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2-D ANALYTICAL SOLUTION OF SOLUTE TRANSPORT WITH DECAY-TYPE INPUT SOURCE ALONG GROUNDWATER

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Abstract

The stabilization of groundwater resources in excellent quality is crucial for both the environment and human societies. To examine the contaminant concentration pattern of infinite and semi-infinite aquifers, mathematical models provide accurate descriptions. The two-dimensional model for a semi-infinite heterogeneous porous medium with temporally dependent and space-dependent (degenerate form) dispersion coefficients for longitudinal and transverse directions is derived in this study. The Laplace Integral Transform Techniques (LITT) is used to find analytical solutions. The dispersion coefficient is considered the square of the velocity which represents the seasonal variation of the year in coastal/tropical regions. To demonstrate the solutions, the findings are presented graphically. Figures are drawn for different times for a function and discussed in the result and discussion section. It is also concluded that a two-dimensional model is more useful than a one-dimensional model for assessing aquifer contamination.

Keywords: 2-D Advection-dispersion equation; Aquifer, Heterogeneity; Pollution; Laplace transform.

I. Introduction

The increasing concern over water source contamination stems from our society's increased reliance on groundwater. As everyone knows, groundwater aquifer pollution is a major issue that currently affects practically the entire nation and may even extend to the entire world. In general, the need for reliable sources of clean, unpolluted fresh water increases along with the global population. The well-known issue of mine water being pumped and deposited into the current drainage system, contaminating the water. Water is used extensively in coal preparation and beneficiation operations, and the water is contaminated when tailings are discharged into streams. Currently, the only ways to eliminate the contamination are by removing the source of the contamination and extracting the chemicals that are already present in the environment. The analytical solution for solute movement for heterogeneous/homogeneous and layered medium followed by various litterateurs ([XXX], [XXIX], [III], [XVII], [VI], [XXII]) were found through the transport equations with constant coefficients, prescribed with boundary and initial conditions and are solved for multi-dimensional (one, two and three) temporally/spatially domains. These solutions apply to semi-infinite/infinite domains.

Solute movement in porous media mathematical models can be used to characterize the polluted medium. The literature has numerous references to and discussions of both homogeneous and heterogeneous solute transport. J. Crank [XI] provided a one-dimensional analytic model for the point source. The model was later expanded into three dimensions by Baetsle [XIV]. Freeze and Cherry [XXIII] established that dispersion theory and dispersion is proportional to the n^{th} power of the velocity, $1 \leq n \leq 2$. Others have explained that the dispersion is directly proportional to velocity i.e. n = 1 (Rumer, [XXV]). Güven et al. [XVIII] offered a foundation for comprehending the phenomenon of scale-dependent dispersion within a deterministic framework. Inouchi et al. [XIII] state that a numerical model and an approximate analytical solution have been provided for studying seawater intrusion in a confined aquifer that takes tide effects into account. Solutions of the convection-diffusion equation with decay term for periodic input conditions through a semi-infinite domain have been attained by Logan and Zlotnik [XII]. Nevertheless, the constant coefficient equation for solute movement is appropriate for use in transport equations in heterogeneous systems. Analytical solutions of 2-D and 3-D for ADE with temporally and spatially dependent dispersion problems have been obtained by Yadav et al. ([XXVII], [XXVIII]). Various sets of position-temporally coefficients of the advection-diffusion equation have been reduced into constant coefficients to get the analytical solutions for semi-infinite and an infinite domain ([II], [VIII], [IV], [XXVI], [V], [XXIV], [XVI], [IX], [XXI]).

The advection-diffusion equation has been applied in recent years to several hydro-environment research issues. Jaiswal et al. [XIX] presented an analytical and fuzzy solution of solute transport along groundwater. Y. Sun et al. [XXXI] presented Lie group solutions of advection-diffusion equations. Because they are solved by the Lie group approach, they demonstrated its promise. The proposed closed-form method provides a simple substitute for numerical calculations in the analysis of mass transfer in many complex physical and industrial processes. The heterogeneous semi-infinite medium (water/air) under consideration in this work is not originally solute-free, that is, it is not clean. The origin is thought to be the source of exponential decay, and the concentration gradient is thought to be zero at the medium's end. In a heterogeneous semi-infinite shallow aquifer with an exponentially decreasing source of input concentration, the concentration level of contaminants is predicted in this research. To obtain a physical comprehension of the situation in both longitudinal and transverse dimensions, this study considers the influence of multiple parameters.

II. Mathematical model: Analytical Solution

The linear advection-diffusion partial differential equation for a two-dimensional, horizontal, isotropic, and heterogeneous medium can be expressed generally as follows:

$$\frac{\partial}{\partial x}\left(D_x(x,t)\frac{\partial c}{\partial x} - u(x,t)c\right) + \frac{\partial}{\partial y}\left(D_y(y,t)\frac{\partial c}{\partial y} - v(y,t)c\right) = \frac{\partial c}{\partial t} \tag{1}$$

where c denotes the pollutant/solute concentration at a space point x at time t, $D_x(x,t), D_y(y,t)$ reflects the solute dispersion and $u(x,t)$and $v(y,t)$ are the velocity of the medium transporting the solute particles. At time zero i.e. initially, the medium is not pure; certain pollutants may be present. It is assumed that one source is an exponential source of the decay type, with zero gradients at the other end. The initial and boundary conditions for the problem are:

$$c(x,y,t) = c_i \;;\; t = 0,\; x \geq 0,\; y \geq 0 \tag{2}$$

$$c(x,y,t) = c_0\, exp(-qt)\,; x = 0\,, y = 0, t > 0 \tag{3}$$

$$\frac{\partial c}{\partial x} = 0,\; \frac{\partial c}{\partial y} = 0;\; x \to \infty, y \to \infty,\; t \geq 0 \tag{4}$$

The pollutant decay concentration is q whose dimension is inverse of time [T^{-1}].

Unsteady flow in heterogeneous medium

Since there may initially be some pollutants that depend on a specific time function in a practical situation, seepage velocity is thought to be both specially and temporally dependent on decay-type exponential function due to the heterogeneous medium in the current challenge. The expression for velocity and dispersion in the degenerate form are considered (P. Singh et. al. [XX], [XXII]) as :

$$u(x,t) = u_0 f_1(mt)(1+ax) \; and \; D_x(x,t) = D_{x_0} f_1^2(mt)(1+ax)^2,$$

$$v(y,t) = v_0 f_1(mt)(1+by) \; and \; D_y(y,t) = D_{y_0} f_1^2(mt)(1+by)^2 \tag{5}$$

where a and b the parameter of heterogeneity with dimension inverse of space variable, the unsteadiness parameter is m, and the dimension of m is inverse of the time variable t. D_{x_0}, D_{y_0} and u_0, v_0 refers to initial dispersion and velocity whose dimensions are $[L^2 T^{-1}]$ and $[LT^{-1}]$ respectively. Using the expressions of dispersion coefficient and flow velocity from Eq. (5), and Eq. (1), the ADE will become,

$$\frac{\partial c}{\partial t} = \frac{\partial}{\partial x}\left(D_{x_0} f_1^2(mt)(1+ax)^2 \frac{\partial c}{\partial x} - u_0 f_1(mt)(1+ax)c\right),$$

$$+\frac{\partial}{\partial y}\left(D_{y_0} f_1^2(mt)(1+by)^2 \frac{\partial c}{\partial y} - v_0 f_1(mt)(1+by)c\right) \tag{6}$$

or $$\frac{1}{f_1(mt)}\frac{\partial c}{\partial t} = \frac{\partial}{\partial x}\left(D_{x_0} f_1(mt)(1+ax)^2 \frac{\partial c}{\partial x} - u_0(1+ax)c\right)$$

$$+\frac{\partial}{\partial y}\left(D_{y_0} f_1(mt)(1+by)^2 \frac{\partial c}{\partial y} - v_0(1+by)c\right) \tag{7}$$

A new time variable, T (J. Crank, [X]) with decay type exponential function is taken for $f_1(mt) = exp(-mt)$,

$$T_1 = \int_0^t f_1(mt)\, dt \tag{8}$$

The dimension of T_1 is the same as t. The value of $T_1 = 0$ for $t = 0$ by taking a suitable form of $f_1(mt)$ so that the conditions are unaffected by a new variable T_1. (Jaiswal et al., [VII]; Kumar et al, [I]) introduced a space variable as,

$$X = \int_0^x \frac{1}{(1+ax)} dx \quad or \quad X = \frac{1}{a} log(1+ax) \tag{9}$$

$$Y = \int_0^y \frac{1}{(1+by)} dy \quad or \quad Y = \frac{1}{b} log(1+by) \tag{10}$$

Using equation (8-10), equation (7) may be written as,

$$\frac{\partial c}{\partial T_1} = \left[D_{x_0} f_1(mt)\frac{\partial^2 c}{\partial X^2} + aD_{x_0} f_1(mt)\frac{\partial c}{\partial X} - u_0\frac{\partial c}{\partial X} - au_0 c\right].$$

$$+\left[D_{y_0} f_1(mt)\frac{\partial^2 c}{\partial Y^2} + bD_{y_0} f_1(mt)\frac{\partial c}{\partial Y} - v_0\frac{\partial C}{\partial Y} - bv_0 c\right] \tag{11}$$

Initial and boundary conditions in new variables (X, Y, T_1) are,

$$c(X, Y, T_1) = c_i\, ; T_1 = 0,\ X, Y \geq 0 \tag{12}$$

$$c(X, Y, T_1) = c_0(1 - qT_1)\ at\ X = 0, Y = 0, T_1 > 0 \tag{13}$$

$$\frac{\partial c}{\partial X} = 0, \frac{\partial c}{\partial Y} = 0\, ; X \to \infty, Y \to \infty,\ T_1 \geq 0 \tag{14}$$

Now introducing the new independent variable

$$\eta = X + Y \tag{15}$$

Using equation (15), equations (11-14) may be written as,

$$\frac{\partial c}{\partial T_1} = \left(D_{x_0} + D_{y_0}\right) f_1(mt) \frac{\partial^2 c}{\partial \eta^2} + \left(aD_{x_0} + bD_{y_0}\right) f_1(mt) \frac{\partial c}{\partial \eta}$$
$$-(u_0 + v_0)\frac{\partial c}{\partial \eta} - (au_0 + bv_0)c \tag{16}$$

$$c(\eta, T_1) = c_i\, ; T_1 = 0,\ \eta \geq 0 \tag{17}$$

$$c(\eta, T_1) = c_0(1 - qT_1)\ at\ \eta = 0, T_1 > 0 \tag{18}$$

$$\frac{\partial c}{\partial \eta} = 0; \eta \to \infty,\ T_1 \geq 0 \tag{19}$$

To remove the first-order decay term From Eq. (16)

$$c(\eta, T_1) = C\ exp(-au_0 - bv_0)T_1 \tag{20}$$

Then Eq. (16) becomes

$$\frac{\partial C}{\partial T_1} = D_0 f_1(mt) \frac{\partial^2 C}{\partial \eta^2} - U_0 f_2(mt) \frac{\partial C}{\partial \eta} \tag{21}$$

where $D_0 = D_{x_0} + D_{y_0}$ and $U_0 = u_0 + v_0$, $f_2(mt) = 1 - \lambda f_1(mt)$ and $\lambda = \frac{aD_{x_0} + bD_{y_0}}{U_0}$.

Initial and boundary conditions in new variables are

$$C(\eta, T_1) = c_i\, ; T_1 = 0,\ \eta \geq 0 \tag{22}$$

$$C(\eta, T_1) = c_0(1 - mT_1) exp(au_0 + bv_0)T_1\ at\ \eta = 0, T_1 > 0 \tag{23}$$

$$\frac{\partial C}{\partial \eta} = 0\, ; \eta \to \infty,\ T_1 \geq 0 \tag{24}$$

Taking another space and time transformations,

$$\xi = \frac{f_2(mt)}{f_1(mt)} \eta \tag{25}$$

$$T = \int_0^t {f_2}^2(mt) dt \tag{26}$$

or $T = (1/m)[-log(1 - mT_1) + (\lambda^2/2)\{1 - (1 - mT_1)^2\} - 2\lambda\{1 - (1 - mT_1)\}]$ (27)

The time frame T_1 has to be expressed in a new time frame T by using equations (26) and (27) as,

$$T_1 = \gamma_1 T, \text{ where } \gamma_1 = (1 - \lambda)^{-2} \tag{28}$$

Then Eq. (21) becomes

$$\frac{\partial C}{\partial T} = D_0 \frac{\partial^2 C}{\partial \xi^2} - U_0 \frac{\partial C}{\partial \xi} \tag{29}$$

Initial and boundary conditions in new variables are,

$$C(\xi, T) = c_i \,; T = 0, \ \xi \geq 0 \tag{30}$$

$$C(\xi, T) = c_0(1 - q\gamma_1 T)exp(au_0 + bv_0)\gamma_1 T \ at \ \xi = 0, T > 0 \tag{31}$$

$$\frac{\partial C}{\partial \xi} = 0 \,; \xi \to \infty, \ T \geq 0 \tag{32}$$

The analytical solution of the advection-diffusion equation with prescribed initial and boundary conditions is,

$$c(x, y, t) = c_i - \frac{c_i}{2}(C_1' + D_1') + \frac{c_0}{2}(C_1 + D_1)$$

$$- \frac{c_0 q \gamma_1}{4\beta}\left\{\left(2\beta T - \xi\sqrt{\frac{1}{D_0}}\right)C_1 + \left(2\beta T + \xi\sqrt{\frac{1}{D_0}}\right)D_1\right\}, \tag{33}$$

where

$$C_1 = exp\left(\frac{U_0}{2D_0}\xi - \beta\sqrt{\frac{1}{D_0}}\ \xi\right)erfc\left(\frac{\xi}{2}\sqrt{\frac{1}{D_0 T}} - \beta\sqrt{T}\right);$$

$$D_1 = exp\left(\frac{U_0}{2D_0}\xi + \beta\sqrt{\frac{1}{D_0}}\ \xi\right)erfc\left(\frac{\xi}{2}\sqrt{\frac{1}{D_0 T}} + \beta\sqrt{T}\right);$$

$$C_1' = exp\left(\frac{U_0}{2D_0}\xi - \alpha\sqrt{\frac{1}{D_0}}\ \xi\right)erfc\left(\frac{\xi}{2}\sqrt{\frac{1}{D_0 T}} - \alpha\sqrt{T}\right);$$

$$D_1' = exp\left(\frac{U_0}{2D_0}\xi + \alpha\sqrt{\frac{1}{D_0}}\ \xi\right)erfc\left(\frac{\xi}{2}\sqrt{\frac{1}{D_0 T}} + \alpha\sqrt{T}\right);$$

$$\beta^2 = A + \frac{{U_0}^2}{4D_0}; \quad A = (au_0 + bv_0)\gamma_1; \quad \alpha^2 = {U_0}^2/4D_0; \qquad \xi = \frac{f_2(mt)}{f_1(mt)}\eta;$$

$$\eta = X + Y; \quad X = \frac{1}{a}log(1 + ax); \qquad Y = \frac{1}{b}log(1 + bx);$$

$$f_2(mt) = 1 - \lambda f_1(mt); \qquad \lambda = \frac{aD_{x_0} + bD_{y_0}}{U_0}; \qquad T = T_1/\gamma_1;$$

$$\gamma_1 = (1 - \lambda)^{-2}; \qquad T_1 = \int_0^t f_1(mt)dt; \qquad f_1(mt) = exp(-mt).$$

III. Result and Discussions

Analytical solutions are obtained for the 2-D advection-diffusion equation by Eq. (33). Utilizing the input values, the concentration values (c) for Eq. (33) are assessed as: reference concentration $(c_0) = 1$, initial velocity along longitudinal (u_0) = 0.21

(km/year) and transverse (v_0) = 0.021 (km/year), initial dispersivity (D_{x_0}) = 0.43 (km^{-2}/year), and (D_{y_0}) = 0.043 (km^{-2}/year), heterogeneity parameter (a) = 0.15 (km^{-1}) and (b) = 0.15 (km^{-1}),, unsteady parameter (m) = 0.1 (year^{-1}), and (q) = 0.1 (year^{-1}). Initial source concentration $(c_i) = 0.1$. Concentration attenuation with time and space for $x = 0\ to\ 1\ (km)$ and for $y = 0\ to\ 1\ (km)$ are discussed, at time $t\ (year)$= 1.5, 3.5 and 5.5.

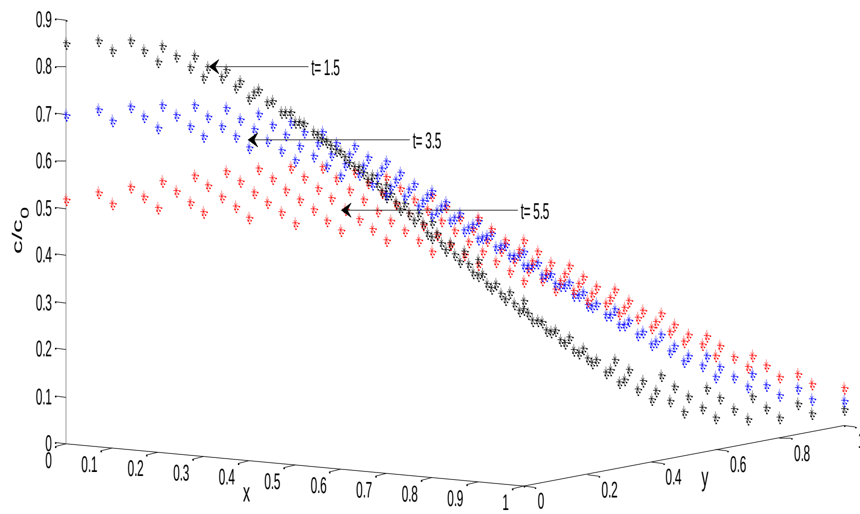

Fig. 1 Solute concentration distribution along 2-D groundwater surface at various times

The distribution of concentration illustrated in Fig. 1 is drawn for decelerating flow $f_1(mt) = exp(-mt)$. The concentration pattern with various times is predicted in Fig.1 and shows the distribution of contaminants with increasing space position. It is also observed from Fig. 1, that the concentration (c) at the origin diminishes concerning time.

Fig. 2 is drawn for exponentially accelerating and decelerating source functions. In both Fig. 1 and Fig. 2, the diffusion of exponential decelerating source concentration with position and time is faster than that of exponential increasing source. It is clear that accelerating/decelerating exponential sources are much more suitable for tropical regions whereas for non-tropical regions; linear and constant input sources are good.

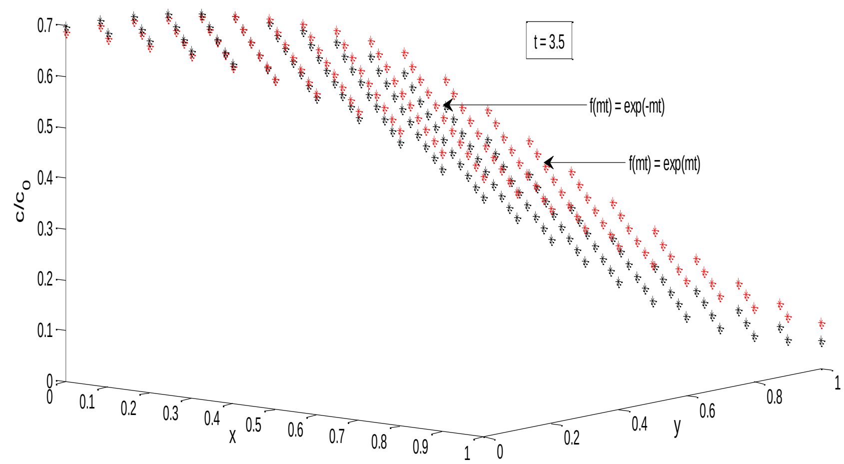

Fig. 2 Solute concentration distribution along 2-D groundwater surface for two exponential functions at one time

We conclude that if there is no source i.e. if the medium (aquifer/air) is solute-free then its concentration decreases much faster than in the presence of the source due to diffusion and velocity which are of variable coefficient.

IV. Conclusion

In the present paper, derived the two-dimensional (2-D) model for a semi-infinite heterogeneous porous media with a time and space-dependent (degenerate form) dispersion coefficient for both longitudinal and transverse directions. For analytical solutions, the Laplace Integral transform techniques (LITT) are employed. In surface water, groundwater, and other open-source regions, flow velocities are different in all directions. For the transverse direction, the valves of various parameters were taken one-tenth of the longitudinal direction. Since dispersion is faster along the flow velocity rather than other flow direction. This situation is shown in both figures. 2-D mathematical model is very useful for predicting seasonal variation and environmental pollution problems. In coastal and tropical environments, the dispersion coefficient is regarded as the square of the velocity, which symbolizes the seasonal change of the year. Analytical solutions validate the numerical solutions. Since each numerical code is created with defined ends of the space domain, such as MATLAB pdepe and others like FEFLOW, MODFLOW, and MT3DMS, it is determined that these codes are appropriate to provide the numerical solution of the given problem.

Conflict of Interest:

The authors declare that there are no conflicts of interest regarding this paper.

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