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STOCHASTIC ANALYSIS OF A TWO-UNIT STANDBY AUTOCLAVE SYSTEM WITH INSPECTION AND VARYING DEMAND

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Abstract

The authors propose a stochastic analysis of a two-unit standby autoclave system with variable demand and inspection in this work. The autoclave system consists of a primary and a redundant unit, a dynamic demand profile, and an inspection mechanism to evaluate their condition. Inspection lowers the possibility of unanticipated failures and avoids financial loss. Best of our knowledge, many of the studies assume that the unit in cold standby mode is always reliable. This hypothesis lacks practical justification. Practically, its performance deteriorates due to environmental issues (dust, moisturizer, etc.). The authors found the same when visiting a ghee manufacturing plant in Punjab. Additionally, weather fluctuations also affect the production (as demand for ghee is higher in winter as compared to summer). Behaviour of redundant units is an intriguing aspect of this study and demonstrates its uniqueness. Thus, the authors explored two-unit standby autoclave systems subject to inspection on standby units with fluctuating demand. In the model, the main autoclave directly goes under repair when it fails, but the redundant autoclave undergoes inspection afterward beyond the determined redundant time, to check its possibility for repair or replacement. Replacement means a change of subparts, like Gear Box, etc., in an autoclave to put it in a working state instantly. Inspection adversely affects the system's reliability. Therefore, the statistical inference under the proposed innovation shows better results and a significant balance between the reliability and economy of the given stochastic system using the semi-Markov process (SMP) and regeneration point technique (RPT). By studying various scenarios about repair prices, inspection frequency, and fluctuating demand patterns, this research provides vital insights into the most effective approaches for handling redundant units and preserving system functionality.

Keywords: Statistical Model, Stochastic Systems, Reliability, Availability, Manufacturing, Innovation

I. Introduction

A widely recognized approach to enhancing the availability and reliability of a system is redundancy. When an active component fails, a standby redundancy occurs. It means a component that isn't being used becomes active. The purpose of these systems is to guarantee that essential services, processes, or operations can carry on without interruption in the event of unpredicted breakdown, interruptions, or downtime. There exists a rich body of work on redundant systems such as hot, warm, and cold. Taneja et al. [XVI] worked on a sugar mill, having three identical units. Their system can be shut down, during the shortage of raw material. Otherwise, the system will work at full capacity. Thereafter, Ram et al. [XIV] investigated the reliability of a two-unit standby system. Initially, one unit was operative and the rest was kept on standby mode. Their system may also fail due to improper starting of the system, the reason behind it was untrained and inexperienced system analysts. The system was examined by supplementary variable technique and Laplace transformation. Additionally, Malhotra and Taneja [IX-X] analyzed a two-unit cold standby system by assuming both units may be operative simultaneously due to increased demand. Furthermore, the Author worked on a comparative study of a cable manufacturing plant by taking the concept demand is not constant. They used regenerative processes and semi-Markov processes [IV] to find numerous measures of the system's effectiveness. According to Levitin et al. [VIII] redundancy is a widely applied technique to achieve high reliability. They also discussed how its failure made reliability non-monotonic and affected other parameters. Also, the elements of the system are assumed to be non-repairable. Furthermore, Zhang et al. [XXI] devoted to studying exponential stability of the system and evaluated profit. They took identical, standby, and repairable systems. Thereafter Yang et al. [XX] took a system having M primary and S spare units. All the units were repairable. They were not considering waiting space, hence failed unit was repaired immediately. The matrix-analytic method is used to compute the steady-state availability. Laplace transform technique used for MTTF.

Moreover, Levitin et al. [VII] discussed the factors that affect industrial systems such as deterioration, corrosion, etc. To overcome such a situation fixed planning is performed to renew the worn element by using redundant elements to enhance the functioning successfully. Additionally, Gao et al. [II] studied warm and cold standby systems with an unreliable repair facility. The life of both the components was supposed to be exponentially distributed random variables. They executed preventive maintenance and repair according to idle time and failure of the unit respectively. Markov process approach was adopted to solve the equations. Thereafter, Wang et al. [XVII] took warm standby and non-identical systems. They considered two types of failure, firstly hardware failure and secondly human error failure. The failed unit goes under repair immediately. After repair unit was assumed to be as good as the new one. Yang et al. [XX] evaluated the reliability of the Markov model, by using a numerical approach based on the Runge–Kutta method. Kamal et al. [VI] analyzed the cost-benefit of a system having two dissimilar units, by considering one unit with high quality and the other having low quality. A failed unit can be repaired or replaced. The main unit kept in priority being its high quality. Kakkar et al. [V] worked on reliability using wolf optimization .The authors developed a nature-inspired algorithm to operate the system smoothly or in a good manner. Shekhar et al. [XV] analyzed the reliability of multi-unit systems having several failures, degradation, random delays, and probabilistic imperfections. Also, they discussed how their impact on production and the system's performance. Stochastic behaviour is used to examine problems systematically. According to Juybari et al., [III] mixed redundancy is the most powerful technique to enhance the reliability of a system. Also, all the components are under environmental shocks and may deteriorate by internal or external shocks. Malhotra [XI, XII, XIII] discussed the reliability of a standby system with varying demands where redundant units required some activation time to start. To the best of our knowledge, none of the extensive literature on reliability considered the concept of varied production due to varied seasons and cold standby can be damaged in this mode simultaneously. Hence, there is a massive gap. The authors will try to fill it.

Given the explanations and evidence revealed above, the motive of the study is to conduct a stochastic analysis of a two-unit cold standby system. The central part of the research is the Autoclave machine (main unit) with an identical cold standby unit. Here, working the standby unit always must; otherwise, there will be huge production loss—our primary focus is on the standby unit. Additionally, both units must be operational to meet the increased demand for ghee during the winter. If not, the system will operate at a lower capacity. We applied the inspection technique on the standby unit only to check its feasibility. After that, the technician will decide whether it will be repaired or replaced, as replacement is only a subpart of the autoclave that is applicable. Here, replacement is instant. If the central unit fails, then it goes directly for repair only. Production in the winter season is high as compared to summer. The authors gathered actual data from the visited company to demonstrate the results. Statistical Inferences of the system, such as mean time to system failure and other measures, are evaluated using SMP and regenerative techniques. This research is company-based. The authors developed a general model and thus can benefit any plant/company where such a model fits.

II. Mathematical Model

The authors observed two identical autoclaves in a visited Ghee manufacturing plant (Fig.1). One works; the other is on cold standby (state S0). When the operating autoclave fails, it undergoes repair, and the standby starts working (state S1). The worker/system analyzer observes that the standby autoclave (SA) may also be subject to failure after exceeding the maximum redundancy time. If SA fails, an inspection of the failed SA (state S2) occurs to determine its feasibility for repair or replacement. The user analyzes that replacing the autoclave means changing only subparts like Gear Box to make the system active instantly. After the replacement of

SA, the system moves to state S0. In another case, SA undergoes repair (state S1). An inspection on SA may still be happening (state S4), and the working autoclave may fail. The failed autoclave (FA) waits for repair as there is a single repairman for inspection, repair, or replacement. State S4 has two possibilities: SA needs reserve, FA undergoes repair (state S1), or SA goes under repair, and FA waits for repair (state S5). From S5, the system moves to state S1 if SA starts working and FA undergoes repair. If the working unit fails and FA is still under repair, the system may proceed to state S3. Various other assumptions are:

- There is statistical independence between the random variables.
- The device functions perfectly after every repair.
- The Autoclave unit's failure rate has an exponential distribution.
- Assuming all random variables, including inspection and repair/replacement rates, have an exponential distribution.

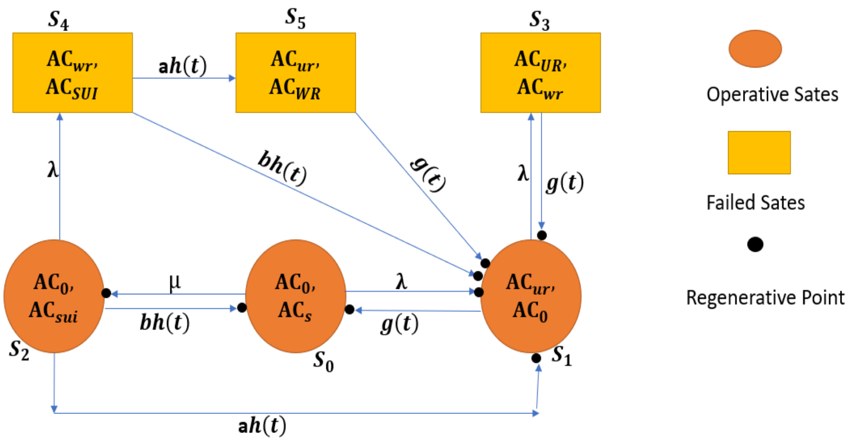

Fig. 1. State Transition Diagram of Model

III. Solution Procedure

Transition Probabilities

The Time-dependent probabilities are as follows:

$$dQ_{01}(t) = \lambda e^{-(\lambda+\mu)t}dt, \qquad dQ_{02}(t) = \mu e^{-(\lambda+\mu)t}dt,$$

$$dQ_{10}(t) = g(t)e^{-\lambda t}dt, \qquad dQ_{13}(t) = \lambda e^{-\lambda t}\bar{G}(t)dt,$$

$$dQ_{20}(t) = bh(t)e^{-\lambda t}dt, \qquad dQ_{21}(t) = ah(t)e^{-\lambda t}dt,$$

$$dQ_{24}(t) = \lambda e^{-\lambda t}\bar{H}(t)dt, \qquad dQ_{31}(t) = g(t)dt,$$

$$dQ_{41}(t) = bh(t)dt, \qquad dQ_{45}(t) = ah(t)dt,$$

$$dQ_{51}(t) = g(t)dt, \qquad dQ_{11}^{3}(t) = d[Q_{13}(t) ⓢ Q_{31}(t)],$$
$$dQ_{21}^{4}(t) = d[Q_{24}(t) ⓢ Q_{41}(t)],$$
$$dQ_{2,1}^{4,5}(t) = d[Q_{24}(t) ⓢ Q_{45}(t) ⓢ Q_{51}(t) \qquad (1)$$

Simple probabilistic considerations yield the following expressions for the non-zero elements:

$$p_{ij} = Q_{i,j}(\infty) = \int_0^\infty dQ_{i,j}(t)dt = \tilde{Q}_{i,j}(0) = \int_0^\infty q_{i,j}(t)dt \qquad (2)$$
$$p_{ij} = \lim_{s\to 0} q^*{}_{ij}(s) \qquad (3)$$

Taking the Laplace Transform of equation (1), we get

$$p_{01} = \frac{\lambda}{\lambda+\mu} \qquad p_{02} = \frac{\mu}{\lambda+\mu}$$
$$p_{10} = g^*(\lambda), \qquad p_{13} = 1 - g^*(\lambda),$$
$$p_{20} = bh^*(\lambda), \qquad p_{21} = ah^*(\lambda),$$
$$p_{24} = 1 - h^*(\lambda), \qquad p_{31} = g^*(0),$$
$$p_{41} = bh^*(0), \qquad p_{45} = ah^*(0),$$
$$p_{51} = g^*(0)\ , \qquad p_{11}^{3} = 1 - g^*(\lambda),$$
$$p_{21}^{4} = b\,[1 - h^*(\lambda)] \qquad p_{2,1}^{4,5} = a\,[1 - h^*(\lambda)], \qquad (4)$$

Using these probabilities,

$$p_{01} + p_{02} = 1, \qquad p_{10} + p_{13} = 1,$$
$$p_{20} + p_{21} + p_{24} = 1, \qquad p_{31} = 1,$$
$$p_{41} + p_{45} = 1, \qquad p_{51} = 1,$$
$$p_{10} + p_{11}^{3} = 1 \qquad p_{20} + p_{21} + p_{21}^{4} + p_{2,1}^{4,5} = 1 \quad (5)$$

Mean sojourn time (μ_i) in the state S_i are

$$\mu_0 = \frac{1}{\lambda+\mu}, \qquad \mu_1 = \frac{[1 - g^*(\lambda)]}{\lambda},$$
$$\mu_2 = \frac{[1 - h^*(\lambda)]}{\lambda}, \qquad \mu_3 = -g^{*\prime}(0),$$
$$\mu_4 = -h^{*\prime}(0), \qquad \mu_5 = -g^{*\prime}(0),$$
$$\mu_1' = [\tfrac{1}{\lambda} - g^{*\prime}(0)][1 - g^*(\lambda)$$
$$\mu_2' = [\tfrac{1}{\lambda} - h^{*\prime}(0) - ag^{*\prime}(0)][1 - h^*(\lambda) \qquad (6)$$

Also $\quad m_{i,j} = \int_0^\infty td\{Q_{i,j}(t)\} = -q_{i,j}^{*\prime}(0)$

Using the above, we get

$$m_{01} = \frac{\lambda}{(\lambda+\mu)^2}, \qquad m_{02} = \frac{\mu}{(\lambda+\mu)^2},$$

$m_{10} = -g^{*'}(\lambda)$, $m_{13} = \frac{1}{\lambda}[1-g^{*}(\lambda)] + g^{*'}(\lambda)$,

$m_{10} = -g^{*'}(\lambda)$, $m_{20} = -bh^{*'}(\lambda)$,

$m_{21} = -ah^{*'}(\lambda)$, $m_{24} = \frac{1}{\lambda}[1-h^{*}(\lambda)] + h^{*'}(\lambda)$,

$m_{31} = -g^{*'}(0)$, $m_{41} = -bh^{*'}(0)$

$m_{45} = -ah^{*'}(0)$, $m_{51} = -g^{*'}(0)$

$m_{11}^{3} = [\frac{1}{\lambda} - g^{*'}(0)][1-g^{*}(\lambda)] + g^{*'}(\lambda)$,

$m_{21}^{4} = b\,[\frac{1}{\lambda} - h^{*'}(0)][1-h^{*}(\lambda)] + bh^{*'}(\lambda)$,

$$m_{2,1}^{4,5} = a\,[\frac{1}{\lambda} - h^{*'}(0) - g^{*'}(0)][1-h^{*}(\lambda)] + ah^{*'}(\lambda) \quad (7)$$

The sum of the unconditional mean times starting from the state 'i' are:

$m_{01} + m_{02} = \mu_0$, $m_{10} + m_{13} = \mu_1$,

$m_{20} + m_{21} + m_{24} = \mu_2$, $m_{31} = \mu_3$,

$m_{41} + m_{45} = \mu_4$, $m_{51} = \mu_5$,

$m_{10} + m_{11}^{3} = \mu_1$

$$m_{20} + m_{21} + m_{21}^{4} + m_{2,1}^{4,5} = \mu_2 \quad (8)$$

MTSF

In determining MTSF, assume the failed states as absorbing states. Recursive relations for $\phi_i(t)$ are:

$$\phi_0(t) = Q_{01}(t)Ⓢ\phi_1(t) + Q_{02}(t)Ⓢ\phi_2(t),$$

$$\phi_1(t) = Q_{10}(t)Ⓢ\phi_0(t) + Q_3(t),$$

$$\phi_2(t) = Q_{20}(t)Ⓢ\phi_0(t) + Q_{21}(t)Ⓢ\phi_1(t) + Q_{24}(t), \quad (9)$$

On taking L.S.T and solving for $\tilde{Q}_0(s)$, we have

$$\tilde{Q}_0(s) = \frac{N(s)}{D(s)},$$

$$N(s) = \tilde{Q}_{0,1}(s)\tilde{Q}_{1,3}(s) + \tilde{Q}_{0,2}(s)[\tilde{Q}_{1,3}(s)\tilde{Q}_{2,1}(s) + \tilde{Q}_{2,4}(s),$$

And $D(s) = 1 - \tilde{Q}_{0,1}(s)\tilde{Q}_{1,0}(s) - \tilde{Q}_{0,2}(s)[\tilde{Q}_{1,0}(s)\tilde{Q}_{2,1}(s) + \tilde{Q}_{2,0}(s)],$

$$R^{*}(s) = \frac{1-\tilde{\phi}_0(s)}{s} \quad (10)$$

where R*(s) is the Laplace transform of R(t).

$$MTSF(T) = \lim_{s\to 0}\frac{1-\tilde{\phi}_0(s)}{s} = \frac{N}{D} \quad (11)$$

Here

$$\mathrm{N} = \mu_0 + [p_{01} + p_{02}\ p_{21}]\mu_1 + p_{02}\mu_2,$$
$$\mathrm{D} = 1 - p_{01}\ p_{10} - p_{02}[p_{10}\ p_{21} + p_{20}],$$

Steady State Availability

$$A_i(t) = M_i(t) + \sum_j q_{i,j}{}^k(t) © A_j(t),\ i \neq j$$

The recursive relations for the system availability are:

$$A_0(t) = M_0(t) + q_{01}(t) © A_1(t) + q_{02}(t) © A_2(t),$$
$$A_1(t) = M_1(t) + q_{10}(t) © A_0(t) + q_{11}^3(t) © A_1(t),$$
$$A_2(t) = M_2(t) + q_{20}(t) © A_0(t) + [q_{21}(t) + q_{21}^4(t) + q_{2,1}^{4,5}(t)] © A_1(t) \quad (12)$$

Here,$M_0(t) = e^{-(\lambda+\mu)t}$,$M_1(t) = e^{-\lambda t}\bar{G}(t)$, $M_2(t) = e^{-\lambda t}\bar{H}(t)$

After taking Laplace and solving for $A_0{}^*(s)$, we have

$A_0{}^*(s) = \frac{N(s)}{D(s)}$, Here

$$N(s) = M_0{}^*(s)[1 - q_{11}^3{}^*(s)] + M_1{}^*(s)\left[q_{01}{}^*(s) + q_{02}{}^*(s)\left\{q_{21}{}^*(s) + q_{21}^4{}^*(s) + q_{2,1}^{4,5}{}^*(s)\right\}\right] + M_2{}^*(s)q_{02}{}^*(s)[1 - q_{11}^3{}^*(s)]$$

and

$$D(s) = [1 - q_{11}^3{}^*(s)][1 - q_{02}{}^*(s)q_{20}{}^*(s)] - q_{01}{}^*(s)q_{10}{}^*(s) - q_{02}{}^*(s)q_{10}{}^*(s)\ [q_{21}{}^*(s) + q_{21}^4{}^*(s) + q_{2,1}^{4,5}{}^*(s)$$

Using the above, the availability is given by

$$A(\infty) = \lim_{s \to 0} sA_0{}^*(s) = \frac{N}{D} \quad (13)$$

Here

$$N = p_{10}\mu_0 + [1 - p_{02}p_{20}]\mu_1 + p_{02}p_{10}\mu_2,$$
$$D = p_{10}\mu_0 + [1 - p_{02}p_{20}]\mu_1{}' + p_{02}p_{10}\mu_2{}',$$

Due to Inspection of Units:

Recursive relations for $B_i^I(t)$ are as follows:

$$B_0^I(t) = q_{01}(t) © B_1^I(t) + q_{02}(t) © B_2^I(t),$$
$$B_1^I(t) = q_{10}(t) © B_0^I(t) + q_{11}^3(t) © B_1^I(t),$$
$$B_2^I(t) = W_2^I(t) + q_{20}(t) © B_0^I(t) + [q_{21}(t) + q_{21}^4(t) + q_{21}^{4,5}(t)] © B_1^I(t) \quad (14)$$

$W_2^I(t)$ is the chance the server occupies in the inspection unit.

Here $W_2^I(t) = e^{-\lambda t}\bar{H}(t) + (\lambda e^{-\lambda t} © 1)\bar{H}(t)$

After taking Laplace of equations, solve for ${B_0}^{I^*}(s)$

we have ${B_0}^{I^*}(s) = \frac{N^I(s)}{D}$, Here

$$N^I(s) = {W_2}^{I^*}(s){q_{02}}^*(s)[1 - q*_{11}^{3}(s)],$$
$$D = p_{10}\mu_0 + [1 - p_{02}p_{20}]\mu_1' + p_{02}p_{10}\mu_2',$$

By using this, the busy time of the server due to inspection is:

$$\boldsymbol{B^I(\infty) = \lim_{s \to 0} sB_0{}^{I^*}(s) = \frac{N^I}{D}} \tag{15}$$

$N^I = {W_2}^{I^*}(0)p_{02}p_{10}$, D is already mentioned.

Due to the repair of the unit:

Recursive relations for $B_i^R(t)$ are given by

$B_i^R(t) = W_f(t) + \sum_j {q_{i,j}}^k(t) © B_i^R(t)$, i ≠ j

Proceeding as above, the busy time of the server due to repair in steady state is:

$$B^R(\infty) = \lim_{s \to 0} s\, {B_0}^{R^*}(s) = \frac{N^R(s)}{D} \tag{16}$$

Here

$$N^R(s) = {W_1}^{R^*}(0)\,\{1 - p_{02}p_{20}\},$$
$$D = p_{10}\mu_0 + [1 - p_{02}p_{20}]\mu_1' + p_{02}p_{10}\mu_2',$$

Expected Number of Inspections of The Unit

The recursive relations are given by

$$I_0(t) = Q_{01}(t)ⓢI_1(t) + Q_{02}(t)ⓢ[1 + I_2(t)],$$
$$I_1(t) = Q_{10}(t)ⓢI_0(t) + Q_{11}^3(t)ⓢI_1(t),$$
$$I_2(t) = Q_{20}(t)ⓢI_0(t) + [Q_{21}(t) + Q_{21}^4(t) + Q_{2,1}^{4,5}(t)]ⓢI_1(t) \tag{17}$$

After Taking LST and solving for $\tilde{I_0}(s)$.

we have $\tilde{I_0}(s) = \frac{N^I(s)}{D}$, Here

$N^I(s) = \tilde{Q}_{02}(s)[1 - \tilde{Q}_{11}^3(s)]$, D is already defined.

The expected number of inspections of the unit is given by

$$I(\infty) = \lim_{s \to 0} s\,\tilde{I_0}(s) = \frac{N^I(s)}{D}, \tag{18}$$

Here $N^I(s) = p_{02}p_{10}$

and D is already defined.

Expected Number of Repairs of The Unit

The following recursive relations for$R_i(t)$:

$$R_0(t) = Q_{01}(t)ⓢR_1(t) + Q_{02}(t)ⓢR_2(t),$$
$$R_1(t) = Q_{10}(t)ⓢ[1 + R_0(t)] + Q_{11}^3(t)ⓢ[1 + R_1(t)],$$
$$R_2(t) = Q_{20}(t)ⓢR_0(t) + [Q_{21}(t) + Q_{21}^4(t)]ⓢR_1(t) + Q_{2,1}^{4,5}(t)ⓢ[1 + R_1(t)] \quad (19)$$

After taking the above LST, solve for $\tilde{R}_0(s)$, we have

$$\tilde{R}_0(s) = \frac{N^R(s)}{D},$$

Here

$$N^R(s) = \tilde{Q}_{01}(s)[\tilde{Q}_{10}(s) + \tilde{Q}_{11}^3(s)] + \tilde{Q}_{02}(s)[\tilde{Q}_{10}(s) + \tilde{Q}_{11}^3(s)][\tilde{Q}_{21}(s) + \tilde{Q}_{21}^4(s)] + \tilde{Q}_{02}(s)\tilde{Q}_{2,1}^{4,5}(s)[1 + \tilde{Q}_{10}(s)]$$

and D is already defined.

Hence, the expected number of repairs for the unit is:

$$R(\infty) = \lim_{s\to 0} s\tilde{R}_0(s) = \frac{N^R(s)}{D}, \quad (20)$$

$N^R(s) = 1 - p_{02}[p_{20} - p_{10}p_{2,1}^{4,5}]$, D is already defined.

Expected Number of Replacements of The Unit

The recursive relations $R_i^c(t)$ are as follows:

$$R_0^c(t) = Q_{01}(t)ⓢ R_1^c(t) + Q_{02}(t)ⓢ R_2^c(t),$$
$$R_1^c(t) = Q_{10}(t)ⓢR_0^c(t) + Q_{11}^3(t)ⓢ R_1^c(t),$$
$$R_2^c(t) = Q_{20}(t)ⓢ[1 + R_0^c(t)] + [Q_{21}(t) + Q_{2,1}^{4,5}(t)]ⓢR_1^c(t) + Q_{21}^4(t)ⓢ[1 + R_1^c(t)] \quad (21)$$

After taking the LST, solve for $\tilde{R}_0^c(s)$, we have $\tilde{R}_0^c(s) = \frac{N^c(s)}{D(s)}$,

$$N^c(s) = \tilde{Q}_{02}(s)[1 - \tilde{Q}_{11}^3(s)][\tilde{Q}_{20}(s)\tilde{Q}_{2,1}^4(s)],$$

D(s) is already defined.

Hence, The Expected number of replacements per unit time to cold standby failure is given by

$$R^c(\infty) = \lim_{s\to 0} s\tilde{R}_0^C(s) = \frac{N^C}{D}, \quad (22)$$

$N^C = p_{02}p_{10}[p_{20} + p_{2,1}^4]$, D is already defined.

The Profit

Profit= Total Revenue Generated Expenses occurred

$$P = (C_0A) - (C_1B^I + C_2B^R + C_3\,I + C_4\,R + C_5R^C), \tag{23}$$

Here

C_0 = revenue per active unit.

C_1 = cost per unit when an attendant is busy with inspection.

C_2 = cost per unit for the time the server occupied while the unit was under repair.

C_3 = cost of inspection per unit.

C_4 = cost of repair of the unit.

C_5 = cost of the expected number of replacements of the unit.

Particular Case

Consider that the random variables included in the model follow an exponential distribution with other parameters. Let the PDF of all random variables be given as

$h(t) = \alpha e^{-\alpha t}$ and $g(t) = \beta e^{-\beta t}$

α is the inspection rate of the redundant unit and β is the repair rate of the operative unit. Using these values, the probabilities and results are evaluated. (See Annexure)

IV. Results and analysis

The authors studied the effect of various parameters on the two-unit standby autoclave system. Only real-time data was used to support the findings of this study. Initially, parameters are taken as a=0.25, b=0.75, α=0.08, β=0.06, μ=0.007, λ=0.001(rates are per hr), and various costs $C_0 = 20000, C_1 = 200,\ C_2 = 600,\ C_3 = 180,\ C_4 = 1000,\ C_5 = 1200$ (costs are in Indian rupees).

Table 2: MTSF v/s failure rate (λ) for varied repair rate (β)

λ	MTSF		
	β=0.06/hr	β=0.11/hr	β=0.16/hr
0.003	2433.333	2982.768	3280.828
0.005	1283.333	1627.007	1830.814
0.007	826.5306	1070.244	1225.011
0.009	590.2778	774.4582	897.9389
0.011	449.4949	594.4302	695.9717
0.013	357.6923	475.0783	560.3666

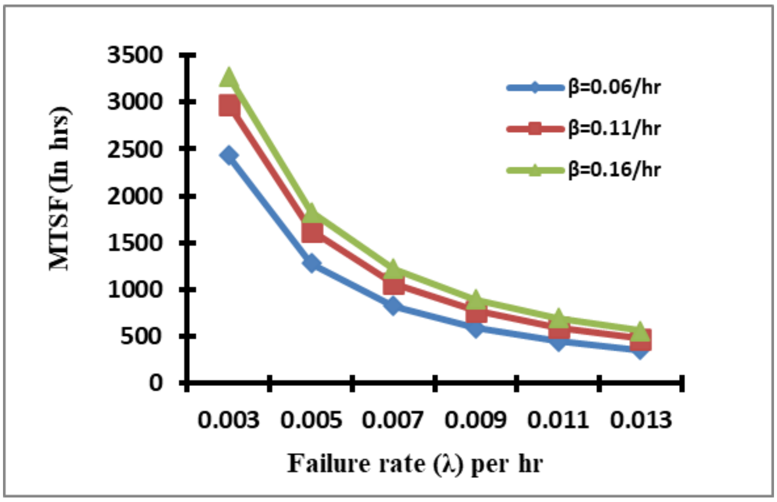

Fig. 2. MTSF V/S failure rate (λ) for varied repair rate (β)

Figure 2 makes it abundantly evident that when the failure rate of the operative unit rises from 0.003 to 0.013 results in a drop in the MTSF of the system. Furthermore, with higher values of repair rate (β) from 0.06 to 0.16 per hr respectively, MTSF shows an upward trend concerning basic parameters. Additionally, even the most minor changes in failure rates significantly decline. The graph was constructed by assuming other parameters such as a=0.25, b=0.75, α=0.08, β=0.06, μ=0.007 per hr.

Table 3: Availability v/s failure rate (λ) for varied repair rate (β)

λ	Availability		
	β=0.06/hr	β=0.11/hr	β=0.16/hr
0.005	0.986301	0.992212	0.993676
0.007	0.978405	0.988427	0.990918
0.009	0.969388	0.984294	0.988031
0.011	0.95941	0.979835	0.98502
0.013	0.948617	0.97507	0.981885
0.015	0.937143	0.970022	0.978632

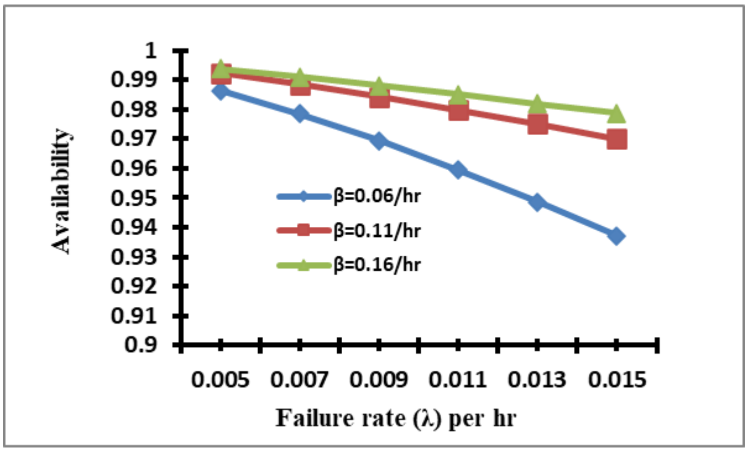

Fig. 3. Availability v/s failure rate (λ) for varied repair rate (β)

High-reliability systems can function continuously, but proper maintenance will ensure availability. However, a machine with low reliability may fail often, but proper maintenance and repair rates can boost its availability. Figure 3 provides clear and convincing evidence that the system's availability rapidly increases as the repair rate (β) slightly rises, it declines with a raised failure rate of the main unit (λ) from 0.005 to 0.015 per hr. Additionally, it is observed that there is a hike in the availability of the system with the growth of repair rate Rest parameters are taken as a=0.25, b=0.75, μ=0.007, ϒ=0.01, λ=0.01, α= 0.08 per hr.

Table 4: Profit v/s revenue per unit time (C0) for varied inspection cost (C3)

C0	Profit		
	C3= INR 1000	C3=INR 10000	C3= INR 19000
20000	-219.38648	-246.8266507	-274.2668194
25000	-123.52842	-150.9685907	-178.4087594
30000	-27.670362	-55.11053079	-82.55069951
35000	68.1876979	40.74752914	13.30736042
40000	164.045758	136.6055891	109.1654204
45000	259.903818	232.463649	205.0234803

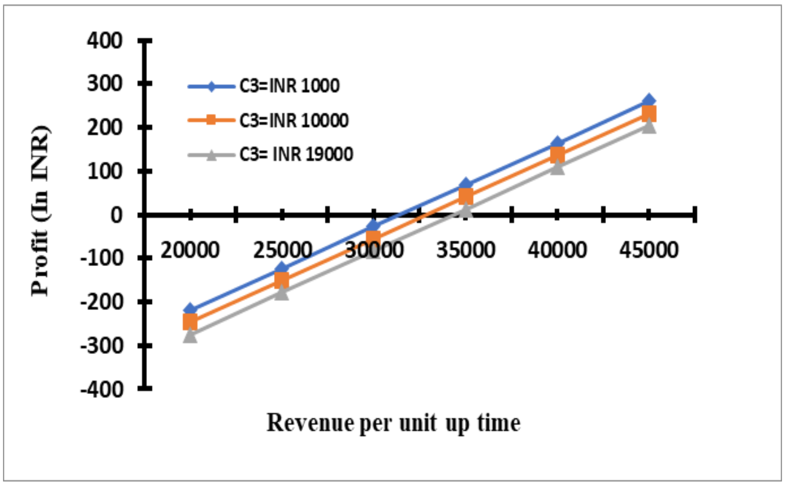

Fig. 4. Profit v/s revenue per unit time (C0) for varied inspection cost (C3)

Figure 4. exhibits profit is defined as income earned after deducting expenses, not revenue. It demonstrates that reducing costs or increasing revenue can boost the plant's or company's net profit. The graph illustrates how system profitability is impacted by revenue and inspection expenses. Profit shows an upward trend with lower values of inspection costs. Cut-off points help to estimate the minimum cost of inspection, to get maximum profit. It results in the company being more profitable if fewer inspections occur. Here, the values are assumed as $C_1 = 200,\ C_2 = 600,\ C_3 = (1000{,}10000{,}19000),\ C_5 = 120$ (Costs are in Indian rupees). Moreover, a=0.25, b=0.75, μ=0.007, α= 0.08 per hr, β= 0.00305 per hr, λ= 0.159 per hr.

Table 5: Profit v/s failure rate (λ) for varied revenue per unit time (C0)

λ	Profit		
	C0= INR 800	C0= INR 900	C0= INR 1000
0.001	363.5001	450.1061	536.712
0.002	184.2389	257.7357	331.2324
0.003	46.43534	109.0137	171.592
0.004	-57.7313	-3.85363	50.02403
0.005	-137.138	-90.1554	-43.1723
0.006	-198.689	-157.215	-115.74

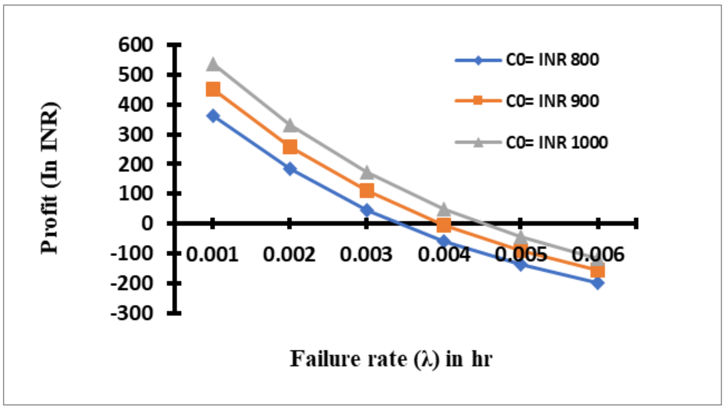

Fig. 5. Profit v/s failure rate (λ) for varied revenue per unit time (C0)

The assertion is clearly illustrated in Figure 5 that the system is more profitable for lower values of failure rate and higher values of revenue per unit of time. Increased revenue per unit of time and a decreased failure rate are two factors that directly affect profitability. Thus, the graph illustrates how raising income and enhancing system reliability jeopardize the system's financial performance. It assists businesses in determining how best to deploy resources to maximize profits. Cut-off points fix the upper limit of the failure rate; beyond this limit, the profit gets negative, and the company faces financial loss.

$C_0 = 20000,\ C_1 = 200,\ C_2 = 600,\ C_3 = 180,\ C_4 = 1000,\ C_5 = 1200$ (Costs are in Indian rupees). Moreover, a=0.25, b=0.75, μ=0.007, α= 0.08 per hr, β= 0.0032, λ= 0.001 (all rates are in per hr).

V. Conclusion

In the proposed study, a two-unit cold standby autoclave system with inspection and varying demand due to seasonal fluctuations (summer/winter) has developed. The model's uniqueness is that standby autoclave (SA) performance deteriorates due to environmental reasons. The redundant unit's behaviour is an intriguing aspect of this study. Moreover, the demand for ghee varied due to seasonal fluctuations (as demand was high in winter as compared to summer). The standby must remain operative constantly to avoid delays/financial loss. Additionally, both units must be operational to meet the increased demand for ghee during the winter. If not, the system will operate at a lower capacity. Thus, inspecting the standby autoclave becomes mandatory when it fails. Inspection adversely affects the system's reliability. The authors gather data from the visited company to determine the cut-off points to help decide when the model is profitable. The plotted graphs show that the system may not be beneficial if the inspection cost increases. Cut-off points help to fix the inspection/ cost. Thus, the system analyzer can set the repairer's inspection/product and visiting charges, to gain profit from the revenue generated by the company. MTSF demonstrates how long the system/part lasts before breaking and needs to be repaired. It has been observed that availability increases for higher values of repair rates but decreases as the value of failure rate increases. The ability of a graph to inform decisions, allocate resources efficiently, and emphasize the value of revenue generation and reliability for a system's financial success makes it significant when it shows that a system is more profitable for lower failure rates and higher revenue per unit time. It is an effective visual tool for comprehending a system's economic dynamics. Since the model is general, any plant or business with a similar circumstance can utilize it.

Conflict of Interest

There is no conflict of interest regarding this article..

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