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OPTIMAL INVENTORY DECISIONS FOR DETERIORATING ITEMS WITH ALL-UNITS DISCOUNT UNDER FUZZY ENVIRONMENT

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Abstract

The proposed inventory model has been developed for deteriorating items subject to all-unit discount in a fuzzy environment. Considering demand as price dependent, holding cost depends on time, and purchase cost depends on order size. The inventory parameters, such as ordering cost, holding cost, and demand rate, are all represented as triangular fuzzy numbers to capture the uncertainty in the system. The objective of the model is to determine the optimal time length, selling price, and order quantity to maximize the total profit function. Numerical examples are carried out to validate the models. Sensitivity analysis is performed to check the effect of fuzzy parameters on profit function and decision variables to get further insights. Results stated that a fuzzy model works better than a crisp model, and an all-units discount policy helps in maximizing a retailer's profit. It allows for flexibility and adaptability, leading to a potential increase in revenue.

Keywords: Deterioration, All-units discount, Graded mean integration method, Price dependent demand, Time dependent holding cost, Triangular fuzzy number,

I. Introduction

It's a tough world out there for businesses trying to attract more customers. With so much competition, companies need to really understand what their customers want and also consider that. Every day, product demand can fluctuate. Until now, many inventory models were designed by assuming different types of demand with other inventory-related parameters. The demand rate is not always constant but depends on time, selling price, stock level, etc. Generally, demand for fruits, vegetables, seafood, and fast food in hotels depends on price. Customers prefer to purchase products based on selling price; a lower selling price increases the demand

rate. Therefore, in this paper, we considered a realistic assumption, a selling price-dependent demand rate.
Another realistic assumption is that holding cost is increasing as a function of time. It is not always constant but varies according to storage time. Some products, such as fruits, vegetables, perfumes, etc., lose their freshness and quality over time. So, to keep products fresh for a long time, holding costs will increase, which depends on time.

In realistic situations, deterioration is another indispensable parameter in an inventory system. Products like foods, fruits, vegetables, perfumes, radioactive substances, etc., lose their freshness, quality, and characteristics after some time. So, there is no use for such deteriorated items. This will cause financial loss. The term is known as deterioration. So, deterioration cannot be ignored. Hence, to reduce holding costs, rate of deterioration, bring in more customers, boost sales, or clear out old stock, manufacturers used discount strategies such as all-units discounts, trade credit discounts, seasonal discounts, etc. All-units discount, i.e., discount policy on per-unit purchase cost based on order quantity. For a lower selling price, demand will increase, so many orders will be placed by customers. In this situation, an all-units discount will be offered to the customers based on order quantity. So, this strategy increases popularity in the market, attracts more customers, and also increases profits.

Nowadays, Business growth is very fast, and its impact on the economy is huge. In the real world, it's impossible to pinpoint all the variables with absolute certainty, as there are often inaccuracies or uncertainties at play. Uncertainty is everywhere. Some parameters of an inventory system are not always known but may be imprecise. For new products, past data on demand and inventory costs are not available for such estimation. It is very difficult for decision-makers to decide the exact annual demand and costs of inventory. For example, we say that the ordering cost is '200$'. But in practice, it is about $200, not exactly $200. The term 'about' describes uncertainty. Hence, to solve the inventory issue, fuzzy set theory gives more precise results. Consequently, it becomes more suitable to address such inventory problems through fuzzy set theory. That's why the fuzzy concept is used in this model. Considering all these realistic situations, a proposed fuzzy model is formulated.

In this study, the ideas proposed by Khan *et al.* [XIII] are extended further using fuzzy concepts. Considering fuzzy set theory, this paper removes the limitation of Khan *et al.* [XIII] by introducing holding cost, demand, and ordering cost as fuzzy numbers. The proposed fuzzy inventory model is formulated for deteriorating items assuming price-dependent demand, time-varying holding cost, and order size-dependent purchase cost under an all-units discount. Because of fuzzy demand, order quantity also becomes fuzzy. For defuzzification of the profit function, a graded mean integration method is used, and then the corresponding optimal cycle time, selling price, and order quantities are derived for maximization of the profit function. The model numerical example is presented. Sensitivity analysis is carried out to check the effect of fuzzy parameters.

The rest portion of the proposed model is presented as follows: Literature review related to the proposed model presented in Section II. Preliminaries for the triangular fuzzy number are presented in Section III. In Section IV, notations and assumptions are introduced for the model. Mathematical models are presented in Section V. A solution algorithm is constructed in Section VI. A numerical example is found in Section VII. The sensitivity analysis with regard to fuzzy parameters is found in Section VIII. At last, a conclusion and future research plan are found in Section IX.

II. Literature Review

Alfares and Ghaithan [I] developed an inventory model under quantity discounts where demand depends on price and holding cost depends on time. Shaikh *et al.* [XXVI] extended the study of Alfares and Ghaithan [I], considering the shortage effect with price and stock-dependent demand. Khan *et al.* [XIII] extended the study of Shaikh *et al.* [XXVI], considering variable type per unit selling price. Kristiyani and Daryanto [XIV] presented a study on the relationship between discount and total carbon emissions. Limansyah and Lesmono [XVI] formulated an inventory model under an all-units discount for probabilistic demand and expiration date. Taleizadeh and Pentico [XXIX] developed an EOQ model with an all-units discount and partial back-ordering. Shah *et al.* [XXIII] formulated price and stock-dependent demand rates and deteriorating items along with greening efforts under an all-units discount.

Fuzzy set theory is introduced by Zadeh [XXXI] to investigate the practical issue by the use of membership functions and fuzzy numbers. Bellman and Zadeh [II] introduced fuzzy set theory for solving decision-making processes. Zadeh [XXXII] has shown that fuzzy numbers work better than probability theory when it comes to new products and seasonal commodities. Jaggi *et al.* [XI] and Sharmila and Uthayakumar [XXVIII] formulated a fuzzy inventory model for deteriorating items with shortages under fully backlogged. Sahoo *et al.* [XXII] developed an inventory model with deterioration, exponential demand, and salvage value under a fuzzy environment. Shah and Soni [XXIV] discussed a continuous review inventory model that has been analyzed with fuzzy price-dependent demand to determine an (r, Q) policy that minimizes the cost function. Maragatham and Lakshmidevi [XVII] presented a model with shortages and price-dependent demand under an uncertain environment. Roy [XIX] developed a fuzzy inventory model for time-dependent deterioration and holding costs with price-dependent demand. Saha [XX] framed a fuzzy inventory model for price-dependent demand with deteriorating items in a supply chain management system. Saha and Chakrabarti [XXI] discussed a supply chain inventory model for deteriorating items with price-dependent demand in a fuzzy environment. Kumar and Kumar [XV] developed a fuzzy inventory model for deteriorating items with linearly time-dependent demand and shortages under partially backlogged. Shaikh *et al.* [XXV] presented a fuzzy inventory model for a deteriorating item with permissible delay in payments; the demand depends on the selling price and the frequency of the advertisement. Varadharajan and Sangma [XXX] formulated a fuzzy EOQ inventory model for deteriorating items with price-dependent demand with shortages. Rani and Kumar [XVIII] developed a fuzzy inventory model for deteriorating items with a selling price-dependent demand and shortages underfully backlogged. Shaikh and Gite [XXVII] developed a fuzzy inventory model for deteriorating items with variable production and selling price-dependent demand under inflation. Indrajitsingha [X] presented a fuzzy inventory model for deteriorating items with selling price-dependent demand and shortages under partially backlogged with a rate dependent on the duration of waiting time up to the arrival of the next order. Garai *et al.* [VIII] extended the study of Alfares and Ghaithan [I] in a fully fuzzy environment. Bera and Maiti [III] constructed a fuzzy EOQ model under all-units discount (AUD) and incremental quantity discount (IQD) for multi-items. Huang *et al.* [IX] presented a fuzzy supply chain integrated inventory model with quantity discounts and an unreliable process.

III. Preliminaries

Several authors, e.g., Dubois and Prade [VII], introduced an important work on the concept of fuzzy numbers. Fuzzy number is defined by Kaufmann and Gupta [XII] as a subset of R that is convex and normal. Without loss of generality, this paper assumed all fuzzy parameters as normal triangular fuzzy numbers (TFNs).

Triangular Fuzzy Number

A triangular fuzzy number (TFN) is defined by the following membership function.

$$\mu_{\tilde{Y}}(x) = \begin{cases} 0 & , \quad y - \Delta_1 \leq x \\ \frac{x - y + \Delta_1}{\Delta_1} & , \quad y - \Delta_1 \leq x \leq y \\ 1 & , \quad x = y \\ \frac{y + \Delta_2 - x}{\Delta_2} & , \quad y \leq x \leq y + \Delta_2 \\ 0 & , \quad x \geq y + \Delta_2 \end{cases}$$

Where, $y,\ \Delta_1,\ \Delta_2,\ x \in R$ and $y - \Delta_1 < y < y + \Delta_2$.

Graded Mean Integration Representation

The graded Mean Integration Method (GMIM) introduced by Chen and Hsieh [V, VI] is based on the integral value of the graded mean h-level of a generalized fuzzy number. This approach considers quality as the key element of each level of support within a generalized fuzzy number to accurately represent a fuzzy number. The graded mean integration of TFN $\tilde{Y} = (y - \Delta_1, y, y + \Delta_2)$ is

$$G(\tilde{Y}) = y + \frac{1}{6}(\Delta_2 - \Delta_1)$$

Fuzzy Arithmetical Operations under Function Principle

Chen [IV] introduced the function principle to compute the fuzzy arithmetical operations of TFNs. If $\tilde{X} = (x_1, x_2, x_3)$ and $\tilde{Y} = (y_1, y_2, y_3)$ are two TFNs where, $x_1 < x_2 < x_3$ and $y_1 < y_2 < y_3$; $x_i,\ y_i \in R$ for $i\ =\ 1, 2, 3$. Then fuzzy arithmetical operations under the function principle are defined as follows:

Addition: $\tilde{X} \oplus \tilde{Y} = (x_1 + y_1, x_2 + y_2, x_3 + y_3)$
Subtraction: $\tilde{X} \ominus \tilde{Y} = (x_1 - y_3, x_2 - y_2, x_3 - y_1)$
Multiplication: $\tilde{X} \otimes \tilde{Y} = (x_1 y_1, x_2 y_2, x_3 y_3)$
Division: $\tilde{X} \oslash \tilde{Y} = \left(\frac{x_1}{y_3}, \frac{x_2}{y_2}, \frac{x_3}{y_1}\right)$

Scalar Multiplication: For $k \in R, k \otimes \tilde{X} = \begin{cases} (kx_1, kx_2, kx_3), k \geq 0 \\ (kx_3, kx_2, kx_1), k < 0 \end{cases}$

IV. Notations and Assumptions

The following notations and assumptions are used for the proposed model. Some additional notations will be inserted later when they are needed.

Notations

u(t)	Stock level at any given time t, in units
α	Constant part of the demand rate
β	Variable part of the demand rate
A	Ordering cost, in dollars
θ	Deterioration rate
U	Order-size per replenishment cycle, in units
A	Constant part of holding cost, in dollars
B	Variable part of holding cost, in dollars
c_j	Purchasing cost per unit, in dollars
$NP(s,T)$	Net profit of retailer in unit time, in dollars
$\tilde{\alpha}$	Fuzzy demand constant
$\tilde{A}$	Fuzzy ordering cost
$\tilde{a}$	Fuzzy part of holding cost
U_f	Order quantity in fuzzy setting
$\widetilde{NP}(s,\ T)$	The total profit in fuzzy sense
$G(\tilde{Y})$	Defuzzified fuzzy number $\tilde{Y}$ by GMIM
Decision variables	
T	Replenishment cycle time
S	Selling price, in dollars

Assumptions

- The demand is a linear and decreasing function of the per-unit selling price s. i.e., $D(s) = \alpha - \beta s$, $\alpha - \beta s \geq 0$, $\alpha, \beta > 0$. Here, α is a TFN $\tilde{\alpha} = (\alpha - \Delta_{\alpha_1},\ \alpha,\ \alpha + \Delta_{\alpha_2})$ where $0 < \Delta_{\alpha_1} < \alpha$ and $0 < \Delta_{\alpha_2} < \alpha$ and $\Delta_{\alpha_1}$and Δ_{α_2} are determined by the decision maker.
- The replenishment rate is infinite, and the lead time is zero.
- The deterioration rate θ $(0 < \theta << 1)$ is constant and depends on the stock level.
- Replacement of the deteriorated products and shortages are not allowed.
- Per unit holding cost is a linearly increasing function of storage period and per unit purchase cost c_j . i.e., $H(t) = c_j\,(a + bt)$. Here a is a TFN $\tilde{a} = (a - \Delta_{a_1},\ a,\ a + \Delta_{a_2})$ where $0 < \Delta_{a_1} < a$ and $0 < \Delta_{a_2} < a$ and Δ_{a_1} and Δ_{a_2} are determined by the decision maker.
- Ordering cost A is a TFN $\tilde{A} = (A - \Delta_{A_1},\ A,\ A + \Delta_{A_2})$ where $0 < \Delta_{A_1} < A$ and $0 < \Delta_{A_2} < A$ and Δ_{A_1} and Δ_{A_2} are determined by the decision maker.

- The per unit purchase cost (c_j) is a decreasing step function of order size (U) under the all-units quantity discount:

$$C(U) = c_j, \quad q_j \leq U < q_{j+1} \quad \text{where} \quad c_1 > c_2 > \cdots > c_n$$

V. Mathematical Models

Crisp Model

Following the Khan et al. [XIII] model initially, at $t = 0$, the retailer buys U units of a deteriorating item. During the cycle time T, the stock decreases gradually due to the demand and deterioration. At the end of each cycle time T, the stock level reaches zero. The rate of decrease in inventory level $u(t)$ is dependent on the rate of demand and deterioration. Therefore, the behavior of the stock level $u(t)$ at any given time t is presented by Fig. 1 and can be expressed by the differential equation with the boundary conditions $u(0) = U$ and $u(T) = 0$.

$$\frac{du(t)}{dt} + \theta u(t) = -(\alpha - \beta s), \qquad 0 < t \leq T \tag{1}$$

The solution of the first-order differential equation (1), i.e., the inventory level $u(t)$ at any time t is,

$$u(t) = \frac{\alpha - \beta s}{\theta}\left[e^{\theta(T-t)} - 1\right] \tag{2}$$

Using the initial condition $u(0) = U$, from equation (2), total order quantity is given by:

$$U = \frac{\alpha - \beta s}{\theta}\left(e^{\theta T} - 1\right) \tag{3}$$

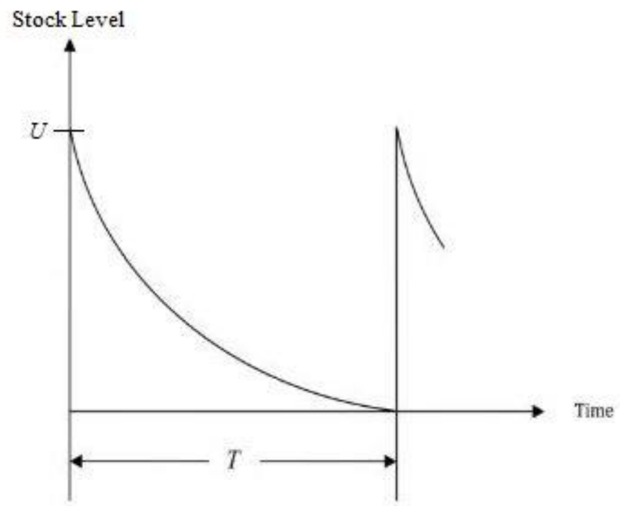

Fig. 1. Graphical representation of stock level concerning time

Sales Revenue (SR) = $s\int_0^T D(s)dt = s(\alpha - \beta s)T$

Ordering cost (OC) = A

Holding Cost (HC) = $c_j\int_0^T (a + bt)u(t)dt = \frac{c_j(\alpha-\beta s)}{\theta}\left\{\left(\frac{a}{\theta} + \frac{b}{\theta^2}\right)\left(e^{\theta T} - \theta T - 1\right) - \frac{bT^2}{2}\right\}$

Purchase Cost (PC) = $c_j U = \frac{c_j(\alpha-\beta s)}{\theta}\left(e^{\theta T} - 1\right)$

The net profit function $NP(s,T)$ includes SR, OC, HC, and PC. Therefore, net profit per unit of time can be expressed as

$$NP(s,T) = \frac{1}{T}[SR - OC - PC - HC]$$

$$= \frac{1}{T}\begin{bmatrix} s(\alpha - \beta s)T - A - \dfrac{c_j(\alpha - \beta s)}{\theta}\left(e^{\theta T} - 1\right) \\ -\dfrac{c_j(\alpha - \beta s)}{\theta}\left\{\left(\dfrac{a}{\theta} + \dfrac{b}{\theta^2}\right)\left(e^{\theta T} - \theta T - 1\right) - \dfrac{bT^2}{2}\right\} \end{bmatrix} \tag{4}$$

In the profit function, the input parameters and decision variables are described as crisp. In real-life situations, some parameters of the inventory system are not always known. Some parameters may be imprecise. Therefore, utilizing fuzzy set theory to address inventory issues can provide more accurate outcomes. In this study, we shall present input parameters (A, α, and a) as TFNs. The fuzzy model for this crisp model is derived in the next section.

Fuzzy Model

In this section, we assumed that ordering cost, demand rate, and holding cost are fuzzy variables to deal with reality more effectively. Here, ordering cost $\tilde{A}$, demand constant $\tilde{\alpha}$, and holding costs constant $\tilde{a}$ are non-negative TFNs, therefore the net profit function defined in equation (4) becomes TFN. So, the fuzzy net profit function can be expressed as

$\widetilde{NP}(s,\ T) = \left(NP_1(s,T),\ NP_2(s,T), NP_3(s,T)\right)$. Here, $NP_i(s,T)$ ($i = 1,2,3$) are real-valued functions satisfying the condition $NP_1(s,T) \leq NP_2(s,T) \leq NP_3(s,T)$.

Using fuzzy arithmetical operations under the function principle Chen [IV], $NP_i(s,T)$ ($i = 1,2,3$) are expressed as follows:

$$NP_1(s,T) = \frac{1}{T}\begin{bmatrix} s\left(\alpha - \Delta_{\alpha_1} - \beta s\right)T - \left(A + \Delta_{A_2}\right) - \frac{c_j\left(\alpha + \Delta_{\alpha_2} - \beta s\right)}{\theta}\left(e^{\theta T} - 1\right) \\ -\frac{c_j\left(\alpha + \Delta_{\alpha_2} - \beta s\right)}{\theta}\left(\frac{a + \Delta_{a_2}}{\theta} + \frac{b}{\theta^2}\right)\left(e^{\theta T} - \theta T - 1\right) + \frac{c_j\left(\alpha - \Delta_{\alpha_1} - \beta s\right)bT^2}{2\theta} \end{bmatrix}$$

$$NP_2(s,T) = \frac{1}{T}\begin{bmatrix} s(\alpha - \beta s)T - A - \frac{c_j(\alpha - \beta s)}{\theta}\left(e^{\theta T} - 1\right) \\ -\frac{c_j(\alpha - \beta s)}{\theta}\left(\frac{a}{\theta} + \frac{b}{\theta^2}\right)\left(e^{\theta T} - \theta T - 1\right) + \frac{c_j(\alpha - \beta s)bT^2}{2\theta} \end{bmatrix}$$

$$NP_3(s,T) = \frac{1}{T}\begin{bmatrix} s\left(\alpha + \Delta_{\alpha_2} - \beta s\right)T - \left(A - \Delta_{A_1}\right) - \frac{c_j\left(\alpha - \Delta_{\alpha_1} - \beta s\right)}{\theta}\left(e^{\theta T} - 1\right) \\ -\frac{c_j\left(\alpha - \Delta_{\alpha_1} - \beta s\right)}{\theta}\left(\frac{a - \Delta_{a_1}}{\theta} + \frac{b}{\theta^2}\right)\left(e^{\theta T} - \theta T - 1\right) + \frac{c_j\left(\alpha + \Delta_{\alpha_2} - \beta s\right)bT^2}{2\theta} \end{bmatrix}$$

Using the graded mean integration method, the net profit function in fuzzy nature is given by

$$G\left(\widetilde{NP}(s,\,T)\right) = \frac{1}{6}\left(NP_1(s,T) + 4\,NP_2(s,T) + NP_3(s,T)\right)$$

$$= NP(s,T) - \frac{\Delta_{A_2} - \Delta_{A_1}}{6T} + \frac{\Delta_{\alpha_2} - \Delta_{\alpha_1}}{6T}\left[sT - \frac{c_j}{\theta}\left\{e^{\theta T} - 1 + \left(\frac{a}{\theta} + \frac{b}{\theta^2}\right)\left(e^{\theta T} - \theta T - 1\right) - \frac{bT^2}{2}\right\}\right] - \frac{c_j(\alpha - \beta s)}{6T\theta^2}\left(e^{\theta T} - \theta T - 1\right)\left(\Delta_{a_2} - \Delta_{a_1}\right) - \frac{c_j}{6T\theta^2}\left(e^{\theta T} - \theta T - 1\right)\left(\Delta_{\alpha_2}\Delta_{a_2} + \Delta_{\alpha_1}\Delta_{a_1}\right) \quad (5)$$

The order quantity in a fuzzy sense is given by

$$U_f = \frac{\alpha - \beta s}{\theta}\left(e^{\theta T} - 1\right) + \frac{\Delta_{\alpha_2} - \Delta_{\alpha_1}}{6\theta}\left(e^{\theta T} - 1\right) \quad (6)$$

VI. Solution Algorithm

A solution algorithm for a feasible solution is discussed below:

Step-1: Initialized j = n and $G_{max}\left(\widetilde{NP}(s,\,T)\right) = 0$

Step-2: Substitute all the values of the known parameters. Evaluate decision variables T and s using necessary conditions, i.e., $\frac{\partial G\left(\widetilde{NP}(s,T)\right)}{\partial T} = 0$ and $\frac{\partial G\left(\widetilde{NP}(s,T)\right)}{\partial s} = 0$.

Step-3: Substitute values of T and s in equation (6) and evaluate the order size U_f.

- Verify if $U_f \in \left(q_j, q_{j+1}\right)$, then the solution is feasible and evaluate $G_j\left(\widetilde{NP}(s,\,T)\right)$ from equation (5). If $G_{max}\left(\widetilde{NP}(s,\,T)\right) < G_j\left(\widetilde{NP}(s,\,T)\right)$ then take $G_{max}\left(\widetilde{NP}(s,\,T)\right) = G_j\left(\widetilde{NP}(s,\,T)\right)$.
- If $U_f \notin \left(q_j, q_{j+1}\right)$, then the solution is not feasible.

Step-4:

- If $j > 0$, then set $j = j - 1$ and go to Step-2.
- If $j = 0$, then go to Step-5.

Step-5:

- If more than one feasible solution exists, choose $G_j\left(\widetilde{NP}(s,\,T)\right)$ for all values of j that satisfy the feasibility condition. Take $G_{max}\left(\widetilde{NP}(s,\,T)\right) = \max\left\{G_j\left(\widetilde{NP}(s,\,T)\right) / j \text{ satisfy the feasibility condition}\right\}$, this value of j represents the optimal feasible solution.
- If only one feasible solution exists, then consider the obtained solution as the optimal feasible solution. $G_{max}\left(\widetilde{NP}(s,\,T)\right)$ represents maximum net profit.

Step-6: The concavity of the net profit function is proved by the graph.

VII. Numerical Example

To further analyze the proposed fuzzy inventory model, a numerical example is provided. Considering the parameters having the following values: A = $1000, α = 600, a = $2, β = 2.5, b = $1, θ = 0.06, c_1 = $9, c_2 = $8.7, c_3 = $8.4, q_1 = 1, q_2 = 150, q_3 = 500. Δ_{A_1} = $50, Δ_{A_2} = $100, Δ_{α_1} = 50, Δ_{α_2} = 100, Δ_{a_1} = $0.8, and Δ_{a_2} = $1.

By applying the solution algorithm and using Maple2020, Table 1 outlines the optimal solutions for both crisp and fuzzy models, including optimal selling price, cycle time, order quantity, and net profit.

Table 1: Optimal solutions of crisp and fuzzy models

Model	s^*	T^*	Order Quantity	Net Profit
Crisp	$127.1800	0.5717	164.0431	$30071.6846
Fuzzy	$128.8138	0.5581	162.4770	$30923.3544

From Table 1, it is observed that the net profit of the retailer for the fuzzy model is higher than the crisp model. Also, the order quantity of the fuzzy model is less than the crisp model. This is because the selling price and demand of the fuzzy model are high, and the holding cost and ordering cost of the fuzzy model are lower compared to the crisp model. The concavity of the net profit function against selling price (s) and cycle time (T) for the fuzzy model is presented in Fig. 2.

If the all-units discount policy is removed by taking a fix per unit purchase cost of $9, then the optimal solution for this example becomes s^* = $129.0136, T^* = 0.5498, U_f = 159.7584, and $G\left(\widetilde{NP}(s^*, T^*)\right)$ = $30780.8705. Here, the retailer's net profit and order quantity decrease, and the selling price increases. This is because if any discount policy is not available, then the retailer reduces the ordering cost and tries to increase profit by increasing a suitable selling price.

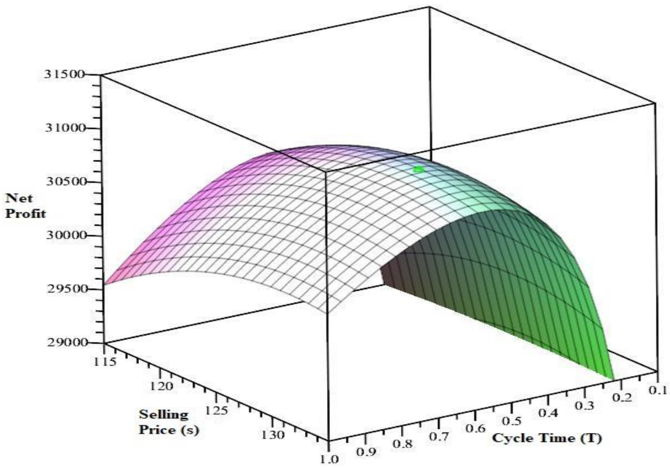

Fig. 2. The concavity of the net profit function against *s* and *T*

VIII. Sensitivity Analysis

In this part of the study, we delved into a sensitivity analysis to explore how the fuzzy parameters impact the best values for cycle length, selling price, order quantities, and total profit in the inventory system. This investigation involved adjusting the fuzzy parameters by increments of 10% (-20%, -10%, +10%, +20%) while keeping all other factors constant. This approach allowed us to observe how changes in the fuzzy parameters affected the overall performance of the inventory system.

Table 2: Effect of $\left(\Delta_{A_1},\ \Delta_{A_2}\right)$ on the optimal solutions.

$\left(\Delta_{A_1},\ \Delta_{A_2}\right)$	T^*	s^*	U_f	$G\left(\widetilde{NP}(s^*,\ T^*)\right)$
-20%	0.5576	$128.8114	162.3527	$30926.3421
-10%	0.5578	$128.8126	162.4149	$30924.8479
10%	0.5583	$128.8150	162.5391	$30921.8614
20%	0.5585	$128.8162	162.6012	$30920.3690

From Table 2, it is noted that if impreciseness in ordering costs increases, $T^*$and U_f will increase, but net profit $G\left(\widetilde{NP}(s^*,\ T^*)\right)$ will decrease, and there are no significant changes in s^*. It is obvious that higher ordering costs result in an increase in cycle time and lead to a decrease in profit. Another observation is that an increase in order size and selling price will not for all time ensure earning more profit. So, it is not advisable to always increase order size and selling price.

Table 3: Effect of $(\Delta_{\alpha_1}, \Delta_{\alpha_2})$ on the optimal solutions.

$(\Delta_{\alpha_1}, \Delta_{\alpha_2})$	T^*	s^*	U_f	$G\left(\widetilde{NP}(s^*, T^*)\right)$
-20%	0.5605	$128.4936	162.6945	$30744.1216
-10%	0.5593	$128.6537	162.5852	$30833.6596
10%	0.5569	$128.9740	162.3700	$31013.2057
20%	0.5557	$129.1342	162.2640	$31103.2135

From Table 3, it is noted that if impreciseness in demand increases, $T^*$and U_f will decrease but s^* and net profit $G\left(\widetilde{NP}(s^*, T^*)\right)$ will increase. It is observed that parameters $\Delta_{\alpha_2}$and Δ_{α_1} have more impact on profit function. Hence, to enhance profit, it is suggested to the retailer to use different marketing strategies to increase customers' demand. If the demand increases, the retailer can earn more profit by raising the appropriate selling price.

Table 4: Effect of $(\Delta_{a_1}, \Delta_{a_2})$ on the optimal solutions.

$(\Delta_{a_1}, \Delta_{a_2})$	T^*	s^*	U_f	$G\left(\widetilde{NP}(s^*, T^*)\right)$
-20%	0.5604	$128.8184	163.1608	$30939.3933
-10%	0.5592	$128.8161	162.8179	$30931.3655
10%	0.5569	$128.8116	162.1381	$30915.3601
20%	0.5558	$128.8093	161.8012	$30907.3825

From Table 4, it is noted that if impreciseness in holding cost increases, T^*, U_f, and net profit $G\left(\widetilde{NP}(s^*, T^*)\right)$ will decrease, and there are no significant changes in s^*. Net profit decreases due to higher holding costs. So, decision-makers need to improve their storage facilities to reduce the total holding cost.

IX. Conclusion and Future Research Plan

The proposed inventory model is designed for deteriorating products with an all-units discount in an uncertain environment. In many research studies, the costs of inventory and demand have been assumed to be constant. However, in reality, these costs should be seen as uncertain rather than fixed due to various factors that can cause fluctuations. For example, ordering costs can vary based on transportation costs and other factors that change over time. Fluctuations in shipping fees, mailing charges, inspection fees, and telephone charges can all impact the ordering cost. Similarly, holding costs can fluctuate based on factors like storage costs, depreciation costs, opportunity costs, and employee costs. Changes in labor salaries and warehouse rents can also lead to changes in holding costs. In addition, demand can vary due to factors such as seasonality, competitors, trends, promotions, weather conditions, natural disasters, political instability, or disruptions in the supply chain. To handle the uncertainty in these factors, fuzzy numbers are used to represent demand, holding costs, and ordering costs. The graded mean integration method is applied for the defuzzification of the profit function. This innovative approach aims to optimize decision-making in a complex and uncertain environment. One numerical example is conducted to compare the profit of the fuzzy model and the crisp model. From a numerical example, we conclude that the net profit obtained by the fuzzy model is 2.83% more than the crisp model. This percentage is proportional to the change of uncertainty. Another discovery is that the retailer earns more profit with the implementation of the all-units discount strategy. From the sensitivity analysis, we observed that the retailer's total profit per unit is more sensitive to the demand rate and less sensitive to ordering costs and holding costs. Therefore, the retailer has tried to increase customers' demand and reduce associated costs using different marketing strategies. If uncertainty is removed by taking fuzzy parameters $\Delta_{A_1} = \Delta_{A_2} = \Delta_{\alpha_1} = \Delta_{\alpha_2} = \Delta_{a_1} = \Delta_{a_2} = 0$, then this fuzzy model will be converted into the crisp model.

For future study, the proposed model can be extended to consider different demand forms, considering shortages, different forms of deterioration rate, using other defuzzification methods, introducing different discount policies, etc.

Conflict of Interests:

The authors have no relevant financial or non-financial interests to disclose.

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