Then Molodtsov [VII] first defined soft set in 1999. Through this soft set, it is possible to explain the topics of game theory, Riemann integration, theory of measurement, and smoothness of functions in more detail. Nazmul and Samanta [VIII] defined and studied Soft Topological Groups and Normal Soft Topological Groups in 2010.

Also, Shabir and Naz [XIII], Tanay and Kandemir [XV] completed research on soft topological space and separation axioms. Saleh et al [XIV] defined the relationship between soft topological space and soft ditopological space, which opened the door to further research. Later, Roy and Samanta [X] defined and discussed fuzzy soft topology over the initial universe set. Karal and Ahmed [V] defined the notion of a mapping on classes of fuzzy soft sets. the notions of fuzzy soft $R_1$ are developed by Ruhul Amin and Raihanul Islam [XI]. Hannan Miah and Ruhul Amin [III] and were also introduced the features of pairwise $\alpha - T_0$ spaces in supra fuzzy bitopology.

Moreover, Ruhul Amin et al [XII] introduced and discussed the fuzzy soft $T_0$ topological spaces in a quasi-coincidence sense.

In this paper, we give four new definitions of fuzzy soft α -T_1 topological space using the fuzzy soft points concept. Based on the definitions, we present some important lemmas and theorems, along with their proofs. To illustrate the non-implications we have taken the help of some examples. Moreover, we have discussed in detail the hereditary, productive, and projective properties with proof. Finally, we discuss that our given notions are preserved under bijective, fuzzy soft open, and fuzzy soft continuous mappings.

## II. Preliminaries

We adopt the following definitions and results from various papers which have simplified and enriched the presentation of our paper.

**Definition 2.1: [**VII**]** A pair (F, E) over an initial universe X is called a soft set if we have a function F such that E to P(X) where P(X) is the collection of all subsets of X. Let A subset of the set of parameters E. We say that (F, E) is a soft set over an initial universe X iff by $F : A \to P(X)$ is a mapping such that

$$F(e) = \begin{cases} F(e) \neq \emptyset \text{ if } e \in A \\ F(e) = \emptyset \text{ if } e \notin A \end{cases}$$

**Definition 2.2: [**X**]** Suppose the set of all fuzzy sets in $X$ is $I^X$ ($I = [0,1]$). A pair $(F, A)$ is said to be a fuzzy soft set over the universe $X$ iff $F$ is a function given by $F: A \to I^X$ defined as

$$F(e) = \begin{cases} F(e) \neq 0_X \text{ if } e \in A \\ F(e) = 0_X \text{ if } e \notin A \end{cases}$$

where $0_X$denotes empty fuzzy set in $X$. The family of all fuzzy soft sets over $X$ is denoted by FSS($X$,). Moreover, a fuzzy soft set $(F, A)$ over $X$ can be expressed by the set of ordered pairs $(F, A) = \{(e, F(e)) : e \in A, F(e) \in I^X\}$.

**Definition 2.3: [VI**]  A fuzzy soft set $f_A \in FSS(X, E)$ is said to be a fuzzy soft point if

$$f_A(e) = \begin{cases} f_A(e) \neq 0_X \text{ if } e \in A \\ f_A(e) = 0_X \text{ if } e \notin A \end{cases}$$

The family of all fuzzy soft points in *X,* denoted by *FSP (X, E).*

**Definition 2.4: [II]** Two fuzzy soft sets are said to be fuzzy soft quasi- coincident if and only if for$(F, E), (G, E) \in FSS(X, E)$, there are $e \in E, x \in X$ such that $F(e)(x) + G(e)(x) > 1$. It is denoted by $(F, E)q(G, E)$. If not quasi-coincident, then $F(e)(x) + G(e)(x) \leq 1$ for all $x \in X$ and denoted by $(F, E)\bar{q}(G, E)$. If $x_\alpha^e$ is a fuzzy soft point and $(G, E)$ is a fuzzy soft set, then $x_\alpha^e(q(F, E)$ means $\alpha + G(e)(x) > 1$.

**Definition 2.5: [X]** Let τ be a class of fuzzy soft sets over a initial universe X with a set of parameters E. Then τ is called a fuzzy soft topology if the following three properties satisfy:

i) $0_E, 1_E \in \tau$, where $0_E(e) = 0$ and $1_E(e) = 1$ for all e ∈ E

ii) The union of any members of $\tau$ again in $\tau$

iii) The intersection of any two members of $\tau$ again belongs to $\tau$.

The triple($X$, $\tau$,$E$) is said to be a fuzzy soft topological space over the initial universe $X$, and members of $\tau$ are called fuzzy open soft sets in (X, $\tau$, E). FOS(X, $\tau$,E), or FOS(X) will denote the set of all fuzzy open soft sets.

**Definition 2.6: [XVI]** The product of two fuzzy soft sets is denoted by $F_A \times G_B$, (where $F_A \in FSS(X,E)$ and $G_B \in FSS(Y,K)$ ) is a fuzzy soft set over X × Y and is defined by $(F_A \times G_B)(e,k) = F_A(e) \times G_B(k)$; $\forall(e,k) \in E \times K$ and for all $(x,y) \in X \times Y$ we have,

$$(F_A \times G_B)(e,k)(x,y) = \big(F_A(e) \times G_B(k)\big)(x,y) = \min\{F_A(e)(x), G_B(k)(y)\}.$$

**Definition 2.7: [I]** For two the families $FSS(X,E)$, $FSS(Y,K)$ of all the fuzzy soft sets over $X$ and $Y$ respectively and the parameters set $E, K$ for the universe $X$ and $Y$ respectively let $u : X \to Y$ and $p : E \to K$ be two maps. Suppose that $f_{up} : FSS(X,E) \to FSS(Y,K)$ is a fuzzy soft mapping from $X$ to $Y$.

*(a) The image of* $(F,A) \in FSS(X,E)$ under the fuzzy soft mapping $f_{up}$ is a fuzzy soft set over Y, denoted by $f_{up}(F,A)$ and is defined by

$$f_{up}(F,A)(k)(y) = \begin{cases} sup\{u(x) = y\}\, sup\{p(e) = k\}\}\, F_A\,(e)(x); if\ u^{-1}\,(y) \neq \emptyset\ and\ p^{-1}(k) \neq \emptyset \\ 0; \end{cases}$$

otherwise

(b) The inverse image of $(G,B) \in FSS(Y,K)$ under the fuzzy soft mapping $f_{up}$is a fuzzy soft set over $X$, denoted by $f_{up}^{-1}\,(G,B)$ and is defined as

$$f_{up}^{-1}(G,B)(e)(x) = (G,B)\big(p(e)\big)\big(u(x)\big); \forall e \in E, x \in X.$$

**Definition 2.8:** [IX] A fuzzy soft mapping $f_{up} : (X, \tau_1, E) \longrightarrow (Y, \tau_2, K)$ is called

(1) fuzzy soft continuous if $f^{-1}{}_{up}(g_E) \in \tau_1$, for all $g_E \in \tau_2$,

(2) fuzzy soft open if $f_{up}(f_E) \in \tau_2$ for all $f_E \in \tau_1$

(3) fuzzy soft homeomorphism if $f_{up}$ is fuzzy soft bijective, fuzzy soft continuous, and fuzzy soft open.

**Definition 2.9:** [IV] Two fuzzy soft points $f_e = x_\alpha^e$ and $g_e = y_\beta^e$ are said to be distinct if and only if $x \neq y$.

## III. Definition and Properties of Fuzzy Soft α-$T_1$ Space

**Definition 3.1** Suppose $(X, \tau, E)$ is a fuzzy soft topological space and $\alpha \in I_1$, then

(a) $(X, \tau, E)$ is a fuzzy soft $\alpha - T_1(i)$ space if and only if for any pair $x_r^e, y_r^e \in FSP\ (X, E)$ with $x \neq y$, there exists $(F, E), (G, E) \in \tau$ such that $F\ (e)(x) = 1, F\ (e)(y) \leq \alpha$ and $G(e)(x) \leq \alpha, G(e)(y) = 1$.

(b) $(X, \tau, E)$ is a fuzzy soft $\alpha - T_1(ii)$ space if and only if for any pair $x_r^e, y_r^e \in FSP\ (X, E)$ with $x \neq y$, there exists $(F, E) \in \tau$ such that $F\ (e)(x) = 0, F\ (e)(y) > \alpha$ and there exists $(G, E) \in \tau$ such that $G(e)(x) > \alpha, G(e)(y) = 0$.

(c) $(X, \tau, E)$ is a fuzzy soft $\alpha - T_1(iii)$ space if and only if for any pair $x_r^e, y_r^e \in FSP\ (X, E)$ with $x \neq y$, there exists $(F, E) \in \tau$ such that $0 \leq F\ (e)(y) \leq \alpha < F\ (e)(x) \leq 1$ and there exists $(G, E) \in \tau$ such that $0 \leq G(e)(x) \leq \alpha < G(e)(y) \leq 1$.

(d) $(X, \tau, E)$ is a fuzzy soft $\alpha - T_1(iv)$ space if and only if for any pair $x_r^e, y_r^e \in FSP\ (X, E)$ with $x \neq y$, there exists $(F, E) \in \tau$ such that $F\ (e)(x) \neq$ $F\ (e)(y)$ and there exists $(G, E) \in \tau$ such that $G(e)(x) \neq G(e)(y)$.

**Lemma 3.1** If $(X, \tau, E)$ is a fuzzy soft topological space and $\alpha \in I_1$, then the following implications are true:

(a) $(X, \tau, E)$ is a $FS\alpha - T_1(i)$ implies $(X, \tau, E)$ is a $FS\alpha - T_1(iii)$ implies $(X, \tau, E)$ is a $FS\alpha - T_1(iv)$.

(b) $(X, \tau, E)$ is a $FS\alpha - T_1(ii)$ implies $(X, \tau, E)$ is a $FS\alpha - T_1(iii)$ implies $(X, \tau, E)$ is a $FS\alpha - T_1(iv)$.

**Proof of (a):** Suppose $(X, \tau, E)$ is a fuzzy soft topological space and $(X, \tau, E)$ is a $FS\alpha - T_1(i)$. We have to prove that $(X, \tau, E)$ is a $FS\alpha - T_1(iii)$.

Let $x_r^e, y_r^e$ be fuzzy soft points in $(X, E)$ with $x \neq y$. Since $(X, \tau, E)$ is a $FS\alpha - T_1(ii)$ space,

for $\alpha \in I_1$ by definition, there exists $(F, E) \in \tau$ such that $F\ (e)(x) = 1, F\ (e)(y) \leq \alpha$ and there exists $(G, E) \in \tau$ such that $G(e)(x) \leq \alpha, G(e)(y) = 1$, which shows that there exists $(F, E) \in \tau$ such that $0 \leq F\ (e)(y) \leq \alpha < F\ (e)(x) \leq 1$ and there exists $(G, E) \in \tau$ such that $0 \leq G(e)(x) \leq \alpha < G(e)(y) \leq 1$.

Hence, by definition (c), $(X, \tau, E)$ is a $FS\alpha - T_1(iii)$.

Suppose $(X, \tau, E)$ is a $FS\alpha - T_1(iii)$. Then for $x_r^e, y_r^e \in FSP\ (X, E)$ with $x \neq y$, there exists $(F, E) \in \tau$ such that $0 \leq F\ (e)(y) \leq \alpha < F\ (e)(x) \leq 1$ i.e. $F\ (e)(x) \neq F\ (e)(y)$ and there exists $(G, E) \in \tau$ such that $0 \leq G(e)(x) \leq \alpha < G(e)(y) \leq 1$ i.e. $G(e)(x) \neq G(e)(y)$.

Hence, by definition (d), $(X, \tau, E)$ is a $FS\alpha - T_1(iv)$.

**Proof of (b):** Suppose $(\boldsymbol{X}, \boldsymbol{\tau}, \boldsymbol{E})$ be a fuzzy soft topological space and $(\boldsymbol{X}, \boldsymbol{\tau}, \boldsymbol{E})$ is a $\boldsymbol{FS\alpha - T_1(ii)}$. We have to prove that $(\boldsymbol{X}, \boldsymbol{\tau}, \boldsymbol{E})$ is a $\boldsymbol{FS\alpha - T_1(ii)}$.Let $\boldsymbol{x_r^e}$, $\boldsymbol{y_r^e}$ be fuzzy soft points in $(\boldsymbol{X}, \boldsymbol{E})$ with $\boldsymbol{x \neq y}$. Since $(\boldsymbol{X}, \boldsymbol{\tau}, \boldsymbol{E})$ is a $\boldsymbol{FS\alpha - T_1(ii)}$ space, for $\boldsymbol{\alpha} \in \boldsymbol{I_1}$ by definition, there exists $(\boldsymbol{F}, \boldsymbol{E}) \in \boldsymbol{\tau}$ such that $\boldsymbol{F}(\boldsymbol{e})(\boldsymbol{x}) = \boldsymbol{0}, \boldsymbol{F}(\boldsymbol{e})(\boldsymbol{y}) > \boldsymbol{\alpha}$ which implies that $\boldsymbol{0} \leq \boldsymbol{F}(\boldsymbol{e})(\boldsymbol{y}) \leq \boldsymbol{\alpha} < \boldsymbol{F}(\boldsymbol{e})(\boldsymbol{x}) \leq \boldsymbol{1}$ and there exists $(\boldsymbol{G}, \boldsymbol{E}) \in \boldsymbol{\tau}$ such that $\boldsymbol{G}(\boldsymbol{e})(\boldsymbol{x}) > \boldsymbol{\alpha}, \boldsymbol{G}(\boldsymbol{e})(\boldsymbol{y}) = \boldsymbol{0}$ which implies that $\boldsymbol{0} \leq \boldsymbol{G}(\boldsymbol{e})(\boldsymbol{x}) \leq \boldsymbol{\alpha} < \boldsymbol{G}(\boldsymbol{e})(\boldsymbol{y}) \leq \boldsymbol{1}$.

Hence, by definition (c), $(X, \tau, E)$ is a $FS\alpha - T_1(iii)$ and therefore $(X, \tau, E)$ is a $FS\alpha - T_1(iv)$.

The non-implications among $FS\alpha - T_1(i), FS\alpha - T_1(ii), FS\alpha - T_1(iii)$ and $FS\alpha - T_1(iv)$ are shown in the following examples:

**Example:** Let $X = \{x, y\}, E = \{e\}$ and $(F, E), (G, E) \in FSS(X, E)$be given by $F(e)(x) = 0.4, F(e)(y) = 0.8$ and $G(e)(x) = 0.8, G(e)(y) = 0.4$. Consider the fuzzy soft topology $\tau$ on $(X, E)$ generated by $\{\emptyset, X, (F, E), (G, E)\}$ where, $(F, E) = \{F(e)(x) = 0.4, F(e)(y) = 0.8, G(e)(x) = 0.8, G(e)(y) = 0.4\}$. For $\alpha = 0.9$, we can easily shown that $(X, \tau, E)$ is a $FS\alpha - T_1(iv)$ but $(X, \tau, E)$ is not a $FS\alpha - T_1(iii)$. It can also be shown that $(X, \tau, E)$ is not a $FS\alpha - T_1(i)$ and $(X, \tau, E)$ is not a $FS\alpha - T_1(ii)$.

**Example:** Let $X = \{x, y\}, E = \{e\}$ and $(F, E), (G, E) \in FSS(X, E)$ be given by $F(e)(x) = 0.4, F(e)(y) = 0.8$ and $G(e)(x) = 0.8, G(e)(y) = 0.4$. Consider the fuzzy soft topology $\tau$ on $(X, E)$ generated by $\{\emptyset, X, (F, E), (G, E)\}$ where,$(F, E) = \{F(e)(x) = 0.4, F(e)(y) = 0.8, G(e)(x) = 0.8, G(e)(y) = 0.4\}$.

For $\alpha = 0.7$ we have,

$0 \leq F(e)(x) \leq 0.7 < F(e)(y) \leq 1$ and $0 \leq G(e)(y) \leq 0.7 < G(e)(x) \leq 1$.

This according to the definition, $(X, \tau, E)$ is a $FS\alpha - T_1(iii)$ but $(X, \tau, E)$ is not a $FS\alpha - T_1(i)$. It can also be shown that $(X, \tau, E)$ is not a $FS\alpha - T_1(ii)$.

**Example:** Let $X = \{x, y\}, E = \{e\}$ and $(F, E), (G, E) \in FSS(X, E)$ be given by $F(e)(x) = 1, F(e)(y) = 0.6$ and $G(e)(x) = 0.6, G(e)(y) = 1$. Consider the fuzzy soft topology $\tau$ on $(X, E)$ generated by $\{\emptyset, X, (F, E), (G, E)\}$ where,

$(F, E) = \{F(e)(x) = 1, F(e)(y) = 0.6, G(e)(x) = 0.6, G(e)(y) = 1\}$.

For $\alpha = 0.7$ we have,

$F(e)(x) = 1, F(e)(y) \leq 0.7 \; and \; G(e)(x) \leq 0.7, G(e)(y) = 1$

This according to the definition, $(X, \tau, E)$ is a $FS\alpha - T_1(i)$ but $(X, \tau, E)$ is not a $FS\alpha - T_1(ii)$.

**Example:** Let $X = \{x, y\}, E = \{e\}$ and $(F, E), (G, E) \in FSS(X, E)$ be given by $F(e)(x) = 0, F(e)(y) = 0.7$ and $G(e)(x) = 0.7, G(e)(y) = 0$. Consider the fuzzy soft topology $\tau$ on $(X, E)$ generated by $\{\emptyset, X, (F, E), (G, E)\}$ where, $(F, E) = \{F(e)(x) = 0, F(e)(y) = 0.7, G(e)(x) = 0.7, G(e)(y) = 0\}$.

For $\alpha = 0.3$, it is clear that $(X, \tau, E)$ is a $FS\alpha - T_1(ii)$ but not a $FS\alpha - T_1(i)$.

**Lemma 3.2** Let $(X, \tau, E)$ is a fuzzy soft topological space and $\alpha, \beta \in I_1$ with $0 \leq \alpha \leq \beta < 1$, then

*(a)* $(X, \tau, E)$ is a $FS\alpha - T_1(i)$ implies $(X, \tau, E)$ is a $FS\beta - T_1(i)$.

*(b)* $(X, \tau, E)$ is a $FS\beta - T_1(ii)$ implies $(X, \tau, E)$ is a $FS\alpha - T_1(ii)$.

*(c)* $(X, \tau, E)$ is a $FS0 - T_1(ii)$ implies $(X, \tau, E)$ is a $FS0 - T_1(iii)$.

**Proof:** Suppose $(\boldsymbol{X}, \boldsymbol{\tau}, \boldsymbol{E})$ be a fuzzy soft topological space and $(\boldsymbol{X}, \boldsymbol{\tau}, \boldsymbol{E})$ is a $\boldsymbol{FS\alpha - T_1(i)}$. We have to prove that $(\boldsymbol{X}, \boldsymbol{\tau}, \boldsymbol{E})$ is a $\boldsymbol{FS\beta - T_1(i)}$.

Let $x_r^e, y_r^e$ be fuzzy soft points in $(X, E)$ with $x \neq y$. Since $(X, \tau, E)$ is a $FS\alpha - T_1(i)$ space, for $\alpha \in I_1$ by definition, there exists $(F, E) \in \tau$ such that $F(e)(x) = 1, F(e)(y) \leq \alpha$ and there exists $(G, E) \in \tau$ such that $G(e)(x) \leq \alpha, G(e)(y) = 1$. This shows that there is $(F, E) \in \tau$ so that $F(e)(x) = 1$, $F(e)(y) \leq \beta$, since $0 \leq \alpha \leq \beta < 1$ and there is $(G, E) \in \tau$ so that $G(e)(x) \leq \beta, G(e)(y) = 1$, since $0 \leq \alpha \leq \beta < 1$. Hence, by definition, $(X, \tau, E)$ is a $FS\beta - T_1(i)$. Suppose $(X, \tau, E)$ is a $FS\beta - T_1(ii)$. Then for $x_r^e, y_r^e \in FSP(X, E)$ with $x \neq y$, there exists $(F, E) \in \tau$ such that $F(e)(x) = 0, F(e)(y) > \beta$, which implies that $F(e)(x) = 0, F(e)(y) > \alpha$, since $0 \leq \alpha \leq \beta < 1$. Thus there is $(G, E) \in \tau$ so that $G(e)(x) > \beta, G(e)(y) = 0$, since $0 \leq \alpha \leq \beta < 1$. Hence by definition, $(X, \tau, E)$ is a $FS\alpha - T_1(ii)$.

**Example:** Let $X = \{x, y\}, E = \{e\}$ and $(F, E), (G, E) \in FSS(X, E)$ be given by $F(e)(x) = 1, F(e)(y) = 0.7$ and $G(e)(x) = 0.7, G(e)(y) = 1$. Consider the fuzzy soft topology $\tau$ on $(X, E)$ generated by $\{\emptyset, X, (F, E), (G, E)\}$ where, $(F, E) = \{F(e)(x) = 1, F(e)(y) = 0.7, G(e)(x) = 0.7, G(e)(y) = 1\}$.

For $\alpha = 0.4$ and $\beta = 0.9$; $(X, \tau, E)$ is a $FS\beta - T_1(i)$ but $(X, \tau, E)$ is not a $FS\alpha - T_1(i)$.

**Example:** Let $X = \{x, y\}, E = \{e\}$ and $(F, E), (G, E) \in FSS(X, E)$ be given by $F(e)(x) = 0, F(e)(y) = 0.5$ and $G(e)(x) = 0.5, G(e)(y) = 0$. Consider the fuzzy soft topology $\tau$ on $(X, E)$ generated by $\{\emptyset, X, (F, E), (G, E)\}$ where, $(F, E) = \{F(e)(x) = 0, F(e)(y) = 0.5, G(e)(x) = 0.5, G(e)(y) = 0\}$.

For $\alpha = 0.4$ and $\beta = 0.6$; $(X, \tau, E)$ is a $FS\alpha - T_1(ii)$ but $(X, \tau, E)$ is not a $FS\beta - T_1(ii)$.

In the following theorem, we shall show that our notions satisfy good extension property.

**Theorem 3.1** Let $(X, \tau, E)$ be a fuzzy soft topological space. Consider the following statements:

1. $(X, \tau, E)$ be a soft $T_1$ topological space.
2. $(X, \omega(\tau), E)$ be a $FS\alpha - T_1(i)$ space.
3. $(X, \omega(\tau), E)$ be a $FS\alpha - T_1(ii)$ space.
4. $(X, \omega(\tau), E)$ be a $FS\alpha - T_1(iii)$ space.
5. $(X, \omega(\tau), E)$ be a $FS\alpha - T_1(iv)$ space.

The following implication are true

(a) $(\mathbf{1}) \Leftrightarrow (\mathbf{2}) \Rightarrow (\mathbf{4}) \Rightarrow (\mathbf{5})$

(b) $(\mathbf{3}) \Rightarrow (\mathbf{4}) \Rightarrow (\mathbf{5})$.

**Proof :** Firstly, we want to show that $(\mathbf{1}) \Leftrightarrow (\mathbf{2})$. Suppose that $(\boldsymbol{X}, \boldsymbol{\tau}, \boldsymbol{E})$ is soft $\boldsymbol{T_1}$ topological space. Now to prove that $(\boldsymbol{X}, \boldsymbol{\omega}(\boldsymbol{\tau}), \boldsymbol{E})$ is $\boldsymbol{FS\alpha - T_1(i)}$ space, let $\boldsymbol{x_r^e}, \boldsymbol{y_s^e}$ be fuzzy points in $(\boldsymbol{X}, \boldsymbol{E})$ with $\boldsymbol{x \neq y}$. Since $(\boldsymbol{X}, \boldsymbol{\tau}, \boldsymbol{E})$ is $\boldsymbol{T_1}$ soft topological space we have, there exists $(\boldsymbol{U}, \boldsymbol{E}), (\boldsymbol{V}, \boldsymbol{E}) \in \boldsymbol{\tau}$ such that $\boldsymbol{x_r^e} \in (\boldsymbol{U}, \boldsymbol{E}), \boldsymbol{y_s^e} \notin \boldsymbol{U}, \boldsymbol{E})$ and $\boldsymbol{y_s^e} \in (\boldsymbol{V}, \boldsymbol{E}), \boldsymbol{x_r^e} \notin (\boldsymbol{V}, \boldsymbol{E})$.

Using soft lower semi-continuous (lsc) we get,

$1_{(U,E)}, 1_{(V,E)} \in \omega(\tau)$ and $1_U(e)(x) = 1, 1_U(e)(y) = 0, 1_V(e)(y) = 1,$

$$1_V(e)(x) = 0; \forall e \in E.$$

Then $1_U(e)(x) = 1$

and $1_U(e)(y) = 0$

$\Rightarrow 1_U(e)(y) + r \leq 1$

$\Rightarrow 1_U(e)(y) \leq 1 - r = \alpha$

$\Rightarrow 1_U(e)(y) \leq \alpha$

Also,

$1_V(e)(x) = 0$

$\Rightarrow 1_V(e)(x) + r \leq 1$

$\Rightarrow 1_V(e)(x) \leq 1 - r = \alpha$

$\Rightarrow 1_V(e)(x) \leq \alpha$ and $1_V(e)(y) = 1$

It follows that there exists $1_{(U,E)}, 1_{(V,E)} \in \omega(\tau)$ such that $1_U(e)(x) = 1, 1_U(e)(y) \leq$
$\alpha$ and $1_V(e)(x) \leq \alpha, 1_V(e)(y) = 1$.

Hence, we have $(X, \omega(\tau), E)$ is a $FS\alpha - T_1(i)$ space. Thus $(1) \Rightarrow (2)$ holds.

Conversely,

Let $(X, \omega(\tau), E)$ be a fuzzy soft topological space and $(X, \omega(\tau), E)$ is $FS\alpha - T_1(i)$. Now to show that $(X, \tau, E)$ is soft $T_1$ topological space, suppose $x, y$ points in X with $x \neq y$. Since $(X, \omega(\tau), E)$ is $FS\alpha - T_1(i)$ soft topological Space, we have, for any fuzzy soft points $x_r^e, y_r^e$ in $(X, E)$ there exists $(F, E) \in \tau$ such that $F(e)(x) = 1, F(e)(y) \leq \alpha$ and there exists $(G, E) \in \tau$ such that $G(e)(x) \leq \alpha, G(e)(y) = 1$.
Now,

$F(e)(x) = 1$

$\Rightarrow F(e)(x) + r > 1, \forall e \in E, r \in (0, 1]$

$\Rightarrow F(e)(x) > 1 - r = \alpha$

$\Rightarrow F(e)(x) > \alpha$

$\Rightarrow x \in (F, E) - 1(\alpha, 1]$

And $F(e)(y) \leq \alpha$

$\Rightarrow y \notin (F, E) - 1(\alpha, 1]$

Also, $(F, E) - 1(\alpha, 1] \in \tau$

Similarly, we can show that $y \in (G, E) - 1(\alpha, 1], x \notin (G, E) - 1(\alpha, 1]$. Hence, $(X, \tau, E)$ is soft $T_1$ topological space. Thus $(2) \Rightarrow (1)$ holds. Hence, $(1) \Leftrightarrow (2)$.

Again,

Since $(X, \omega(\tau), E)$ be a $FS\alpha - T_1(i)$, for $\alpha \in I$, by definition there exists $1_{(U,E)}, 1_{(V,E)} \in \omega(\tau)$ such that $1_U(e)(x) = 1, 1_U(e)(y) \leq \alpha$ and $1_V(e)(x) \leq \alpha, 1_V(e)(y) = 1$ which shows that there exists $1_{(U,E)} \in \omega(\tau)$ such that $0 \leq 1_U(e)(y) \leq \alpha < 1_U(e)(x) \leq 1$ and there exists $1_{(V,E)} \in \omega(\tau)$ such that $0 \leq 1_V(e)(x) \leq \alpha < 1_V(e)(y) \leq 1$.

Hence, $(X, \omega(\tau), E)$ is a $FS\alpha - T_1(iii)$. Thus $(2) \Rightarrow (4)$ holds.

Suppose $(X, \omega(\tau), E)$ is a $FS\alpha - T_1(iii)$. Then for $\alpha \in I$, by definition there exists $1_{(U,E)}, 1_{(V,E)} \in \omega(\tau)$ such that $0 \leq 1_U(e)(y) \leq \alpha < 1_U(e)(x) \leq 1$ i.e. $1_U(e)(x) \neq 1_U(e)(y)$ and $0 \leq 1_V(e)(x) \leq \alpha < 1_V(e)(y)$ 1 i.e. $1_V(e)(x) \neq 1_V(e)(y)$.

Hence by definition, $((X, \omega(\tau), E)$ is a $FS\alpha - T_1(iv)$. Thus $(4) \Rightarrow (5)$ holds. Therefore, $(1) \Leftrightarrow (2) \Rightarrow (4) \Rightarrow (5)$.

**Proof of (b):** First we show that $(\mathbf{3}) \Rightarrow (\mathbf{4})$. Since $(\boldsymbol{X}, \boldsymbol{\omega}(\boldsymbol{\tau}), \boldsymbol{E})$ be a $\boldsymbol{FS\alpha} - \boldsymbol{T_1(ii)}$ space. Then by definition, for $\boldsymbol{\alpha} \in \boldsymbol{I_1}$, there exists $\mathbf{1}_{(\boldsymbol{U},\boldsymbol{E})}, \mathbf{1}_{(\boldsymbol{V},\boldsymbol{E})} \in \boldsymbol{\omega}(\boldsymbol{\tau})$ such that $\mathbf{1}_{\boldsymbol{U}}(\boldsymbol{e})(\boldsymbol{x}) = \mathbf{0}, \mathbf{1}_{\boldsymbol{U}}(\boldsymbol{e})(\boldsymbol{y}) > \boldsymbol{\alpha}$ and $\mathbf{1}_{\boldsymbol{V}}(\boldsymbol{e})(\boldsymbol{x}) > \boldsymbol{\alpha}, \mathbf{1}_{\boldsymbol{V}}(\boldsymbol{e})(\boldsymbol{y}) = \mathbf{0}$ which implies that there exists $\mathbf{1}_{(\boldsymbol{U},\boldsymbol{E})} \in \boldsymbol{\omega}(\boldsymbol{\tau})$ such that $\mathbf{0} \leq \mathbf{1}_{\boldsymbol{U}}(\boldsymbol{e})(\boldsymbol{y}) \leq \boldsymbol{\alpha} < \mathbf{1}_{\boldsymbol{U}}(\boldsymbol{e})(\boldsymbol{x}) \leq \mathbf{1}$ and there exists $\mathbf{1}_{(\boldsymbol{V},\boldsymbol{E})} \in \boldsymbol{\omega}(\boldsymbol{\tau})$ such that $\mathbf{0} \leq \mathbf{1}_{\boldsymbol{V}}(\boldsymbol{e})(\boldsymbol{x}) \leq \boldsymbol{\alpha} <$

$\mathbf{1}_V\,(e)(y) \,\leq\, \mathbf{1}$.

Hence by definition, $(\mathbf{X}, \boldsymbol{\omega}(\boldsymbol{\tau}\,), \mathbf{E})$ is a $\mathbf{FS\alpha - T_1(iii)}$ and therefore $(\mathbf{X}, \boldsymbol{\omega}(\boldsymbol{\tau}\,), \mathbf{E})$ is a $\mathbf{FS\alpha - T_1(iv)}$. Hence, $(\mathbf{3}) \Rightarrow (\mathbf{4}) \Rightarrow (\mathbf{5})$.

Now we show that our notions obey the hereditary property in the following theorem.

**Theorem 3.2** Let $(\mathbf{X}, \boldsymbol{\tau}, \mathbf{E})$ be a fuzzy soft topological space. If $\mathbf{A} \subseteq \mathbf{X}, \mathbf{t_A} = \{(\mathbf{F_A}, \mathbf{E}) = (\mathbf{F}, \mathbf{E}) \cap \mathbf{A} : (\mathbf{F}, \mathbf{E}) \in \boldsymbol{\tau}\}$ then

$(\mathbf{X}, \boldsymbol{\tau}, \mathbf{E})\mathbf{is\ FS\alpha - T_1(j)} \Rightarrow (\mathbf{A}, \mathbf{t_A}, \mathbf{E})\mathbf{is\ FS\alpha - T_1(j)}$ where $\mathbf{j = i, ii, iii, iv}$.

**Proof :** Suppose that $(X, \tau, E)$ is $FS\alpha - T_1(i)$. To show that $(A, t_A, E)$ is $FS\alpha - T_1(i)$, suppose $x_r^e, y_s^e$ be fuzzy soft points in $(A, E)$ with $x \neq y$. Since given that $A \subseteq X$, these fuzzy soft points belong to $(X, E)$. Also, since $(X, \tau, E)$ is $FS\alpha - T_1(i)$ space, we have for $\alpha \in I_1$, there exists $(F, E), (G, E) \in \tau$ such that $F\,(e)(x) = 1, F\,(e)(y) \leq \alpha$ and $G(e)(x) \leq \alpha, G(e)(y) = 1$.

For $A \subseteq X$, we have $(F, E) \cap A = (F_A, E)$.

Now,

$$F\,(e)(x) = 1$$

$$\Rightarrow F\,(e)(x) \cap A(x) = 1, x \in A \subseteq X, e \in E$$

$$\Rightarrow ((F, E) \cap A)(e)(x) = 1$$

$$\Rightarrow (F_A, E)(e)(x) = 1$$

And

$$F\,(e)(y) \leq \alpha$$

$$\Rightarrow F\,(e)(y) \cap A(y) \leq \alpha, y \in A \subseteq X, e \in E$$

$$\Rightarrow ((F, E) \cap A)(e)(y) \leq \alpha$$

$$\Rightarrow (F_A, E)(e)(y) \leq \alpha$$

Similarly, we can show that $(G_A, E)(e)(x) \leq \alpha$ and $(G_A, E)(e)(y) = 1$.

It follows that there exists $(F_A, E) \in t_A$ such that $(F_A, E)(e)(x) = 1, (F_A, E)(e)(y) \leq \alpha$ and there exists $(G_A, E) \in t_A$ such that $(G_A, E)(e)(x) \leq \alpha,\ (G_A, E)(e)(y) = 1$.

Hence by definition, $(A, t_A, E)$ is $FS\alpha - T_1(i)$.

In the same way, we can can easily profe the theorem for $j = ii, iii, iv$.

In the next two theorems, we will discuss and prove that our new notions satisfy productive property as well as projective property.

**Theorem 3.3** Suppose $(X_i, \tau_i, E_i), i \in \Lambda$ be a fuzzy soft topological spaces and

$X = \prod_{i\in\Lambda} X_i,\ E = \prod_{i\in\Lambda} E_i$ and $\tau$ be the fuzzy soft topology on $(X, E)$, then for all $i \in \Lambda$, $(X_i, \tau_i, E_i)$ is $FS\alpha - T_1(j)$ if and only if $(X, \tau, E)$ is $FS\alpha - T_1(j)$ where $j = i, ii, iii, iv$

**Proof:** Suppose for all $\boldsymbol{i} \in \boldsymbol{\Lambda}, (\boldsymbol{X_i}, \boldsymbol{\tau_i}, \boldsymbol{E_i})$ is $\boldsymbol{FS\alpha - T_1(i)}$ space. Now, we want to show that (X, τ, E) is FSα-$T_1$(j) where j=i. Let $\boldsymbol{x_r^e}, \boldsymbol{y_s^e}$ be fuzzy points in $(\boldsymbol{X}, \boldsymbol{E})$ with $\boldsymbol{x} \neq \boldsymbol{y}$. Then $(\boldsymbol{x_i})_r^e, (\boldsymbol{y_i})_s^e$ are fuzzy soft points with $\boldsymbol{xi} \neq \boldsymbol{yi}$ for some $\boldsymbol{i} \in \boldsymbol{\Lambda}$. Since $(\boldsymbol{X_i}, \boldsymbol{\tau_i}, \boldsymbol{E_i})$ is $\boldsymbol{FS\alpha - T_1(i)}$, there exists $(\boldsymbol{F_i}, \boldsymbol{E_i}), (\boldsymbol{G_i}, \boldsymbol{E_i}) \in \boldsymbol{\tau}$ such that $\boldsymbol{F_i(e_i)(x_i) = 1}, \boldsymbol{F_i(e_i)(y_i) \leq \alpha}$ and $\boldsymbol{G_i(e_i)(x_i) \leq \alpha}, \boldsymbol{G_i(e_i)(y_i) = 1}$.

But we have $PX_i(x) = x_i,\ PX_i(y) = y_i, qE_i(e) = e_i$.

Now, $F_i(e_i)(x_i)$=1 $\Longrightarrow F_i\big(qE_i(e)\big)\big(PX_i(x)\big) = 1, \forall\, x_i \in X_i, e_i \in E_i$

$\Longrightarrow (F_i{\circ}qE_i(e)\ \big(PX_i(x)\big) = 1$

$\Longrightarrow (F_i{\circ}qE_i{\circ}\ PX_i)(e)(x) = 1$

And $F_i(e_i)(y_i) \leq \alpha \ \Rightarrow\ F_i\big(qE_i(e)\big)\big(PX_i(y)\big) \leq \alpha, \forall y_i \in X_i, e_i \in E_i$

$\Rightarrow\ (F_i \circ qE_i)(e)(PX_i(y)) \leq \alpha$

$\Rightarrow\ (F_i \circ qE_i{\circ}\ PX_i)(e)(y) \leq \alpha$

Similarly, we can show that $(G_i{\circ}qE_i{\circ}PX_i)(e)(x) \leq \alpha$, $(G_i{\circ}qE_i{\circ}PX_i)(e)(y) = 1$.
It follows that there exists $((F_i{\circ}qE_i{\circ}PX_i, E)$, $(G_i{\circ}qE_i{\circ}PX_i, E) \in \tau_i$ such that

$(F_i{\circ}qE_i{\circ}PX_i)(e)(x) = 1$, $(F_i{\circ}qE_i{\circ}PX_i)(e)(y) \leq \alpha$ and

$(G_i{\circ}qE_i{\circ}PX_i)(e)(x) \leq \alpha$, $(G_i{\circ}qE_i{\circ}PX_i)(e)(y) = 1$.

Hence, $(X, \tau, E)$ is $FS\alpha - T_1(i)$.
Conversely, Suppose that $(\mathrm{X}, \tau, \mathrm{E})$ is $\mathrm{FS\alpha - T_1(i)}$. We have to prove that $(\mathrm{X_i}, \tau_\mathrm{i}, \mathrm{E_i}), \mathrm{i} \in \Lambda$ is $\mathrm{FS\alpha - T_1(i)}$. Let $\mathrm{a_i}$ be a fixed element in $\mathrm{X_i}$. Let $\mathrm{A_i} = \{\mathrm{x} \in \mathrm{X} = \prod_{\mathrm{i}\in\Lambda} \mathrm{X_i} : \mathrm{x_j} = \mathrm{a_j}$ for some $\mathrm{i} \neq \mathrm{j}\}$. Then $\mathrm{A_i}$ is a subset of X and hence $(\mathrm{A_i}, \tau\mathrm{A_i}, \mathrm{E_i})$ is a subspace of $(\mathrm{X}, \tau, \mathrm{E})$. Since $(\mathrm{X}, \tau, \mathrm{E})$ is $\mathrm{FS\alpha - T_1(i)}$, so $(\mathrm{A_i}, \tau\mathrm{A_i}, \mathrm{E_i})$ is $\mathrm{FS\alpha - T_1(i)}$. Now we have, $(\mathrm{A_i}, \mathrm{E_i})$ is homeomorphic image of $(\mathrm{X_i}, \mathrm{E_i})$. Thus, it is proved that for all $\mathrm{i} \in \Lambda, (\mathrm{X_i}, \tau_\mathrm{i}, \mathrm{E_i})$ is $\mathrm{FS\alpha - T_1(i)}$ space.
In the same way, we can prove the others.

**Theorem 3.4** Suppose $(\mathrm{X}, \tau_1, \mathrm{E})$ and $(\mathrm{Y}, \tau_2, \mathrm{K})$ be two fuzzy soft topological spaces. If $\mathrm{u} : \mathrm{X} \to \mathrm{Y}, \mathrm{p} : \mathrm{E} \to \mathrm{K}$ be bijective fuzzy open maps and suppose that fuzzy soft mapping $\mathrm{f_{up}} : \mathrm{FSS(X, E)} \to \mathrm{FSS(Y, K)}$ be a one-one, onto and fuzzy soft open map, then $(\mathrm{X}, \tau_1, \mathrm{E})$ is $\mathrm{FS\alpha - T_1(j)} \Rightarrow (\mathrm{Y}, \tau_2, \mathrm{K})$ is $\mathrm{FS\alpha - T_1(j)}$, where $\mathrm{j} = \mathrm{i, ii, iii, iv}$

**Proof:** We prove only j = ii here. Other proofs are the same. Suppose (X, τ1, E) be a fuzzy soft topological space and $(\mathrm{X}, \tau_1, \mathrm{E})$ is $\mathrm{FS\alpha - T_1(ii)}$. For $(\mathrm{Y}, \tau_2, \mathrm{K})$ is $\mathrm{FS\alpha - T_1(ii)}$, let $\mathrm{x_r'^k}, \mathrm{y_s'^k}$ be fuzzy points in (Y, K) with $\mathrm{x'} \neq \mathrm{y'}$ . Since $\mathrm{f_{up}}$ is onto and so u, p are onto, then there exists $\mathrm{x, y} \in \mathrm{X}$ with $\mathrm{u(x) = x'}, \mathrm{u(y) = y'}$ and there exists $\mathrm{e} \in \mathrm{E}$ with $\mathrm{p(e) = k}, \forall \mathrm{k} \in \mathrm{K}$. Also $\mathrm{x_r^e}, \mathrm{y_s^e}$ are fuzzy points in (X, E) with $\mathrm{x} \neq \mathrm{y}$ as $\mathrm{f_{up}}$ is one-one.

Again, since $(\mathrm{X}, \tau_1, \mathrm{E})$ is $\mathrm{FS\alpha - T_1(ii)}$ space, there exists $(\mathrm{F, E}), (\mathrm{G, E}) \in \tau_1$ such that

$\mathrm{F\,(e)(x) = 0, F\,(e)(y) > \alpha}$ and $\mathrm{G(e)(x) > \alpha, G(e)(y) = 0}$. We have,

$f_{up}(F,E)(k)(x') = \sup\{u(x) = x'\}\sup\{p(e) = k\}F(e)(x)$

$\Rightarrow f_{up}(F,E)(k)(x') = F(e)(x)$, for some x

and

$f_{up}(F,E)(k)(y') = \sup\{u(y) = y'\}\sup\{p(e) = k\}F(e)(y)$

$\Rightarrow f_{up}(F,E)(k)(y') = F(e)(y)$, for some y

Also since $f_{up}$ is a soft open map then $f_{up}(F,E) \in \tau_2$ as $(F,E) \in \tau_1$. Now, $F(e)(x) = 0$

$\Rightarrow f_{up}(F,E)(k)(x') = 0, \forall x \in X, e \in E, \forall x' \in Y, k \in K$

and $F(e)(y) > \alpha$

$\Rightarrow f_{up}(F,E)(k)(y') > \alpha, \forall y' \in Y, k \in K$

Similarly, we can show that $f_{up}(G,E)(k)(x') > \alpha$ and $f_{up}(G,E)(k)(y') = 0$.

It follows that there exists $f_{up}(F,E), fup(G,E) \in \tau_2$ such that $f_{up}(F,E)(k)(x') = 0$, $f_{up}(F,E)(k)(y') > \alpha$ and $f_{up}(G,E)(k)(x') > \alpha$, $f_{up}(G,E)(k)(y') = 0$. Hence, $(Y,\tau_2,K)$ is $FS\alpha - T_1$(ii).

**Theorem 3.5** Let $(X,\tau_1,E)$ and $(Y,\tau_2,K)$ be two fuzzy soft topological spaces. Let $u : X \to Y, p : E \to K$ be one-one and continuous maps and hence a fuzzy soft mapping $fup : FSS(X,E) \to FSS(Y,K)$ be an injective and fuzzy soft continuous map, then $(Y,\tau_2,K)$ is $FS\alpha - T_1(j) \Rightarrow (X,\tau_1,E)$ is $FS\alpha - T_1(j)$, where $j = i, ii, iii, iv$.

**Proof:** Suppose $(Y,\tau_2,K)$ be a fuzzy soft topological space, and $(Y,\tau_2,K)$ is $FS\alpha - T_1$(iii). For $(X,\tau_1,E)$ is $FS\alpha - T_1$(iii), let $x_r^e, y_s^e$ be fuzzy soft points in $(X,E)$ with $x \neq y$. Then there is $e \in E$ such that $p(e) = k$, for all $k \in K$ and also $(u(x))_r^x, (u(y))_s^k$ are fuzzy soft points in $(Y,K)$ with $u(x) \neq u(y)$ as $f_{up}$ is injective.

Again, since $(Y,\tau_2,K)$ is $FS\alpha - T_1$(iii) space, there exists $(F,K),(G,K) \in \tau_2$ such that $0 \le (F,K)\big(u(y)\big)(k) \le \alpha < (F,K)(u(x))(k) \le 1$ and

$0 \le (G,K)(u(x))(k) \le \alpha < (G,K)(u(y))(k) \le 1$.

Now,

$0 \le (F,K)(u(y))(k) \le \alpha < (F,K)(u(x))(k) \le 1$

$\Rightarrow 0 \le (F,K)(u(y))(p(e)) \le \alpha < (F,K)(u(x))(p(e)) \le 1, x,y \in X, k \in K, e \in E$

$\Rightarrow 0 \le (F,K)(p(e))(u(y)) \le \alpha < (F,K)(p(e)(u(x)) \le 1$

$\Rightarrow 0 \le f_{up}^{-1}(F,K)(e)(y) \le \alpha < f_{up}^{-1}(F,K)(e)(x) \le 1$

Similarly, we can prove that $0 \le f_{up}^{-1}(G,K)(e)(x) \le \alpha < f_{up}^{-1}(G,K)(e)(y) \le 1$.

Now since, $f_{up}^{-1}$ is soft continuous map and $(F,K),(G,K) \in \tau_2$ then $f_{up}^{-1}(F,K), f_{up}^{-1}(G,K) \in \tau_1$.

It follows that there exists $f_{up}^{-1}(F,K), f_{up}^{-1}(G,K) \in \tau_1$ such that $0 \leq f_{up}^{-1}(F,K)(e)(y) \leq \alpha < f_{up}^{-1}(F,K)(e)(x) \leq 1$

and $0 \leq f_{up}^{-1}(G,K)(e)(x) \leq \alpha < f_{up}^{-1}(G,K)(e)(y) \leq 1$.

Hence, $(X, \tau_1, E)$ is $FS\alpha - T_1(iii)$.

Now, we want to show that if $(Y, \tau_2, K)$ is $FS\alpha - T_1(iv)$ then $(X, \tau_1, E)$ is $FS\alpha - T_1(iv)$. For this, let $x_r^e, y_s^e$ be fuzzy soft points in $(X, E)$ with $x \neq y$. Then there exists $e \in E$ such that $p(e) = k, \forall k \in K$ and also $(u(x)_r^x), (u(y)_s^k)$ are fuzzy soft points in $(Y, K)$ with $u(x) \neq u(y)$ as $f_{up}$ is one-one.

Again, since $(Y, \tau_2, K)$ is $FS\alpha - T_1(iv)$ space, there exists $(F,K), (G,K) \in \tau_2$ such that $(F,K)(u(x))(k) \neq (F,K)(u(y))(k)$ and $(G,K)(u(x))(k) \neq (G,K)(u(y))(k)$.

Now,

$$(F,K)(u(x))(k) \neq (F,K)(u(y))(k)$$

$\Rightarrow (F,K)(u(x))(p(e)) \neq (F,K)(u(y))(p(e)), x, y \in X, k \in K, e \in E$

$\Rightarrow (F,K)(p(e))(u(x)) \neq (F,K)(p(e)(y))(u(y))$

$$\Rightarrow f_{up}^{-1}(F,K)(e)(x) \neq f_{up}^{-1}F,K)(e)(y)$$

Similarly, we can prove that $f_{up}^{-1}(G,K)(e)(x) \neq f_{up}^{-1}(G,K)(e)(y)$.

Now since, $f_{up}$ is a soft continuous map and $(F,K), (G,K) \in \tau_2$ then $f_{up}^{-1}(F,K), f_{up}^{-1}(G,K) \in \tau_1$

It follows that there exists $f_{up}^{-1}(F,K)$, $f_{up}^{-1}(G,K) \in \tau_1$ such that $f_{up}^{-1}(F,K)(e)(x) \neq f_{up}^{-1}(F,K)(e)(y)$ and $f_{up}^{-1}(G,K)(e)(x) \neq f_{up}^{-1}(G,K)(e)(y)$.

Hence, $(X, \tau_1, E)$ is $FS\alpha - T_1(iv)$.

## IV. Conclusion

Nowadays, many mathematicians have extensively studied soft set theory, which was introduced by Molodtsov [VI] and the obtained results have been successfully applied in real life, playing an important role in solving many uncertainty problems. In the present work, we have continued to study the properties of fuzzy soft topological spaces. We introduce some new concepts of fuzzy soft $T_1$ separation in fuzzy soft topological spaces using the fuzzy soft points concept. To expand this work, shortly, we will focus more on applying our results to different branches of topology. We hope that the findings in this paper will help researchers enhance and promote further study on fuzzy soft topology.

**Conflict of Interest:**

There is no conflict of interest regarding this article

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