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ANALYSIS OF STRUCTURAL CHANGES OF THE BALANCE SHEET ECOLOGICAL-ECONOMIC MODELS OF THE "INPUT-OUTPUT" TYPE

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Abstract

The purpose of the research is to develop methods, algorithms, and computational procedures of analysis, and solution (subsolution) of problems during changes at the stages of ecological and economic modelling of processes. Metaheuristics are proposed, which take into account the experimentally obtained knowledge about the properties of the model. A computational experiment was conducted to analyze the properties of the improved "input-output" model (linear system) using the method of basic matrices. This method has several software implementations of the corresponding algorithms in "exact" and "long" numbers. It includes the ability to both solve the problem (from beginning to end) and resolve the problem with changes in the model (without re-solving at first). Hence, using the example of calculations based on the speed criterion, decision-making on choosing the "best" algorithm for solving the problem is demonstrated.

Keywords: Basic Matrices, Ecological and Economic System, Method of Exact Calculations, Sustainable Development, Ill-Conditioned System of Linear Equations.

I. Introduction

It is widely recognized that the development of the economy should be balanced with the state of ecology and comply with legislation. In this regard, the mathematical models should justify such a balance. All this outlines the complexity of the problem (as a multi-criterion) and indicates the urgency of continuing research in this direction.

To develop based on the analysis, taking into account the experience of modelling ecological and economic processes, the developed methods and calculation algorithms in previous studies, to translate into the practical plane of experiments on typical (according to structural properties) problems. To propose new approaches to evaluating the effectiveness of computational procedures (according to various criteria) for choosing the "best" solution scheme or sub-solution of the problem.

The following tasks are set:

- outline the typical properties of the elements of the basic model (as a linear system);
- build model examples based on the identified typical properties of models, in particular, poorly conditioned ones;
- to propose variants of computing algorithms that were chosen as basic based on the results of the analysis for the experiment;
- to analyze the efficiency of calculations according to algorithms according to the criteria;
- construction of metaheuristics for choosing an effective calculation algorithm and determination of the guaranteed number of changes in model constraints in which it is expedient to solve the problem further (without resolving the problem first);
- to investigate the computational properties of MBM implementation algorithms for various quantitative changes in the model, for example, in the rows of the model (without resolution and with additional solution);
- to build a graphic and tabular interpretation of the results of the experiment.

The object of research in the work is mathematical models of ecological and economic processes.

The subject of research are methods, algorithms, and computational procedures for analyzing the properties of mathematical models of linear systems.

II. Materials and Methods

The structural analysis of a linear system (input-output) during modelling is an important component of studies such as Li [V] Kudin [XX], Onyshchenko [XII], and Voloshin [XIV]. This is due to several factors:

Contradiction of evaluation criteria of component models of ecological and economic processes (economic development, evaluation of the state of ecology, motivation of the activity of individuals and groups). In particular, consideration of the role in the decision-making circuit during modelling. Taking into account the interests of individual individuals and various groups that exist and interact in the ecosystem (environment) under strict legal restrictions, international agreements, etc.

For example, to reduce gas emissions, the advanced economies (China, India, etc.) conducted an extensive study [XXV, III, XI, XXIV, VII, XIII]. These studies are mainly conceptual and theoretical in nature and do not allow to determination of specific quantitative values of the implementation of certain mechanisms. However, they facilitate studying this problem prom the mathematical modelling perspective. We can name macroeconomic models based on Keynes's theory, for example, the Oxford and Dri-Wefa models [XVII].

In the study by Nishi [XIX], the optimal distribution of responsibility according to the Kyoto Protocol was analyzed using the decision-making theory and the apparatus of fuzzy sets. Based on this forecast, the agents of the agreement can make an appropriate decision on cooperation. In the development of the proposed mechanisms for the distribution of quotas, according to the authors, it is expedient to consider the vague statements of models for the distribution of collective costs, since the parameters introduced in the formation of individual potential incomes of agents are empirical, and therefore inaccurate.

Another class of ecological and economic models, that allows to study of the balance of the general system, can be called models of inter-industry balance, which study the mutual influence of the structure of the economy on the surrounding natural environment. This class of models includes the interdisciplinary Leontiev-Ford model and its generalization [IV].

Applied models of general equilibrium are built on this ideology [XXIII, XVIII]. Such models are the most widely used, which allow considering the economic interrelationships in the economic system with maximum completeness, based on a voluminous array of statistical data.

- Contradiction of calculation efficiency criteria. Multi-criteria evaluation of the quality of calculations by methods and algorithms – by accuracy, speed, consumption of resources, for example, PC memory. By their nature, the mentioned criteria are contradictory, and, naturally, even industrially developed methods have different effectiveness when evaluated according to them on specific mathematical models. Then it is expedient to develop special decision-making procedures (formation of priority based on criteria) in conditions of multicriteria to justify the choice of method and algorithm for solving the problem. Often, for this, the analysis of the results of experiments on typical mathematical models to obtain additional information ("knowledge") is actively taken into account. Nowadays, the direction of application of heuristics (plausible reasoning) and metaheuristics (combination and integration of algorithms into each other) based on the methods of modern evolutionary computations [VI, XV] to increase the efficiency of calculations is actively developing.

Metaheuristics are grounded on the properties of the studied phenomenon, mathematical modelling, and methods for increasing the efficiency of calculations. Quality criteria for building metaheuristics can be accuracy, calculation speed, resource consumption, etc.

In this study, the main emphasis will be on taking into account the structural properties of evolving (changing) models when building metaheuristics.

- Structural features of the mathematical model of the linear system, for example, the matrix of restrictions (percentage filled with non-zero elements) or the conditioning of the system significantly affect the efficiency of calculations [X, XII, IX]. This determines the development of an original organization of calculations [VIII, XXI].

- Inadequacy of the mathematical model and ecological and economic process.

Accordingly, a mathematical model of an ecological and economic system is the calculation of continuous improvements from the initial model to the final one, by the efficiency criterion. In terms of a linear system (matrix model), such a system can structurally change various components of the model (element, row, column, group of rows or columns, "expand" ("narrow") spatially by inclusion-exclusion of blocks of sub-matrices. In general, such a system is more convenient for improvements Further improvement of the model to achieve greater adequacy of the process model can be achieved by including weakly nonlinear inclusions in the model elements.

It should be noted that models of ecological and economic processes (as linear systems) have exactly a block (cellular) structure – quadrants of the matrix of constraints. In particular, the classic scheme of inter-industry balance in the first quadrant contains inter-industry flows that correspond to functional-structural industry connections.

It is known that the most effective computational procedures and industrial implementations have been developed for linear models, for example, variants of the simplex method, which are conditionally divided into methods applied to a direct problem and a dual problem. According to the Gaussian elimination algorithm, the original model can be equivalently performed to the model of a simple structure (with a diagonal matrix of constraints); the number of calculations declines with each iteration. It should be noted that such a transformation of the initial model somewhat limits the post-calculation analysis when there are changes in the model.

The basis of the proposed method of basis matrices and its corresponding algorithms is the idea of a basis matrix formed by linearly independent rows of the matrix of constraints. The process of iterative "growing" (replacing) constraints of the auxiliary system with relaxed constraints of the main system [VIII, XXI] is carried out. The method of basic matrices (MBM) can be applied to transform the model of a simple structure by inserting one of the original system constraints. In other words, the transition to the original model occurs. Consequently, the value of the rank is set to establish the inverse matrix and verify the conditionality.

MBM algorithms are used when analyzing the impact of changes in the linear system of the model. Partial studies on the analysis of the impact of such changes on the properties of a linear system (without resolving the problem) were carried out by Wibawa et al. [XXII] and Grylitska [I], – element, row, column of the constraint matrix. This approach is aimed at iterative improvement of the model – achieving process adequacy.

Adequacy of the mathematical reference model and the computer model. The reality of the application when performing computer implementation of algorithms on a PC is as follows: in general, the property of commutativity is lost when performing calculations. This "imposes" additional restrictions on the organization of calculations according to the algorithm. When developing a universal high-precision method for solving a wide class of linear problems, the problems of large dimensions, incorrectness (sensitivity to inaccuracies), ill-conditioned systems, etc., did not "go away" [V, XXI]. In this regard, for various types of poor conditionality or the structure of constraint matrices, it is necessary to adapt the solution algorithm to ensure the quality of calculations [XII, XIV, XV].

We can make sure that the information about the value of the conditioning number (or its estimate), as a factor controlling the correctness of calculations, currently remains in the field of further research [XXV]. In particular, the availability of control over the conditioning of the system, and the accumulation of calculation errors when solving such inconvenient problems is an important necessary, and integral component of the computational process. The presence of an estimate of the conditioning number during the calculations is an important component, as it "gives a signal" (indicates) the correctness of the calculations.

To check and control the properties of solution algorithms (testing this class of problems), several algorithms, programs, and model problems with ill-conditioned constraint matrices have been developed, for example, BLAS (Basic Linear Algebra Subroutines) subroutines [III]. It is known that the Hilbert matrix belongs to such test matrices. The development of mathematical methods, algorithms, and software for performing basic operations has gained significant attention, which is attributed to the need to perform operations with rational numbers and avoid errors [XI, XVI, XVII]. The use of rational arithmetic for direct methods of solving SLAR eliminates the computational error and makes it possible to focus on the properties of the model itself.

III. Results

III.i. The Method of Basic Matrix of Linear Ecology-Economic System Analyses

Let's set a task based on a balanced "input-output" model – to calculate the expenses for fulfilling the restrictions in accordance with the Paris Agreement. Solving the task implements dealing with the complex fundamental issues of contemporary science. The list of which should include, for example, the development of a safe and valid method of forecasting the environmental conditions parameters and its quality criteria, capable of quantitative measurement of the public needs and wants satisfaction, especially in purity and natural variety; creation of the scientifically grounded methods of determining the economic loss caused by the environmental pollution; construction of the interrelation model system for the different components of natural complexes, taking into account the natural and anthropogenies factors and conditions.

Within the research, it is required to take into account the expenses for performing the greenhouse gas emission restrictions in the structure of the core production branches, like this in the Leontief-Ford model [XX, XII, XIV]:

$$\begin{cases} x_1 = A_{11}x_1 + A_{12}x_2 + Cy_2 + y_1, \\ x_2 = A_{21}x_1 + A_{22}x_2 - y_2, \end{cases}, \qquad (1)$$

The first formula of the given model reveals the economic balance – the allocation of the branch production gross output to the main or auxiliary productive consumption, the final consumption of the main production, and the expenses, associated with performing the obligations under the terms of Paris Agreement.

The second formula shows the physical balance of the greenhouse gas as the emissions total, caused by the activity of the main or auxiliary production and their undestroyed production volume.

The two separate classes of subsystems serve as the basis for building the whole inter-branch system. The differentiation of the procedure's reconciliation required the allocation and reallocation analyses to calculate the newly-created cost of material production. This, in turn, demonstrated the accumulation fund within the disbursed national income and consumption fund. They involve the actual volume of goods and services. The implementation of the binding procedures for the trends of production and non-production spheres development allowed to use of such a system as the means of analyses for the social and economic whole societies development.

To evaluate the technological structure within the model (1), the rational way should be the application of the provided by Yang and Deb [XXIV] the solution algorithm for defining the transformed matrix structures based on the method of basic matrix. For this aim, by [VIII] the model (1) should be written in the following way:

$$Au = C, \qquad (2)$$

Where

$A = \begin{pmatrix} E_1 - A_{11} & -A_{12} \\ -A_{21} & E_2 - A_{22} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$, $u = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = (u_1, u_2, ..., u_m)^T$ – m -measurement vector, x_1, x_2 – "subvector" u, $C = \begin{pmatrix} E_1 & C \\ 0 & -E_2 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$, E_1, E_2 – block unity matrix of relative dimension, 0 – block zero matrix.

We should also go into the system, disturbed (at matrix elements A_{11}, A_{12}, A_{21}, A_{22} and C) concerning the system of linear algebraic equation (2) like as:

$$\bar{A}u = \bar{C}, \qquad (3)$$

Where $\bar{A}$, $\bar{C}$ – are the relevant disturbed matrix and vector-column. It is assumed that the perturbation in the system (3) is performed for the 1st line ($k = 1$), not imposing limitations on the generality. Thus, we get the following step-by-step algorithm:

Step 1. To find the solution of u_0 outgoing system (2) and its reverse matrix A^{-1}.

Step 2. To disturb the matrix A within the elements of the 1st line 1st like as $\bar{a}_i = a_i + a_i'$, $\bar{c}_i = c_i + c_i'$, $i = n$.

Step 3. To measure the coefficient $\bar{\alpha}_{kk} = a_k e_k + a'_k e_k = 1 + a'_k e_k$, where e_k – matrix column A^{-1}.

Step 4. To evaluate
$\bar{\Delta}_k = (a_k + a_k')u_0 - (c_k + c_k') = (a_k u_0 - c_k) + (a_k' u_0 - c_k') = \Delta_k + \Delta_k' = \Delta_k'$.

Step 5. To calculate $\lambda = -\frac{\overline{\Delta}_k}{\overline{\alpha}_{kk}}$.

Step 6. To find the new vector column $\overline{e}_k = \lambda e_k$.

Step 7. To form the new solution based on the ratio $\overline{u}_0 = u_0 + \overline{e}_k$.

In MBM [XX, XII, XIV], finding the value of the rank, establishing its completeness, a scheme for restoring its completeness, and solvability analysis are included. Implementations have been developed in the environment of various software products, with various scenarios of calculations, in particular in exact numbers, and the "build-up" of the mechanism for taking into account vagueness in the presentation of the model is provided.

-There is an obvious need for the development of existing and the development of new methods and algorithms, which would provide for the possibility of analyzing linear systems according to a single methodology. This applies, in particular, to models applicable in the future when solving SLAR with an ill-conditioned constraint matrix, for various scenarios of presentation of model elements (mantissa length, order values, etc.). At the same time, the properties of the model during changes are calculated using different calculation scenarios. The possibilities of the influence of these methods and algorithms on the key parameters should be calculated to determine the leading element of the iteration, rank completeness, and the conditions for resistance of magnitude. This also applies to methods of analysis and processing of transient situations ("near zero").

-When analyzing the properties of the linear ecology-economic system (an improved "input-output" model), several works by the authors the method of basic matrices (MBM) was used, which has several implementations of algorithms in MATLAB and C++ both in exact (rational) and long numbers (with mantissa and order of different lengths) when presenting model elements. Model elements have several special limitations. For example, the system may be poorly conditioned. MBM includes the possibility of both solving a problem (using one of the algorithms) and analyzing its conditioning during the iterative process. The modelling process is a procedure consisting of a certain procedure, which is used to determine the structure of the task and identify changes to ensure the adequacy of the process. In the method of basic matrices, it is possible to take into account the impact of changes on the solution during the modelling process without resolving the problem first. That is, only refinement of the found solution is carried out according to the changed elements of the model. It was established that the computational complexity of performing MBM iterations is not constant. In particular, there is no linear dependence on the iteration number.

-Modelling, as the process of obtaining an adequate mathematical model, is essentially iterative. That is the result of a series of refinements (improvements) of the values of the model parameters (from the previously achieved level), i.e., further solving the problem with structural changes in the part of the model, for example, rows or columns of the constraints matrix. On the one hand, it is necessary to analyze the impact of quantitative changes on structural properties of model constraints such as rank, dimension, etc. On the other hand, it is important to establish a quantitative limit of one-time changes in the model, in which the expediency of the final solution of the problem remains (without re-solving it first). The mentioned concept of modelling has been studied in several scientific works [XX, XII, XIV].

In previous studies [XX, XII, XIV], attention was focused on the construction of methods and corresponding algorithms for solving the problem (with changes in the model) according to various computational schemes. Therefore, this article aims to perform experimental studies on a linear model using developed and implemented algorithms to determine changes in the mathematical model with additional solutions.

This determines the relevance of conducting experimental studies to establish the permissible limit of the number of changes (when refining the model) at which it is advisable to carry out additional solutions (without resolving the problem at first).

III.ii. Description of the Computer Experiment

Performing the computing experiment on research of ecology-economic balanced models at changes (direct task) the following aspects were considered:

- The impact of changes in the basic model on the system property should be defined directly.
- The elements of the model matrix were within the limits [0, 1].
- Changes in the model could produce necessary conditions for the system.
- Changes in the remoulded model could generate negative elements of the model.
- The restriction matrix became depraved.

Therefore, it is essential to analyze the minor changes in the model's impact on the system properties, which correlate with the technological changes of the real system. Thus, the analysis facilitated determining the critical percentage of the changes in the model that were required to solve the task. The basic matrix method was used to carry out the performance analysis. The method went through a wide range of approbation grounded on the research of linear model of variable dimension.

At performing the computing experiment (at provision of the model elements), the classes of the long whole numbers (longint3) and rational numbers (longrat3) [II] have been taken as a basis; and the program, based on the C++ software language, has been worked out for the performance of the algorithms, provided at research, and provided at research computation scheme with the use of both, exact arithmetic, and with the help of standard class of long double real numbers, and a minimal interface of the user for setting the parameters and computation scheme selection:

- mbm_slar_long_double/mdm_new – program for performing computations with long double types;
- mbm_slar_rational/mdm_new – program for performing computations in exact numbers (with longrat3 types).

Thus, the Gauss-Seidel method was applied, choosing the maximal element. When conducting the computing experiment, the classes of the long whole numbers (longint3) and rational numbers (longrat3) [XX] were selected as a basis. The program, based on the C++ software language, was created to process the algorithms. As a result, the computation scheme was generated with the help of exact arithmetic and long double real numbers, with a minimal interface of the user:

- mbm_slar_long_double/mdm_new – program for performing computations with long double types;
- mbm_slar_rational/mdm_new – program for performing computations in exact numbers (with longrat3 types).

In particular, during the experiment, the Gauss-Seidel method was used with the choice of the maximal element, the method of the basic matrix (MBM), and the "extra-computing" algorithm was performed (at replacement of the system restriction in the matrix line).

Based on the developed program, a computational experiment was organized. The restriction matrix disturbance impact on the model characteristics has been examined. For example, the disturbance of one of the matrix lines A has been accomplished in the following way:

$$\bar{a}_i = a_i + a_i', \ i = 1, \ a_i' = (\frac{(-1)^i}{2}, i = \overline{1,10}), \tag{4}$$

III.iii. Computer experiment results

As the experiment demonstrated, it is essential to analyze the model properties, identifying the computational properties of the algorithms following the speed criterion. The importance of evaluating the efficiency of calculations according to various criteria when making decisions on choosing the best calculation algorithm for the task for further research was confirmed.

The results of algorithm application to the result recalculation in the cases of lines (series of lines) disturbance at model elements provision in data types: long double numbers are given in Table 1.

Table 1 : The results of applying the algorithm to recalculate the solution in cases of perturbation of rows in double types

Number of perturbed lines			Solution *u*					Solution recalculation time, in nanoseconds			
1	2.99839	1.75135	6.7416	3.35945	-.452502	2.0625	0.163369	-1.14836	-3.82441	0.154731	5720
2	2.12216	1.27738	7.61853	-2.34673	0.458626	3.11335	0.750534	0.0173935	4.44607	0.123979	5440
3	-2.9922	-0.489451	-5.95765	8.10848	1.85397	3.41601	2.84854	6.10529	-0.6286	2.76166	5960
4	-3.46628	4.98704	39.8093	-2.95933	-1.00615	17.6235	6.6371	10.8209	21.1179	-2.36218	5880

5	-35.0955	-7.71836	-58.605	64.4357	8.02833	7.11615	18.9283	47.0108	-30.192	12.6991	5640
6	-162.704	-52.649	-387.062	345.897	38.2322	-3.8495	71.9137	189.647	-203.677	64.7024	5640
7	-148.609	-47.6007	-347.72	314.859	34.9001	-2.6109	65.0629	173.48	-183.898	61.4886	5800
8	-488.004	-167.074	-1220.13	1011.63	116.756	-42.3945	193.528	496.389	-621.539	210.837	6080
9	-523.397	-180.77	-1315.39	1070.57	125.699	-62.2434	189.345	543.518	-680.791	222.888	7640
10	-895.477	-310.399	-2328.67	1809	196.755	-151.841	347.071	987.137	-1216.89	384.712	6760

The exact numbers are given in Table 2.

Table 2 : The results of the algorithm application for "re-computation" of the result in cases of line disturbance in the types of exact numbers

Lines, being disturbed (quantity)	1	2	3	4	5	6	7	8	9	10
Time of the result re-computation, in milliseconds	110080	299811	595616	1049330	1565483	2343675	3162645	4146848	5082008	6436250

The results of algorithm application to the result recalculation in the cases of lines (series of lines) disturbance at model elements provision in data types: long double is graphically presented (Figure 1-2).

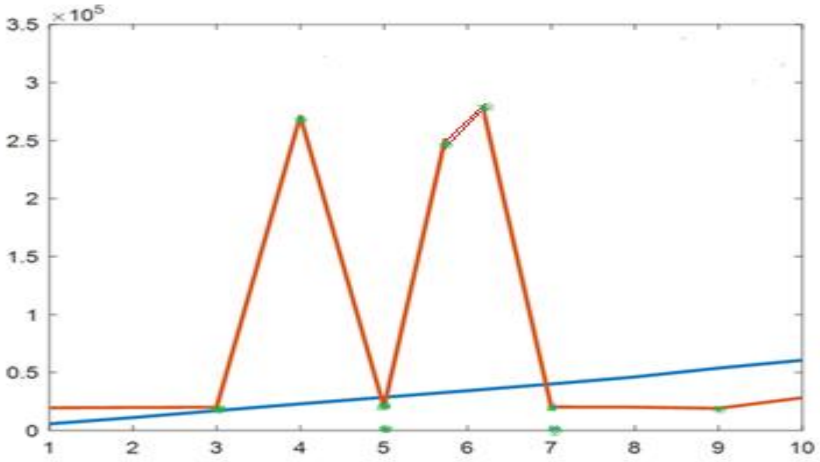

Fig 1. The dependence of the result re-computation time from the number of disturbed lines at solving in numbers of "double" type

Figure 1-3 – "red line" of MBM application to "the whole" system, and "blue" – consistent application of the algorithm for taking into account changes in the model.

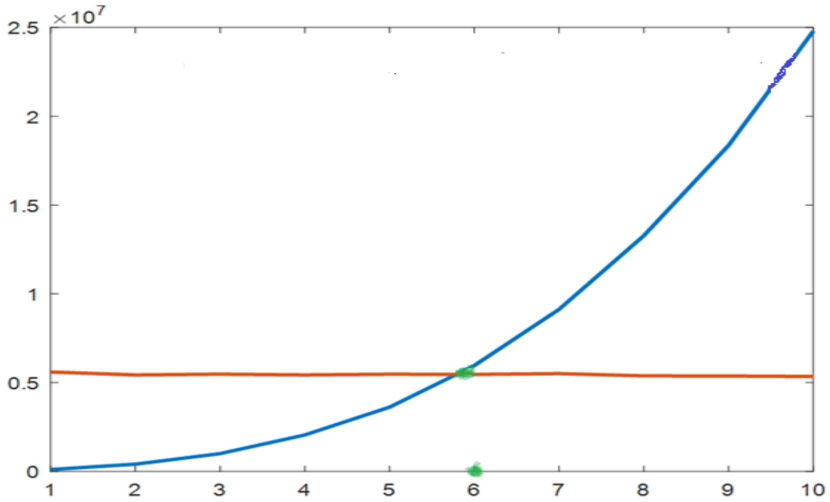

Fig. 2. The dependence of the result re-computation time from the number of disturbed lines at providing the elements in exact numbers for formula (1)

As you can see from these figures, the experiment was carried out for the dimension of model 10, the quality criterion of speed calculations was chosen, and three algorithms were used for evolutionary calculations (subsolution). Typical structural properties of linear systems (well and poorly-conditioned) were considered. As a result, it was established that it was reasonable to conduct the pre-solution with algorithms for changes in 5 (50%), 6 (60%), and 7 (70%) rows. In other words, for changes in 5 (50%) lines, where 5=min {5, 6, 7}, the calculations of the additional solution effectiveness using the 3rd algorithm are preserved. Of course, with changes, for example, in 7 (70%)

lines, the expediency of further solving is preserved only for the 3-rd algorithm (with a loss of speed).

IV. Conclusion

The Leontief-Ford model (taking into account the expenses for performing restrictions under the Paris Agreement) has been chosen as the basic ecology-economic model for the computing experiment. The analyses of the change's impact on the ecology-economic models and balancing the ecological and economic components remain important (prospective) tasks. The consideration of such changes caused the development of new algorithms for the analyses of the impact on the system properties. In particular, the new computation schemes on setting quantitative frameworks for the changes in models (till "saturation"), at which the most effective should be the application of the extra-solving (without re-solving of the task primarily). The real improvement of the system could cause different changes in the restriction matrix for the ecology-economic model, namely, the lines change, the change of columns and elements, etc. Algorithms for change analysis are iterative procedures for sequential transformation of a known initial solution by performing the same type of iterations, for instance, MBM. Establishing the upper quantitative level of such iterations (preserving the computation efficiency) could essentially influence the quality of the computing in general.

In this regard, it is possible to conduct further research, focusing on the following aspects:

- decision-making procedures for choosing a computational scheme for solving the problem (involving expert evaluation methods), as a multi-criteria method for the formation of preferences according to criteria;

- predictive procedures for evaluating the effectiveness of algorithms on different dimensions of the problem (in reality, we have a limited experiment for a specific dimension);

- the quantitative limits of expediency in further solving the problem (without re-solving) should be determined by agreement based on a set of efficiency criteria on the dimensions of the problems solved in the future.

The construction of metaheuristics for increasing the efficiency of evolutionary calculations (with changes) according to several criteria will be considered later.

Conflict of interest:

The author declares that there was no conflict of interest regarding this paper.

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