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COMPARATIVE ANALYSIS OF A REDUNDANT SYSTEM SUBJECT TO INSPECTION OF A MANUFACTURING PLANT

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Abstract

The present paper is a comparative analysis of a two-unit autoclave system in a manufacturing plant. Most of the studies have been done by considering standby units to remain as good as new ones in this mode, but practically they may be corrupted by any environmental issues. This fact makes us concerned about the standby unit. Two stochastic models were developed based on such concern. Model 1 is constructed based on basically two possibilities; firstly, the standby unit is inspected after a fixed amount of time to check its feasibility. Secondly, either it will be repaired or replaced. Replacement is instant. Model 2 is constructed based on the same assumptions but replacement is not instant, it takes some random amount of time to be replaced. Stochastic analysis uses Markov processes to investigate how these dynamic factors interact and impact system profitability, availability, and dependability. By studying various scenarios about repair prices, replacement costs, inspection frequency, and fluctuating demand patterns, this research provides vital insights into the most effective approaches for handling redundant units and preserving system functionality. The results guide managing complex decision-making processes for safeguarding and maximizing system functionality, which has practical ramifications for sectors and systems that depend on redundancy to guarantee continuity and reliability.

Keywords: Stochastic Model, Reliability, semi-Markov Process, Regenerative Point Technique, Varied Production, comparative analysis, Innovation.

I. Introduction

The literature on reliability is becoming more and more rich day-by-day as a large number of researchers are making a lot of contributions in the field by incorporating some new ideas/concepts/studies. There exists a rich body of work on redundant systems such as hot, warm, and cold. In the field of reliability research, it is common practice to assume that a cold standby unit remains as good as new or never failed in this mode. For instance, Taneja et al. [XV] worked on a sugar mill, having three identical units with varied production. Their system can be shut down, during the shortage of raw material. Otherwise, the system will work at full capacity. Additionally, Malhotra and Taneja [VIII- XI] analyzed a two-unit cold standby system by assuming both units may be operative simultaneously due to increased demand. Furthermore, the authors worked on a comparative study of a cable manufacturing plant by taking the concept that demand is not constant. They used regenerative processes and semi-Markov processes to find numerous measures of the system's effectiveness. According to Levitin et al. [VI] redundancy is a widely applied technique to achieve high reliability. They also discussed how its failure made reliability non-monotonic and affected other parameters. Also, the elements of the system are assumed to be non-repairable. Thereafter Yang et al. [XVIII] took a system having M primary and S spare units. All the units were repairable. They were not considering waiting space; hence the failed unit was repaired immediately. The matrix-analytic method is used to compute the steady-state availability. Laplace transform technique used for MTTF. Gao et al. [I] studied warm and cold standby systems by assuming repair is not reliable. They executed preventive maintenance and repair according to idle time and failure of the unit respectively. Markov process approach was adopted to solve the equations. Thereafter, Wang et al. [XVI] took a warm standby and non-identical system. They considered two types of failure, firstly hardware failure and secondly human error failure. The failed unit goes under repair immediately. After that, the repair unit was assumed to be as good as the new one. Additionally, Malhotra [XI] developed a two-unit redundant system where one unit was in working mode while the other was cold redundant by enchanting the activation time. After that, Malhotra et al. [XII] worked on a hot standby system with varying demands. In this model, identical standby units remain operative from an initial state. Kakkar et al. [III-IV] analyzed the cost-benefit of a system having three redundant units, by considering the concept with correlated and repair times . They used the concept of repairing or inspection according to the type of failure. The main unit kept its high quality. On the other hand, some authors agree that it can be corrupted in this mode. For instance, Ram et al. [XIIII] investigated the reliability of two-unit standby systems. Initially, one unit was operative and the rest was kept on standby mode. Their system may also fail due to improper starting of the system, the reason behind it was untrained and inexperienced system analysts. Additionally, Wang et al. [XVII] studied redundant allocation problems with degrading components. The main concern is cold standby units suffering from performance degradation when exposed to extreme standby environments for long-term storage. Moreover, Levitin et al. [XVII] discussed the factors which affect industrial systems such as deterioration, corrosion, etc. To overcome such a situation fixed planning is performed to renew the worn element by using redundant elements to enhance the functioning successfully. Manocha et al. studied the possibility that redundant systems may be corrupted and inoperable due to non-use of it for a long time. The authors considered random inspection to handle the situation. Thereafter, Kumar et al. [V] worked on two identical unit systems. Authors assume a single unit is sufficient to operate the system smoothly. The rest can be failed due to unused or any other environmental issues. The repairman will inspect the failed unit and then decide whether it will be repaired or replaced. According to Juybari et al. [II] mixed redundancy is a most powerful technique to enhance the reliability of a system. Also, assume all the components are under environmental shocks and may deteriorate by internal or external shocks. Also, Shekhar et al. [XIV] evaluated the reliability of multi-unit systems having several failures, degradation, random delays, and probabilistic imperfections. Also, they discussed their impact on production and the system's performance. Given the explanations and evidence revealed above, two major observations are given below: If demand varies then standby units remain as good as new ones or never corrupted. If standby may deteriorate due to environmental shocks but they were not considering demand is variable.

To the best of our knowledge, none of the extensive literature on reliability considered the concept of varied demand due to varied seasons and cold standby can be damaged in this mode simultaneously. On visiting the plant, the candidates observed that cold standby may get corrupted in this mode due to environmental issues. Hence, there is a massive gap. The authors will try to fill it.

In the present paper, the candidate analyses a comparison between two stochastic models for a two-unit cold standby system. The central part of the research is the Autoclave machine (main unit) with an identical cold standby unit. Here, working the standby unit always must; otherwise, there will be huge production loss—our primary focus is on the standby unit. On visiting the plant, the candidate observed there is variation in demand due to varied seasons. Demand for ghee is high in winter as compared to summer. Mainly in winter working as a standby unit is a must, to fulfill increased demand. We applied the inspection technique on the standby unit only to check its feasibility. After that, the technician will decide whether it will be repaired or replaced, as replacement is only a subpart of the autoclave that is applicable. Model 1 assumes replacement is instant whereas Model 2 consists of replacement that takes some random amount of time. If the central unit fails, then it goes directly for repair only. Our study is based on actual data collected from Industry, Punjab, India.

II. Mathematical Model(s)

- ❖ The system has two identical Autoclave units. Initially, one unit is functioning and the rest is kept in standby mode.
- ❖ The standby unit is subjected to failure after exceeding the maximum redundancy time, an upper time limit for which the team remains perfect in the cold standby mode.
- ❖ If the central unit of the autoclave fails, it directly goes under repair. In contrast, the cold standby unit goes for inspection after a fixed amount of time to check the feasibility of repair/ replacement.
- ❖ There is a single repairman who attends the system immediately whenever there is a need to perform any action of inspection and repair/replacement.

- **Replacement in model 1 is instant whereas in model 2 replacement takes some random amount of time. Replacement of a sub-part of autoclave (like Gear Box) is considered.**
- The inspection and repair/replacement times follow arbitrary distributions, whereas the failure time of the unit follows an exponential distribution.
- The switches are perfect and instantaneous.
- After each repair, the unit works as new.
- All the random variables are statistically independent.

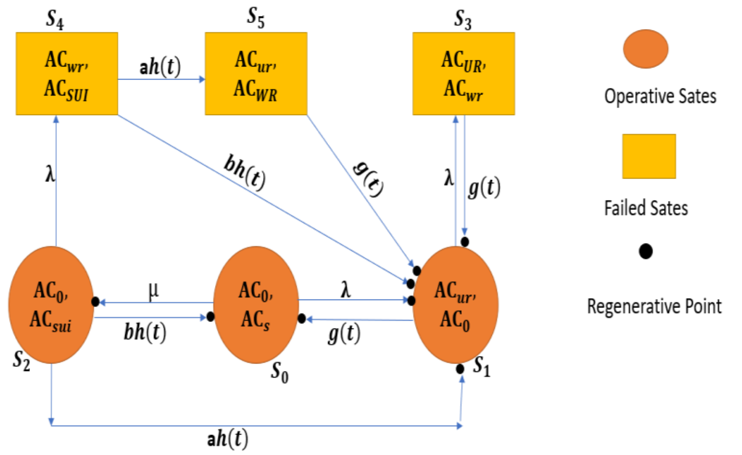

Fig. 1. State Transition Diagram of Model

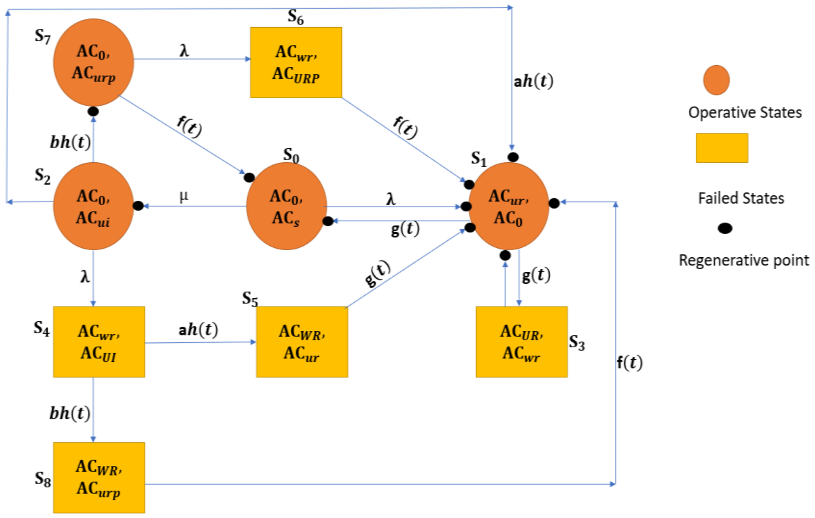

Fig. 2. State Transition Diagram of Model

III. Solution Procedure

Transition Probabilities and Mean Sojourn Times

Simple probabilistic considerations yield the following expressions for the non-zero elements:

$$p_{ij} = Q_{i,j}(\infty) = \int_0^{\infty} dQ_{i,j}(t)dt = \tilde{Q}_{i,j}(0) = \int_0^{\infty} q_{i,j}(t)dt \text{ and } p_{ij} = \lim_{s\to 0} q^*_{ij}(s). \quad (1)$$

For Model 1:

$$p_{01} = \frac{\lambda}{\lambda+\mu},\ p_{02} = \frac{\mu}{\lambda+\mu},\ p_{10} = g^*(\lambda),\quad p_{13} = 1 - g^*(\lambda),\ p_{20} = b\,h^*(\lambda),$$
$$p_{21} = a\,h^*(\lambda),\quad p_{24} = 1 - h^*(\lambda),\quad p_{31} = g^*(0),\quad p_{41} = b\,h^*(0),$$
$$p_{45} = a\,h^*(0),\quad p_{51} = g^*(0),\quad p_{11}^{3} = 1 - g^*(\lambda),\quad p_{21}^{4} = b\,[1 - h^*(\lambda)],$$
$$p_{2,1}^{4,5} = a\,[1 - h^*(\lambda)], \quad (2)$$

By these probabilities, it can be verified that

$$p_{01} + p_{02} = 1,\quad p_{10} + p_{13} = 1,\quad p_{20} + p_{21} + p_{24} = 1,\quad p_{31} = 1,$$
$$p_{41} + p_{45} = 1, p_{51} = 1, p_{10} + p_{11}^{3} = 1, p_{20} + p_{21} + p_{21}^{4} + p_{2,1}^{4,5} = 1. \quad (3)$$

For Model 2:

$$p_{01} = \frac{\lambda}{\lambda+\mu},\quad p_{02} = \frac{\mu}{\lambda+\mu},\quad p_{10} = g^*(\lambda),\quad p_{13} = 1 - g^*(\lambda),$$
$$p_{27} = b\,h^*(\lambda), p_{21} = a\,h^*(\lambda), p_{24} = 1 - h^*(\lambda),\ p_{31} = g^*(0),$$
$$p_{48} = b\,h^*(0),\ p_{45} = a\,h^*(0),\ p_{51} = g^*(0),\ p_{61} = f^*(0),\ p_{70} = f^*(\lambda),$$
$$p_{76} = 1 - f^*(\lambda), p_{81} = f^*(0),\ p_{11}^{3} = [1 - g^*(\lambda)]g^*(0),$$
$$p_{21}^{4,5} = a\,[1 - h^*(\lambda)]g^*(0)h^*(0), p_{21}^{4,8} = b[1 - h^*(\lambda)]g^*(0)f^*(0),$$
$$p_{71}^{6} = [1 - f^*(\lambda)]f^*(0), \quad (4)$$

By these probabilities, it can be verified that

$$p_{01} + p_{02} = 1,\quad p_{10} + p_{13} = 1,\quad p_{21} + p_{24} + p_{27} = 1,\quad p_{31} = 1,$$
$$p_{45} + p_{48} = 1,\ p_{51} = 1,\ p_{61} = 1,\quad p_{70} + p_{76} = 1,\quad p_{81} = 1,$$
$$p_{10} + p_{11}^{3} = 1, p_{21} + p_{27} + p_{21}^{4,5} + p_{21}^{4,8} = 1, p_{70} + p_{71}^{6} = 1, \quad (5)$$

The mean sojourn times (μ_i) in the state S_i are:

For Model 1:

$$\mu_0 = \frac{1}{\lambda+\mu},\ \mu_1 = \frac{[1 - g^*(\lambda)]}{\lambda},\ \mu_2 = \frac{[1 - h^*(\lambda)]}{\lambda},\ \mu_3 = -g^{*\prime}(0),\ \mu_4 = -h^{*\prime}(0),$$
$$\mu_5 = -g^{*\prime}(0), \mu_1' = [\tfrac{1}{\lambda} - g^{*\prime}(0)][1 - g^*(\lambda)]$$
$$\mu_2' = [\tfrac{1}{\lambda} - h^{*\prime}(0) - a g^{*\prime}(0)][1 - h^*(\lambda)],$$

For Model 2:

$$\mu_0 = \frac{1}{\lambda+\mu},\ \mu_1 = \frac{[1-g^*(\lambda)]}{\lambda},\ \mu_2 = \frac{[1-h^*(\lambda)]}{\lambda},\ \mu_3 = -g^{*'}(0),\ \mu_4 = -h^{*'}(0),$$
$$\mu_5 = -g^{*'}(0), \mu_6 = -f^{*'}(0),\qquad \mu_7 = \frac{[1-f^*(\lambda)]}{\lambda},\qquad \mu_8 = -f^{*'}(0),$$
$$\mu_1{}' = [\tfrac{1}{\lambda} - g^{*'}(0)][1 - g^*(\lambda)], \mu_2{}' = [\tfrac{1}{\lambda} - h^{*'}(0) - ag^{*'}(0)][1 - h^*(\lambda)],$$
$$\mu_7{}' = [\tfrac{1}{\lambda} - f^{*'}(0)][1 - f^*(\lambda), \qquad (6)$$

Measures of System Effectiveness:

Mean Time to System Failure

In determining MTSF, assume the failed states as absorbing states. Recursive relations $\phi_i(t)$ are:

For Model 1:

$$\phi_0(t) = Q_{01}(t)ⓢ\phi_1(t) + Q_{02}(t)ⓢ\,\phi_2(t),$$
$$\phi_1(t) = Q_{10}(t)ⓢ\phi_0(t) + Q_3(t),$$
$$\phi_2(t) = Q_{20}(t)ⓢ\phi_0(t) + Q_{21}(t)ⓢ\,\phi_1(t) + Q_{24}(t), \qquad (7)$$

For Model 2:

$$\phi_0(t) = Q_{01}(t)ⓢ\phi_1(t) + Q_{02}(t)ⓢ\,\phi_2(t),$$
$$\phi_1(t) = Q_{10}(t)ⓢ\phi_0(t) + Q_{13}(t),$$
$$\phi_2(t) = Q_{21}(t)ⓢ\phi_1(t) + Q_{24}(t) + Q_{27}(t)ⓢ\,\phi_7(t)$$
$$\phi_7(t) = Q_{70}(t)ⓢ\phi_0(t) + Q_{76}(t), \qquad (8)$$

On taking LST of the relation (7) and (8) and solving for $\tilde{Q}_0(s)$, we get

$$MTSF(T) = \lim_{s\to 0}\frac{1-\tilde{\phi}_0(s)}{s} = \frac{N_i}{D_i}\ \ (i = 1,2)\ ,$$
$$N_1 = \mu_0 + [p_{01} + p_{02}\ p_{21}]\mu_1 + p_{02}\,\mu_2,$$
$$D_1 = 1 - p_{01}\ p_{10} - p_{02}[p_{10}\,p_{21} + p_{20}],$$
$$N_2 = \mu_0 + [p_{01} + p_{02}\ p_{21}]\mu_1 + p_{02}\,\mu_2 + p_{02}\,p_{27}\,\mu_7$$
$$D_2 = 1 - p_{01}\ p_{10} - p_{02}\,p_{27}\,p_{70} - p_{02}\,p_{10}\,p_{21}$$

Steady State Availability

$$A_i(t) = M_i(t) + \textstyle\sum_j q_{i,j}{}^k(t)©A_j(t),\ i \neq j \qquad (9)$$

For Model 1:

$$A_0(t) = M_0(t) + q_{01}(t)©A_1(t) + q_{02}(t)©A_2(t),$$

$$A_1(t) = M_1(t) + q_{10}(t)©A_0(t) + q_{11}^{3}(t)©A_1(t),$$

$$A_2(t) = M_2(t) + q_{20}(t)©A_0(t) + [q_{21}(t) + q_{21}^{4}(t) + q_{2,1}^{4,5}(t)]©A_1(t) \quad (10)$$

For Model 2:

$$A_0(t) = M_0(t) + q_{01}(t)©A_1(t) + q_{02}(t)©A_2(t),$$

$$A_1(t) = M_1(t) + q_{10}(t)©A_0(t) + q_{11}^{3}(t)©A_1(t),$$

$$A_2(t) = M_2(t) + [q_{21}(t) + q_{21}^{4,5}(t) + q_{21}^{4,8}(t)]©A_1(t) + q_{27}(t)©A_7(t),$$

$$A_7(t) = M_7(t) + q_{70}(t)©A_0(t) + q_{71}^{6}(t)©A_1(t), \quad (11)$$

Here,

$$M_0(t) = e^{-(\lambda+\mu)t},\ M_1(t) = e^{-\lambda t}\bar{G}(t),\ \ M_2(t) = e^{-\lambda t}\bar{H}(t), M_7(t) = e^{-\lambda t}\bar{F}(t)$$

Taking LT of above relations (10) and (11) and solving for $A_0{}^*(s)$, we get steady state Availability

$$A(\infty) = \lim_{s\to 0} s\, A_0{}^*(s) = \frac{N_i}{D_i}\ \ (i = 1,2)\,, \quad (12)$$

$$N_1 = p_{10}\mu_0 + [1 - p_{02}p_{20}]\mu_1 + p_{02}p_{10}\mu_2,$$

$$D_1 = p_{10}\mu_0 + [1 - p_{02}p_{20}]\mu_1{}' + p_{02}p_{10}\mu_2{}' \ \textbf{(model 1)}$$

$$N_2 = p_{10}\mu_0 + [1\text{-}p_{02}p_{27}p_{70}]\mu_1 + p_{02}p_{10}\,[\mu_2 + p_{27}\,\mu_7],\ \textbf{(model 2)}$$

$$D_2 = p_{10}\mu_0 + [1 - p_{02}p_{27}p_{70}]\mu_1{}' + p_{02}p_{10}[\,\mu_2{}' + p_{27}\mu_7{}'],$$

Busy Period Analysis for Server

Due to Inspection of Unit:

Recursive relations for $B_i^I(t)$ are as follows:

For Model 1:

$$B_0^I(t) = q_{01}(t)©B_1^I(t) + q_{02}(t)©B_2^I(t),$$

$$B_1^I(t) = q_{10}(t)©B_0^I(t) + q_{11}^{3}(t)©B_1^I(t),$$

$$B_2^I(t) = W_2^I(t) + q_{20}(t)©\, B_0^I(t) + [q_{21}(t) + q_{21}^{4}(t) + q_{21}^{4,5}(t)]©\, B_1^I(t) \quad (13)$$

For Model 2:

$$B_0^I(t) = q_{01}(t)©B_1^I(t) + q_{02}(t)©B_2^I(t),$$

$$B_1^I(t) = q_{10}(t)©B_0^I(t) + q_{11}^{3}(t)©B_1^I(t),$$

$$B_2^I(t) = W_2^I(t) + [q_{21}(t) + q_{21}^{4,5}(t) + q_{21}^{4,8}(t)]©\, B_1^I(t) + q_{27}(t)©\, B_7^I(t),$$

$$B_7^I(t) = q_{70}(t)©B_0^I(t) + q_{71}^{6}(t)©B_1^I(t) \quad (14)$$

Here, $W_2^I(t) = e^{-\lambda t}\bar{H}(t) + (\lambda e^{-\lambda t}©1)\bar{H}(t)$

Taking LT of above relations (13) and (14) and solving for ${B_0}^{I^*}(s)$, we have the busy time of the server due to inspection is given below:

$$B^I(\infty) = \lim_{s\to 0} s\, {B_0}^{I^*}(s) = \frac{N^I(s)}{D^I(s)} \tag{15}$$

$N^I = {W_2}^{I^*}(0)p_{02}p_{10}$, D_1 is already mentioned. **(For model 1)**

$N^I = {W_2}^{I^*}(0)p_{02}p_{10}$, D_2 is already mentioned. **(For model 2)**

Due to the repair of the unit:

Recursive relations for $B_i^R(t)$ are given by

$B_i^R(t) = W_f(t) + \sum_j {q_{i,j}}^k(t) © B_i^R(t)$, i ≠ j

For Model 1:

The recursive relations $B_i^R(t)$ are:

$$B_0^R(t) = q_{01}(t) © B_1^R(t) + q_{02}(t) © B_2^R(t),$$
$$B_1^R(t) = W_1^R(t) + q_{10}(t) © B_0^R(t) + q_{11}^3(t) © B_1^R(t),$$
$$B_2^R(t) = q_{20}(t) © B_0^R(t) + \left[q_{21}(t) + q_{21}^4(t) + q_{2,1}^{4,5}(t)\right] © B_1^R(t) \tag{16}$$

For Model 2:

The Recursive Relations $B_i^R(t)$ are as follows:

$$B_0^R(t) = q_{01}(t) © B_1^R(t) + q_{02}(t) © B_2^R(t),$$
$$B_1^R(t) = W_1^R(t) + q_{10}(t) © B_0^R(t) + q_{11}^3(t) © B_1^R(t),$$
$$B_2^R(t) = \left[q_{21}(t) + q_{21}^{4,5}(t) + q_{21}^{4,8}(t)\right] © B_1^R(t) + q_{27}(t) © B_7^R(t), \tag{17}$$
$$B_7^R(t) = q_{70}(t) © B_0^R(t) + q_{71}^6(t) © B_1^R(t)$$

Where $W_1^R(t) = e^{-\lambda t}\bar{G}(t) + \left(\lambda e^{-\lambda t} © 1\right)\bar{G}(t)$

Taking LT of above relations (16) and (17) and solving for them ${B_0}^{R^*}(s)$, we get the busy time of the server due to repair is given below:

$$B^R(\infty) = \lim_{s\to 0} s\, {B_0}^{R^*}(s) = \frac{N^R}{D}, \tag{18}$$

$N^R(s) = {W_1}^{R^*}(0)\{1 - p_{02}p_{20}\}$, D_1 is already mentioned. **(For model 1)**

$N^R = {W_1}^{R^*}(0)\{1 - p_{02}p_{27}p_{70}\}$, D_2 is already mentioned. **(For model 2)**

Due to Replacement:

For Model 2:

Recursive relations for $B_i^{RP}(t)$ are given by

$B_i^{RP}(t) = W_f(t) + \sum_j {q_{i,j}}^k(t) © B_i^{RP}(t)$, i ≠ j follows:

$$B_0^{RP}(t) = q_{01}(t) © B_1^{RP}(t) + q_{02}(t) © B_2^{RP}(t),$$
$$B_1^{RP}(t) = q_{10}(t) © B_0^{RP}(t) + q_{11}^3(t) © B_1^{RP}(t),$$
$$B_2^{RP}(t) = \left[q_{21}(t) + q_{21}^{4,8}(t) + q_{21}^{4,5}(t)\right] © B_1^{RP}(t) + q_{27}(t) © B_7^{RP}(t),$$
$$B_7^{RP}(t) = W_7^{RP}(t) + q_{70}(t) © B_0^{RP}(t) + q_{71}^6(t) © B_1^{RP}(t) \tag{19}$$

Here $W_7^{RP}(t) = e^{-\lambda t}\bar{F}(t) + \left(\lambda e^{-\lambda t} © 1\right)\bar{F}(t)$,Taking LT of equations (19) and solving for ${B_0}^{RP^*}(s)$, solving for ${B_0}^{RP^*}(s)$, we have

$$ {B_0}^{RP^*}(s) = \frac{N^{RP}(s)}{D(s)}, $$

Where, $N^{RP}(s) = {W_7}^{RP^*}(s) q^*{}_{02}(s) q^*{}_{27}(s)\,[1 - q^{*3}_{11}(s)]$, D_2 is already mentioned.

(For model 2)

Expected Number of Inspections of The Unit

For Model 1:

The recursive relations are given by

$$ I_0(t) = Q_{01}(t) \circledS I_1(t) + Q_{02}(t) \circledS [1 + I_2(t)], $$

$$ I_1(t) = Q_{10}(t) \circledS I_0(t) + Q_{11}^{3}(t) \circledS I_1(t), $$

$$ I_2(t) = Q_{20}(t) \circledS I_0(t) + \left[Q_{21}(t) + Q_{21}^{4}(t) + Q_{2,1}^{4,5}(t)\right] \circledS I_1(t) \quad (20) $$

For Model 2:

The recursive relations are given by

$$ I_0(t) = Q_{01}(t) \circledS I_1(t) + Q_{02}(t) \circledS [1 + I_2(t)], $$

$$ I_1(t) = Q_{10}(t) \circledS I_0(t) + Q_{11}^{3}(t) \circledS I_1(t), $$

$$ I_2(t) = \left[Q_{21}(t) + Q_{21}^{4,5}(t) + Q_{21}^{4,8}(t)\right] \circledS I_1(t) + Q_{27}(t) \circledS I_7(t), $$

$$ I_7(t) = Q_{70}(t) \circledS I_0(t) + Q_{71}^{6}(t) \circledS I_1(t) \quad (21) $$

Taking LST of equations (20,21) solving for $\tilde{I_0}(s)$, we get the expected number of inspections of the unit is given by

$$ I(\infty) = \lim_{s \to 0} s\,\tilde{I_0}(s) = \frac{N^I(s)}{D} \quad (22) $$

$N^I = p_{02}p_{10}$, D_1 is already mentioned. **(For model 1)**

$N^I = p_{02}p_{10}$, D_2 is already mentioned. **(For model 2)**

Expected Number of Repairs of The Unit:

The recursive relations for $R_i(t) = \sum_j Q_{ij}^{k}(t) \circledS (R_j(t) + T_j(t))$ are as follows:

For Model 1:

The following recursive relations for $R_i(t)$:

$$ R_0(t) = Q_{01}(t) \circledS R_1(t) + Q_{02}(t) \circledS R_2(t), $$

$$ R_1(t) = Q_{10}(t) \circledS [1 + R_0(t)] + Q_{11}^{3}(t) \circledS [1 + R_1(t)], $$

$$ R_2(t) = Q_{20}(t) \circledS R_0(t) + [Q_{21}(t) + Q_{21}^{4}(t)] \circledS R_1(t) + Q_{2,1}^{4,5}(t) \circledS [1 + R_1(t)] \quad (23) $$

For Model 2:

The following recursive relations for $R_i(t)$:

$$R_0(t) = Q_{01}(t)\circledS R_1(t) + Q_{02}(t)\circledS R_2(t),$$

$$R_1(t) = Q_{10}(t)\circledS[1 + R_0(t)] + Q_{11}^{3}(t)\circledS[1 + R_1(t)],$$

$$R_2(t) = [Q_{21}(t) + Q_{21}^{4,8}(t)]\circledS R_1(t) + Q_{21}^{4,5}(t)\circledS[1 + R_1(t)] + Q_{27}(t)\circledS R_7(t),$$

$$R_7(t) = Q_{70}(t)\circledS R_0(t) + Q_{71}^{6}(t)\circledS R_1(t). \quad (24)$$

Taking LST of equations (23, 24) and solving for them $\tilde{R}_0(s)$, we get the expected number of repairs for the unit mentioned below:

$$R(\infty) = \lim_{s\to 0} s\tilde{R}_0(s) = \frac{N^R}{D}, \quad (25)$$

$N^R\,(s) = 1 - p_{02}\,[p_{20} - p_{10}p_{2,1}^{4,5}\,]$, D_1 is already mentioned. **(For model 1)**

$N^R\,(s) = 1 - p_{02}\,[p_{20}p_{70} - p_{10}p_{21}^{4,5}]$, D_2 is already mentioned. **(For model 2)**

Expected Number of Replacements of The Unit

For Model 1:

The recursive relations $Rp_i(t)$ are as follows:

$$R_0^c(t) = Q_{01}(t)\circledS\, R_1^c(t) + Q_{02}(t)\circledS\, R_2^c(t),$$

$$R_1^c(t) = Q_{10}(t)\circledS R_0^c(t) + Q_{11}^{3}(t)\circledS\, R_1^c(t),$$

$$R_2^c(t) = Q_{20}(t)\circledS[1 + R_0^c(t)] + [Q_{21}(t) + Q_{2,1}^{4,5}(t)]\circledS R_1^c(t) + Q_{21}^{4}(t)\circledS[1 + R_1^c(t)] \quad (26)$$

For Model 2:

The recursive relations $Rp_i(t)$ are as follows:

$$Rp_0(t) = Q_{01}(t)\circledS Rp_1(t) + Q_{02}(t)\circledS Rp_2(t),$$

$$Rp_1(t) = Q_{10}(t)\circledS Rp_0(t + Q_{11}^{3}(t)\circledS Rp_1(t,$$

$$Rp_2(t) = [Q_{21}(t) + Q_{2,1}^{4,5}(t)]\circledS Rp_1(t) + Q_{21}^{4,8}(t)\circledS[1 + Rp_1(t)] + Q_{27}(t)\circledS[Rp_7(t)], \quad (27)$$

Taking the LST of equations (26, 27) and solving for $\tilde{R}p_0(s)$, The Expected number of replacements per unit time to cold standby failure is given by

$$R^c\,(\infty) = \lim_{s\to 0} s\tilde{R}_0^c(s) = \frac{N^{RP}}{D} \quad (28)$$

$N^{RP} = p_{02}p_{10}\,[p_{20} + p_{2,1}^{4}\,]$, D_1 is already mentioned. **(For model 1)**

$N^{RP} = p_{02}p_{10}[p_{27} + p_{21}^{4,8}]$, D_2 is already mentioned. **(For model 2)**

Profit Analysis

The profit acquired from the system model in a steady state is below.

Profit= Total Revenue Generated - Total Expenses occurred

$P_1 = (C_0A) - (C_1B^I + C_2B^R + C_3\ I + C_4\ R + C_5R^C)$, **(For model 1)**

$P_2 = (C_0A) - (C_1B^I + C_2B^R + C_3B^{RP} + C_4I + C_5R + C_6RP)$ **(For model 2)**

Here

C_0= Revenue per active unit of the system.

C_1 = cost per unit period for which the server is busy inspecting the unit.

C_2 = cost per unit for the time the server occupied while the unit was repaired

C_3=cost per unit for the time the server occupied while the unit was replaced

C_4 = cost per unit inspection of the unit

C_5 = cost per unit repair of a unit

C_6 = cost per unit replacement of standby (only sub-part of Autoclave like Gear Box) unit

And A, I, R, B^I, B^R, B^C are already mentioned above.

Particular Case

Let random variables included in the model follow an exponential distribution with different parameters. Let the PDF of all the random variables given as:

$h(t) = \alpha e^{-\alpha t}$,$g(t) = \beta e^{-\beta t}$and $f(t) = \gamma e^{-\gamma t}$Where $\alpha =$ Inspection Rate,$\beta =$ Repair Rate, and γ= Replacement Rate of Standby Autoclave. Using these values, the probabilities and results are evaluated. (See Annexure)

III. Results and discussion

The authors studied the effect of various parameters on the two-unit standby autoclave system. Only real-time data was used to support the findings of this study.

Table 1: MTSF V/S Failure Rate of Main Unit for Varied Repair Rate.

λ	MTSF (M1-M2)		
	β=0.02/hr	β=0.06/hr	β=0.1/hr
0.001	3599.17743	7575.541591	9430.927659
0.002	1358.34207	3160.911644	4128.050634
0.003	714.388581	1787.587246	2428.814715
0.004	435.948684	1151.704889	1618.253847
0.005	290.40404	799.4871654	1156.363056
0.006	205.191511	582.9299522	864.7303204

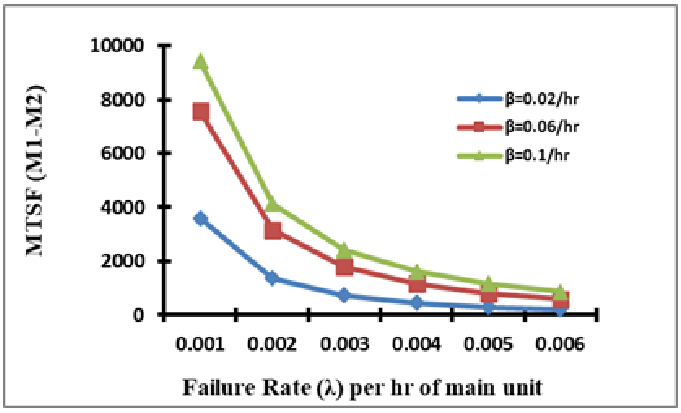

Fig. 2. MTSF V/S Failure Rate of Main Unit for Varied Repair Rate.

The impact of many parameters on the MTSF of both Model 1 and Model 2 of the system is depicted in Figure 2. As λ increases, the mean time to system breakdown reduces. When the other parameters remain fixed, the graph displays an increasing trend with an increased value of β. The values assumed as, a=0.25, b=0.75, α=0.8, β= (0.02, 0.06, 0.1)/hr, µ=0.007/hr, λ= (0.001, 0.002, 0.003, 0.004, 0.005, 0.006) /hr., ϒ=0.01/hr. It demonstrates the typical lifespan of the machine/part or how long it lasts before breaking and needs to be repaired.

Table 2: **MTSF V/S Failure Rate of Main Unit for Varied Inspection Rate.**

λ	MTSF (M1-M2)		
	α=0.008/hr	α=0.08/hr	α=0.8/hr
0.002	170.012043	1234.904627	1981.051728
0.003	106.720289	664.8618247	1010.720883
0.004	74.6328822	413.1072294	605.8244884
0.005	55.2882279	279.1854354	399.3055556
0.006	42.4623116	199.6148108	280.4617911
0.007	33.4410487	148.6809468	206.2537072

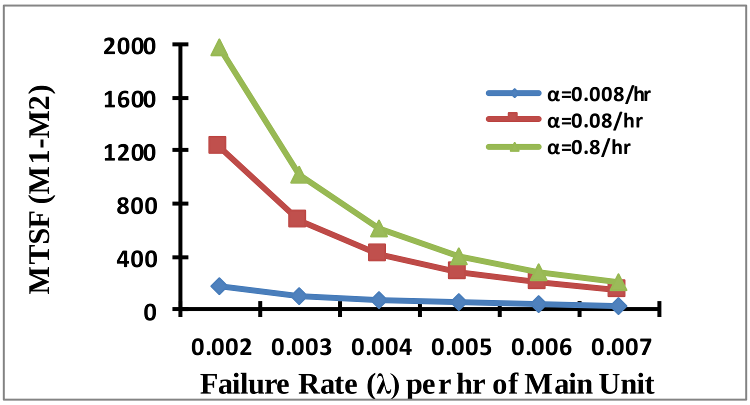

Fig. 3. MTSF V/S Failure Rate of Main Unit for Varied Inspection Rate

Figure 3 depicts that a higher inspection rate indicates a shorter inspection time. This suggests that a system model with an increased inspection feature stays more dependable even at a respectably low unit failure rate. The graph plotted by taking values as, a=0.25, b=0.75, α= (0.02, 0.03, 0.04)/hr, β= 0.02/hr, μ=0.007/hr, λ= (0.002, 0.003, 0.004, 0.005, 0.006, 0.007)/hr., ϒ=0.01/hr. Hence figure 3 demonstrates that the MTSF lowers as the values of λ increase from (0.002 to 0.007)/hr. With a larger value of α= (0.008, 0.08, 0.8)/hr, the graph exhibits a rising trend, which is consistent with the fixed values of other parameters. However, as the value of λ increases, this rate of rise decreases slightly. This indicates that a system model featuring an enhanced inspection function maintains greater dependability despite a comparatively low rate of unit failure. Additionally, it illustrates how a decrease in repair or replacement durations contributes to the enhanced dependability of the system.

Table 3: Availability V/S Failure Rate of Main Unit for Varied Inspection Rate.

λ	Availability (A1-A2)		
	α=0.001/hr	α=0.002/hr	α=0.003/hr
0.001	0.00646538	0.013265101	0.01803538
0.002	0.00844889	0.019038446	0.027189122
0.003	0.00939383	0.022099705	0.032378966
0.004	0.00993839	0.023881647	0.035471232
0.005	0.01028621	0.024957876	0.037322976
0.006	0.01052137	0.025601123	0.038381105

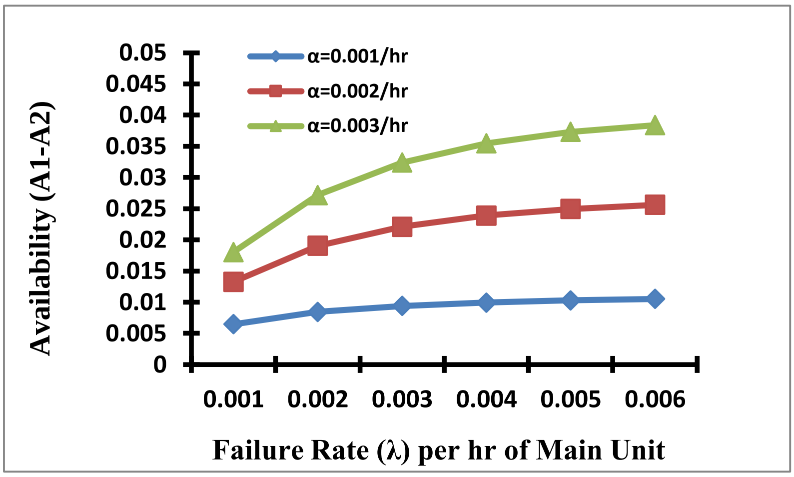

Fig. 4. Availability V/S Failure Rate of Main Unit for Varied Inspection Rate

A highly dependable system can run continuously, but its availability will drop if not maintained correctly. On the other hand, a machine with low reliability may experience numerous failures. Its availability may increase with inspection. Fig. 4 shows the system availability decreases as the value of the failure rate of the main unit increases over time. Moreover, the system availability increases if the inspection rate increases. Thus, inspection techniques on such systems could make that system more available. This analysis has been done by considering values as, a=0.25, b=0.75, α= (0.001, 0.002, 0.003)/hr, β= 0.02/hr, μ=0.007/hr, λ= (0.001, 0.002, 0.003, 0.004, 0.005, 0.006)/hr., ϒ=0.01/hr

Table 4: Availability V/S Inspection Rate for Varied Repair Rate.

α	Availability (A1-A2)		
	β=0.001/hr	β=0.002/hr	β=0.003/hr
0.009	-0.0049819	0.001923055	0.008592759
0.011	-0.0058797	0.000979288	0.007951582
0.013	-0.0065949	0.000179967	0.007353777
0.015	-0.0071763	-0.00049814	0.006815333
0.017	-0.0076575	-0.00107729	0.006336123
0.019	-0.0080619	-0.00157598	0.005910819

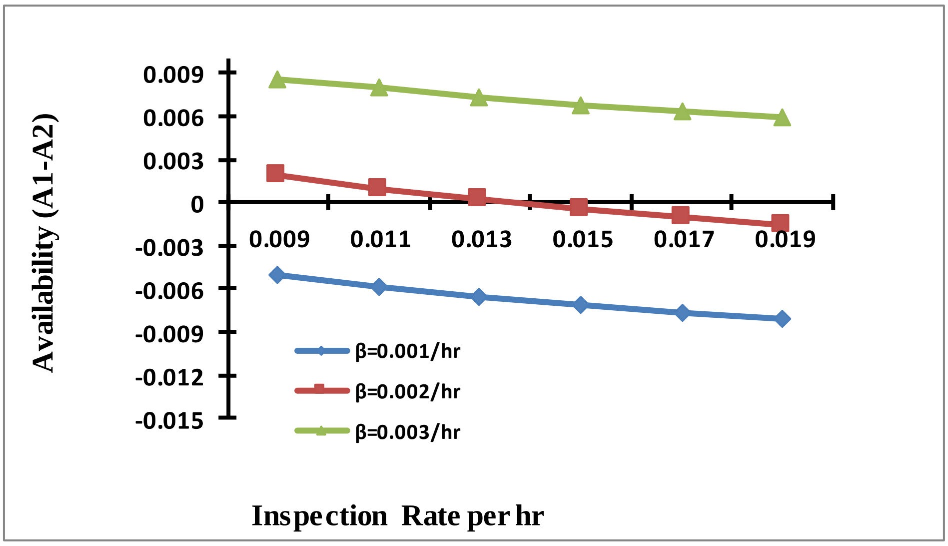

Fig. 5. Availability V/S Inspection Rate for Varied Repair Rate.

Figure 5 exhibits the effect of various parameters on the availability of model 1 and model 2 together. By using repair techniques, a system's availability can be increased. The system may become more available with minor changes to repair rates. Also, it suggests that to keep the system operational, more repairs should be made if the unit breaks down frequently than should be inspected. This graph is constructed by taking parameters as, a=0.25, b=0.75, α= (0.009, 0.011, 0.013, 0.015, 0.017, 0.019)/hr, β= (0.001, 0.002, 0.003)/hr, μ=0.007/hr, λ= 0.001/hr., ϒ=0.01/hr.

Table 5: Profit V/S Revenue for Varied Inspection Rate.

C0	Profit (P1-P2)		
	α=0.001/hr	α=0.002/hr	α=0.003/hr
15000	2766.27032	3840.568884	4303.4011
20000	3452.4752	4790.816316	5357.444178
25000	4138.68008	5741.063747	6411.487255
30000	4824.88496	6691.311179	7465.530332
35000	5511.08984	7641.55861	8519.57341
40000	6197.29472	8591.806042	9573.616487

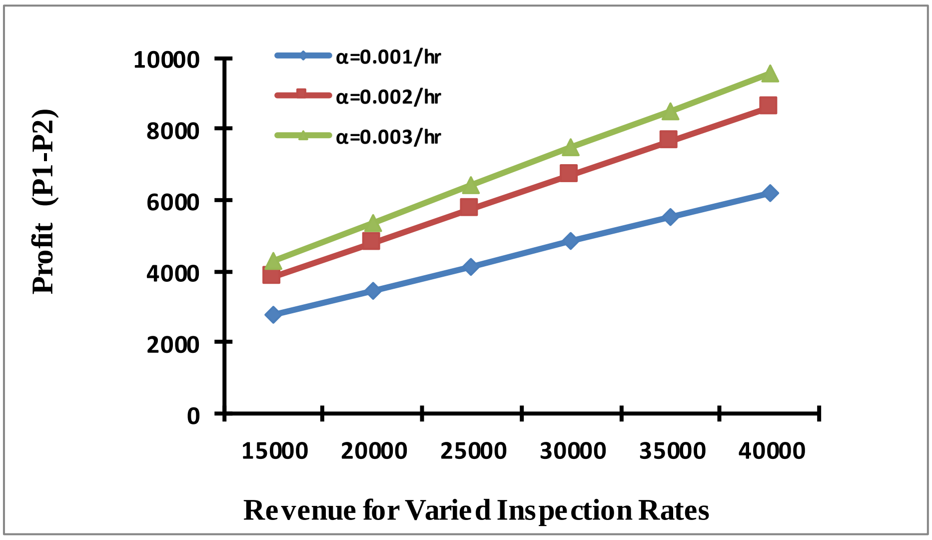

Fig. 6. Profit V/S Revenue for Varied Inspection Rate

Figure 6 shows how revenue and inspection rate changes affect the system's profitability. Furthermore, if the inspection rate increases over time, the profit that the system generates will increase as well. As a result, the use of inspection methods for such systems has the potential to make those systems more accessible and profitable.

Table 6: Profit V/S Revenue for Varied Inspection Cost.

C0	Profit (P1-P2)		
	C1= INR 200	C1=INR 1200	C1= INR 2200
18000	-215.39108	-367.886877	-520.382671
24000	-84.701187	-237.196981	-389.692774
30000	45.9887098	-106.507084	-259.002877
36000	176.678607	24.18281305	-128.312981
42000	307.368503	154.8727099	2.376916312
48000	438.0584	285.5626067	133.0668132

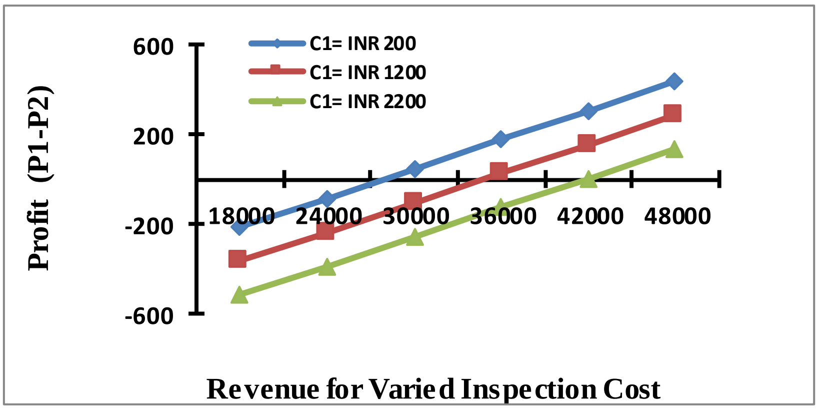

Fig. 7. Profit V/S Revenue for Varied Inspection Cost.

Figure 7 reveals that if the expected inspection rate increases, the profit gets diminished. It means inspection increases safety but reduces profit. Cut-off points help to decide the inspection cost to gain profit from the revenue generated by the company It results in the company being more profitable if fewer inspections occur.

Table 7: Profit V/S Revenue for Varied Repair Cost.

C0	Profit (P1-P2)		
	C2= INR 600	C2=INR 1000	C2= INR 1400
18000	-207.87463	-254.05589	-300.237148
24000	-77.184734	-123.365993	-169.547252
30000	53.5051628	7.32390407	-38.8573547
36000	184.19506	138.0138009	91.83254216
42000	314.884956	268.7036977	222.522439
48000	445.574853	399.3935946	353.2123358

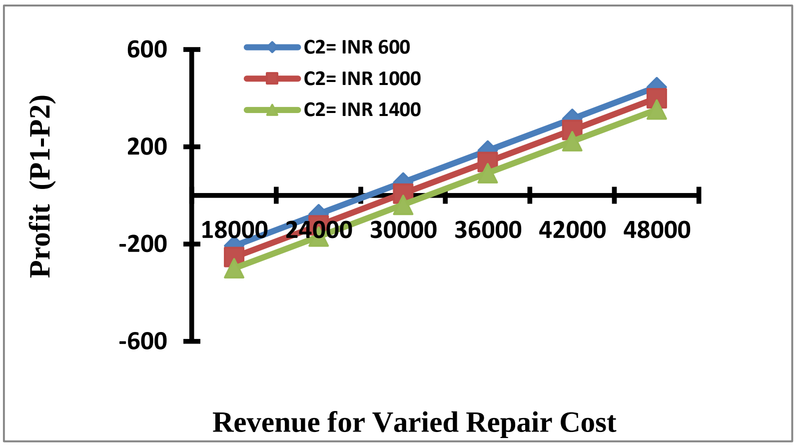

Fig. 8. Profit V/S Revenue for Varied Inspection Cost.

Although revenue is the income generated before costs, profit is the income obtained after paying all expenses. Figure 8 shows the plant/company's net profit can increase if the cost/revenue is optimized. The findings indicate that a decline in profit occurs if the expected price of repairs is increased. The meaning of this is that repairs enhance safety but diminish profits. It is possible to estimate the cost of repairs using cut-off points, which allows the company to make a profit from the revenue

V. Conclusions

The comparison of graphs demonstrates how different parameters have an effect on MTSF, availability, and profit on both models at the same time. These findings demonstrate that, over time, system availability decreases as the failure times of both units decrease. However, with constant failure rate values, system availability improves as corrective activity duration decreases. The repair rate improves availability more than the inspection rate at increasing operating unit failure rates. This suggests that if the unit fails frequently, repair helps more than inspection to maintain the system operational.

Also, graphs enlighten the impact of different parameters on both models' profits. larger values of inspection, repair, and replacement rates still make the system more economical, but larger failure rates put the system in a different scenario. For various parameter values, the cut-off points for revenue per unit up time for the system models. In each of these scenarios, the graph reveals that the system model generates a profit.

Therefore, this article suggests that reliability experts focus on the time it takes to replace the failed unit and analyze the working unit following a failure to limit system downtime. It is critical to evaluate failing units immediately and replace them if needed for the cold standby system to be profitable. It can be deduced from this; that the first model's performance should be carefully monitored to keep it at a high level. Figures (2-8) demonstrate the results of comparing the two models concerning the MTSF, availability, and profit. These comparison graphs show that the difference in performance measures goes on increasing costs and rates of remedial actions. But the availability and profitability of model 1 is greater than model 2, Hence model 1 is better according to our costs and rates of the company.

Conflict of Interest

There was no relevant conflict of interest regarding this paper.

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