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SYNCHRONIZATION: IDENTICAL AND NON-IDENTICAL INVESTIGATION OF RUCKLIDGE SYSTEM

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Abstract

This article explores the impact of synchronization, both identical and non-identical supporting systems with six different Co-efficient Matrices, on the Rucklidge chaotic system. Two paired chaotic systems are proposed to synchronize using the Active Control Algorithm (ACA). Six sets of different control functions originating from identical and non-identical Master/Drive systems. All synchronizing design demonstrates that six sets of different control functions are always perfectly applied and chaotic systems are significantly synchronized with six different co-efficient Matrices. Parameters are similar across identical pairings of chaotic systems however must be different for non-identical pairs. The feasibility and efficacy of synchronizing the state variables are derived from the error dynamics coefficient matrix. We analyze the effectiveness of synchronized identical and non-identical approaches to explore which control functions would provide better results. The non-identical pair is formed utilizing the Harb-Zohdy chaotic system with a unique initial value. In addition, numerical simulations are offered to validate and expand upon the theoretical findings.

Keywords: Rucklidge system, Synchronization, Identical pair, Non-identical pair, Co-efficient Matrices, Active Control Algorithm

I. Introduction

Synchronized nonlinear systems are significantly adjusted in a wide range of real-world dynamical problems. Ever since the initial explanation of coupled synchronization techniques by Pecora L. M. and Carroll T. L. in the year 1990 [XI], either identical or non-identical synchronization has attracted attention and interest for its useful specialty and application in science, engineering, economics, neural networks [V], chemical systems [XVI], biology [XVII], weather models [XVIII], circuits [XIX], secure communications, laser physics, ecological systems, finance[XX]. Some concepts of chaotic system synchronization are described as the selection of synchronization modality, whether is it identical or non-identical synchronization is chosen depending on how the control laws reduce error at an astonishing rate. Active control of structures increasingly derives from the enormous field of knowledge that constitutes "control," including its theories, procedures, and technologies. The usual way to control a system is to describe it mathematically in a sufficient way to be able to control its most significant parameters.

By establishing the suggested error dynamics co-efficient matrix can be able to create an active controller that facilitates synchronization between the state variables of two pairs of identical and non-identical chaotic systems [XII]. The stability of the synchronization of the non-linear identical and non-identical Rucklidge system is proven in this article. Systems should be designed to be as similar as practicable in real-world applications. Additionally, in actual devices, parameter fluctuations might lead to desynchronization. The identical nature of the systems under consideration is a frequent characteristic of chaotic synchronization. Parameters cannot be constructed with complete consistency in real-world applications. For this reason, it seems more practical to study models with non-identical parameters to synchronize real-world systems. Since the natural system has various parameters, non-identical couples of both chaotic systems are more beneficial. In this article, the abilities of active control for breaking the limit of "controlling two identical systems" and apply these techniques to synchronize two non-identical systems, on the Rucklidge chaotic system.

Kocamaz et al. [XXIII] studied continuous time nonlinear Rucklidge systems to manage chaos with a single controller and simplify construction. Karthikeyan, R. and Sundarapandian, V. [XV] synchronized hybrid chaos synchronization of identical and non-identical W. 4-scroll systems but they don't compare identical and non-identical synchronization to figure out which is better. To accomplish global synchronization of identical double and single-well Duffing-Van der Pol (DVP) oscillators, and non-identical DVP oscillators, Njah, A. N., and U. E.Vincent. [I] employed the active control approach. El-Dessoky et al. [XXV] explored the S-M chaotic system with asynchronous control by feedback and proved the presence of its local Hopf bifurcation. Nishad [VI] demonstrated how phase portrait inspection of fractional derivatives for several satellite systems is generated and displayed with varied parameters. Wang et al. [XXI] examined time-frame prolonged fractional-order financial systems with chaotic properties and the suggested active control strategy synchronizes the fractional time-frame prolonged and time-varying order financial systems. Liu et al. [XIII] presented and examined a 3D autonomous chaotic system that may create one- and two-scroll chaotic attractors with adequate parameters. Tang, Jianeng [X] considered Chaos synchronization of time-frame prolonged fractional-order chaotic systems, and their fraction order and time delays are studied. Kocamaz et al. [VIII] explore three distinct methods of controlling the chaotic system of continuous time Shimizu-Morioka when the characteristics of the system are unknown. Pehlivan et al. [IX] studied Chaotic synchronization and masking communication circuits that may use the Rucklidge scheme. Wei, Z. and Zhang, X. [XXVII] scientifically studies the fractional-order Shimizu-Morioka model. Vaidyanathan, S. [XXII] explore controlling Rucklidge chaotic systems globally using sliding mode in order to achieve double convection. Different control signals for the Rucklidge oscillator were developed by Marwan, M. et al. [XIV] employing sliding, adaptive, and backstepping control strategies. To determine whether the nonlinear dynamical system is stable for the given controller, the Lyapunov theory is used. With the justification in aforesaid consideration, we realized the profitable application of Rucklidge chaotic system all over the globe.

This research study explores into:

- This article utilizes ACA controllers and describes what kind of supporting system should be used in couple synchronization for the original systems to achieve global asymptotic synchronization.
- Our main objective is to figure out the format of considering supporting systems and co-efficient matrices for reducing error from couple synchronization, whether it be in an identical or non-identical process.
- More than one control function is possible for one chaotic system when co-efficient Matrices are chosen in various categories.
- To achieve this, bidirectional synchronization is required in the entire system to determine the most advantageous direction for the field of synchronization.
- An appropriate control function can be derived from any system's synchronization procedures, whether identical or non-identical. It will be effective in a chaotic natural system for adjusting the supporting systems.
- Comparing identical and non-identical six control functions, one can readily observe which controller is more effective than the other controller. So all control functions are important for the researcher.

The sequence that follows is the structure of this paper: In section **2**, we set our model, and present dynamical analysis deals with Rucklidge and Harb-Zhody chaotic systems with different parameters. The theory and design of the Active Control Algorithm(ACA) are discussed in Section **3** through the Master-Drive (MS-DS) procedure. Section **4** explains the synchronization procedure of the Rucklidge system, which is characterized by its identical and non-identical nature with some graphical remarks given. Section **5** explores the outcomes of numerical simulations, specifically focusing on the comparison of indicated results. Ultimately, within the confines of section **6**, we are to elaborate on several sets of concluding remarks.

II. Dynamical analysis of chaotic system

This article describes a dynamic description of the Rucklidge and the Harb-Zohdy dynamic systems. The chaotic attractor of the Rucklidge and the Harb-Zohdy dynamic systems on the $(x-y-z)$ plane, which is shown in the 3-D graphical portrayal for the Master/Drive systems.

II.i. The Rucklidge dynamical systems with chaotic behavior

In the present procedure, we clarify the chaotic nature with respect to the time of the

Rucklidge system (1) which is defined in ([II]). At its point of equilibrium, the Rucklidge system consists of: $E_1 = (0,0,0)$, $E_{2,3} = \left(0, \sqrt{a}, a\right)$

$$\begin{aligned}\dot{x} &= -b_1 x - a_1 y - yx \\ \dot{y} &= x \\ \dot{z} &= -z + y^2\end{aligned} \qquad (1)$$

where we define x, y, z as variables and a_1, b_1 as parameters that represent the state of variables and the positive constant. If we set the parameters at $a_1 = 6.7$, $b_1 = 2$, and initial conditions $X(0) = (5,8,4,); \forall X = (x, y, z)$ for the system (1), then chaotic behavior is shown in the phase portrait. For identical synchronization, we use Rucklidge chaotic systems as both Master/Drive systems whereas in non-identical systems that of as the Master system. The Rucklidge system's (1) 3-dimensional projections on the $(x-y-z)$ plane space are illustrated in Figure 1.

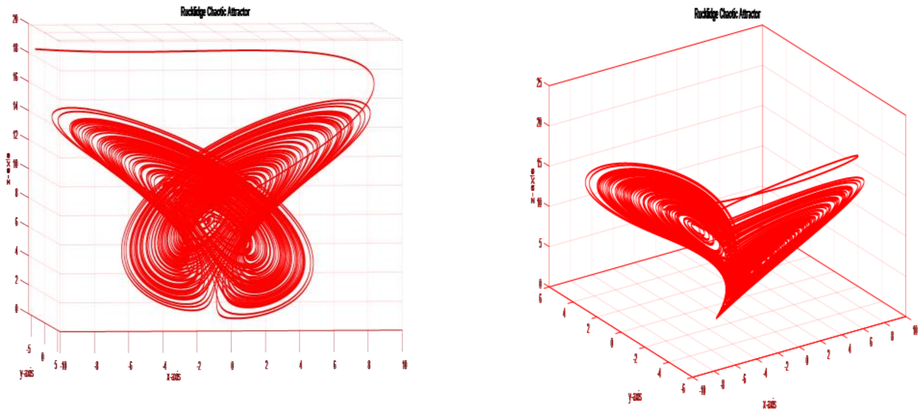

Fig. 1. The Rucklidge chaotic attractor demonstrates 3-D phase portrait.

II.ii. The Harb-Zohdy(HZ) chaotic system

The non-identical pair is formed utilizing the Harb-Zohdy chaotic system. So we clarify the chaotic nature with respect to the time of the Harb-Zohdy system (2) which is defined in ([XXIV]):

$$\begin{aligned}\dot{x}_2 &= -z_2 \\ \dot{y}_2 &= x_2 - y_2 \\ \dot{z}_2 &= -px_2 + {y_2}^2 + qz_2\end{aligned} \qquad (2)$$

where we define x, y, z as variables and p, q as parameters that represent the state of variables and the positive constant. If we set the parameters at $p = 3.1$, $q = 0.5$, this study demonstrates that the system (2) is chaotic. Numerical simulations are to be performed using the following initial conditions for the system (2): $X(0) = (0.05, 0.01, 0.06,); \forall X = (x, y, z)$. Figure (2) demonstrate the Harb-Zohdy system's (3) 3-D projections on the $(x-y-z)$ space projections, respectively.

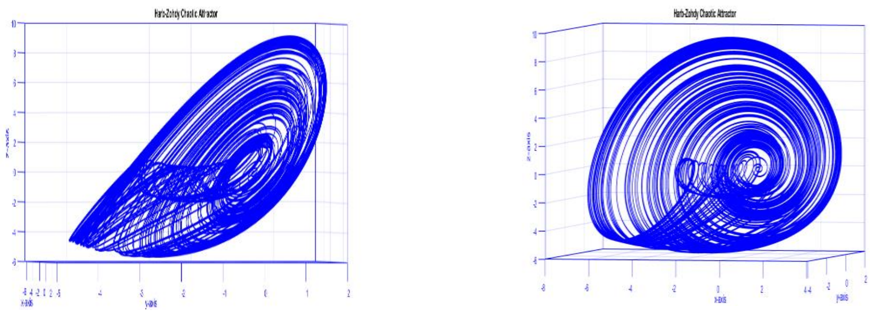

Fig. 2. The Harb-Zohdy chaotic attractor demonstrates a 3-D phase portrait.

The findings of these dynamic investigations offer an apparent representation for providing the pattern of divergence or convergence of infinitesimally proximate trajectories, in addition to the assessment of the initial parameters of the chaotic system as proposed by Rucklidge and Harb-Zohdy.

III. The Formulation of ACA Procedure

ACA for synchronization performs effectively when all parameters in an MS-DS system are identical and non-identical. Consider the (MS-DS) system already exists as follows:

$$\begin{aligned}\dot{x} &= px + h(x)\\ \dot{y} &= qy - h(y) + u(t)\end{aligned} \tag{3}$$

where both systems' respective state vectors are denoted by $x = \left(X_1; \forall X = (x, y, z)\right)^T \in R^n$; $y = \left(Y_2; \forall Y = (x, y, z)\right)^T \in R^n, (p, q) \in R^n \times R^n$ matrices that remain constant throughout the given context. These matrices are specifically designated as coefficient matrices. The functions $h(x)$ and $h(y)$ can be characterized as sequentially nonlinear functions. Additionally, the control function $u(t)$, represented as $u(t) = \left(u_i(t); \forall i = (1, ..., n)\right)^T$ depends on state factors and necessitates computation.

Corollary: Synchronization Conditions for Stabilized Systems (SCSS)

Creating a feedback controller u is essential to the design process. Equation (3) shows that the two interconnected chaotic systems exhibit global asymptotic synchronization, as signified by the measure of $\lim_{t \to \infty} \|Y(t) - X(t)\| = \lim_{t \to \infty} \|e_i\| = 0, \forall e_i(0) \in \left(R^n, X\right)$, if a suitable controller $u(t)$ satisfies the criterion. There are time sets in $0 \le t_0 \le t$ for the continuous system controlling, where t_0 is considerable time for the control effort activation.

Definition: Error Dynamics Formulation (EDF):

Based on the complex arrangement of the Master-Drive system, as represented by equation (3), it can be calculated that the error function, denoted by e_i, is to be computed by subtracting the value of x_i from y_i . The synchronization error is expressed in the following approach:

$$\begin{aligned}\dot{e} &= \dot{y} - \dot{x} \\ &= (q-p)e + \big(h(y) - h(x)\big) + u(t) \\ &= (q-p)e + F\big(x(t), y(t), e(t)\big) + u(t)\end{aligned} \tag{3}$$

Theorem: Control Function Formulation (CFF)

There is a working definition of a controller:

$$v(t) = -F(x(t), y(t), e(t)) + u(t)$$

The potential elimination of the non-linear component can be achieved through the utilization of a controller $u(t)$ in the absence of e in the system (5). As in

$$u(t) = v(t) - F(x(t), y(t), e(t)) \tag{4}$$

$$u(t) = (q-p)y(t) - (q-p)x(t) - F(x(t), y(t), e(t)) + v(t)$$

Where $q - p = S_1 \in R^{n\times n}$ is the matrices structure's linear controller and controls its effect. Sub-controller operator $v(t)$ provides this controller.

Given the scalar matrix $v(t) = -S_2 e(t)$, where $e(t)$ represents the error variables, we can derive the following equation by integrating calculations (3) and (4):

$$\dot{e} = S_1 e + v(t) \tag{5}$$

When $v(t)$ represents a scalar matrix containing error variables and $v(t) = -S_2 e(t)$ then equation (5) can be reformulated as follows:

$$\dot{e} = (S_1 + S_2)e = Se \tag{6}$$

The zero solution of the closed-loop system equation (6) is globally asymptotically stable if and only if all eigenvalues $S \in R^{n\times n}$ have negative real parts. The Gershgorin theorem [VII] and Linear control theory [IV] prove that any system matrix eigenvalue $S \in R^{n\times n}$ is negative if and only if the matrix is strictly diagonally dominant then the two chaotic systems in equation (3) are globally asymptotically full period synchronized.

IV. Synchronization Procedure for Rucklidge System

IV.i. Identical Synchronization of Rucklidge System:

The MS Rucklidge system's dynamical model:

$$\begin{aligned}\dot{x}_1 &= -b_1 x_1 + a_1 y_1 - y_1 z_1 \\ \dot{y}_1 &= x_1 \\ \dot{z}_1 &= -z_1 + {y_1}^2\end{aligned} \tag{7}$$

The DS Rucklidge system's dynamical model:

$$\begin{aligned}\dot{x}_2 &= -b_1 x_2 + a_1 y_2 - y_2 z_2 + u_1 \\ \dot{y}_2 &= x_2 + u_2 \\ \dot{z}_2 &= -z_2 + {y_2}^2 + u_3\end{aligned} \tag{8}$$

where $u_{i=(1,2,3)}$, a_1, b_1 are adequately defined and specified as control functions and parameters. We apply the definition (EDF) to estimate the error dynamics. The analysis of state errors between the DS (8) and the MS (7) requires the efficient use of

$$e_X = X_2 - X_1, \forall X = x, y.z \tag{9}$$

By applying the definitions (EDF) in Equation (9) the error dynamics derived as:

$$\begin{aligned}\dot{e}_x &= -b_1 e_x + a_1 e_y - y_2 z_2 + y_1 z_1 + u_1 \\ \dot{e}_y &= e_x + u_2 \\ \dot{e}_z &= -e_z + e_y(y_2 + y_1) + u_3\end{aligned} \tag{10}$$

The error dynamics equation can be obtained by utilizing the definition (CFF)

$$\begin{aligned}v_1 &= u_1 - y_2 z_2 + y_1 z_1 \\ v_2 &= u_2 \\ v_3 &= u_3\end{aligned} \tag{11}$$

The equation (12) governing the dynamics of error, assumes an entirely different state as,

$$\begin{aligned}\dot{e}_x &= -b_1 e_x + a_1 e_y + v_1 \\ \dot{e}_y &= e_x + v_2 \\ \dot{e}_z &= -e_z + e_y(y_2 + y_1) + v_3\end{aligned} \tag{12}$$

For the control $v_{i=(1,2,3)}$, there exists a wide range of potential groups. A constant matrix, denoted as A, has been developed to govern the error dynamics (12) in correspondence with the zero solution of the closed-loop system Equation (12), such that

$$[vi, \forall i = 1,2,3]^T = A[e_X, \forall X = x, y, z]^T \tag{13}$$

where A is a constant-value matrix. To stabilize the error system, the matrix A must be carefully selected to guarantee the feedback system possesses eigenvalues with negative real components. Selection of the matrix is A_i where (i = 1, 2, 3), in the following procedure [III, XXVIII]:

Case 1 In this case, we take A_1 matrix as follows

$$A_1 = \begin{pmatrix} 0 & a_1 & 0 \\ -1 & -a_1 & 0 \\ 0 & -(y_2 + y_1) & -a_1 \end{pmatrix}$$

and the control function is

$$\begin{aligned} u_1 &= -a_1 e_x + y_2 z_2 - y_1 z_1 \\ u_2 &= -e_x - a_1 e_y \\ u_3 &= -(y_2 + y_1) e_y - a_1 e_z \end{aligned}$$

with the corresponding eigenvalues are $\lambda_1 = (-b_1, -a_1, -1 - a_1)$

Case 2 In this case, we take A_1 matrix as follows

$$A_2 = \begin{pmatrix} -1 & -a_1 & 0 \\ -1 & -b_1 & 0 \\ 0 & -(y_2 + y_1) & -b_1 \end{pmatrix}$$

and the control function is

$$\begin{aligned} u_1 &= -e_x - a_1 e_y + y_2 z_2 - y_1 z_1 \\ u_2 &= -e_x - b_1 e_y \\ u_3 &= -(y_2 + y_1) e_y - b_1 e_z \end{aligned}$$

with the corresponding eigenvalues are $\lambda_2 = (-1 - b_1, -b_1, -1 - b_1)$

Case 3. In this case, we proceed A_3 matrix for verifying the control function

$$A_3 = \begin{pmatrix} -b_1 & -a_1 & 0 \\ -1 & -1 & 0 \\ 0 & -(y_2 + y_1) & 0 \end{pmatrix}$$

If we consider A_3, and we input (13) into equation (12), we can obtain

$$\dot{e} = Ke$$

$$\begin{pmatrix} \dot{e}_x \\ \dot{e}_y \\ \dot{e}_z \end{pmatrix} = \begin{pmatrix} -b_1 & a_1 & 0 \\ -1 & 0 & 0 \\ 0 & (y_2 + y_1) & -1 \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} + A_3 \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix}$$

Where

$$B_3 = \begin{pmatrix} -b_1 & a_1 & 0 \\ 1 & 0 & 0 \\ 0 & (y_2 + y_1) & -1 \end{pmatrix}$$

$$\Rightarrow K = B_3 + A_3$$

$$K = \begin{pmatrix} k_1 & 0 & 0 \\ 0 & k_2 & 0 \\ 0 & 0 & k_3 \end{pmatrix}$$

In this specific selection, the eigenvalues of the system (12) with **Case 3** are $\lambda_3 = (-b_1, -a_1, -1-a_1)$. It can be shown, using the principles of stability theory for linear systems, that the selected action causes the error states (e_x, e_y, e_z) to be decreased to zero when the time variable t approaches infinity. As a result, the control system represented by the control function is able to correctly synchronize the two systems and the control function is

$$\begin{aligned} u_1 &= -b_1 e_x - a_1 e_y + y_2 z_2 - y_1 z_1 \\ u_2 &= -e_x - e_y \\ u_3 &= -(y_2 + y_1) e_y \end{aligned}$$

IV.ii. Non-Identical Synchronization of Rucklidge System:

The MS Rucklidge system's dynamical model:

$$\begin{aligned} \dot{x}_1 &= -b_1 x_1 + a_1 y_1 - y_1 z_1 \\ \dot{y}_1 &= x_1 \\ \dot{z}_1 &= -z_1 + y_1^{\ 2} \end{aligned} \tag{14}$$

The Harb-Zohdy is exactly characterized as a drive dynamical model:

$$\begin{aligned} \dot{x}_2 &= -z_2 + u_1 \\ \dot{y}_2 &= x_2 - y_2 + u_2 \\ \dot{z}_2 &= p x_2 + y_2^{\ 2} + q z_2 + u_3 \end{aligned} \tag{15}$$

A similar way of identical procedure is to apply to estimate the error dynamics. The efficient use of equation (9) as follows:

$$\begin{aligned} \dot{e}_x &= -b_1 e_x + a_1 e_y - e_z + b_1 x_2 - a_1 y_2 - z_1 + y_1 z_1 + u_1 \\ \dot{e}_y &= e_x - e_y - y_1 + u_2 \\ \dot{e}_z &= p e_x + p x_1 + e_y (y_2 + y_1) + q e_z + q z_1 + z_1 + u_3 \end{aligned} \tag{16}$$

The error dynamics equation can be obtained by utilizing the definition (CFF)

$$\begin{aligned} v_1 &= b_1 x_2 - a_1 y_2 - z_1 + y_1 z_1 + u_1 \\ v_2 &= -y_1 + u_2 \\ v_3 &= p_1 x_1 + q z_1 + z_1 + u_3 \end{aligned} \tag{17}$$

The equation(18) governing the dynamics of error, assumes an entirely different state as,

$$\begin{aligned} \dot{e}_x &= -b_1 e_x + a_1 e_y - e_z + v_1 \\ \dot{e}_y &= e_x - e_y + v_2 \\ \dot{e}_z &= p e_x + e_y (y_2 + y_1) + q e_z + v_3 \end{aligned} \tag{18}$$

For the control $v_{i=(1,2,3)}$, there exists a wide range of potential groups. A constant matrix, denoted as A, has been developed to govern the error dynamics (18) in correspondence with the zero solution of the closed-loop system Equation (18), such that

$$[vi, \forall i = 1,2,3]^T = A[e_X, \forall X = x, y, z]^T \tag{19}$$

where A is a constant-value matrix. To stabilize the error system, the matrix A must be carefully selected to guarantee the feedback system possesses eigenvalues with negative real components. Selection of the matrix is A_i where $i = (1,2,3)$ in the following procedure:

Case 4. In this case, we take A_1 matrix as follows

$$A_1 = \begin{pmatrix} 0 & -a_1 & 1 \\ -1 & -b_1 & 0 \\ -p & -(y_2 + y_1) & -q^2 \end{pmatrix}$$

and the control function is

$$\begin{aligned} u_1 &= -a_1 e_y + e_z - b_1 x_2 + a_1 y_2 + z_1 - y_1 z_1 \\ u_2 &= -e_x - b_1 e_y + y_1 \\ u_3 &= -pe_x - e_y(y_2 + y_1) - q^2 e_z - px_1 - qz_1 - z_1 \end{aligned}$$

with the corresponding eigenvalues are $\lambda_1 = (-b_1, -1 - b_1, -q^2 + q)$

Case 5. In this case, we take A_2 matrix as follows

$$A_2 = \begin{pmatrix} -1 & -a_1 & 1 \\ -1 & -p & 0 \\ -p & -(y_2 + y_1) & -2q \end{pmatrix}$$

and the control function is

$$\begin{aligned} u_1 &= -e_x - a_1 e_y + e_z - b_1 x_2 + a_1 y_2 + z_1 - y_1 z_1 \\ u_2 &= -e_x - pe_y + y_1 \\ u_3 &= -pe_x - e_y(y_2 + y_1) - 2qe_z - px_1 - qz_1 - z_1 \end{aligned}$$

with the corresponding eigenvalues are $\lambda_2 = (-1 - b_1, -1 - p_1, -1 - b_1)$

Case 6. In this case, we proceed A_3 matrix for verifying the control function

$$A_3 = \begin{pmatrix} -b_1 & -a_1 & 1 \\ -1 & 0 & 0 \\ -p & -(y_2 + y_1) & -2q \end{pmatrix}$$

If we consider A_3, and we input (19) into equation (18), we can obtain

$$\dot{e} = Ke$$

$$\begin{pmatrix} \dot{e}_x \\ \dot{e}_y \\ \dot{e}_z \end{pmatrix} = \begin{pmatrix} -b_1 & a_1 & -1 \\ 1 & -1 & 0 \\ p & (y_2 + y_1) & q \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} + A_3 \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix}$$

Where

$$B_3 = \begin{pmatrix} -b_1 & a_1 & -1 \\ 1 & -1 & 0 \\ p & (y_2 + y_1) & q \end{pmatrix}$$

$$\Rightarrow K = B + A$$

$$K = \begin{pmatrix} k_1 & 0 & 0 \\ 0 & k_2 & 0 \\ 0 & 0 & k_3 \end{pmatrix}$$

In this specific selection, the eigenvalues of the system (18) with **Case 6** are $\lambda_3 = (-2b_1, -1, -q)$. It can be shown, using the principles of stability theory for linear systems, that the selected action causes the error states (e_x, e_y, e_z) to be decreased to zero when the time variable t approaches infinity. As a result, the control system represented by the control function is able to correctly synchronize the two systems and the control function is

$$\begin{aligned} u_1 &= -b_1 e_x - a_1 e_y + e_z - b_1 x_2 + a_1 y_2 + z_1 - y_1 z_1 \\ u_2 &= -e_x + y_1 \\ u_3 &= -p e_x - e_y (y_2 + y_1) - 2q e_z - p x_1 - q z_1 - z_1 \end{aligned}$$

IV.iii. Numerical Simulations

The numerical simulations utilize the fourth-order Runge-Kutta method, utilizing an initial time step of 0.01 on the time grid. This method is applied to solve the two kinds of dynamical equations (7) and (8), while integrating the nonlinear controller of **Case 1,2,3** for the identical Rucklidge systems. In the same way, another two kinds of dynamical equations (14) and (15) are solved, while integrating the nonlinear controller of **Case 4,5,6** for the non-identical Rucklidge systems. The parameters associated with the systems (7), (8), (14) and (15) are being systematically specified as

$$a_1 = 6.7,\ b_1 = 2,\ p = 3.1,\ q = 0.5$$

We are going to start the execution of the MS and DS procedures, taking into consideration the prevailing initial circumstances

$$\begin{pmatrix} x_1(0) \\ y_1(0) \\ z_1(0) \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \\ 4 \end{pmatrix} \text{ and } \begin{pmatrix} x_2(0) \\ y_2(0) \\ z_2(0) \end{pmatrix} = \begin{pmatrix} 0.05 \\ 0.01 \\ 0.06 \end{pmatrix}$$

The illustration is presented in **Fig.4.** and **Fig.6.** indicates the synchronization phenomenon is to be identified in the Rucklidge systems (7) and (8), which are characterized by their identical nature. The illustration is presented in Figure 8 and **Fig.10.** indicates the synchronization phenomenon is to be identified in the Rucklidge system (14) and Harb-Zohdy system (15), which are characterized by their non-identical nature.

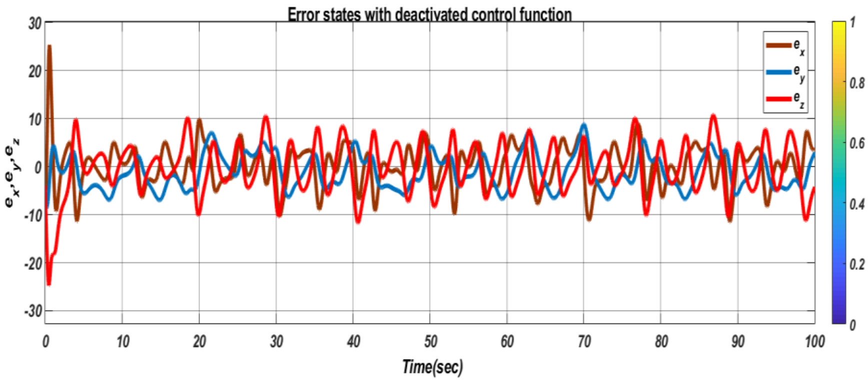

Fig. 3. Error states time-frame before synchronization.

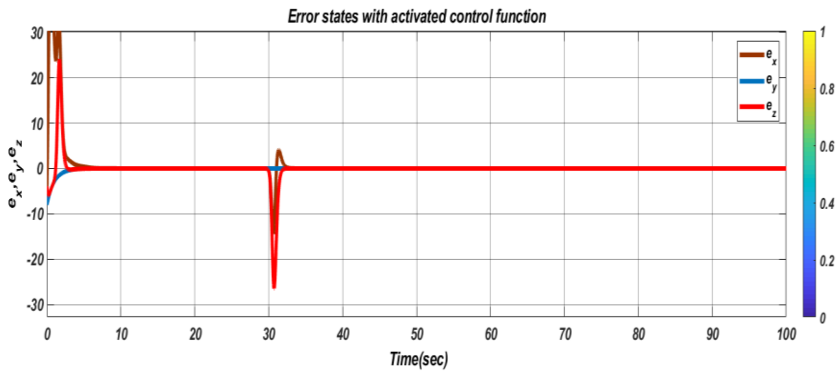

Fig. 4. Synchronized Error states in time-frame of identical synchronization

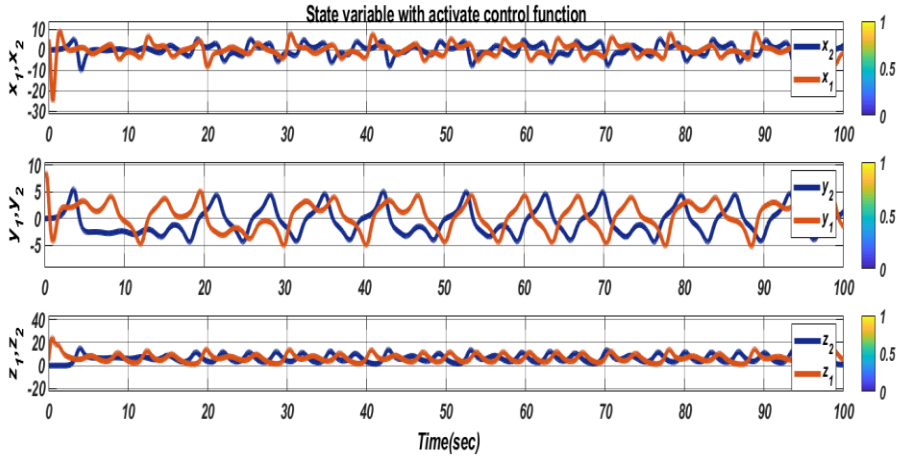

Fig. 5. The state's time frame before identical synchronization.

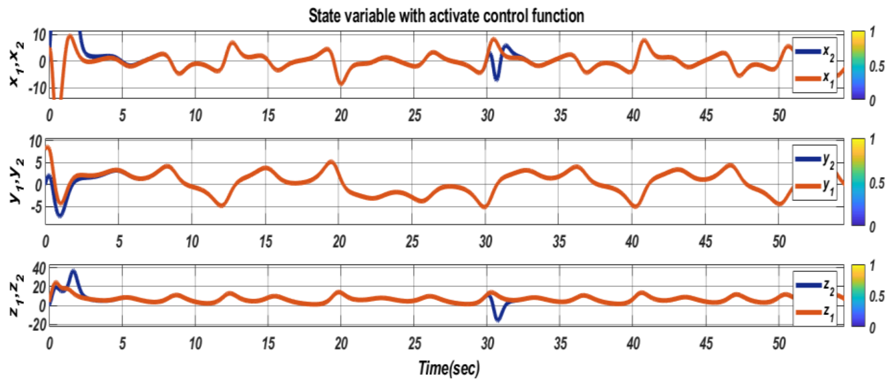

Fig. 6. The synchronized states time-frame of identical synchronization

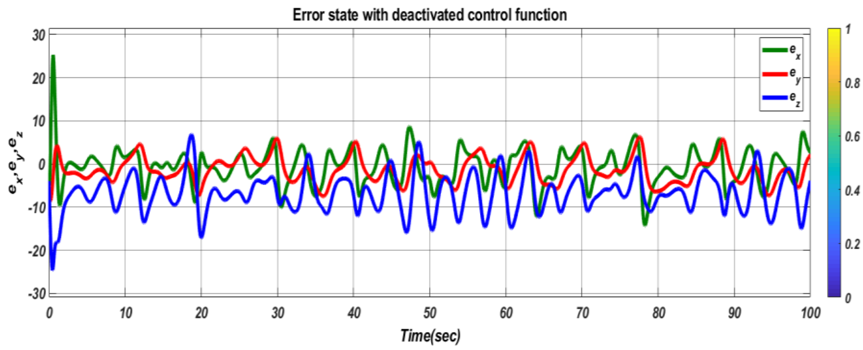

Fig. 7. Error states time-frame of prior to non-identical synchronization.

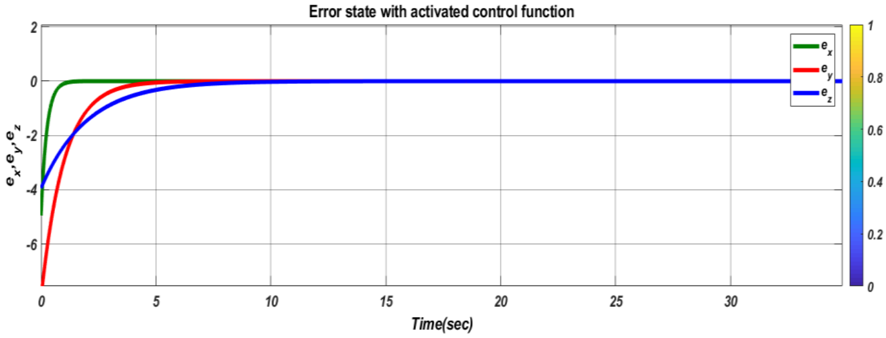

Fig. 8. Synchronized Error states in time-frame of non-identical synchronization.

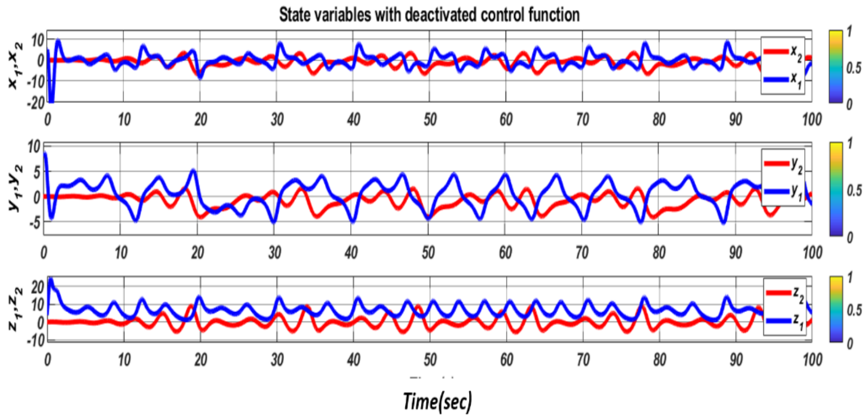

Fig. 9. The state's time frame before non-identical synchronization.

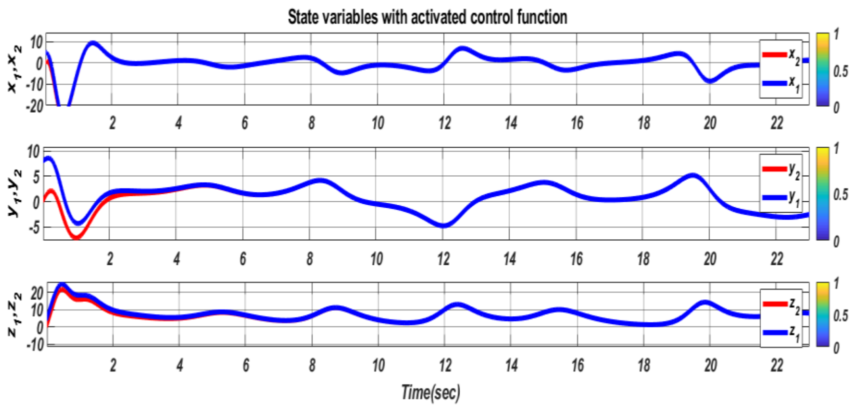

Fig. 10. The synchronized states time-frame of non-identical synchronization

V. Comparing identical and non-identical strategies with result discussion

Previously, synchronization was only possible between two identical systems, but the active control method has been extended to work with any pair of systems. That means it is possible in a non-identical pair of systems. **Fig.11.** and **Fig.12.** exhibit edge comparisons illustrating the impact of two distinct approaches on the synchronization time of a given MS-DS Rucklidge chaotic system, both in cases of identical and non-identical configurations. From **Fig.11(a)** and **Fig.11(b).**, we get in the Rucklidge system identical synchronization to reduce the error at an astonishing rate within 2 seconds. So if we want any real state domain adjusted with the Rucklidge system then we should apply identical synchronization with **Case 1.** and **Case 2.** Control functions.

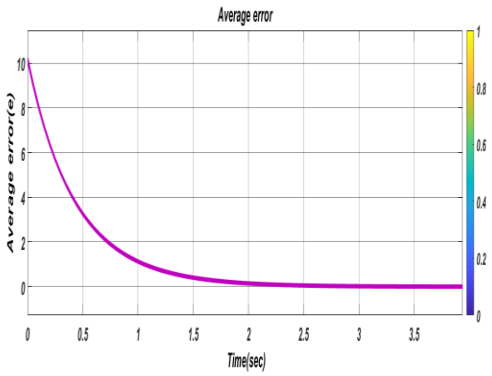

Fig. 11(a). Average error with A_1 co-efficient in the time frame for an identical process.

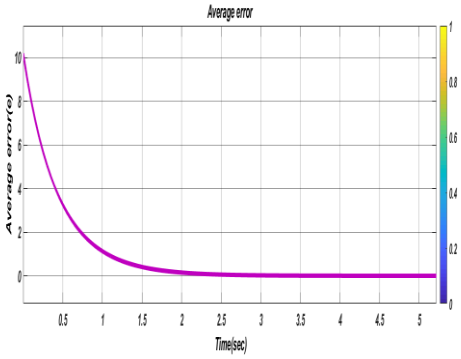

Fig. 11(b). Average error with A_2 co-efficient in the time frame for an identical process.

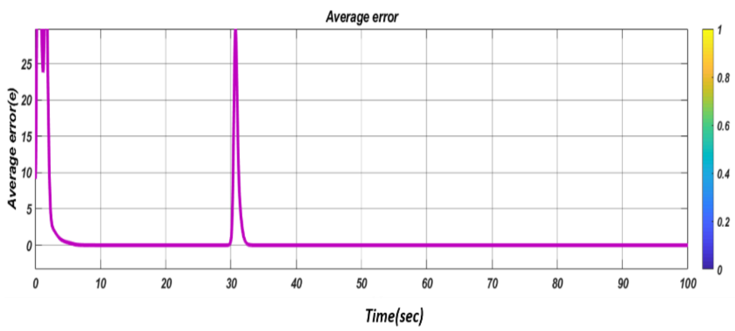

Fig. 11(c). Average error with A_3 co-efficient in the time frame for an identical process.

The utilization of the average error (e) serves as a dynamic analysis tool to assess the synchronization performance when the control function is stimulated at t = 0, as depicted in the average error time frame. In order to validate the synchronization act, we ascertain the average error associated with the state variables of the error dynamics, as designated below:

$$e = \sqrt{e_x^2 + e_y^2 + e_z^2}$$

Comparing identical and non-identical six control functions, one can readily observe that which controller is more effective than other controller. Every control function are important for researcher. Because for the synchronization in any system considerable appropriate control function is so significant matters.

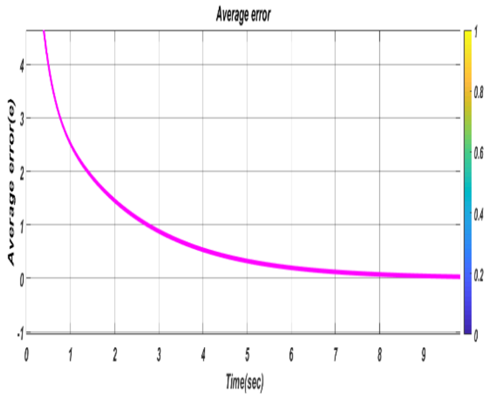

Fig. 12(a). Average error with A_1 co-efficient in the time frame for non-identical processes.

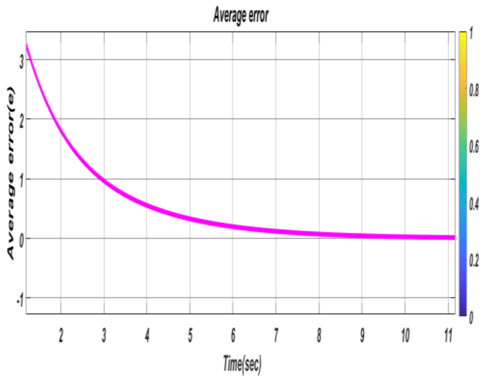

Fig. 12(b). Average error with A_2 co-efficient in the time frame for non-identical processes.

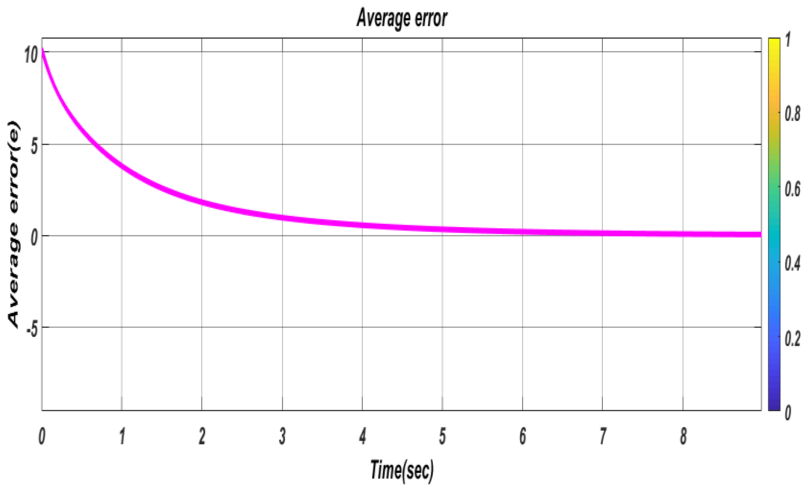

Fig. 12(c). Average error with A_3 co-efficient in the time frame for non-identical processes.

VI. Conclusion

In this article, we have explored six different control function sets with six different sets of coefficient matrices to synchronize the chaotic behaviors of the Rucklidge system. It is found that the identical procedure can be synchronized more effectively rather than the non-identical procedure. The efficiency of the aforesaid techniques are calculated numerically through the simulation.

The following outcomes have been summarized:

- The controller's design is conducted by a couple of systems where the original system and supporting system are entitled as the master system and the drive system.
- In the identical design, the master system and the drive system are Rucklidge systems with the same parameters.
- In the non-identical design, the Rucklidge system acts as the master system, and the Harb-Zohdy system acts as the drive system with different parameters.
- Identical design approach better result that skillfully connects the choice of A_1, A_2 co-efficient matrices with **Case 1** and **Case 2** control functions.
- Certainly, every control function can assure global stability, tracking, and transient performance for nonlinear systems.

Conflict of Interest:

The author declares that there was no conflict of interest regarding this paper.

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