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UNVEILING THE EFFICIENCY OF THE TGR WEIGHTED METHOD IN SOLVING PHYSICAL DISTRIBUTION PROBLEMS

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Abstract

In today's highly competitive world, the distribution of products plays a major role which makes it an important optimization problem related to determining the transportation route to transport a certain amount of products from supply points to demand points with minimum total transportation cost. This paper aims to introduce a new method to find the best and quick initial basic feasible solution for both balanced and unbalanced transportation problems. The proposed method always gives either optimal value or nearest to optimal value which is illustrated with two numerical illustrations i.e. one balanced and one unbalanced transportation problem. Also, the comparison of the results with some existing methods is also discussed.

Keywords. Transportation Problems, Physical Distribution Problem, Optimal Solution, Initial Basic Feasible Solution.

I. Introduction

The delivery of raw materials from the point of beginning to the final utilization point at the lowest price become a major requirement in today's highly competitive world for the development of communication, information technologies, production sectors etc. Shipment of products corresponds to one- third to two-thirds of the total logistics cost [XXIV]. Transportation cost always plays a major role in many companies. Therefore, transporting the products efficiently became a brittle problem for the companies. One of the most application-based categories of linear programming problems is transportation problem (TP). TP is also known as the physical distribution problem which deals with the distribution of products (raw or finished) from several origin points to numerous destinations. The main objective of TP is to transport the products at minimum total cost along with meeting the requirements at the destinations. TP was first modeled by Hitchcock [V] and the first solution methods for TP were developed by Dantzig [VI] and Charnes et al. [1].

To start finding the optimal solution of TP, there is a requirement for a suitable initial basic feasible solution (IBFS). The value of IBFS has a big influence on calculation time for optimal solution and hence on solution time. Therefore, there is a need for the best IBFS which must be closest to the optimal solution.

Also to find the optimal solution, generally preferred methods are Stepping Stone and Modified Distribution Methods (MODI) [XI].

In the literature, many researchers [XII, XIII, VIII, XI] have developed methods to find initial solutions for TP. Putcha et. al. [IV] pointed out the drawbacks of the Northwest Corner Rule Method and Russell Method and proposed a method to overcome their drawbacks. Korukoglu and Balli [XV] proposed an improved Vogel's approximation method. They chose the three highest penalties instead of one and then calculated the cost for these three allocations. Ahmed et. al. [XVII] developed a technique named row distribution indicator and column distribution indicator for finding IBFS. Ahmed et. al. [XVIII] proposed an Incessant Allocation Method in which they allocate the cell having minimum cost in the cost matrix and then adjust the supply and demand in a particular manner. Gupta and Anupam [VII] and Gupta et. al. [IX] proposed a method in which they reduce the cost matrix in such a manner that each row and column must have at least one zero. For each cell having zero value, they calculated some particular values and allocated the cell having the largest value. Prajwal et. al. [III] proposed two methods; one is the Continuous Allocation Method which is a sequential approach. First allocation has been made to the cell having the smallest cost and then moved row-column-wise to the least cost cell in the row or column of the currently allocated cell. The second method is the Supply Demand Reparation Method in which they select a row or column which has the highest value in supply and demand and then choose the least cost cell from this row or column for allocation. Karagul and Sahin [XIV] proposed a method named the Karagul-Sahin Approximation Method. They tested their method on twenty-four problems and compared it with six initial solution methods.

In this paper, we have introduced a novel method i.e. Tania-Gourav-Renu Weighted Method to find the best and quickest IBFS of both balanced and unbalanced TP. This new technique is explained completely with two numerical illustrations i.e. one balanced and one unbalanced problem. The results are tested on ten problems [2, 22, 23, 20, 10, 19, 27, 26, 15] and are compared pictorially and in tabular form with existing methods i.e. North West Corner Method (NWCM), Least Cost Method (LCM), Vogel's

Approximation Method (VAM), Karagul-Sahin Approximation Method (KSAM) and MODI Method.

II. Preliminaries

In this section, we are highlighting the classical transportation problem and the key terminologies associated with it.

II.i. Transportation Problem.

The objective of the TP is to minimize the transportation cost of a given commodity from several manufacturing facilities (sources or origins) to several stores (destinations). Every source has a limited supply and each destination has a demand to be satisfied. The maximum number of goods that can be sent from every source is limited while the number of goods that need to be shipped to the store must be satisfied. The shipping cost from the origin to the destination is directly proportional to the number of goods shipped. Let us illustrate a typical transportation problem as given in Figure 2.1. Suppose that m manufacturing units supply some goods to n warehouses. Let manufacturing facility $i(i = 1,2,\dots,m)$ produce a_i units and the warehouse $j(j = 1,2,\dots,n)$ require b_j units. The unit cost of shipping from i^{th}manufacturing facility to j^{th} warehouse is c_{ij} . The decision variable x_{ij} is the amount being shipped from the manufacturing facility i to j warehouse.

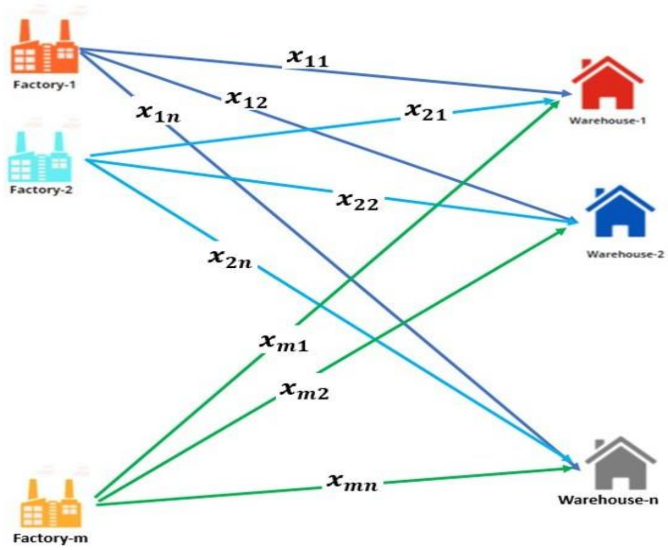


Fig. 2.1 – Classical Transportation Problem

Mathematically, the problem can be stated as:

$$\text{Minimize } z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

subject to the constraints

$$\sum_{i=1}^{m} x_{ij} = a_i \qquad i = 1,2.\dots\dots m$$

$$\sum_{j=1}^{n} x_{ij} = b_j \qquad j = 1,2.\dots\dots n$$

And $x_{ij} \geq 0$ for all $i = 1,2,\dots,m,\ \ j = 1,2.\dots,n.$

The problem can be stated in the tabular form 1 as given below

Table 1: Tabular Representation of the Transportation Problem

Source	**Destination**				
	D_1	D_2	$\cdots$	D_n	Availability (a_i)
S_1	c_{11}	c_{12}	$\cdots$	c_{1n}	(a_1)
S_2	c_{21}	c_{22}	$\cdots$	c_{2n}	(a_2)
.	.	.	...	.	(a_m)
Demand (b_j)	(b_1)	(b_2)	...	(b_n)	

II.ii. Key Terminologies [16].

(i) Feasible Solution (FS)

A set of non-negative values $x_{ij},\ i = 1,2,\dots,m,\ j = 1,2.\dots,n.$ that satisfies the constraints is called a feasible solution to the transportation problem.

Basic Feasible Solution (BFS)

A feasible solution that contains not more than $m + n - 1$ non-negative allocations is called a basic feasible solution to the transportation problem.

Optimal Solution (OS)

A feasible solution (not necessarily basic) is said to be optimal if it minimizes the total transportation cost.

Balanced Transportation Problem

If the sum of the supplies of all the sources is equal to the sum of the demands of all the destinations, then the problem is called a balanced transportation problem.

This is represented mathematically as:

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

(ii) Unbalanced Transportation Problem

If the sum of the supplies of all sources is not equal to the sum of the demands of all destinations, then the problem is called an unbalanced transportation problem.

This is represented mathematically as:

$$\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$$

(iii) Reduced Matrix

A matrix in which each row and each column has at least one zero element is known as a reduced matrix. It is obtained by locating the smallest element in each row of the given cost matrix and then subtracting the same from each element of that row. Likewise, locate the smallest element in each column of the matrix and subtract the same from each element of that column.

III. Proposed algorithm: Tania-Gourav-Renu weighted method (TGR weighted method)

Step 1. Check whether the TP is balanced or unbalanced. If unbalanced, then first balance it.

Step 2. Reduce the given cost matrix to a reduced matrix as defined in Section 2.2.

Step 3. In the reduced matrix, mark the cell with the largest cost. Select the zeroes in the corresponding row and column of the largest cost element.

Step 4. For all the cells (i, j) having marked zeroes, find the weighted value.

Weighted Value,

$$w_{ij} = \frac{\begin{array}{c}\text{sum of all other cost elements in the corresponding} \\ \text{row and column of the given cost matrix}\end{array}}{\text{Number of cost added}}$$

Step 5.

• If there is only one cell with the largest weighted value, select it and make the necessary allocation.

• If there is more than one cell with the same largest weighted value, then choose the cell which has the lowest quantity to allocate.

Step 6. If the resultant matrix is reduced matrix go to **Step 3** otherwise go to **Step 2** and repeat the process till all the allocations are done.

IV. Numerical illustration

We have applied the proposed algorithm to both balanced and unbalanced transportation problems.

IV.i. Illustration B_1. The algorithm designed in this manuscript is explained in the following balanced transportation problem (Table 2) taken from the work of Thamaraiselvi and Santhi [2].

In the problem stated, apply **Step 2** of the proposed algorithm, and the row-reduced matrix is obtained as in Table 3.

Apply **Step 2** of the proposed algorithm on the row-reduced matrix given in Table 3 and the reduced matrix is obtained as in Table 4.

Table 2: Illustration B_1 (Balanced Problem)

	D_1	D_2	D_3	D_4	Supply
S_1	2	3	7	10	26
S_2	1	3	6	4	24
S_3	5	2	3	3	30
Demand	17	23	28	12	

Table 3: Row-reduced matrix of Illustration B_1

	D_1	D_2	D_3	D_4	Supply
S_1	0	1	5	8	26
S_2	0	2	5	3	24
S_3	3	0	1	1	30
Demand	17	23	28	12	

Table 4: Reduced Matrix of Illustration B_1

	D_1	D_2	D_3	D_4	Supply
S_1	0	1	4	7	26
S_2	0	2	4	2	24
S_3	3	0	0	0	30
Demand	17	23	28	12	

Using **Step 3** of the proposed algorithm, cell (1, 4) is selected having the largest cost element, $c_{14} = 7$. The weighted values for the corresponding zeroes marked by the colored cells (1,1) and (3, 4), as shown in Table 5, are calculated as in Eq. (1-2) using **Step 4** of the proposed algorithm as shown below:

$$w_{11} = \frac{c_{12}+c_{13}+c_{14}+c_{21}+c_{31}}{5} = \frac{3+7+10+1+5}{5} = 5.2 \quad (1)$$

$$w_{34} = \frac{c_{31}+c_{32}+c_{33}+c_{14}+c_{24}}{5} = \frac{5+2+3+10+4}{5} = 4.8 \quad (2)$$

Table 5: Matrix with weighted value for the first allocation

	D_1	D_2	D_3	D_4	Supply
S_1	**17** 0	1	4	7	26
S_2	0	2	4	2	24
S_3	3	0	0	0	30
Demand	17	23	28	12	

Table 6: Modified Matrix after first allocation

	D_2	D_3	D_4	Supply
S_1	1	4	7	9
S_2	2	4	2	24
S_3	0	0	0	30
Demand	23	28	12	

Cell (1, 1) is selected corresponding to the largest weighted value, $w_{11} = 5.2$. We allocate 17 to the cell (1, 1) as shown in Table 5 which fully satisfies the demand of the first destination leading to the removal of the first column as depicted in Table 6.

It is observed that the matrix given in Table 6 is not a reduced matrix. So, we apply **Step 2** of the proposed algorithm and obtain the matrix as given in Table 7.

Table 7 : Matrix with weighted value for second allocation

	D_2	D_3	D_4	Supply
S_1	**9** 0	3	6	9
S_2	0	2	0	24
S_3	0	0	0	30
Demand	23	28	12	

Using **Step 3** of the proposed algorithm, cell (1, 4) is selected having the largest cost element, $c_{14} = 6$. The weighted values for the corresponding zeroes marked by the colored cells (1, 2), (2, 4), and (3, 4), as shown in Table 7, are calculated as in Eqs. (3-5) using **Step** 4 of the proposed algorithm as shown below:

$$w_{12} = \frac{c_{13}+c_{14}+c_{22}+c_{23}}{4} = \frac{7+10+3+2}{4} = 5.5 \quad (3)$$

$$w_{24} = \frac{c_{22}+c_{23}+c_{14}+c_{34}}{4} = \frac{3+6+10+3}{4} = 5.5 \quad (4)$$

$$w_{24} = \frac{c_{34}+c_{33}+c_{14}+c_{24}}{4} = \frac{2+3+10+4}{4} = 4.75 \quad (5)$$

Cells (1, 2) and (2, 4) are selected corresponding to the largest weighted value, 5.5. Using **Step 5**, we allocate 9 to the cell (1, 2) as shown in Table 7 which fully satisfies the supply of the first source leading to the removal of the first row as depicted in Table 8.

Table 8 : Modified Matrix after second allocation

	D_2	D_3	D_4	Supply
S_2	0	2	**12** 0	24
S_3	0	0	0	30
Demand	14	28	12	

Using **Step 3** of the proposed algorithm, cell (2, 3) is selected having the largest cost element, $c_{23} = 2$. The weighted values for the corresponding zeroes marked by the colored cells (2, 2), (2, 4) and (3, 3), as shown in Table 8, are calculated as in Eqs. (6-8) using **Step 4** of the proposed algorithm as shown below.

$$w_{22} = \frac{c_{23}+c_{24}+c_{32}}{3} = \frac{6+4+2}{3} = 4 \quad (6)$$

$$w_{24} = \frac{c_{22}+c_{23}+c_{34}}{3} = \frac{3+6+3}{3} = 4 \quad (7)$$

$$w_{33} = \frac{c_{32}+c_{34}+c_{23}}{3} = \frac{2+3+6}{3} = 3.7 \quad (8)$$

Cell (2, 2) and (2, 4) are selected corresponding to the largest weighted value, 4. Using **Step 5**, we allocate 12 to the cell (2, 4) as shown in Table 8 which fully satisfies the demand of the fourth destination leading to the removal of the fourth column as depicted in Table 9.

Using **Step 3** of the proposed algorithm, cell (2, 3) is selected having the largest cost element, $c_{23} = 2$. The weighted values for the corresponding zeroes marked by the colored cells (2, 2) and (3, 3), as shown in Table 9, are calculated as in Eqs. (9-10) using **Step 4** of the proposed algorithm as shown below.

$$w_{22} = \frac{c_{23}+c_{32}}{2} = \frac{6+2}{2} = 4 \quad (9)$$

$$w_{33} = \frac{c_{32}+c_{23}}{2} = \frac{6+2}{2} = 4 \quad (10)$$

Cells (2, 2) and (2, 3) are selected corresponding to the largest weighted value, $w_{22} = 4$. Using **Step 5**, we allocate 12 to the cell (2, 2) as shown in Table 9 which fully satisfies the supply of the second source leading to the removal of the second row as depicted in Table 10.

Table 9 – Modified Matrix after the third allocation

	D2	D3	Supply
S2	**12** / 0	2	12
S3	0	0	30
Demand	14	28	

In Table 10, all the cells have zero cost. So, accordingly, we make the necessary allocations. The final allocations obtained are shown in Table 11.

Table 10: Modified Matrix after the fourth allocation

	D2	D3	Supply
S3	2 / 0	28 / 0	30
Demand	2	28	

Table 11: Final Allocations of Illustration B_1

	D1	D2	D3	D4	Supply
S1	**17** / 2	**9** / 3	7	10	26
S2	1	**12** / 3	6	**12** / 4	24
S3	5	**2** / 2	**28** / 3	3	30
Demand	17	23	28	12	

The initial basic feasible solution of Illustration B_1 is $x_{11} = 17, x_{12} = 9, x_{22} = 12, x_{24} = 12, x_{32} = 2$ and $x_{33} = 28$. The transportation cost is 2 × 17 + 3 × 9 + 3 × 12 + 4 × 12 + 2 × 2 + 3 × 28 = 233.

IV.ii. Illustration U_1. We have applied the algorithm to an unbalanced transportation problem [18] as given in Table 12.

Step 1 of the proposed algorithm is applied to Illustration U_1 by adding a dummy column D_4 with demand 6 as shown in Table 13.

In the above matrix, apply **Step 2** of the proposed algorithm and the reduced matrix is obtained as in Table 14.

Using **Step 3** of the proposed algorithm, cells (2, 2), (3, 1), and (4, 3) are selected having the largest cost element, C_{22} C_{31} and $C_{43} = 3$.

Table 12: Illustration U_1 (Unbalanced Problem)

	D_1	D_2	D_3	Supply
S_1	4	3	2	10
S_2	5	6	1	8
S_3	6	4	3	5
S_4	3	5	4	6
Demand	7	12	4	

Table 13: Illustration U_1 converted to a Balanced Transportation Problem

	D_1	D_2	D_3	D_4	Supply
S_1	4	3	2	0	10
S_2	5	6	1	0	8
S_3	6	4	3	0	5
S_4	3	5	4	0	6
Demand	7	12	4	6	

Table 14: Reduced Matrix of Illustration U_1

	D_1	D_2	D_3	D_4	Supply
S_1	1	0	1	0	10
S_2	2	3	0	0	8
S_3	3	1	2	0	5
S_4	0	2	3	0	6
Demand	7	12	4	6	

The weighted values for the corresponding zeroes marked by the colored cells (1, 2), (2, 3), (2, 4), (3, 4), (4, 1) and (4, 4), as shown in Table 15, are calculated as in Eqs. (11-16) using Step 4 of the proposed algorithm as shown below.

$$w_{12} = \frac{c_{11}+c_{13}+c_{14}+c_{22}+c_{32}+c_{42}}{6} = \frac{4+2+0+6+4+5}{6} = 3.5 \quad (11)$$

$$w_{23} = \frac{c_{21}+c_{22}+c_{24}+c_{13}+c_{33}+c_{43}}{6} = \frac{5+6+0+2+3+4}{6} = 3.3 \quad (12)$$

$$w_{24} = \frac{c_{21}+c_{22}+c_{23}+c_{14}+c_{34}+c_{44}}{6} = \frac{5+6+1+0+0+0}{6} = 2 \quad (13)$$

$$w_{34} = \frac{c_{31}+c_{32}+c_{33}+c_{14}+c_{24}+c_{44}}{6} = \frac{6+4+3+0+0+0}{6} = 2.1 \quad (14)$$

$$w_{41} = \frac{c_{11}+c_{21}+c_{31}+c_{42}+c_{43}+c_{44}}{6} = \frac{4+5+6+5+4+0}{6} = 4 \quad (15)$$

$$w_{44} = \frac{c_{41}+c_{42}+c_{43}+c_{14}+c_{24}+c_{34}}{6} = \frac{5+5+4+0+0+0}{6} = 2 \quad (16)$$

Table 15: Matrix with weighted value for the first allocation

	D_1	D_2	D_3	D_4	Supply
S_1	1	0	1	0	10
S_2	2	3	0	0	8
S_3	3	1	2	0	5
S_4	**6** 0	2	3	0	6
Demand	7	12	4	6	

Cell (4, 1) is selected corresponding to the largest weighted value, $w_{41} = 4$. We allocate 6 to the cell (4, 1) as shown in Table 15 which fully satisfies the supply of the fourth source leading to the removal of the fourth row as depicted in Table 16.

Table 16: Modified Matrix obtained after the first allocation

	D_1	D_2	D_3	D_4	Supply
S_1	1	0	1	0	10
S_2	2	3	0	0	8
S_3	3	1	2	0	5
Demand	1	12	4	6	

It is observed that the matrix given in Table 16 is not a reduced matrix. So, we apply **Step 2** of the proposed algorithm and obtain the matrix as given in Table 17.

Using **Step 3** of the proposed algorithm, cell (2, 2) is selected having the largest cost element, $c_{22} = 3$.

Table 17 – Matrix with weighted value for second allocation

	D_1	D_2	D_3	D_4	Supply
S_1	0	0	1	0	10
S_2	1	3	**4** 0	0	8
S_3	2	1	2	0	5
Demand	1	12	4	6	

The weighted values for the corresponding zeroes marked by the colored cells (1, 2), (2, 3), and (2, 4), as shown in Table 17, are calculated as in Eqs. (17-19) using **Step 5** of the proposed algorithm as shown below.

$$w_{12} = \frac{c_{11}+c_{13}+c_{14}+c_{22}+c_{32}}{5} = \frac{4+2+0+6+4}{5} = 3.2 \tag{17}$$

$$w_{23} = \frac{c_{21}+c_{22}+c_{24}+c_{13}+c_{33}}{5} = \frac{5+6+0+2+3}{5} = 3.2 \tag{18}$$

$$w_{24} = \frac{c_{21}+c_{22}+c_{23}+c_{14}+c_{34}}{5} = \frac{5+6+1+0+0}{5} = 2.4 \tag{19}$$

Cells (1, 2) and (2, 3) are selected corresponding to the largest weighted value, w_{12} and $w_{23} = 3.2$. Using **Step 5**, we allocate 4 to the cell (2, 3) as shown in Table 17 which fully satisfies the demand of the third destination leading to the removal of the third column as depicted in Table 18.

Table 18: Modified Matrix after second allocation

	D_1	D_2	D_4	Supply
S_1	0	**10** 0	0	10
S_2	1	3	0	4
S_3	2	1	0	5
Demand	1	12	6	

Using **Step 3** of the proposed algorithm, cell (2, 2) is selected having the largest cost element, $c_{22} = 3$. The weighted values for the corresponding zeroes marked by the colored cells (1, 2) and (2, 4), as shown in Table 18, are calculated as in Eqs. (20-21) using **Step 4** of the proposed algorithm as shown below.

$$w_{12} = \frac{c_{11}+c_{14}+c_{22}+c_{32}}{4} = \frac{4+0+6+4}{4} = 3.5 \tag{20}$$

$$w_{24} = \frac{c_{21}+c_{22}+c_{14}+c_{34}}{4} = \frac{5+6+0+0}{4} = 2.7 \tag{21}$$

Cell (1, 2) is selected corresponding to the largest weighted value, $w_{12} = 3.5$. Using **Step 5**, we allocate 10 to the cell (1, 2) as shown in Table 18 which fully satisfies the supply of the first source leading to the removal of the first row as depicted in Table 19.

Table 19: Modified Matrix after the third allocation

	D_1	D_2	D_4	Supply
S_2	1	3	0	4
S_3	2	1	0	5
Demand	1	2	6	

Again, we reduce this matrix using **Step 2** of the proposed algorithm before making further allocations as shown in Table 20.

Table 20: Reduced Matrix

	D_1	D_2	D_4	Supply
S_2	**1** 0	2	0	4
S_3	1	0	0	5
Demand	1	2	6	

Using **Step 3** of the proposed algorithm, cell (2, 2) is selected having the largest cost element, $c_{22} = 2$. The weighted values for the corresponding zeroes marked by the colored cells (2, 1), (2, 4), and (3, 2), as shown in Table 20, are calculated as in Eqs. (22-24) using Step 4 of the proposed algorithm as shown below.

$$w_{21} = \frac{c_{22}+c_{24}+c_{31}}{3} = \frac{6+0+6}{3} = 4 \quad (22)$$

$$w_{24} = \frac{c_{21}+c_{22}+c_{34}}{3} = \frac{5+6+0}{3} = 3.6 \quad (23)$$

$$w_{32} = \frac{c_{31}+c_{34}+c_{22}}{3} = \frac{6+0+6}{3} = 4 \quad (24)$$

Cells (2, 1) and (3, 2) are selected corresponding to the largest weighted value, w_{21} and $w_{32} = 4$. Using **Step 5**, we allocate 1 to the cell (2, 1) as shown in Table 20 which fully satisfies the demand of the first destination leading to the removal of the first column as depicted in Table 21.

Table 21: Modified Matrix after the fourth allocation

	D_2	D_4	Supply
S_2	2	0	4
S_3	**2** 0	0	5
Demand	2	6	

Using **Step 3** of the proposed algorithm, cell (2, 2) is selected having the largest cost element, $c_{22} = 2$. The weighted values for the corresponding zeroes marked by the colored cells (2, 4) and (3, 2), as shown in Table 21, are calculated as in Eqs. (25-26) using **Step 4** of the proposed algorithm as shown below.

$$w_{24} = \frac{c_{22}+c_{34}}{2} = \frac{6+0}{2} = 3 \tag{25}$$

$$w_{32} = \frac{c_{22}+c_{34}}{2} = \frac{6+0}{2} = 3 \tag{26}$$

Now both the cells (2, 4) and (3, 2) correspond to same weighted value. Using **Step 5**, we allocate 2 to the cell (3, 2) as shown in Table 21 which fully satisfies the demand of the second destination leading to the removal of the second column as depicted in Table 22.

Table 22: Modified Matrix after the fifth allocation

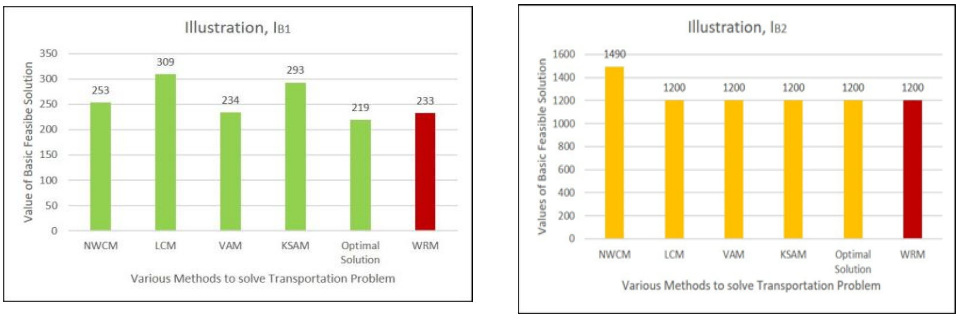
	D_4	Supply
S_2	**3** 0	3
S_3	**2** 0	3
Demand	6	

In Table 22, all the cells have zero cost. So, accordingly, we make the necessary allocations. The final allocations obtained are shown in Table 23.

The initial basic feasible solution of Illustration U_1 is $x_{12} = 10, x_{21} = 1, x_{23} = 4, x_{24} = 3, x_{32} = 2, x_{34} = 3$ and $x_{41} = 6$.The transportation cost is 3 × 10 + 5 × 1 + 1 × 4 + 0 × 3 + 4 × 2 + 3 × 6 = 65.

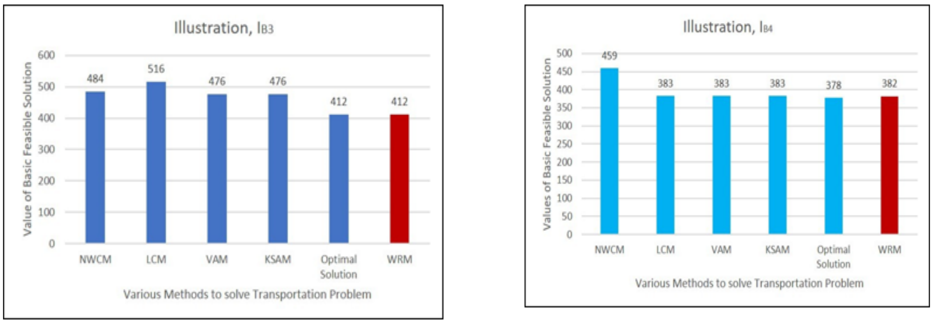
Table 23: Final Allocations of Illustration U_1

	D_1	D_2	D_3	D_4	Supply
S_1	4	10 3	2	0	10
S_2	1 5	6	4 1	3 0	8
S_3	6	2 4	3	3 0	5
S_4	6 3	5	4	0	6
Demand	7	12	4	6	



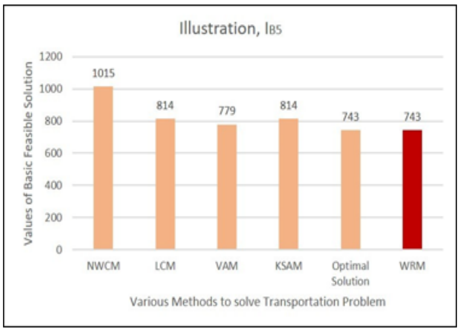
(a) (b)

Fig. 4.2. Pictorial Comparison of results for balanced TPs in (a) $IB1$, (b) $IB2$

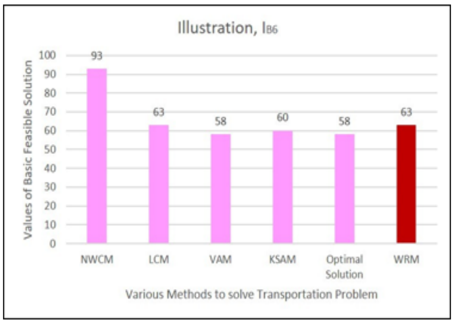


(a) (b)

Fig. 4.3. Pictorial Comparison of results for balanced TPs in (a) $IB3$, (b) $IB4$



(a)



(b)

Fig. 4.4. Pictorial Comparison of results for balanced TPs in (a) $IB5$ and (b) $IB6$

5. Results and discussion

We have applied the proposed algorithm to various existing TP (both balanced and unbalanced). The comparison of the results obtained by NWCM, LCM, VAM, KSAM, MODI, and TGR Weighted Method (proposed method) are given in Tables 23 and 24 respectively. The pictorial comparison of the results are shown in Figures 4.2, 4.3, 4.4, and 4.5. The percentage deviation of the results obtained by the above-mentioned methods from the optimal solution is depicted in Table 26.

Table 24: Comparison of results for balanced TPs

Illustration	Problem taken from	Problem Size	NWCM	LCM	VAM	KSAM	MODI	Proposed Method
I_{B_1}	Table 2 on Page 7 [2]	3 × 4	253	309	234	293	219	233
I_{B_2}	Example 1 on Page 5697 [23]	3 × 3	484	516	476	1200	412	412
I_{B_3}	Example 2 on Page 5698 [23]	3 × 4	1490	1200	1200	476	1200	1200
I_{B_4}	Example 3.2 on Page 4 [20]	5 × 5	459	383	383	383	378	382
I_{B_5}	Example 1 on Page 4 [10]	3 × 4	1015	814	779	814	743	743
I_{B_6}	Example 1 on Page 1713 [19]	4 × 5	93	63	58	60	58	63

Figure 4.6 shows the pictorial representation of Table 26. It clearly indicates that in NWCM there is no such case when deviation from optimal solution is zero. While in LCM and KSAM there is only one case when the deviation is zero. Likewise, there are two cases when VAM gives an optimal solution. However, using the proposed method i.e. TGR Weighted Method, five out of ten problems discussed have zero deviation from the optimal value and the remaining five problems are nearest to the optimal value.

Table 25: Comparison of results for unbalanced TPs

Illustration	Problem taken from	Problem Size	NWCM	LCM	VAM	KSAM	MODI	Proposed Method
I_{U_1}	Example 4.1 on Page 145 [22]	4 × 3	65	101	77	73	71	65
I_{U_2}	Table 2 on Page 22 [27]	4 × 3	1010	1210	880	980	840	840
I_{U_3}	Example 4.2 on Page 21 [26]	3 × 4	1125	965	1010	1065	960	1060
I_{U_4}	Example 1 on Page 96 [15]	4 × 3	14140	12550	12020	13020	11720	11850

Table 26: Percentage Deviation of results from the optimal solution

Illustration	NWCM	LCM	VAM	KSAM	Proposed Method
I_{B_1}	15.5	41.09	6.84	33.7	6.39
I_{B_2}	24.16	**0**	**0**	**0**	**0**
I_{B_3}	17.47	25.24	15.53	15.53	**0**
I_{B_4}	21.42	1.32	1.32	1.32	1.05
I_{B_5}	36.6	9.55	4.84	9.55	**0**
I_{B_6}	60.34	8.62	**0**	3.44	8.62
I_{U_1}	55.38	18.46	9.23	12.30	**0**
I_{U_2}	20.23	44.04	4.76	16.66	**0**
I_{U_3}	17.18	0.52	5.20	10.93	10.41
I_{U_4}	20.64	7.08	2.55	11.09	1.1

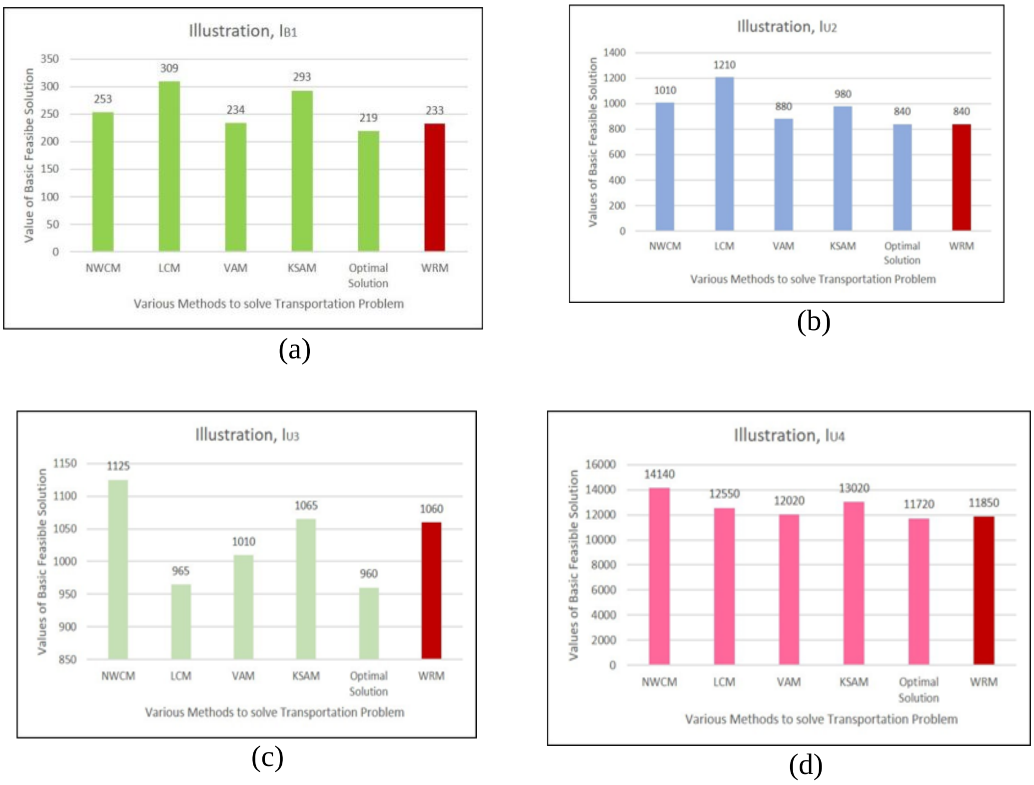


Fig. 4.5. Pictorial Comparison of results for unbalanced TPs in (a) I_{U1}, (b) I_{U2}, (c) I_{U3}, and (d) I_{U4}.

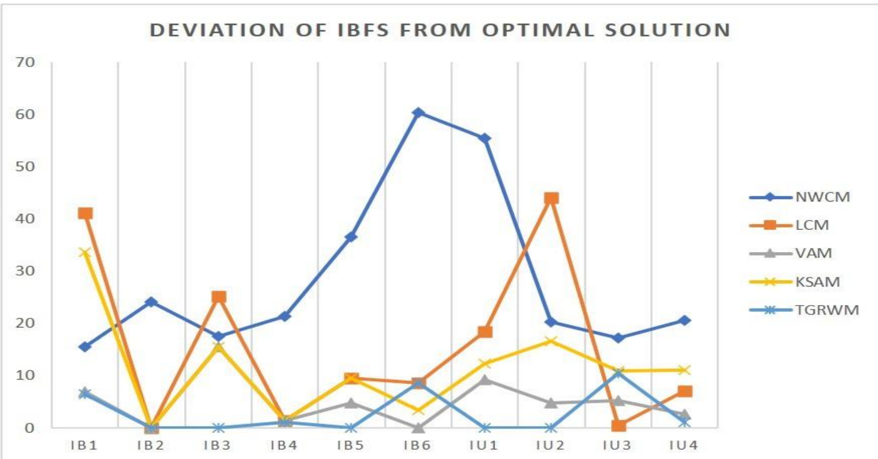


Fig. 4.6. Pictorial Representation of Table 25

6. Conclusion

This paper introduces a new approach called the TGR Weighted Method for solving TP. It can be utilized for both balanced and unbalanced problems. To demonstrate its effectiveness, we solved ten problems (five balanced and five unbalanced). The results show that for five problems, the solution matches the optimal value, while for the remaining five, it is closest to optimal. Additionally, the computational workload of the TGR Weighted Method is significantly lower compared to existing methods. We aim to implement this method in MATLAB with some modifications, further exploring its potential and practical utility. Further research could explore its potential for larger-scale problems and its robustness in diverse scenarios.

Conflict of interest

The authors have no conflicts of interest to declare that are relevant to the content of this article.

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