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MOTION OF NON-NEWTONIAN FLUID BETWEEN TWO CO-AXIAL POROUS CIRCULAR CYLINDER

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Abstract

This paper aims to study the flow of a non-Newtonian fluid of Reiner-Rivlin type between two co-axial porous circular cylinders. The inner cylinder moves with a transient velocity while the outer one is fixed.

Keywords: Co-axial Porous, Non-Newtonian fluid, Transient Velocity.

I. Introduction

Khamrui [III] studied the flow of an ordinary viscous fluid between two co-axial porous circular cylinders when the inner cylinder is oscillating. Flow of non-Newtonian fluid contained in a fixed circular cylinder due to the longitudinal oscillation of a concentric circular cylinder has been investigated by Bagchi [I].

In this paper an attempt has been made to investigate the motion of a Reiner – Rivilin fluid through two porous concentric circular cylinders when the inner cylinder performs a transient movement and the outer one is at rest. The exact solution of this problem has been obtained in terms of Hankel functions and approximate results for the two extreme cases have been derived. Also the authors have obtained the solution for the transient movement of a piston in the non-Newtonian fluid contained in a circular cylinder as a particular case.

We know that a Reiner – Rivilin fluid (Rivilin [IV]) is characterized by the constitutive equations.

$$\tau_{ij} = -p\delta_{ij} + \tau'_{ij}$$
$$\tau'_{ij} = \mu e_{ij} + \mu_c e_{ik} e_{kj} \quad (1)$$

Where μ is coefficient of viscosity, μ_c is the coefficient of cross-viscosity τ_{ij} is the stress tensor, e_{ij} is the ratio of strain tensor and $\delta_{ij} = 1$, i = j and $\delta_{ij} = 0$, i ≠ j.

II. Formulation and Solution of the Problem

Let us consider 'a' and 'b' be the radii of the inner and the outer cylinder respectively (b > a). Considering the cylindrical polar co-ordinates (r, q, z) with z-axis as the common axis of the cylindrical and (u, v, w) as the components of velocity in the directions of r, θ and z increasing respectively, we see that v = 0. We assume that all the physical quantities are independent of θ and z. Then the equations. Of motion are

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r}\right) = \frac{\partial \tau_{rr}}{\partial r} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\tau_{rr} - \tau_{\theta\theta}}{r}$$

and

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r}\right) = \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) + \frac{\partial \tau_{rz}}{\partial z} \qquad (2)$$

and the equation of continuity is

$$\frac{\partial u}{\partial r} + \frac{u}{r} = 0 \qquad (3)$$

Now

$$e_{rr} = 2\frac{\partial u}{\partial r}, e_{\theta\theta} = \frac{2u}{r}, e_{rz} = \frac{\partial w}{\partial r}$$

$$e_{r\theta} = e_{\theta z} = e_{zz} = 0$$

Then on using (1) , we got

$$\tau_{rr} = -p + \mu e_{rr} + \mu_c(e_{rr}^2 + e_{rz}^2), \tau_{\theta\theta} = -p + \mu e_{\theta\theta} + \mu_c e_{\theta\theta}^2,$$

$$\tau_{rz} = \mu e_{rz} + \mu_c e_{rr} e_{rz}, \tau_{zz} - p + \mu_c e_{rz}^2$$

Hence with the help of (3), equations (2) become as

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} = -\frac{1}{P}\frac{\partial p}{\partial r} + \nu_c\left[\frac{1}{r}\frac{\partial}{\partial r}\left\{r\left(\frac{\partial w}{\partial r}\right)^2\right\} + 4\frac{\partial}{\partial r}\left(\frac{\partial u}{\partial r}\right)^2\right] \qquad (4)$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} = \frac{\nu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) + \frac{2\nu_c}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}.\frac{\partial w}{\partial r}\right) \qquad (5)$$

Where ν is the kinematic coefficient of viscosity and ν_c is the kinematic coefficient of cross – viscosity

Here we have considered that the rate of liquid withdrawn at one wall is always equal to the rate of injection of the liquid at the other wall. Then we have on using (3)

$$u = \frac{au_a}{r} = \frac{bu_b}{br} \qquad (6)$$

where u_a and u_b are the radial velocities at the walls of the inner and outer cylinders respectively.

Substituting for u in (5) and considering u_a to be constant, we have

$$\frac{\partial w}{\partial t} + \frac{a u_a}{r} \cdot \frac{\partial w}{\partial r} = \frac{\nu}{r}\frac{\partial}{\partial r}\left(r\,\frac{\partial w}{\partial r}\right) - \frac{2_a u_a \nu_c}{r}\left(\frac{1}{r}\,\frac{\partial w}{\partial r}\right) \quad (7)$$

Let us put

$$w\,(r, t) = f(r) e^{-\,nt} \quad (8)$$

to obtain a solution of equation (7).

Therefore equation (7) is reduced to

$$\left(\frac{2a\,u_a\,\nu_c}{r^2} - \nu\right)\frac{d^2 f}{dr^2} - \left(\frac{2 a u_a \nu_c}{r^3} + \frac{\nu}{r} - \frac{a u_a}{r}\right)\frac{df}{dr} - nf = 0 \quad (9)$$

Putting $\frac{a u_a}{\nu}$ = R (the cross-flow Reynold's number in Newtonian fluid) and $\frac{2 a u_a \nu_c}{\nu}$ = L (a parameter depending on the cross-viscosity coefficient ν_c) in equation (9), we get

$$r(r^2 - L)\,\frac{d^2 f}{dr^2} + \{L + r^2(1 - R)\}\frac{df}{dr} + q^2\, r^3\, f = 0 \quad (10)$$

where $q^2 = \frac{n}{\upsilon}$

Substituting $f = (r^2 - L)^{\frac{R}{4}}.\Psi(r)$ in equation (10) we get

$$r(r^2 - L)\frac{d^2\Psi}{dr^2} + (r^2 + L)\,\frac{d\Psi}{dr} + \left\{q^2 - \frac{R^2}{4(r^2 - L)}\right\} r^3\, \Psi = 0 \quad (11)$$

Further, putting $x = q(r^2 - L)^{\frac{1}{2}}$ we have from equation (11)

$$x^2\,\frac{d^2}{dx^2}\,\Psi + x\frac{d\Psi}{dx} + \left(x^2 - \frac{R^2}{4}\right)\Psi = 0 \quad (12)$$

Which is Bessel's equation of (R/2) th order.

For practical need, we may take the solution of equation (12) as

$$\Psi(r) = A_1\, H^{(1)}_{\frac{R}{2}}(x) + A_2\, H^{(2)}_{\frac{R}{2}}(x),$$

Where $H^{(1)}_{\frac{R}{2}}(x)$ and $H^{(2)}_{\frac{R}{2}}(x)$ are Hankel functions of the first and second kind respectively and A_1, A_2 are constants.

Hence we have

$$f(r) = (r^2 - L)^{\frac{R}{4}}\left[A_1\, H^{(1)}_{\frac{R}{2}}\left(q\sqrt{r^2 - L}\right) + A_2\, H^{(2)}_{\frac{R}{2}}\left(q\sqrt{r^2 - L}\right)\right]$$

Therefore, we have from (8)

$$w\,(r,t) = (r^2 - L)^{\frac{R}{4}} \left[A_1\, H^{(1)}_{\frac{R}{2}}\left(q\,\sqrt{r^2 - L}\right) + A_2\, H^{(2)}_{\frac{R}{2}}\,\left(q\,\sqrt{r^2 - L}\right)\right], e^{-\,nt} \quad (13)$$

The boundary conditions are

$w = We^{-\,nt}$ when r = a

and

w = 0 when r = b (14)

By using (14), we have from (13)

$$(a^2 - L)^{\frac{R}{4}} \left[A_1\, H^{(1)}_{\frac{R}{2}}\left(q\,\sqrt{a^2 - L}\right) + A_2\, H^{(2)}_{\frac{R}{2}}\left(q\,\sqrt{a^2 - L}\right)\right] = W$$

and

$$(b^2 - L)^{\frac{R}{4}} \left[A_1\, H^{(1)}_{\frac{R}{2}}\left(q\,\sqrt{b^2 - L}\right) + A_2\, H^{(2)}_{\frac{R}{2}}\left(q\,\sqrt{b^2 - L}\right) = 0\right] \quad (15)$$

Now, we have from (15)

$$A_1 = \frac{W}{(a^2-L)^{\frac{R}{4}}} \cdot \frac{H^{(2)}_{\frac{R}{2}}\left(q\,\sqrt{b^2-L}\right)}{H^{(1)}_{\frac{R}{2}}\left(q\,\sqrt{a^2-L}\right) H^{(2)}_{\frac{R}{2}}\left(q\,\sqrt{b^2-L}\right) - H^{(1)}_{\frac{R}{2}}\left(q\,\sqrt{b^2-L}\right) H^{(2)}_{\frac{R}{2}}\left(q\,\sqrt{a^2-L}\right)}$$

$$A_2 = \frac{W}{(a^2-L)^{\frac{R}{4}}} \cdot \frac{H^{(1)}_{\frac{R}{2}}\left(q\,\sqrt{b^2-L}\right)}{H^{(1)}_{\frac{R}{2}}\left(q\,\sqrt{a^2-L}\right) H^{(2)}_{\frac{R}{2}}\left(q\,\sqrt{b^2-L}\right) - H^{(1)}_{\frac{R}{2}}\left(q\,\sqrt{b^2-L}\right) H^{(2)}_{\frac{R}{2}}\left(q\,\sqrt{a^2-L}\right)}$$

Substituting the values of A_1 and A_2 in (13), we get

$$w\,(r,t) =$$

$$\frac{W\,(r^2-L)^{\frac{R}{4}}}{\left((a^2-L)^{\frac{R}{4}}\right)}, e^{-nt} \cdot \frac{H^{(1)}_{\frac{R}{2}}\left(q\,\sqrt{r^2-L}\right) H^{(2)}_{\frac{R}{2}}\left(q\,\sqrt{b^2-L}\right) - H^{(1)}_{\frac{R}{2}}\left(q\,\sqrt{b^2-L}\right) H^{(2)}_{\frac{R}{2}}\left(q\,\sqrt{r^2-L}\right)}{H^{(1)}_{\frac{R}{2}}\left(q\,\sqrt{a^2-L}\right) H^{(2)}_{\frac{R}{2}}\left(q\,\sqrt{b^2-L}\right) - H^{(1)}_{\frac{R}{2}}\left(q\,\sqrt{b^2-L}\right) H^{(2)}_{\frac{R}{2}}\left(q\,\sqrt{a^2-L}\right)} \quad (16)$$

By using (6), (4) can be written as

$$-\frac{a^2\, u_a^2}{r^3} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu_c \left[\frac{1}{r}\frac{\partial}{\partial r}\left\{r\left(\frac{\partial w}{\partial r}\right)^2\right\} - \frac{16 a^2\, u_a^2}{r^5}\right]$$

or, $$\frac{1}{\rho}\frac{\partial p}{\partial r} = \nu_c \left[\frac{1}{r}\left(\frac{\partial w}{\partial r}\right)^2 + 2\frac{\partial w}{\partial r}\frac{\partial^2 w}{\partial r^2} - \frac{16\, a^2 u_a^2}{r^5}\right] + \frac{a^2\, u_a^2}{r^3} \quad (17)$$

Substituting for *W* from (16) in (17) and integrating, the pressure p can be determined as a function of r and t.

III. Flow for Small Values of n

In terms of the Bessel functions $J_{\frac{R}{2}}(z)$ and $J_{-\frac{R}{2}}(z)$ we have (Watson [VII])

$$H^{(1)}_{\frac{R}{2}}(z) = \frac{e^{-\frac{R}{2\pi i}} J_{\frac{R}{2}}(z) - J_{-\frac{R}{2}}(Z)}{-i \sin\frac{1}{2} R.\pi}$$

$$H^{(2)}_{\frac{R}{2}}(z) = \frac{e^{\frac{R}{2\pi i}} J_{\frac{R}{2}}(z) - J_{-\frac{R}{2}}(Z)}{-i \sin\frac{1}{2} R.\pi}$$

Therefore

$$\frac{H^{(1)}_{\frac{R}{2}}(q\sqrt{r^2-L})\, H^{(2)}_{\frac{R}{2}}(q\sqrt{b^2-L}) - H^{(1)}_{\frac{R}{2}}(q\sqrt{b^2-L})\, H^{(2)}_{\frac{R}{2}}(q\sqrt{r^2-L})}{H^{(1)}_{\frac{R}{2}}(q\sqrt{a^2-L})\, H^{(2)}_{\frac{R}{2}}(q\sqrt{b^2-L}) - H^{(1)}_{\frac{R}{2}}(q\sqrt{b^2-L})\, H^{(2)}_{\frac{R}{2}}(q\sqrt{a^2-L})}$$

$$= \frac{J_{\frac{R}{2}}(q\sqrt{r^2-L})\, J_{-\frac{R}{2}}(q\sqrt{b^2-L}) - J_{\frac{R}{2}}(q\sqrt{b^2-L})\, J_{-\frac{R}{2}}(q\sqrt{r^2-L})}{J_{\frac{R}{2}}(q\sqrt{a^2-L})\, J_{-\frac{R}{2}}(q\sqrt{b^2-L}) - J_{\frac{R}{2}}(q\sqrt{b^2-L})\, J_{-\frac{R}{2}}(q\sqrt{a^2-L})} \tag{18}$$

Moreover, we have (Watson [4])

$$J_{\frac{R}{2}}(z) = \left(\frac{1}{2}z\right)^{\frac{R}{2}} \left[\frac{1}{\Gamma\left(\frac{R}{2}+1\right)} - \frac{z^2}{4\,\Gamma\left(\frac{R}{2}+2\right)} + \cdots\right]$$

$$= \left(\frac{1}{2}z\right)^{\frac{R}{2}} .\ (a_0 - a_1 z^2 + \cdots)$$

and

$$J_{-\frac{R}{2}}(z) = \left(\frac{1}{2}z\right)^{-\frac{R}{2}} \left[\frac{1}{\Gamma\left(-\frac{R}{2}+1\right)} - \frac{z^2}{4\,\Gamma\left(-\frac{R}{2}+2\right)} + \cdots\right]$$

$$= \left(\frac{1}{2}z\right)^{-\frac{R}{2}} .\ (b_0 - b_1 z^2 + \cdots)$$

where

$$a_0 = \frac{1}{\Gamma\left(\frac{R}{2}+1\right)},\ a_1 = \frac{1}{4\,\Gamma\left(\frac{R}{2}+2\right)},\ b_0 = \frac{1}{\Gamma\left(-\frac{R}{2}+1\right)},\ b_1 = \frac{1}{4\,\Gamma\left(-\frac{R}{2}+2\right)} \tag{19}$$

Here the absolute values of $q\sqrt{r^2-L}$, $q\sqrt{a^2-L}$ and $q\sqrt{b^2-L}$ are small and so, avoiding terms involving powers of q a higher than two, we have

$$J_{\frac{R}{2}}\left(q\sqrt{r^2-L}\right) J_{-\frac{R}{2}}\left(q\sqrt{b^2-L}\right) - J_{\frac{R}{2}}\left(q\sqrt{b^2-L}\right) J_{-\frac{R}{2}}\left(q\sqrt{r^2-L}\right)$$

$$= a_0\, b_0 \left[\left(\frac{r^2-L}{b^2-L}\right)^{\frac{R}{4}} - \left(\frac{b^2-L}{r^2-L}\right)^{\frac{R}{4}}\right] - q^2 \left[\{a_0\, b_1\,(b^2-L) + a_1\, b_0(r^2 - L)\}\left(\frac{r^2-L}{b^2-L}\right)^{\frac{R}{4}} - \{a_0\, b_1\,(r^2-L) + a_1\, b_0(b^2-L)\}\left(\frac{b^2-L}{r^2-L}\right)^{\frac{R}{4}}\right]$$

and

$$\mathrm{J}_{\frac{R}{2}}\left(q\sqrt{a^2-L}\right)\mathrm{J}_{-\frac{R}{2}}\left(q\sqrt{b^2-L}\right) - \mathrm{J}_{\frac{R}{2}}\left(q\sqrt{b^2-L}\right)\mathrm{J}_{-\frac{R}{2}}\left(q\sqrt{a^2-L}\right)$$

$$= a_0\, b_0 \left[\left(\frac{a^2-L}{b^2-L}\right)^{\frac{R}{4}} - \left(\frac{b^2\, L}{r^2-L}\right)^{\frac{R}{4}}\right] - q^2 \left[\{a_0\, b_1\,(b^2-L) + a_1\, b_0(a^2 - L)\}\left(\frac{ar^2-L}{b^2-L}\right)^{\frac{R}{4}} - \{a_0\, b_1\,(a^2-L) + a_1\, b_0(b^2-L)\}\left(\frac{b^2\, L}{r^2-L}\right)^{\frac{R}{4}}\right]$$

Hence with the help of (16) and (18) and putting $q^2 = \frac{n}{\nu}$, we finally got

$$w\,(r,L) = \frac{W\,(r^2-L)^{\frac{R}{4}}\,.\; e^{-nt}}{(a^2-L)^{\frac{R}{4}}}\,.\frac{\left(\begin{array}{c}\left[\left(\frac{r^2\, L}{b^2-L}\right)^{\frac{R}{4}} - \left(\frac{b^2\, L}{r^2-L}\right)^{\frac{R}{4}}\right] - \frac{n}{\nu a_0\, b_0} \\ \left[\begin{array}{c}\{a_0\, b_1\,(b^2-L) + a_1\, b_0\,(r^2-L)\}\left(\frac{r^2\, L}{b^2-L}\right)^{\frac{R}{4}} \\ -\{a_0\, b_1\,(r^2-L) + a_1\, b_0\,(b^2-L)\}\left(\frac{b^2\, L}{r^2-L}\right)^{\frac{R}{4}}\end{array}\right]\end{array}\right)}{\left(\begin{array}{c}\left[\left(\frac{a^2\, L}{b^2-L}\right)^{\frac{R}{4}} - \left(\frac{b^2\, L}{a^2-L}\right)^{\frac{R}{4}}\right] - \frac{n}{\nu a_0\, b_0} \\ \left[\begin{array}{c}\{a_0\, b_1\,(b^2-L) + a_1\, b_0\,(a^2-L)\}\left(\frac{a^2\, L}{b^2-L}\right)^{\frac{R}{4}} \\ -\{a_0\, b_1\,(a^2-L) + a_1\, b_0\,(b^2-L)\}\left(\frac{b^2\, L}{a^2-L}\right)^{\frac{R}{4}}\end{array}\right]\end{array}\right)} \quad (20)$$

IV. Flow for Large Values of n

In this case, $|q|$ is very large and we use the asymptotic formulas (Sommerfeld [VI]).

$$\mathrm{H}^{(1)}_{\frac{R}{2}}\,(z) = \left(\frac{2}{\pi z}\right)^{\frac{1}{2}}\,.\;\; e^{i\left\{z - \frac{1}{2}\pi\left(\frac{1}{2}R + \frac{1}{2}\right)\right\}}$$

and

$$H^{(2)}_{\frac{R}{2}}\,(z) = \left(\frac{2}{\pi z}\right)^{\frac{1}{2}} .\ e^{i\left\{z-\frac{1}{2}\pi\left(\frac{1}{2}R+\frac{1}{2}\right)\right\}}$$

Therefore

$$H^{(1)}_{\frac{R}{2}}\left(q\sqrt{r^2-L}\right) H^{(2)}_{\frac{R}{2}}\left(q\sqrt{b^2-L}\right) - H^{(1)}_{\frac{R}{2}}\left(q\sqrt{b^2-L}\right) H^{(2)}_{\frac{R}{2}}\left(q\sqrt{r^2-L}\right) = \frac{2}{\pi q}\,(r^2-L)^{-\frac{1}{4}}(b^2-L)^{-\frac{1}{4}}\left\{e^{iq\left(\sqrt{b^2-L}-\sqrt{r^2-L}\right)} - e^{-iq\left(\sqrt{b^2-L}-\sqrt{r^2-L}\right)}\right\} \approx -\frac{2}{\pi q}\,(r^2-L)^{-\frac{1}{4}}(b^2-L)^{-\frac{1}{4}} .\ e^{iq\left(\sqrt{b^2-L}-\sqrt{r^2-L}\right)}$$

, since b > r

and

$$H^{(1)}_{\frac{R}{2}}\left(q\sqrt{a^2-L}\right) H^{(2)}_{\frac{R}{2}}\left(q\sqrt{b^2-L}\right) - H^{(1)}_{\frac{R}{2}}\left(q\sqrt{b^2-L}\right) H^{(2)}_{\frac{R}{2}}\left(q\sqrt{a^2-L}\right) = \frac{2}{\pi q}\,(a^2-L)^{-\frac{1}{4}}(b^2-L)^{-\frac{1}{4}}\left\{e^{iq\left(\sqrt{b^2-L}-\sqrt{a^2-L}\right)} - e^{-iq\left(\sqrt{b^2-L}-\sqrt{a^2-L}\right)}\right\} \approx -\frac{2}{\pi q}\,(a^2-L)^{-\frac{1}{4}}(b^2-L)^{-\frac{1}{4}} .\ e^{iq\left(\sqrt{b^2-L}-\sqrt{a^2-L}\right)}$$

since a > r

Hence from (16) and putting $q = \sqrt{\frac{n}{\nu}}$, we get the expression for the velocity as

$$w\,(r,t) = \frac{W\,(r^2-L)^{\frac{R}{4}(R-1)} .\ e^{-nt}}{(a^2-L)^{\frac{R}{4}(R-1)}} .\ \mathrm{Re}\left[e^{i\sqrt{\frac{n}{\nu}}}\left(\sqrt{a^2\,L} - \sqrt{r^2-L}\right)\right]$$

$$w\,(r,t) = \frac{W\,(r^2-L)^{\frac{R}{4}(R-1)} .\ e^{-nt}}{(a^2-L)^{\frac{R}{4}(R-1)}} .\ \cos\left\{\sqrt{\frac{n}{\nu}}\left(\sqrt{r^2\,L} - \sqrt{a^2-L}\right)\right\} \quad (21)$$

Substituting the radial component u = 0 in (6), we get $u_a = u_b = 0$ and consequently R = 0. The motion, in this case, is due to the transient movement of a piston in a fluid contained in a circular cylinder and the cylinders are rigid.

Substituting R = 0 in (16), we get

$$w\,(r,l) = w.e^{-nt} .\left[\frac{H_0^{(1)}\left(q\sqrt{r^2-L}\right) H_0^{(2)}\left(q\sqrt{b^2-L}\right) - H_0^{(1)}\left(q\sqrt{b^2-L}\right) H_0^{(2)}\left(q\sqrt{r^2-L}\right)}{H_0^{(1)}\left(q\sqrt{a^2-L}\right) H_0^{(2)}\left(q\sqrt{b^2-L}\right) - H_0^{(1)}\left(q\sqrt{b^2-L}\right) H_0^{(2)}\left(q\sqrt{a^2-L}\right)}\right] \quad (22)$$

The velocity distribution is given by (22). Here we discuss two cases.

Case I.

For small values of n, w can be obtained from (20). When R → O the right hand side of (20) assume the form $\left(\frac{0}{0}\right)$, so it can be evaluated by L Hospitols's rule.

We know (Sommerfeld) [5]

$$\Gamma'\,(1) = -\gamma, \quad \Gamma'\,(2) = -\gamma + 1$$

γ being Euler's constant. Hence from (19), we have

$$a_0 \rightarrow 1, \quad b_0 \rightarrow 1, \quad a_1 \rightarrow \frac{1}{4}, \quad b_1 \rightarrow \frac{1}{4}$$

and

$$a_0' \rightarrow \frac{1}{2}\gamma\,, \quad b_0' \rightarrow \frac{1}{2}\gamma\,, \quad a_1' \rightarrow \frac{1}{8}(\gamma - 1)\,, \quad b_1' \rightarrow \frac{1}{8}(\gamma - 1)$$

as R → 0, the dashes denoting differentiation with respect to R.

Therefore, when R → 0, we have from (20)l

$$w\,(r,t) = W\,e^{-nt}\,\frac{\left[Log\left(\frac{r^2-L}{b^2-L}\right) - \frac{n}{\vartheta}\left\{4(b^2 - r^2) + \frac{b^2+r^2-2}{4}\,Log\left(\frac{r^2-L}{b^2-L}\right)\right\}\right]}{\left[Log\left(\frac{a^2-L}{b^2-L}\right) - \frac{n}{\vartheta}\left\{4(b^2 - a^2) + \frac{b^2+a^2-2}{4}\,Log\left(\frac{a^2-L}{b^2-L}\right)\right\}\right]} \qquad (23)$$

Case II.

For large values of n, substituting R = 0 in (21), we get

$$w\,(r,t) = W\,\left(\frac{a^2-L}{r^2-L}\right)^{\frac{1}{4}} e^{-nt}\,.\cos\,\left[\sqrt{\frac{n}{\vartheta}}\,\left(\sqrt{r^2 - L} - \sqrt{a^2 - L}\right)\right] \qquad (24)$$

If we put $\vartheta_c = 0$ in each of these results, we get the classical results for ordinary Newtonian viscous fluids.

V. Conclusion

It is clear from (20) and (21) that the velocity gradient is transient in nature for both small and large value of *n* and will die out after a long interval of time.

The velocity distribution for transient movement of a piston has been studied as a particular case in (22) just by substituting the radial component $u = 0$ i.e., $u_a = u_b = 0$ i.e., $R = 0$ in (16). The velocity distributions of the piston have also been found for small and large values of *n* in Case I and Case II respectively. In both the Cases, it is seen that the velocity distribution of the piston is transient in nature and die out after long time.

Conflict of Interest

There is no conflict of interest regarding this article.

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