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AN EFFICIENT APPROACH TO SOLVE TWO-STAGE FUZZY TRANSPORTATION PROBLEM

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Abstract

Transportation problems are one of the most important classes of linear programming problems. This manages a product's transportation from its point of origin to its final destination. The primary objective is to meet destination requirements while minimizing shipping expenses. This work presents a two-stage fuzzy transportation cost-related problem and uses a parametric approach to derive a fuzzy solution. A novel method is suggested to address a two-phase fuzzy transportation issue where the transport cost is expressed in terms of fuzzy trapezoidal figures. This approach is particularly effective because it is easy to comprehend. By supporting decision-makers during the process and offering a simple and cost-effective solution, the suggested strategy assists decision-makers with logistics-related problems.

Keywords: trapezoidal uncertain number; two-stage uncertain transportation problem; optimal transportation cost solution

I. Introduction

Mathematical optimization is a relatively recent subject of study. Because it can handle problems in the real world, it has advanced over the last few decades. Finding and evaluating every solution to choose the best one is the aim of optimization. The foundation of the transportation problem was first identified by Hitchcock [V], who presented his findings about the delivery of goods from several sources to multiple locations. The first significant contribution to the resolution of transportation-related issues was this study. Koopman [XXVII] conducted an independent study on the optimal use of transportation systems. The purpose of this study is to apply the theory of optimal allocation of resources.

The method developed by Tuncay Can was presented by Can and Koçak [XXVI] as a substitute for the methods found in the literature. Hossain et. al. [XIV] introduced an effective algorithm for finding a better initial basic feasible solution to a balanced transportation problem. Hosseini [IV] proposed an efficient algorithm in three cases with its Matlab code, which is forcefully efficient for problems of large sizes. A new adjustment to the VAM was introduced by Hussein and Shiker [VIII] to find an IBFS for transportation problems that was almost optimal. To minimize the cost of transporting a specific amount of products from sources to destinations, numerous authors [XII, XV, XX, XXII, and XXIII] developed new and efficient methods for the solution of the initial basic feasible, which are used in the cases of balanced and unbalanced transportation problems. To determine that the best possible solution for a transportation problem is the most important prerequisite for transportation difficulties, Sathyavathy and Shalini [XVIII] analyzed four distinct proposed techniques (PAM, PHM, PGM, and PQM). An updated approach for the Vogel Approximation Method was given by Singla et al. [XI]; it provides a feasible approach that is always closest to the optimal answer. Choudhary [III] introduced an optimal revised distribution method that is simple to apply to both balanced and unbalanced transportation problems with a maximize or minimize objective function. An algorithm to identify a full tour at the lowest possible cost that does not go over the total cost and duration of the trip has been explained by Mondal and Srivastava [XVI and XVII]. An algorithm known as the zero suffix method was suggested by Gupta et al. [VI] to provide the optimal transportation cost solution for transportation problems without first determining the basic feasible solution. Singh and Singh [VII], the authors, proposed a modified particle transport problem involving three-level time reduction that can be solved using swarm optimization. In these challenges, the path from the origin to the terminal points is separated into three levels to meet the needs of the transport companies and reduce the overall time as much as possible. The suggested method was discovered to be a better substitute for current methods in terms of making logistical choices.

Fuzzy sets and fuzzy logic are the most important mathematical tools for modeling and regulating uncertain systems in industry, human anatomy, and the natural sciences. Due to parameter fluctuations, real-world problems are intrinsically imprecise. The characteristic function can be extended in a way that indicates the membership level of the set for each element in the universal set and allows its values to lie within a specified range. The idea for the fuzzy system was first proposed by Zadeh [XIII] during his seminar work on fuzzy sets in 1965. Fuzzy set theory, which asserts that an object either belongs to a set or does not, can be distinguished from classical set theory. Fuzzy sets are those that permit different membership levels in the interval (0, 1). The decision-maker in the traditional transportation model is presumed to be cognizant of the exact figures for product demand, product capacity, and transportation costs. However, because of unpredictable factors, it might not be possible to correctly collect all parameters in real applications. For the purpose of traffic engineering, fuzzy logic systems were developed. Fuzzy transportation problems are defined as those in which the parameters cost, supply, and demand have fuzzy numbers. Numerous researchers have presented their approaches to solving fuzzy situations in the literature. Fuzzy sets were used by Noora and Karami [I] to represent imprecise data in the development of DEA models. The author presents a method for solving the fuzzy non-racial model using a ranking function.

Real-world issues are typically depicted with numerous goals that simultaneously conflict and have varying measuring scales. Sometimes the destination cannot receive more than the minimum amount because of storage constraints. They are prepared for the second step, which entails obtaining the excess quantity after having consumed a portion of this initial cargo. As a result, there are two processes involved in moving the items from their origin to their destination. The first thing that is transferred from the origin to the destination is the minimal demand for the destination. They are prepared to receive the remaining quantity in the second stage after using up a certain amount of the first shipment. This type of transportation problem is called a two-stage transportation problem. The main objective of the two-stage transportation problem is to carry goods from the point of origin to the destination in two stages while reducing the total cost of transportation for both stages. During these two stages, the flow of items from the source to the destination occurs simultaneously. Sonia and Rita Malhotra [XXV] presented a polynomial bound technique for the two-stage time minimization problem, which aims to minimize the total of the linked transportation time by determining the optimal scheduling of the first and second phases. A novel approach, the zero-point method, was proposed by Pandian and Natarajan [XXI] to solve the transportation problem optimally without the need for optimality test procedures or even simple plausible solutions. A two-stage fixed-charge transportation issue with a heuristic evolutionary algorithm that utilizes properties to direct the algorithm toward better solutions was developed by Calvete et al. [IX]. By converting stocks and demand into trapezoidal fuzzy numbers using parametric approaches, Ghani and Razak [II] optimized the two-stage fuzzy transportation expenses. Ritha and Vinotha [XXX] proposed a method of using geometric planning to find the best compromise solution for the more objective two-stage fuzzy transportation problem. Sudhakar and Navaneetha [XXVIII] proposed a feasible zero suffix method to solve a multi-objective, two-stage fuzzy transportation problem. Hashmi et. al. [XIX] proposed a multi-objective model for a two-stage fixed-charge transportation planning problem. An alternative technique was given by Singh et al. [XXIV] to determine the initial basic feasible solution for unpredictable transportation costs. The first answer is found by defining the grade value at zero expenses. An initial basic feasible approach that is either equivalent to or closest to an optimal solution with less computational labor was given by Singla et al. [X] for transportation cost problems in an uncertain environment. Using a range ranking function that included the current zero point method, zero suffixes method, and increased zero suffixes method, Jaiswal et al. [XXIX] solved a heptagonal number and found that the current zero-point method produces a superior optimal result.

II. Preliminaries

II.i Fuzzy number

The fuzzy set $\mathcal{P}$ on the real line R needs to fulfill certain requirements, such as:

(i) $\mu_P(x)$ must be piecewise continuous.

(ii) At least one a∈R exists where $\mu_P(x) = 1$.

(iii) $\mathcal{P}$ should be convex and normal.

II.ii Trapezoidal uncertain number

$\mathcal{P}$ = (p, q, r, and s), an uncertain set defined on real numbers R is called a trapezoidal uncertain number if its membership function has the following characteristics:

(i) $\mu_P(x)$: X→ {0,1} is continuous.

(ii) $\mu_P(x) = 0\ \forall\ x \in (-\infty, r] \cup [s, \infty)$

(iii) It is strictly increasing on [p, q] and strictly decreasing on [r, s].

(iv) $\mu_P(x) = 1\ \ x \in [p, q]$

II.iii Membership function of trapezoidal fuzzy number

Trapezoidal fuzzy numbers like $\mathcal{P} = (p, q, r, s)$ have a membership function $\mu_{\mathcal{P}}(x)$ that can be represented as follows:

$$\mu_{\mathcal{P}}(x) = \begin{cases} \frac{x-p}{q-p}; & p \leq x \leq q, \\ 1; & q \leq x \leq r \\ \frac{x-r}{s-r}; & r \leq x \leq s \\ 0 & s \geq 0 \end{cases}$$

II.iv. Defuzzification of trapezoidal fuzzy numbers

Ranking mapping is a helpful tool for comparing fuzzy numbers. With R being a set of real numbers and F(R) being the set of all uncertain numbers defined on a set of real numbers, every uncertain number is mapped into a real number by a ranking mapping $\Re : F(R) \longrightarrow R$. Let $\mathcal{P}$ and Q be two fuzzy numbers, then

(i) If $R(\mathcal{P}) \geq R(Q)$ $then$ $\mathcal{P} \geq Q$

(ii) If $R(\mathcal{P}) > R(Q)$ $then$ $\mathcal{P} > Q$

(iii) If $R(\mathcal{P}) = R(Q)$ $then$ $\mathcal{P} = Q$

Let $\mathcal{P} = (p, q, r, s)$ be the trapezoidal fuzzy number, then the rank of $\mathcal{P}$ is

$$R(\mathcal{P}) = \frac{p+q+r+s}{4}.$$

II.v. Algebraic operations on trapezoidal fuzzy numbers

Here, arithmetic operations such as addition, subtraction, etc. between two trapezoidal fuzzy numbers, defined on a set of real numbers *R*, are given.

Let $\mathcal{P}$ = (p_1, q_1, r_1, s_1) and $\mathcal{Q}$ = (p_2, q_2, r_2, s_2) are trapezoidal fuzzy numbers, then

(i) $\mathcal{P} + \mathcal{Q}$ = ($p_1 + p_2$, $q_1 + q_2$, $r_1 + r_2$, $s_1 + s_2$)

(ii) $\mathcal{P} - \mathcal{Q}$ = ($p_1 - s_2$, $q_1 - r_2$, $r_1 - q_2$, $s_1 - p_2$)

(iii) $\alpha * \mathcal{P} = \begin{cases} \alpha p1,\ \alpha q1,\ \alpha r1,\ \alpha s1 & \alpha > 0 \\ \alpha s1,\ \alpha r1,\ \alpha q1,\ \alpha p1 & \alpha < 0 \end{cases}$

III. Two-Stage Fuzzy Transportation Problem

The linear programming formulation and tabular representation of a two-stage fuzzy transportation problem in which transport costs are uncertain are given under:

Table 1: Tabular representation of two-stage fuzzy transportation problem

	D_1	D_2	…	D_n	Supply (s_i)
S_1	$\tilde{c}_{11}$	$\tilde{c}_{12}$	…	$\tilde{c}_{1n}$	a_1
S_2	$\tilde{c}_{21}$	$\tilde{c}_{22}$	…	$\tilde{c}_{2n}$	a_2
⋮	⋮	⋮	⋮	⋮	⋮
S_m	$\tilde{c}_{m1}$	$\tilde{c}_{m2}$	…	$\tilde{c}_{mn}$	a_m
Demand (d_j)	b_1	b_2	…	b_n	

Linear programming formulation of the two-stage fuzzy transportation problem is,

Minimize $Z = Z_1 + Z_2$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij}x_{ij} \le Z_1$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij}y_{ij} \le Z_2$$

$$\sum_{j=1}^{n} x_{ij} \le s_i; \quad \forall i = 1,2,\dots,m \text{ v} \qquad (1)$$

$$\sum_{i=1}^{m} x_{ij} = k_j; \quad \forall j = 1,2,\dots,n \qquad (2)$$

$$\sum_{j=1}^{n} y_{ij} = s_i - \sum_{j=1}^{n} x_{ij}; \quad \forall\ i = 1,2,\dots,m \qquad (3)$$

$$\sum_{i=1}^{m} y_{ij} = d_j - k_j; \quad \forall\ j = 1,2,\dots,n \qquad (4)$$

x_{ij} and y_{ij} are greater than or equal to zero 0, ∀ *i* and *j* where

i is the source index, *j* is the destination index, m is the number of sources, n is the number of destinations, The variables d_j, x_{ij}, y_{ij}, k_j, and c_{ij} represent the number of goods required at destination *j*, the number of goods to be shipped from source *i* to destination j in stage I, the number of goods to be shipped from source *i* to destination *j* in stage II, the maximum storage capacity of destination *j*, and the fuzzy transportation cost of one unit of goods from source *i* to destination *j*, respectively.

IV. Proposed Approach

Step 1: Consider two-stage fuzzy transportation problems whose transportation cost is taken as a fuzzy number.

Step 2(a): Check $s_i < 0$ and $d_j < 0$, then stop.

(b) If $\sum s_i > \sum d_j$ or $\sum s_i < \sum d_j$, balance the transportation problem by introducing a dummy column or row. Take half of the availability and demand in Stage I and remain in Stage II in such a way that the transportation problem obtained should be balanced.

Step 3(a): Determine the penalties by subtracting the minimum and next-to-minimum fuzzy costs by using the ranking function and algebraic operations for each row and each column, i.e.

$p_i = | c_{ih} - c_{ik} |$ and $p_j = | c_{ri} - c_{sj} |$, where p_i and p_j are the penalties of i^{th} row and j^{th} column and $c_{ij} = \Re(\tilde{c}_{ij})$.

(b) The penalty for the associated row or column is zero if more than one cell in that row or column has the minimal cost.

Step 4: Select a row and column, say the r^{th} row and c^{th} column, $p_r = \max\{p_i\}$ and $p_c = \max\{p_j\}$ and allocate the maximum possible quantity $x_{rc} = \min\{a_r, b_c\}$ to this cell. Choose the row or column with the lowest cost cell (u, v) if multiple rows or columns have the highest penalty.

Step 4.1: If more than one minimum cost cell exists in the r^{th} row or c^{th} column, then select the cell (*l*, *t*) in which we can allocate the maximum quantity, i.e. $x_{lt} = \max\{x_{uv}\}$.

Step 5: Update the availability at the sources and demand at the destinations by subtracting the allocated quantity. If a column or row is fully satisfied, remove it. In a situation where both the row and the column are satisfied simultaneously, remove one of them, and assign zero (write ε) supply or zero (write ε) demand to the remaining one.

Step 6: Repeat the steps from Step 3 to Step 5 until all the rows and columns are satisfied.

Step 7: Repeat Step 3 to Step 5 for Stage II by taking the remaining availability and demand.

Step 8: The total minimum fuzzy cost is $\tilde{Z}_1 + \tilde{Z}_2$, where $\tilde{Z}_1$ is the minimum fuzzy cost obtained at stage I and $\tilde{Z}_2$ is the minimum fuzzy cost obtained at stage II.

V. Illustrative Examples

Example 1: Consider the following fuzzy transportation problem, in which the cost of transporting one unit of goods is taken as a trapezoidal fuzzy number.

Table 2: Fuzzy transportation problem

	W_1	W_2	W_3	W_4	W_5	W_6	Supply
F_1	(2,3,2,1)	(4,1,5,2)	(4,6,7,3)	(10,9,12,13)	(6,3,5,2)	(4,0,1,3)	6
F_2	(4,6,1,5)	(8,7,9,4)	(9,8,11,8)	(7,4,5,4)	(12,9,8,11)	(7,5,0,4)	8
F_3	(11,14,13,10)	(35,30,15,20)	(12,9,8,7)	(8,5,7,4)	(20,35,25,24)	(12,15,10,11)	7
F_4	(6,7,9,10)	(7,6,8,7)	(10,12,8,6)	(20,25,30,21)	(12,11,7,10)	(9,10,6,7)	7
Demand	3	5	5	6	4	5	

For Stage I,

we take $a_1 = \frac{6}{2} = 3$, $a_2 = \frac{8}{2} = 4$, $a_3 = \frac{7}{2} = 3.5 = 3$, $a_4 = \frac{7}{2} = 3.5 = 3$

$b_1 = \frac{3}{2} = 1.5 = 1$, $b_2 = \frac{5}{2} = 2.5 = 2$, $b_3 = \frac{5}{2} = 2.5 = 3$, $b_4 = \frac{6}{2} = 3$, $b_5 = \frac{4}{2} = 2$, $b_6 = \frac{5}{2} = 2.5 = 2$

Table 3: Stage I of the fuzzy transportation problem

	W_1	W_2	W_3	W_4	W_5	W_6	Supply
F_1	(2,3,2,1)	(4,1,5,2)	(4,6,7,3)	(10,9,12,13)	(6,3,5,2)	(4,0,1,3)	3
F_2	(4,6,1,5)	(8,7,9,4)	(9,8,11,8)	(7,4,5,4)	(12,9,8,11)	(7,5,0,4)	4
F_3	(11,14,13,10)	(35,30,15,20)	(12,9,8,7)	(8,5,7,4)	(20,35,25,24)	(12,15,10,11)	3
F_4	(6,7,9,10)	(7,6,8,7)	(10,12,8,6)	(20,25,30,21)	(12,11,7,10)	(9,10,6,7)	3
Demand	1	2	3	3	2	2	13

By using Step 3 – Step 5 of proposed approach,

Table 4: After first allocation

	W_1	W_2	W_3	W_4	W_5	W_6	Supply	P1
F_1	(2,3,2,1)	(4,1,5,2)	(4,6,7,3)	(10,9,12,13)	(6,3,5,2)**2**	(4,0,1,3)	~~3~~1	0
F_2	(4,6,1,5)	(8,7,9,4)	(9,8,11,8)	(7,4,5,4)	(12,9,8,11)	(7,5,0,4)	4	0
F_3	(11,14,13 ,10)	(35,30,15,20)	(12,9,8,7)	(8,5,7,4)	(20,35,25,24)	(12,15,10,11)	3	3
F_4	(6,7,9,10)	(7,6,8,7)	(10,12,8,6)	(20,25,30,21)	(12,11,7,10)	(9,10,6,7)	3	1
Demand	1	2	3	3	~~2~~	2	13	
P1	2	4	4	1	6	2		

Hence, we may get all the allocations for stage I as shown in Table 4.

Table 5 – After all allocations in Stage I

	W_1	W_2	W_3	W_4	W_5	W_6	Supply
F_1	(2,3,2,1)	(4,1,5,2) **1**	(4,6,7,3)	(10,9,12,13)	(6,3,5,2) **2**	(4,0,1,3)	1
F_2	(4,6,1,5) **1**	(8,7,9,4)	(9,8,11,8) **1**	(7,4,5,4)	(12,9,8,11)	(7,5,0,4) **2**	4
F_3	(11,14,13, 10)	(35,30,15, 20)	(12,9,8,7)	(8,5,7,4) **3**	(20,35,25, 24)	(12,15,10, 11)	3
F_4	(6,7,9,10)	(7,6,8,7) **1**	(10,12,8,6) **2**	(20,25,30, 21)	(12,11,7,10)	(9,10,6,7)	3
Demand	1	2	3	3	2	2	15

The obtained solution is$x_{12} = 1, x_{15} = 2, x_{21} = 1, x_{23} = 1, x_{26} = 2,\ x_{33} = 0, x_{34} = 3, x_{42} = 1, x_{43} = 2$. The objective value is$\tilde{Z}_1 = (94, 76,72,58)$.

For Stage II

we take $a_1 = 3$, $a_2 = 4$, $a_3 = 4$, $a_4 = 4$, $b_1 = 2$, $b_2 = 3$, $b_3 = 2$, $b_4 = 3$, $b_5 = 2$, $b_6 = 3$

After applying Step 3 to Step 5 of the proposed approach, we get the solution as shown in Table 5.

Table 6: After all allocation in Stage II

	W_1	W_2	W_3	W_4	W_5	W_6	Supply
F_1	(2,3,2,1)	(4,1,5,2) **1**	(4,6,7,3)	(10,9,12,	(6,3,5,2) **2**	(4,0,1,3)	3
F_2	(4,6,1,5) **1**	(8,7,9,4)	(9,8,11,8)	(7,4,5,4)	(12,9,8,1 1)	(7,5,0,4) **3**	4
F_3	(11,14,1 3,10)	(35,30,1 5,20)	(12,9,8,7) **1**	(8,5,7,4) **3**	(20,35,2 5,24)	(12,15,1 0,11)	4
F_4	(6,7,9,10) **1**	(7,6,8,7) **2**	(10,12,8, 6) **1**	(20,25,30 ,21)	(12,11,7, 10)	(9,10,6,7)	4
Dem and	2	3	2	3	2	3	

The obtained solution is $x_{12} = 1, x_{15} = 2, x_{21} = 2, x_{26} = 3, x_{33} = 1, x_{34} = 3, x_{41} = 1, x_{42} = 2, x_{43} = 1$. The objective value is $Z_1 = (107,83,78,72)$.

Therefore, the optimal cost of two stage fuzzy transportation problem is $\tilde{Z} = \tilde{Z}_1 + \tilde{Z}_2$.

i.e. $\tilde{Z} = (94,76,72,58) \oplus (107,83,78,72) = (201,159,150,130)$.

Example 2: Consider an unbalanced fuzzy transportation problem as shown in Table 6.

Table 7: Unbalanced fuzzy transportation problem

	W_1	W_2	W_3	Supply
F_1	(2,3,4,3)	(1,1,1,1)	(4,5,4,3)	300
F_2	(3,2,1,2)	(6,7,3,8)	(8,9,9,10)	500
F_3	(8,9,7,8)	(5,2,3,2)	(3,2,1,2)	400
Demand	250	350	200	

The minimum fuzzy transportation cost is $\tilde{Z} = (2350,1600,1650,1600)$.

VI. Result and Discussion

We solved the two transportation problems (balanced and unbalanced) to illustrate the proposed approach. Also, we solved the same examples with some existing methods. As the solution obtained is a trapezoidal fuzzy number, to compare the results, we are using a ranking function. Table 8 provides a comparison of the acquired results.

Table 8: Result Comparison

Transportation Problem	Balanced/Unbalanced	Gani & Razak [II]	Ritha & Merline [XXX]	Proposed Approach
Example 1	**Balanced**	168	166	160
Example 2	**Unbalanced**	2900	2500	1800

VII. Conclusion

In this paper, a new approach is proposed to solve the problem of transportation when sources or destinations have limited capacity. Two examples are illustrated for the proposed approach, and we also solved these examples with the existing methods. The result comparison highlights the effectiveness and benefits of the suggested approach.

Conflict of Interest

There was no relevant conflict of interest regarding this paper.

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