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A NOVEL CONCEPT OF THE BHATTACHARYYA'S THEOREM: $\sqrt{-(\mathbf{x}^2+\ \mathbf{y}^2)} = -\sqrt{\mathbf{x}^2+\ \mathbf{y}^2}$ TO FIND THE SQUARE ROOT OF ANY NEGATIVE NUMBER INTRODUCING FERMAT'S LAST THEOREM IN REAL NUMBERS WITHOUT USING THE CONCEPT OF COMPLEX NUMBERS

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Abstract

In this paper, the author stated and proved Bhattacharyya's Theorem : $\sqrt{-(x^2+\ y^2)} = -\sqrt{x^2+\ y^2}$*. With the help of this theorem, the author finds the square root of any negative number introducing Fermat's last theorem without using the concept of complex numbers. The author has introduced Fermat's Last Theorem in Bhattacharyya's Theorem to find the square root of any negative number in real numbers in a very simple way. Indeed it is a new invention in mathematics in this era.*

Keywords: Extended form of Pythagoras Theorem, Fermat's Last Theorem, Pythagoras Theorem, Rectangular Bhattacharyya's Co-ordinate System, Theory of Dynamics of Numbers,

I. Introduction

In this paper, the author has developed a new mathematical tool to find the square root of any negative number in real numbers without using the concept of complex numbers. The Bhattacharyya's Theorem: $\sqrt{-(\mathrm{x}^2+\ \mathrm{y}^2)} = -\sqrt{\mathrm{x}^2+\ \mathrm{y}^2}$ is based on "A New Concept of Extended Form of Pythagoras Theorem" [XIII]. The author proved the Bhattacharyya's Theorem by introducing the following new concepts:

1) AN INTRODUCTION TO THEORY OF DYNAMICS OF NUMBERS: A NEW CONCEPT. [IX]

2) A NOVEL CONCEPT OF THE THEORY OF DYNAMICS OF NUMBERS AND ITS APPLICATION IN THE QUADRATIC EQUATION. [X]

3) A NEW CONCEPT TO PROVE, $\sqrt{-1} = -1$ IN BOTH GEOMETRIC AND ALGEBRAIC METHODS WITHOUT USING THE CONCEPT OF IMAGINARY NUMBERS. [XI]

4) AN INTRODUCTION TO RECTANGULAR BHATTACHARYYA'S COORDINATES: A NEW CONCEPT. [XII]

By introducing Farmat's last theorem in Bhattacharyya's theorem the author became successful in finding the square root of any negative numbers in real numbers in a very simple way without using the concept of imaginary numbers. It is a great achievement in mathematics and mathematical sciences.

II. Formulation of the Problem and its Solution:

II.i. Statement of the Bhattacharyya's Theorem

In a right-angled triangle where base = x; altitude = y, and hypotenuse = z then Bhattacharyya's Theorem states that

$$\sqrt{-(x^2 + y^2)} = -\sqrt{x^2 + y^2}$$

Proof:

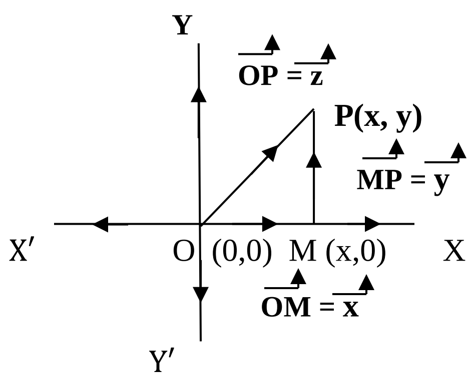

Fig. 1(a)

In a right-angled triangle OMP where, base = x; altitude = y and hypotenuse = z, then according to Pythagoras' Theorem:

$$x^2 + y^2 = z^2 \tag{1}$$

or, $$\sqrt{x^2 + y^2} = \sqrt{z^2} \tag{2}$$

From equation (1) we have

$$-x^2 - y^2 = -z^2 \tag{3}$$

or, $$\sqrt{-x^2 - y^2} = \sqrt{-z^2} \tag{4}$$

Now, let us introduce the Theory of Dynamics of Numbers in both equations (1) and (4) where O(0,0) is the starting point and P(x, y) is the terminating point.

According to the Theory of Dynamics of Numbers from Fig. 1(a) we have

$$\overrightarrow{OM}^2 + \overrightarrow{MP}^2 = \overrightarrow{OP}^2 \tag{5}$$

or, $$\overrightarrow{x}^2 + \overrightarrow{y}^2 = \overrightarrow{z}^2$$

where, $\overrightarrow{OM} = \overrightarrow{x} = +x$, $\overrightarrow{MP} = \overrightarrow{y} = +y$ and $\overrightarrow{OP} = \overrightarrow{z} = +z$

Therefore,

$$\sqrt{\overrightarrow{x^2} + \overrightarrow{y^2}} = \sqrt{\overrightarrow{z^2}}$$

or, $$\sqrt{+x^2 + y^2} = \sqrt{+z^2}$$

or, $$\sqrt{x^2 + y^2} = +z \tag{6}$$

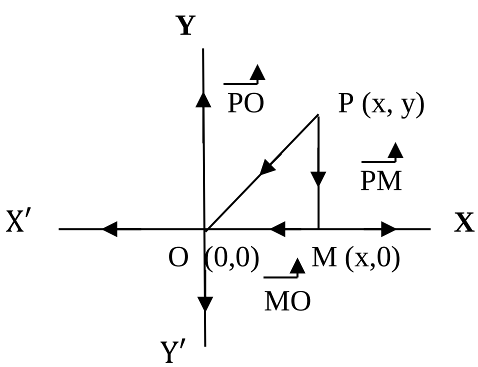

Fig. 1(b)

According to the Pythagoras Theorem from Fig, 1(b) where P (x,y) is the starting point and O (0,0) is the terminating point we have

$$\overset{\uparrow}{\overline{PM}}^2 + \overset{\uparrow}{\overline{MO}}^2 = \overset{\uparrow}{\overline{PO}}^2 \qquad (7)$$

or, $$\sqrt{\overset{\uparrow}{\overline{PM}}^2 + \overset{\uparrow}{\overline{MO}}^2} = \sqrt{\overset{\uparrow}{\overline{PO}}^2}$$

or, $$\sqrt{\overset{\uparrow}{\overline{PM}}^2 + \overset{\uparrow}{\overline{MO}}^2} = \overset{\uparrow}{\overline{PO}} \qquad (8)$$

where, $\overset{\uparrow}{\overline{OM}} = \overset{\uparrow}{\overline{x}} = +x$, $\overset{\uparrow}{\overline{MP}} = \overset{\uparrow}{\overline{y}} = +y$ and $\overset{\uparrow}{\overline{OP}} = \overset{\uparrow}{\overline{z}} = +z$

Now, let us introduce the Theory of Dynamics of Numbers in the coordinate system of Fig. 1 (b) we have

$$\overset{\uparrow}{\overline{MO}}^2 = \overset{\downarrow}{\overline{OM}}^2;\ \overset{\uparrow}{\overline{PM}}^2 = \overset{\downarrow}{\overline{MP}}^2 \text{ and } \overset{\uparrow}{\overline{PO}} = \overset{\downarrow}{\overline{OP}} \qquad (9)$$

or, $$\overset{\uparrow}{\overline{MO}}^2 = \overset{\downarrow}{\overline{x}}^2 = -x^2,\ \overset{\uparrow}{\overline{PM}}^2 = \overset{\downarrow}{\overline{y}}^2 = -y^2 \text{ and } \overset{\uparrow}{\overline{PO}} = \overset{\downarrow}{\overline{z}} = -z \qquad (10)$$

Now, using equation (10) in the equation (8) we have

$$\sqrt{-y^2 - x^2} = -z \qquad (11)$$

or, $$\sqrt{-(x^2 + y^2)} = -z \qquad (12)$$

Therefore, from equation (6) and (12) we have

$$\sqrt{x^2 + y^2} + \sqrt{-(x^2 + y^2)} = +z - z \qquad (13)$$

or, $$\sqrt{x^2 + y^2} + \sqrt{-(x^2 + y^2)} = 0$$

or, $$\sqrt{-(x^2 + y^2)} = -\sqrt{x^2 + y^2}$$

Hence the proof.

II.ii. Application of the Bhattacharyya's Theorem introducing the Fermat's Last Theorem

$$x^n + y^n = z^n \qquad (1)$$

The theorem has not been proved for all values of n. But Fermat's last theorem is true for n = 2, which has been already proved.

So, $$x^2 + y^2 = z^2 \qquad (2)$$

is solvable where $x > 0$, $y > 0$ and $z > 0$

It has been already proven. (G. H. Hardy and E. M. Wright. : 'An Introduction to the Theory of Numbers, Sixth Edition. Page 245 – 247) [III]

From equation (2) we have

$$\sqrt{x^2 + y^2} = \sqrt{z^2} \quad (3)$$

Let us consider $x^2 + y^2 = z^2 = p$ (4)

Where $p > 0$, is a real number.

According to the Bhattacharyya's Theorem, we have

$$\sqrt{-(x^2 + y^2)} = -\sqrt{x^2 + y^2} \quad (5)$$

or, $$\sqrt{-p} = -\sqrt{p} \quad (6)$$

Therefore, we can find the square root of any negative number in real numbers without using imaginary numbers.

III. Application of The Bhattacharyya's Theorem

Example – 1: Prove that $\sqrt{-1} = -1$

<u>Proof</u>:

$$\text{Let, } x = \frac{1}{2} \text{ and } y = \frac{\sqrt{3}}{2} \quad (1)$$

According to the Bhattacharyya's Theorem we have,

$$\sqrt{-(x^2 + y^2)} = -\sqrt{x^2 + y^2} \quad (2)$$

Now putting the values of x and y in equation (2) we have

$$\sqrt{\left[-\left\{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2\right\}\right]} = -\sqrt{\left[\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2\right]}$$

or, $$\sqrt{-\left(\frac{1}{4} + \frac{3}{4}\right)} = -\sqrt{\frac{1}{4} + \frac{3}{4}}$$

or, $$\sqrt{-1} = -\sqrt{1}$$

or, $$\sqrt{-1} = -1$$

<u>Alternative Method</u> :

Let, $x^2 + y^2 = p$ where $p > 0$, a real number

So, $\sqrt{-1} = -\sqrt{1}$

or, $\sqrt{-1} = -1$

Example 2: Prove that $\sqrt{-13} = -\sqrt{13}$

<u>Proof</u>:

Let x = 2 and y = 3 (1)

According to the Bhattacharyya's Theorem we have,

$$\sqrt{-(x^2 + y^2)} = -\sqrt{x^2 + y^2} \quad (2)$$

Now putting the values of x and y in equation (2) we have

$$\sqrt{-(2^2 + 3^2)} = -\sqrt{2^2 + 3^2}$$

or, $\sqrt{-(4 + 9)} = -\sqrt{4 + 9}$

or, $\sqrt{-13} = -\sqrt{13}$

<u>Alternative Method</u> :

Let, $x^2 + y^2 = p$ where $p = 13 > 0$, a real number

so, $\sqrt{-13} = -\sqrt{13}$

Example – 3: Prove that $\sqrt{-100} = -10$

<u>Proof</u>:

Let x = 6 and y = 8 (1)

According to the Bhattacharyya's Theorem we have,

$$\sqrt{-(x^2 + y^2)} = -\sqrt{x^2 + y^2} \quad (2)$$

Now putting the values of x and y in equation (2) we have

$$\sqrt{-(6^2 + 8^2)} = -\sqrt{6^2 + 8^2}$$

or, $\sqrt{-(36 + 64)} = -\sqrt{36 + 64}$

or, $\sqrt{-100} = -\sqrt{100}$

or, $\sqrt{-100} = -10$

<u>Alternative Method</u> :

Let, $x^2 + y^2 = p$ where $p = 100 > 0$, a real number

so, $\sqrt{-100} = -\sqrt{100}$

or, $\sqrt{-100} = -10$

Similarly, we can find the square and square root of any negative number in real numbers without using the concept of complex numbers.

IV. Conclusion

By introducing Fermat's Last Theorem in Bhattacharyya's Theorem it is possible to find the square root of any negative number in real numbers in a very simple way without using the concept of imaginary numbers. Bhattacharyya's Theorem is applicable in any branch of mathematics, science, and technology. Indeed, Bhattacharyya's Theorem will be a milestone for the mathematician of the future generation.

Conflict of Interest:

The authors declare that there are no conflicts of interest regarding this paper.

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