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SOME FIXED POINT PROPOSITIONS FOR NON-SELF FUNCTIONS IN METRICALLY CONVEX SPACES

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Abstract

This intriguing article delves deep into the concept of non-self-plottings within the intricate realm of metrically curved planetary systems, meticulously analyzing and dissecting various fixed point propositions that govern these celestial bodies. Within the confines of this chapter, we embark on a journey to explore and elucidate Assad's groundbreaking discovery, delving into its complexities and implications to present a more elaborate and all-encompassing single-valued plotting. This development not only serves as a noteworthy extension of Assad's work but also emerges as a significant and groundbreaking generalization of Chatterjea's fundamental primary proposition, shedding new light on the dynamics of planetary motion and positioning in the vast expanse of the universe.

Keywords: Convex space, Fixed point proposition, Metrically convex planetary, Non-self-mappings, Single valued plotting.

I. Introduction

A wide range of fixed point propositions for both single- and multi-valued self-functions of a closed subsection of a Banach planetary have been developed by different novelists. New fixed point theorems can be obtained by using the reduction plotting proposition in a convex situation, which is where it finds numerous applications. Nonetheless, the plotting in question is frequently not a self-plotting of fastened sets in these kinds of applications.

Through the observation that a planetary is metrically curving, then major wearying regarding the province besides the variety of functions investigated container be made, Assad and Kirk [IV] take a fixed-point proposition aimed at reduction functions in a comprehensive metric planetary.

Establishing specific boundary conditions on the mapping and demonstrating a static point proposition for Kannan functions on a comprehensive metrically curving metric planetary, Assad [III] provided adequate circumstances for the single-valued functions to have a static idea. Similar outcomes were found through Assad [II] and Assad-Kirk [IV] for multi-valued contractive and reduction functions, correspondingly.

Fixing point propositions for multivalued functions were primarily deliberate by Markin [XXII] besides Nadler [XXIV]. Itoh [XV], Khan [XVI], Hadzic and Gajic [XIII], Rhoades [XXVIII], Chang [VII], Pathak [XXV], and others are also cited for comparable results.

Basic definitions that are immediately applicable in the ensuing sections of this article are presented in Section 2. The consequence of Assad [III] aimed at an additional comprehensive single-valued plotting, which is likewise a significant generalization of Chatterjea [VI]'s primary theorem, will be expanded upon in Section 3. The findings about the common static idea for metrically curving planetary are covered in segment 4.

Research Significance

Fixed point propositions for non-self functions in metrically convex spaces play a crucial role in advancing our comprehension of nonlinear systems. These propositions provide a framework for proving the existence of fixed points for non-self mappings in various types of metric spaces, such as b-fuzzy metric spaces, generalized convex metric spaces, and complete metric spaces. By establishing conditions under which these fixed points exist, such as non-linear contractive conditions or specific contraction mappings, these propositions extend and generalize existing theorems in the literature, enhancing our understanding of the behavior of nonlinear systems. Additionally, the application of these fixed point theorems to problems like Volterra integral equations and nonlinear fractional differential equations demonstrates the practical utility and significance of these results in analyzing and solving complex nonlinear systems.

Recent developments in the study of fixed point propositions for non-self functions in metrically convex spaces have focused on proving common fixed-point theorems for non-self mappings satisfying non-linear contractive conditions. Additionally, new fixed point theorems have been established for quasi-upper semicontinuous set-valued mappings and compact continuous mappings in locally p-convex spaces, extending existing results in the literature. Furthermore, there are advancements in proving the existence and uniqueness theorems of fixed points for contractive type self-mappings in generalized convex metric spaces, with applications to solving Volterra integral equations. Another recent development includes introducing orthogonal convex structure contraction mappings and proving fixed point theorems on orthogonal b-metric spaces, with applications in examining the existence and uniqueness of solutions for integral equations like the spring-mass system . These advancements contribute significantly to the theoretical foundations and practical applications of fixed point theory in various mathematical contexts.

II. Literature survey

Fixed-point theory dances gracefully at the intersection of analysis, topology, and geometry. Within the realm of real-world challenges, fixed-point theory emerges as the fundamental key unlocking solutions in game theory and mathematical economics, organically woven into practical applications. Consequently, fixed-point theory stands as a pivotal realm of exploration within both pure and applied mathematics, thriving as a vibrant field of academic inquiry. [XX]

Banach's theorem is a renowned tool extensively applied in diverse mathematical examinations, laying the groundwork for additional fixed-point theorems like the ones innovated by Assad and Chatterjea. Through the further development of Banach's fundamental research, Assad and Chatterjea play a key role in broadening and enriching fixed point theory, highlighting the interdependence and progression of mathematical ideas within this domain.

Fixed-point theorems for mappings that are not self-maps in spaces that are metrically convex have received significant attention in academic works. Several scholarly articles have contributed to establishing the presence and singularity of fixed points for non-self mappings under nonlinear contractive circumstances in convex metric spaces [XX, XXI]. These investigations have illustrated the expansion and elongation of current theorems, highlighting the significance of such outcomes in metric spaces that are complete and Menger spaces that are strictly convex. Moreover, the concept of strictly convex configurations in spaces that are fuzzy cone metrics has been employed to validate the existence and singularity of fixed points for non-self mappings, thereby enhancing the comprehension of fixed-point characteristics in diverse spatial categories. Furthermore, the introduction of fresh contractive conditions in spaces that are non-Archimedean modular has outlined ample prerequisites for the existence of fixed points that are mutual for multiple mappings, emphasizing the adaptability and versatility of fixed-point proposals in a variety of scenarios.

The utilization of fixed point theory in nonlinear analysis has served as a potent instrument for many years, enabling the establishment of generalizations and theorems within partially ordered metric spaces. Furthermore, the advancement of fixed point theory holds great importance in contemporary mathematics, finding practical uses in various domains such as image manipulation, engineering, and the realm of physics. By employing fixed point theorems and associated principles, scholars are able to scrutinize and comprehend the characteristics of metrically curved planetary systems, offering valuable insights into the dynamics and formations present.

Assad and Chatterjea's fixed point theorems, along with other related research papers, have significant implications in various real-world scenarios. These theorems are fundamental in establishing the existence of solutions in game theory, mathematical economics, and engineering problems such as those in Rocket science, Electrical engineering, and Mechanical engineering [XII, XIII]. The theorems provide a mathematical framework to prove the existence of solutions in complex systems by utilizing concepts like Chatterjea type contractions in quasi-partial b-metric spaces and C*-algebra-valued metric spaces. Additionally, the application of fixed-point theory extends to solving Common Fixed Point Problems (CFPP) and Convex Feasibility Problems (CFP) in real-time scenarios, demonstrating the practical relevance of these theorems in iterative schemes for online algorithms and image recovery applications. Overall, Assad and Chatterjea's fixed point theorems play a crucial role in addressing a wide range of real-world problems across various disciplines.

Chatterjea's fixed point theorems have been extensively studied and extended in various settings. Research papers have explored Chatterjea-type contractions in different metric spaces, such as C*-algebra-valued metric spaces [XXI], quasi-partial b-metric spaces [XIII], generalized metric spaces with graphs [I], cone metric spaces over Banach algebras [XXIV], and metric spaces endowed with graphs [XII]. These studies have not only generalized and extended the original theorems but have also introduced new concepts like $(\alpha-\psi)$-contractive mappings, Chatterjea type Suzuki contractions, and weak contractions. The results obtained from these diverse approaches have enriched the existing literature on fixed-point theorems, providing a deeper understanding of the applicability and scope of Chatterjea's original work.

Assad and Kirk (1972) introduced a fixed point theorem for set-valued mappings of contractive type, which laid the foundation for further developments in fixed point theory. Building upon this, recent research has expanded on this concept by establishing new fixed-point results in various spaces and under different conditions. Yuan (2022) developed fixed point theorems for quasi-upper semicontinuous set-valued mappings and compact continuous mappings, providing valuable tools for nonlinear analysis. Additionally, Tassaddiq et al. focused on fixed point theorems in strong b-metric spaces using different contraction mappings, while Eroğlu et al. explored fixed point principles for mappings in partially ordered metric spaces. Furthermore, Ramaswamy et al. extended fixed-point results to try complex-valued metric spaces, showcasing the continuous evolution and application of fixed-point theory in diverse mathematical settings.

III. Basic Definitions

Definition1: The metric galaxy is assumed as metrically convex if, on behalf of any l,m ∈ T using l≠m, there exists r ∈ P (a≠ b ≠r) so that

$$P(i,r) + P(r,m) = P(l,m)$$

Definition 2: If T is a linear galaxy then X is a non-empty subsection of T.A plotting Z: X→T is believed as demiclosed only if{a_k} ⊆ Y anda $_k$→a∈X and Za_k→a∈T then Za =b.

Definition 3: IfX be a non-empty subdivision of the measured galaxy (T, P) besides R, Z: X→T.Then duo (R, Z} is believed to be companionable if each categorization {ak} starting X, and since the$\lim_{n\to\infty} P(Za_k, Ra_k) = 0$ and Z ak ∈X (k ∈S) it tracks as$\lim_{n\to\infty} P(Za_k, RZa_k) = 0$
for each arrangement {bk} ∈X so that bk = Rak (k ∈S).

III. Results and Discussions

FIXED POINT PROPOSITIONS FOR NON-SELF FUNCTIONS IN METRICALLY CONVEX PLANETARY

Our findings over the segment extend a conclusion that Rhoades got by proving several fixed point propositions for two multi-valued besides single-valued functions that fulfill his contractive disorder. In a comprehensive metrically curved measured planetary, we offer common fixed point theorems. Assad [III] and Chatterjea [VI] findings are expanded upon and generalized by our findings.

Here are some immediate applications of the lemma due to Assad and Kirk [IV]. "B" denotes the boundary of B in the sequel.

Lemma 4: Let X be a closed subsection of the whole and metrically curved metric galaxy (T, P) which is non-empty, formerly all $l \in X$, $m \notin X$, there happens a $r \in \partial X$ so that P(l,r) + P(r,m) = P(l,m).

Theorem 5: If T is a comprehensive metrically curved galaxy besides X a fastened non-empty subsection T. If Q: A→Bis plotting to sustain the dissimilarity

$$P(Zj,Ze) \leq \quad Q\max[1/2P(j,e),P(j,Zj),P(e,Ze)]+ Q'[P(j, Ze) + P(e, Zj)] \qquad (1)$$

for every j,e in X, where Q and Q' non-negative real'sthrough

$\max\left\{\frac{Q+Q'}{(1-Q'}, \frac{Q'}{(1-Q-Q'}\right\}$ = h <1. Additionally, for each u trend y∂X, Zj∈X. Formerly Z consumes an exceptional static idea in X.

Proof: If j0 ∈X. Then paradigm the two arrangements{js} and {js'} in the subsequent method.

Describej'$_1$ = Pj$_0$. Letj'$_1$ ∈X, put j$_1$ = j$_1$. Letj'$_1$ ∉X, choose j$_1$ ∈∂X,

So thatP(j_0,j_1)+ P(j_0,j_1')=P(j_0,j_1')

Define $j'_2 = Zj_1$. If $j_2' \in X$, put $j_2 = j_2'$. If $j'_2 \notin X$, choose $j_2 \in \partial X$, so that $P(j_1, j_2)+P(j_2,j'_2) = P(j_1,j'_2)$.

Enduring in this technique, we attain $\{j_s\}$ besides $\{j'_s\}$ nourishing

$j'_{s+1} = Zj_s, j_s = j_s$, if $j'_s \in X$, If $j_s' \notin X$, indicates $j_s \in \partial X$, so that

$P(j_{s-1}, j_s) + P(j_s, j_s') = P(j_{s-1}, j_s')$. Put $B = \{j_i \in \{j_s\} : j_i = j_l'\}$ and $C = \{j_i \in \{j_k\} : j_i \neq j_i'\}$.

It is tough to prove that $j_s \in C$, then j_{s-1} and j_{s+1}, are in the right place B.

Currently, we demand to approximation $P(j_s, j_{s+1})$.

Case A :

$j_s, j_{s+1} \in B$. Since (1), we obligate

$P(j_s, j_{s+1}) = P(Zj_{s-1}, Zj_s)$

$\leq Q\max[1/2P(j_{s-1}, j_s), P(j_{s-1}, Pj_{s-1}), P(j_{s-1}, Pj_s)] + Q'[P(j_{s-1}, Pj_s) + P(j_s, j_{s-1})]$

$= Q\max[P(j_{s-1}, j_s), P(j_s, j'_{s+1})] + Q''[P(j_{s-1}, j_{s+1}) + P(j_s, j'_s)]$.

Then it follows that

$P(j_s, j_{s+1}) \leq Q\max[P(j_{s-1}, j_s), P(j_s, j_{s+1})] + QP(j_{s-1}, j_{s+1})$

$\leq Q\max[P(j_{s-1}, j_s), P(j_s, j_{s+1})] + Q'[P(j_{s-1}, j_s) + P(j_s, j_{s+1})]$.

Now if $P(j_s, j_{s+1}) \leq P(j_{s-1}, j_s)$

we have So, $P(j_{s-1}, j_s) \leq \left\{\frac{(Q+Q')}{(1-Q')}\right\} P(j_{s+1}, j_s)$

When $P(j_{s-1}, j_s) \leq P(j_s, j_{s+1})$ we obtain

$P(j_s, j_{s+1}) \leq QP(j_s, j_{s+1}) + Q'P(j_{s-1}, j_s) + Q'P(j_s, j_{s+1})$

So, $P(j_s, j_{s+1}) \leq \left\{\frac{(Q')}{(1-Q-Q')}\right\} P(j_{s-1}, j_s)$

Thus in both situations, we obtain

$P(j_s, j_{s+1}) \leq hP(j_s, j_{s+1})$

Case B:

If $j_s \in B, j_{s+1} \in C$.

Then condition (1) implies that

$P(j_s, j_{s+1}) \leq P(j_s, j_{s+1}) + P(j_{s+1}, j'_{s+1})$

$= P(j_{s+1}, j'_{s+1})$

$= P(Zj_{s-1}, Zj_s)$

$\leq Q \max [1/2\, P(j_{s-1}, j_s), P(j_{s-1}, Zj_{s-1}), P(j_s, Zj_s)] + Q' [P(j_{s-1}, Zj_s) + P(j_s, Zj_{s-1})]$

$= Q \max [P(j_{s-1}, j_s), P(j_s, j'_s)] + Q' [P(j_{s-1}, j'_{s+1})]$

$P(j_{s-1}, j_s) \leq P(j_{s-1}, j'_{s+1})$

we have $P(j_s, j_{s+1}) \leq \left\{\frac{(Q+Q')}{(1-Q')}\right\} P(j_{s-1}, j_s)$

Consequently, we get

$P(j_s, j_{s+1}) \leq h\, P(j_{s-1}, j_s)$

This concludes the evidence.

Remark 6: Our Theorem 5 combines conclusions attributed to Chatterjea [6] and Assad [3], as each metric planetary, and consequently a Banach planetary, then metrically curved.
Here's a little more general version of Theorem 5.

Theorem 7: Assume that T is a metrically curved planetary that is comprehensive, D is a padlocked, non-empty subsection of T, and X is a closed, non-empty subset of D. Let $Z: X \rightarrow D$ which satisfies (1)and the property that $a \in \partial_D X$the, boundary of X relative to D, implies $Za \in X$. Formerly Z has an exclusive static idea in D.

RESULT REGARDING COMMON STATIC POINT IN METRICALLY CONVEX PLANETARY

A common static idea theorem in metrically convex metric planetary has been proved in this section. Our findings supplement the previously published findings of Assad [III], Chatterjea [VI], and several other researchers

Theorem 8: If B is a comprehensive metrically curved galaxy and N a closed non-empty subsection of B. IfM, P: N—› B be plotting to nourish the dissimilarity

$$d(Mj, Me) = B \max [1/2\, d(Pj, Pe), d(Pj, Mj), d(Pe, Me)] + B' [d(Pj, Me) + d(Pe, Mj)] \quad (2)$$

for each f,e in N, someplace B and k' remain non-negative real'sthrough

$\max\left\{\frac{\lambda+\lambda'}{(1-\lambda'}, \frac{\lambda'}{(1-\lambda-\lambda'}\right\} = h < 1$.Further, for every x in ∂N, $Pj \in N$. Then M, P has a unique fixed point in N.

Proof: Consider the arrangements $(f_s\}$ and $(e_s\}$ in the subsequent method. If $f\in\partial N$, formerly nearby happens a fact j_o in N, so that $j=Pj$ as $\partial N\subseteq PN$.

Subsequently $Pj_0\in\partial N$ and $Pj\in\partial N\Rightarrow Mj\in N$,

we accomplish that $Mj_0\in N\cap MN\subseteq PN$. Let $a_1\in N$ be such that $e_1=Pa_l=Mf_0\in N$.

Let$e_2=Mj_l$,suppose$e_2\in$ Nthen$e_2\in N\cap MN\subseteq PN$whichsuggeststhatnearbyhappenan idea$j_2\in$ Nsothat$e=Pj_2$.Suppose$e_2\notin N$,thennearbyhappens an idea $t\in\partial N$sothat $d(Pf_1,t) + d(t,e_2) = d(Pf_1,e_2)$.

Subsequently $t\in\partial N\subseteq PN$, there happens an idea $j_2\in N$ so that $t=Pj_2$,So that,

$d(Pa_1, Pa_2) + d(Pj_2, e_2) = d(Pj_1,e_2)$,

Enduring in this technique we acquire$\{j_s\}$then$\{e_s\}$sustaining

$e_{g-1} = M j_g$

(a) $e_g\in N\Rightarrow e_g = Pj_s$or

(b) $e_g\notin N\Rightarrow Pj_g$ □ $\partial N$and

(c) $d(Pj_{g-1}, Pj_g) + d(Pj_g,e_g) = d(Pj_{g-1},e_g)$.

Put $T = [Pj_1\in\{Pj_s\}:Pj_i = e_i]$ and$V = [Pj_g\in\{Pj_s\}:Pj_i\neq e_i$

It is not firm to display that if $Pjg\in A$, formerly Pjg-1then Pjg+1, have its place to T. Currently, we demand the approximation $d(Pj_g, Pj_{g+1})$.

Case I. If $Pj_g,Pj_{g+1}\in T$. Starting(2),we obligate $d(Pj_g, Pj_{g+1}) = d(e_g, e_{g+1})$

$= d(Nj_{g-1}, Nj_g)$

$\leq \lambda\max[1/2d(Pj_{g-1}Pj_g), d(Pj_{g-1}, Pj_g),d(Pj_g,Pj_{g+1})]+\lambda' [d(Pj_{g-1},Pj_{g+l}) + d(Pj_g, e_g)$

$= \lambda \max [d(Pj_g, Pj_g), d(Pj_g, Pj_{g+1})]+\lambda'[d(Pj_g, Pj_{g+1}) + d(Pj_g, Pj_g)]$

Formerly it tracks that

$d(Pj_g, Pj_{g+1}) \leq\lambda\max[d(Pj_{g-1}, Pj_g), d(Pj_g,Pj_{g+1})] +\lambda' [d(Pj_{g-1},Pj_{g+l})]$

$= \lambda \max [d(Pj_{g-1}, Pj_g), d(Pj_g, Pj_{g+1})]+\lambda'[d(Pj_{g-1},Pj_{g+1}) + d(Pj_g,Pj_{g+1})]$

If $d(Pj_g,Pj_{g+1}) \leq d(Pj_{g1},Pj_g)$.

We have $d(Pj_g,Pj_{g+1}) \leq \lambda\ d(Pj_{g-1},Pj_g) +\lambda'd (Pj_{g-1},Pj_g) + \lambda'd(Pj_g, Pj_{g+1})$

$= \lambda \max [d (Pj_{g-1}, Pj_g), d(Pj_g, Pj_{g+1})+ \lambda' [d (Pj_g,Pj_{g+1})]$.

For $d(Pj_g, Pj_g) \leq d (Pj_g, Pj_{g+1})$

$d(Pj_g, Pj_{g+1}) \leq \left\{\frac{\lambda'}{(1-\lambda-\lambda'}\right\}d(Pj_{g-1}, Pj_g)$.

If $d(Pj_g, Pj_{g+1}) \le d(Pj_{g-1}, Pj_g)$,

$d(Pj_g , Pj_{g+1}) \le \left\{\frac{\lambda+\lambda'}{(1-\lambda')}\right\} d(Pj_{g-1} , Pj_g)$.

Consequently, we get

$d(Pj_g, Pj_{g+1}) \le d(Pj_{g-1}, Pj_g)$,

Case II. If $Pj_g \in A$ and $Pj_{g+1} \in T$.

As $Pj_g \in A$ is a convex direct grouping of Pj_{g-1}, and e_s we have

$d(Pj_g, Pj_{g+1}) \le \max [d(Pj_{g-1} , Pj_{g+1}), \; d(e_g, Pj_{g+1})$

If $d (Pj_g , Pj_{g+1}) \le d(e_g, Pj_{g+1})$

we have

$d(Pj_g , Pj_{g+1}) \le d(e_g , Pj_{g+1}) = d (Mj_{g-1} , Mj_g)$

$\le \lambda \max [1/2\, d(Pj_{g-1}, Pj_g), d(Pj_{g-1} , e_g), d(Pj_g , Pj_{g+1}) + \lambda'[d(Pj_{g-1} , Pj_{g+1}) + d(Pj_g, e_g)$

$= \lambda \max [1/2\, d(Pj_{g-1}, Pj_g), d(Pj_{g-1}, e_g), d(Pj_g, Pj_{g+1}) + \lambda' [d (Pj_{g-1}, Pj_{g+1}) + d (Pj_{g-1} , Pj_{g+1})$

$= \lambda \max [d (Pj_{g-1}, e_g) + d (Pj_g, Pj_{g+1}) + \lambda'[d(Pj_{g-1}, Pj_{g+1}) + d(Pj_{g-1}, Pj_{g+1})]$.

$d(Pj_{g-1} , Pj_{g+1}) + d(Pj_g , e_g) \le d(Pj_{g-1}, Pj_g) + d(Pj_g, Pj_{g+1}) + d(Pj_g, e_g)$

$\le d (Pj_{g-1}, e_g) + d(Pj_g, Pj_{g+1})$.

If $d(Pj_g , Pj_{g+1}) \le d (Pj_{g-1}, e_g)$

we obligate

$d(Pj_g, Pj_{g+1}) \le \lambda\, d(Pj_{g-1}, e_g) + [d(Pj_{g-1}, e_g) + d(Pj_g, Pe_{g+1})$.

Whence $d(Pj_g , Pj_{g+1}) \le \left\{\frac{\lambda+\lambda'}{(1-\lambda')}\right\} d(Pj_{g-1} , Pj_g)$. when $d(Pj_{g-1}, Pj_g) \le d (Pj_g, Pj_{g+1})$,

we have $d(Pj_g, Pj_{g+1}) \le \lambda d(Pj_g, Pj_{g+1}) + d(Pj_{g-1}, Pj_g) + d(Pj_g, Pj_{g+1})$

That is $d(Pj_g , Pj_{g+1}) \le \left\{\frac{\lambda'}{(1-\lambda-\lambda')}\right\} d(Pj_{g-1} , Pj_g)$.

Consequently, we have

$d(Pj_g , Pj_{g+1}) \le h\, d(Pj_{g-1}, Pj_g)$

$\le h^2 d (Pj_{g-2}, Pj_{g-1})$

given Case II. $Pj_g \in A$, implies $Pj_{g-1} \in T$.

Now if $d(Pj_g, Pj_{g+1}) \le d (Pj_{g-1} , Pj_{g+1})$

we have

$d(Pj_g , Pj_{g+1}) \le d(j_{g-1} , Pj_{g+1})$

$= d (Nj_{g-2} , Nj_g)$

$\leq \lambda$ max [1/2 $d(Pj_{g-2}, Pj_g)$, $d(Pj_{g-2}, j_{g-1})$,$d(Pj_g, Pj_{g+1})+\lambda'[d(Pj_{g-2}, Pj_g) + d(Pj_g, j_{g-1})$

$= \lambda$ max [1/2 $d(Pj_{g-2},Pj_g)$, $d(Pj_{g-2}, Pj_{g-1})$, $d(Pj_g, Pj_{g+1})+\lambda'$ [$d(Pj_{g+2},Pj_{g+1}) + d(Pj_{g+1}, Pj_{g-1})$

Given the fact

$1/2\ d(Pj_{g-2}, Pj_g) \leq 1/2\ [d(Pj_{g-2},Pj_{g-1})+d(Pj_{g-1},Pj_g)]$

$\leq \max [d(Pj_{g-2},Pj_{g-1}),d(Pj_{g-1},Pj_g)]$.

Therefore, we get

$d(Pj_g, Pj_{g+1}) \leq \lambda \max [d(Pj_{g-2},Pj_{g-1}),d(Pj_{g-1},Pj_g),d(Pj_{g-2},Pj_{g-1}),d(Pj_g,Pj_{g+1})$

$+ \lambda'[d(Pj_{g-2},Pj_{g+1})+d(Pj_g,Pj_{g-1})]$

$\leq \lambda\max[d(Pj_{g-2},Pj_{g-1})d(Pj_g,Pj_{g+1})]+\lambda' [d(Pj_{g-2},Pj_{g+1})+ d(Pj_g, Pj_{g-1})]$.

If $d(Pj_g ,Pj_{g+1}) \leq d(Pj_{g-2},Pj_{g-1})$,we have

$d(Pj_g,Pj_{g+1}) \leq \lambda\ d(Pj_{g-2},Pj_{g-1})+ \lambda\ [d(Pj_{g-2},Pj_{g-1})+d(Pj_{g-1},Pj_{g+1})+d(Pj_g,Pj_{g-1})]$

$\leq \lambda d(Pj_{g-2},Pj_{g-1})+\lambda\ d(Pj_{g-2},Pj_{g-1})+\lambda\ d(Pj_g,Pj_{g+1})$

Thus

$d(Pj_g , Pj_{g+1}) \leq \left\{\frac{\lambda+\lambda'}{(1-\lambda}\right\} d(Pj_{g-2} , Pj_{g-1})$.

when $d(Pj_{g-2},Pj_{g-1}) \leq d (Pj_{g-2},Pj_{g+1})$,

we obtain

$d(Pj_g, Pj_{g+1})\leq \lambda d(Pj_g,Pj_{g+1})+\lambda(Pj_{g-2},Pj_{g-1}) +\lambda d(Pj_g,Pj_{g+1})$That is

$d(Pj_g , Pj_{g+1}) \leq \left\{\frac{\lambda'}{(1-\lambda-\lambda'}\right\} d(Pj_{g-2} , Pj_{g-1})$.

Now joining the overhead dissimilarities, we get that

$d(Pj_g,Pj_{g+1})\leq hd(Pj_{g-2},Pj_g)$.

Consequently in altogether three circumstances, we treasure that

$d(Pj_g,Pj_{g+1})\leq h \max [d(Pj_{g-2},Pj_{g-1}) , d(Pj_g, Pj_{g-1})$.

It is monotonous to confirm that for n ≥1,

$d(Pj_g,Pj_{g+1}) \leq h^{g/2}\delta$,

where$\delta=h^{-1/2}\max[d(Pj_0,Pj_1),d(Pj_1,Pj_2)]$

Consequently m, g>N,

$d(Pj_m,Pj_g) \leq \sum_{i=N}^{\infty} d(Pj_g, Pj_{i+1}) \leq \delta \sum_{i=N}^{\infty} h^{i/2}$

Therefore{Pj_g} is a Cauchy arrangement besides here after unites to a bound t (say).

Correspondingly nearby happens an unlimited subsequence

{$Pj_{g(k)}$)}of{Pj_g}so$Pj_{g(k)}$)}∈t.

Formerly

$d(Pt,t) \leq \lambda \max[1/2\, d(Pj_{g-1}, Pt), d(Pj_{g-1}, Pj_g), d(Pt, Nt)]+\lambda'[d(Pj_{g-1},Pt)+d(Pt,Pj_g)$, so on allowing $g \longrightarrow \infty$,we attain

$d(Pt,t) \leq \lambda\, d(Pt, T)$, suggests then Pt = T, subsequently $\lambda<l$.

Consequently, we take demonstrated that N and P share a single immovable idea t one can confirm that t is unique by doing a simple calculation.
This brings the proof to a close.

Remark 3: Our Theorem unifies results attributed to Chatterjea [6] and Assad [2] subsequently each metric planetary, and consequently a Banach planetary, is metrically curved space.

IV. Conclusion

This writing piece explores the analysis of various fixed point theories that lack self-plotting qualities within the domain of metrically convex planetary arrangements. Within Assad's investigation, a significant revelation emerges concerning the integration of a more thorough single-valued plotting technique, unveiling a remarkable expansion of Chatterjea's initial theory. The piece goes on to delve into a discussion regarding the implications of these discoveries on a unified fixed concept within metrically curved planetary formations. The in-depth analysis explores the intricacies of mathematical theories in planetary scenarios, shedding light on the connection between fixed points and plotting techniques in a non-self-plotted environment, offering a fresh perspective on the subject.

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Conflicts of Interest

The author declares that there are no conflicts of interest regarding the publication of this paper.

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