JOURNAL OF MECHANICS OF CONTINUA AND MATHEMATICAL SCIENCES
www.journalimcms.org

ISSN (Online) : 2454 -7190 Vol.-19, No.-4, April (2024) pp 89-98 ISSN (Print) 0973-8975

ANALYSIS OF METRO NETWORK BY APPLYING GRAPH THEORETICAL NOTIONS

Kamal Jyoti Barman[1], Arijit Mishra[2]

[1]Asst. Prof., Department of Mathematics, PDUAM Dalgaon, India

[2]Asst. Prof., Department of Mathematics, NEF College, Guwahati, India

Email : [1]kamaljyotibarman@yahoo.in, [2]mishraarijit1012@gmail.com

Corresponding Author: **Arijit Mishra**

https://doi.org/10.26782/jmcms.2024.04.00007

(Received: March 03, 2024; Revised: March 24, 2024; Accepted: April 07, 2024)

Abstract

Indian cities are extending and growing very rapidly with the increase in population. As a result, there is a need to implement mass transit systems such as metro rail to meet their day-to-day mobility requirements. In recent years metro rail has grown in many Indian cities. Much like a graph that is made up of vertices and edges, a metro network is composed of stations and a metro route connecting them, where each station represents a vertex and any two vertices are adjacent whenever there is a link (metro route) between them. In this paper, we try to study the structure of a metro network via a graph theoretical approach.

Keywords: Mass transit systems, Metro network, Metro network graph.

I. Introduction

Metro rail is becoming the most popular means of public transport in India with an operational network of 660 km across 12 cities. Metro rail is often built underground (also known as “subway” or “tube”) or elevated above the ground and is suitable for metro cities or metropolitan areas which have large populations. It works with high capacity and frequency for covering shorter distances as compared to trains. The first metro in India started back in the year 1984 in Kolkata. Delhi metro is the second oldest and is the largest and the busiest among all other metros in India. Following the successful operation of Delhi Metro, all other big cities in India have also started to implement metro projects. All the metros in India are operated by autonomous local authorities except the Kolkata metro which is controlled by the Indian Railways.

A metro network is a structure composed of a set of stations and connections among them. A metro network graph is a graph where each station represents a vertex and any two vertices are adjacent whenever there is a link (metro route) between them. In short, a metro network graph is a graphical representation of a metro network with the help of graph theory.

II. Some Preliminary Definitions:

A **graph** G consists of a finite non-empty set $V(G)$ of points/vertices together with a prescribed set $E(G)$ of unordered pairs of distinct points of $V(G)$, where each pair of points denotes an edge.

1. A **subgraph** of G is a graph having all of its points and lines in G.
2. A graph G is said to be **connected** if every pair of points is joined by a path; otherwise, it is said to be **disconnected**.
3. A graph on n vertices is said to be a **complete graph** if every pair of vertices is joined by an edge. It is denoted by K_n.
4. The minimum number of vertices (edges) whose removal from a graph G results in a disconnected or a trivial graph is called the **point-connectivity (line-connectivity)** of G.
5. A **cut point** of a graph is one whose removal increases the number of components, and a **bridge** is such an edge.
6. In a graph G the maximal complete subgraph is called a **clique**. The number of vertices in a clique is called the **clique number**, denoted by $\omega(G)$.
7. The **chromatic number**, $\chi(G)$ is defined as the minimum n for which G has an n-coloring (Assignment of colors to its vertices so that no two adjacent vertices have the same color).

III. Methodological Approach

III.i. Graphical Representation of Metro Network

The metro network can be represented with the help of graph $G = \{ V , E \}$, where V is the set of vertices (nodes) and E set of all edges. The set V includes all the stations present in the network while the set E includes the metro track present between two stations that are directly connected. The graph $G = \{ V , E \}$ is always connected.

Edges between vertex i and vertex j means there are two metro tracks, one for going from station i to station j and another for coming from station j to station i. Thus in the metro network graph edges are considered as non-directional links.

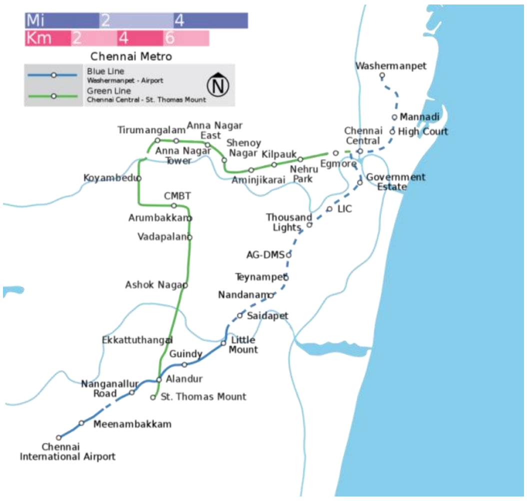

Fig: 3.1. Chennai Metro network

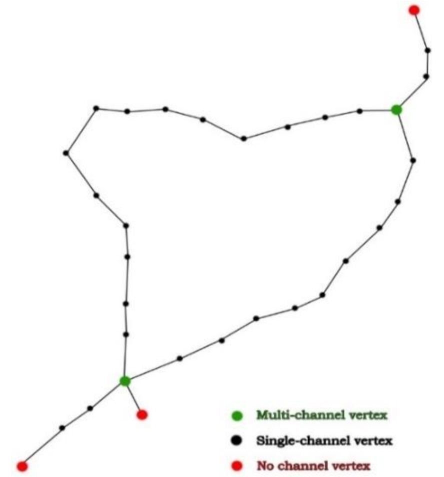

Fig: 3.2. Graphical Representation of Chennai Metro Network

Based on the nature of vertices, they are divided into the following types :

a) Multi-channel vertex.

b) Single-channel vertex.

c) No-channel vertex.

In the case of Multi-channel vertices, we have more than one option to pass through the vertex i.e. it is possible to switch lines (edges) without exiting the system. In Single-channel vertices there is only one way to pass through the vertex and here we cannot switch lines (edges) without exiting the system. Whereas in No-channel vertices we cannot pass through it i.e. they are end vertices with degree one.

III.ii. Indicators to study metro networks

a) Connectivity of network:

The connectivity of the network is expressed by the formula

$$C_N = \frac{Sum\ of\ the\ degrees\ of\ all\ the\ vertices}{Total\ number\ of\ vertices}$$

It is more convenient for passengers to travel to their respective destinations in a metro network having greater connectivity as compared to a metro network having lesser connectivity.

b) Average length between two consecutive stations of the network:

The average length between two consecutive stations of the network is expressed by the formula

$$A_N = \frac{Total\ length\ of\ the\ metro\ network}{Total\ number\ of\ edges\ present\ in\ the\ network}$$

For any two metro networks having equal total length, the network having a lower average length has more stations as compared to the network having a higher average length.

c) Complexity of the network:

The complexity of the network is expressed by the formula

$$Cm_N = \frac{Total\ number\ of\ edges\ present\ in\ the\ network}{Total\ number\ of\ vertices} = \frac{e}{v}$$

For a network having n stations (vertices), the complexity of the network, Cm_N lies between

$$\frac{n-1}{n} \le Cm_N \le \frac{n-1}{2}$$

$Cm_N = \frac{n-1}{n}$ implies that there is no Multi-channel vertex present in the metro network graph and $Cm_N = \frac{n-1}{2}$ implies that every station is connected with every other station in the metro network graph.

d) Extension of the network:

Considering all those vertices of the network only which are either multi-channel or no-channel and treating all the edges lying between two no-channel vertices or two multi-channel vertices or between multi-channel and no-channel vertices as one edge (i.e. Ignoring all single-channel vertices present in the network).

The extension of the network is expressed by the formula

$$E_N = \frac{Possible\ number\ of\ edges - Actual\ number\ of\ edges\ in\ the\ network}{possible\ number\ of\ edges}$$

$$= \frac{e_p - e_A}{e_P}$$

For any metro network, an extension of the network, E_N lies between 0 and 1.
i.e. $0 \le E_N < 1$.

When $E_N = 0$, we say that the network is complete and no further extension is possible.

e) Diameter of the network:

For any two no-channel/end vertices a and b , the distance between a and $b$means the minimum number of edges that we need to cover to reach from a to b. It is denoted by $d(a\ ,b)$.

The diameter of the network is expressed by the formula

$$D_N = Max\ \{\ d(a\ ,b) : where\ a\ and\ b\ are\ any\ two\ no - channel\ vertex\}$$

III.iii. Significance of some graph theoretical terms in metro network

a) Line-connectivity:

For any two metro networks having the same number of stations (vertices) and links (edges), the network having lower line connectivity is simpler as compared to the network having higher line connectivity.

i.e. lower line connectivity => simpler network
higher line connectivity => complex network.

b) Point-connectivity:

For any metro network, the higher point connectivity implies better connection among all the stations present in the network.

c) Bridge:

The presence of a bridge between station x and station y indicates that there is only one path connecting station x with station y and there is no other way of going from station x to station y. As a result, it will cause over crowded of passengers at all those stations which lie between station x and station y.

d) Chromatic number:

The lower chromatic number of the metro network graph implies that the network is simpler and has less connectivity among all its stations.

Whereas a higher chromatic number indicates that the network is complex and all the stations present in the network are well connected.

e) Clique number:

For any two metro networks having the same number of stations (vertices) and links (edges), the network having a higher clique number is more complex as well as has better connectivity among stations as compared to the network having a lower clique number.

IV. Application: A Study of the Indicators for Hyderabad and Bangalore Metro Networks

a) Hyderabad metro network:

Number of vertices (stations) = 57

Number of edges (links) = 57

Total length of the metro network = 69 km

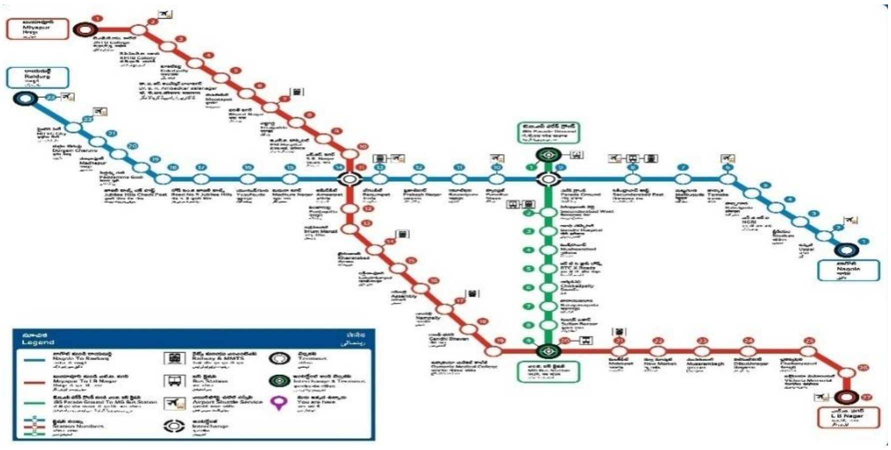

Fig : 4.1. Hyderabad Metro Network

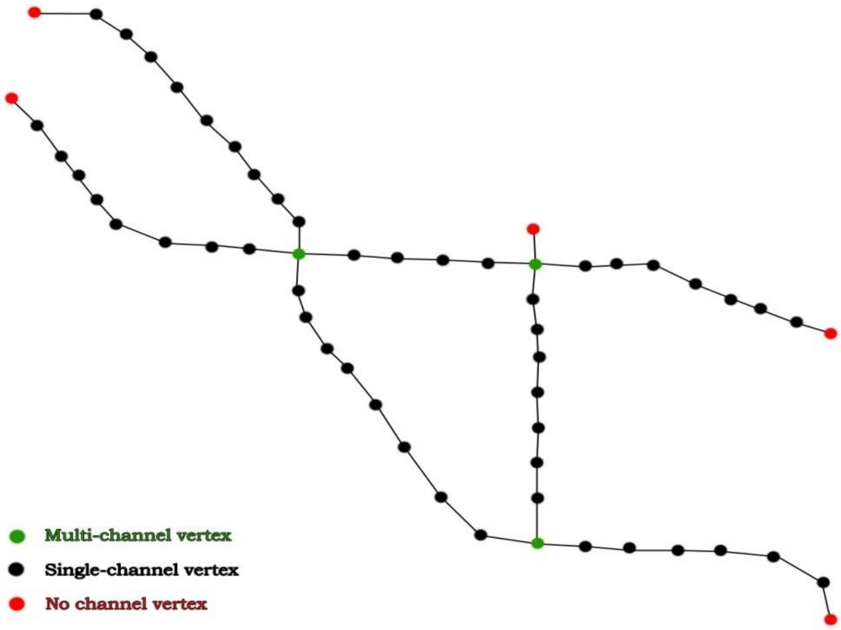

Fig : 4.2. Graphical Representation of Hyderabad Metro Network

Different indicators of the Hyderabad metro network are as follows:

i. Connectivity of network,

$$C_N = \frac{Sum\ of\ the\ degrees\ of\ all\ the\ vertices}{Total\ number\ of\ vertices}$$

$$= \frac{114}{57}$$

$$= 2$$

ii. The average length between two consecutive stations of the network,

$$A_N = \frac{Total\ length\ of\ the\ metro\ network}{Total\ number\ of\ edges\ present\ in\ the\ network}$$
$$= \frac{69}{57}\ km$$
$$= 1.21\ km$$

iii. Complexity of the network,

$$Cm_N = \frac{Total\ number\ of\ edges\ present\ in\ the\ network}{Total\ number\ of\ vertices}$$
$$= \frac{57}{57}$$
$$= 1$$

iv. Extension of the network,

$$E_N = \frac{Possible\ number\ of\ edges - Actual\ number\ of\ edges\ in\ the\ network}{possible\ number\ of\ edges}$$
$$= \frac{28-8}{28}$$
$$= 0.71$$

v. Diameter of the network,

$$D_N = Max\{\,d(a\,,b) : where\ a\ and\ b\ are\ any\ two\ no-channel\ vertex\}$$
$$= 26$$

b. Bangalore metro network:

Number of vertices (stations) = 40

Number of edges (links) = 39

Total length of the metro network = 42.3 km

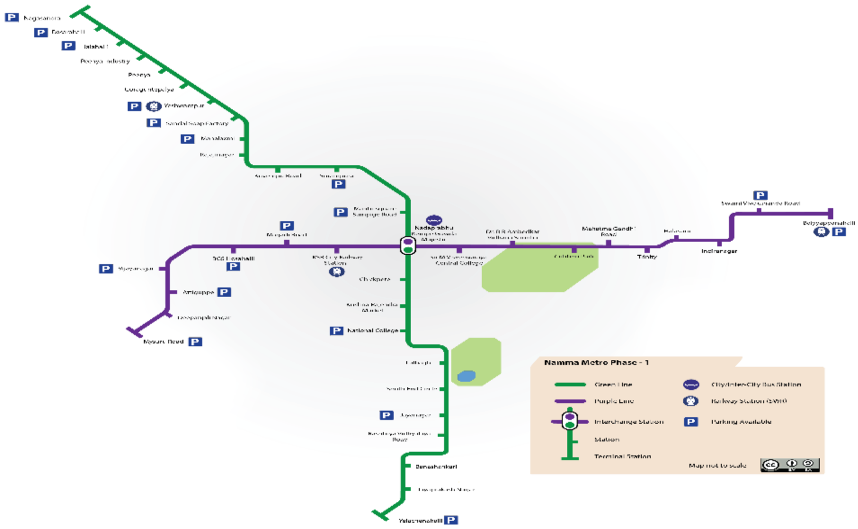

Fig: 4.3. Bangalore Metro Network

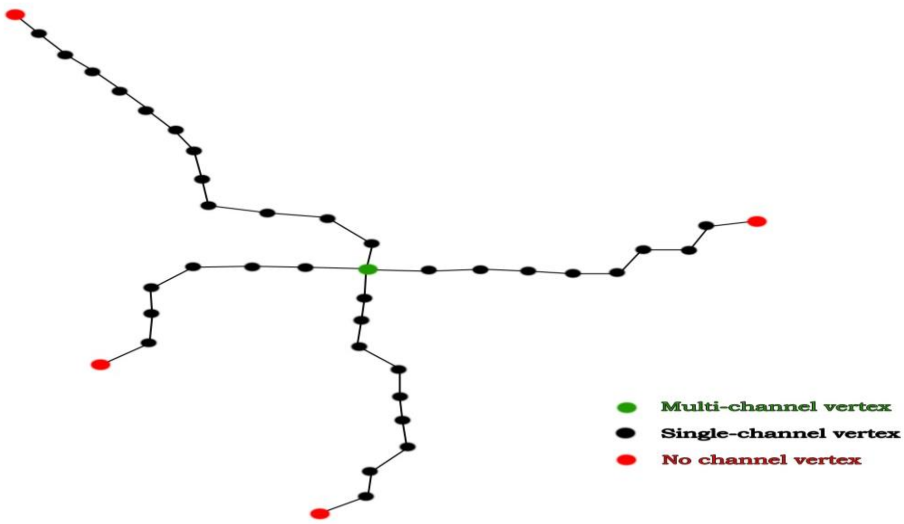

Fig : 4.4. Graphical Representation of Bangalore Metro Network

Different indicators of the Bangalore metro network are as follows:

i. Connectivity of network,

$$C_N = \frac{Sum\ of\ the\ degrees\ of\ all\ the\ vertices}{Total\ number\ of\ vertices}$$

$$= \frac{78}{40}$$

$$= 1.95$$

ii. The Average length of the network ,

$$A_N = \frac{Total\ length\ of\ the\ metro\ network}{Total\ number\ of\ edges\ present\ in\ the\ network}$$
$$= \frac{42.3}{39}\ km$$
$$= 1.08\ km$$

iii. Complexity of the network,

$$Cm_N = \frac{Total\ number\ of\ edges\ present\ in\ the\ network}{Total\ number\ of\ vertices}$$
$$= \frac{39}{40}$$
$$= 0.97$$

iv. Extension of the network,

$$E_N = \frac{Possible\ number\ of\ edges - Actual\ number\ of\ edges\ in\ the\ network}{possible\ number\ of\ edges}$$
$$= \frac{10-4}{10}$$
$$= 0.6$$

v. Diameter of the network,

$$D_N = Max\ \{\ d(a\ ,b)\ : where\ a\ and\ b\ are\ any\ two\ numbers - channel\ vertex\}$$
$$= 23$$

If we compare the various indicators of the Hyderabad and Bangalore metro networks, the following conclusion can be drawn:

i) The connectivity of the Hyderabad metro network is slightly better than the connectivity of the Bangalore metro network.

ii) The average length between two consecutive stations of the Hyderabad metro network is a bit more as compared to the Bangalore metro network and so the Bangalore metro network has slightly less waiting time.

iii) The Hyderabad metro network is slightly more complex than the Bangalore metro network.

iv) With the existing stations only, the Hyderabad metro network is more extendable as compared to the Bangalore metro network.

v) The diameter of the Hyderabad metro network is more than that of the Bangalore metro network.

V. Advantages and Disadvantages of Metro Rail

Advantages:

i) It helps to minimize traffic as it takes cars and buses off the road.

ii) Gets us to our respective destinations at a fast speed with good comfort.

iii) The accident rate in the metro is far less as compared to other means of transport.

iv) Travelers do not have to wait for long.

Disadvantages:

i) Elevated metro lines cause noise pollution.

ii) Huge cost of construction.

iii) High fares as compared to other public transport.

iv) It creates heavy traffic on the road during the construction of metro tracks as well as stations.

VI. Conclusion

The main objective of this paper is to show the importance of graph theoretical notions while studying the properties of metro networks. With the help of different indicators, we were able to examine several interesting statistics regarding metro networks in different cities. Such as by looking at connectivity, we may be able to predict how convenient it is for passengers to travel to their destination in that particular region. The average length of edges gives the idea regarding waiting time and time taken to cover a particular distance in a network. By looking at an extension of the network, we can say to what extent a further extension of the network is possible.

VII. Acknowledgement

The authors would like to thank the Department of Mathematics, Gauhati University, for their support.

Conflict of Interest

The authors declare no conflict of interest regarding this paper.

Reference

I. F. Harary. : 'Graph Theory'. *Addison-Wesley publishing company, Inc.* 1969

II. S. K. Bisen. : 'Graph theory use in transportation problems and railway networks'. *International journal of science and research*, 2017, Vol-6 (5), 1764-1768.

III. S. Stoilova and V. Stoev. : 'An application of graph theory which examines the metro networks'. *Transport Problems*, 2015, vol-10 (2), 35-48.