TWO PHASE SLIP FLOW OF BLOOD IN HEPATIC ARTERY WITH SPECIAL REFERENCE TO HEPATITIS B

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Abstract

In this paper, we have presented a model of two-phased arterial hepatic blood flow in hepaticarteries remote from the heart and proximate to the Liver keeping in view the nature of hepatic blood circulation in the human body. Blood is supposed to be non-Newtonian of the power-law type. Solutions of the constitutive equations are obtained in analytical as well as in numerical forms. The role of hematocrit is explicit in the determination of blood pressure drop in the case of Hepatic disease Hepatitis B.

Keyword: Hepatic Blood Flow, Non-Newtonian power law model, Haematocrit, Blood pressure drop, Hepatitis B

I.   Introduction

The liver is one of the most complex, complicated, and largest human organs. It constitutes about 2.5% of body weight and is a reddish-brown wedge-shaped containing four lobes of unequal size and shape, viz. right and left lobe quadrants and caudate lobe [VI, II]. The lobes are surrounded by a thick capsule overlaid with reflected periforinum. The liver is located in the right upper quadrant of the abdominal cavity and rests just below the diaphragm right of the stomach and overlies the gallbladder. It is connected to two large blood vessels-the hepatic artery and the portal vein. The function of the hepatic artery is to carry oxygen-rich blood from the aorta and that of the portal vein is to carry blood in digested nutrients from the gastrointestinal tract, spleen and pancreas. These blood vessels are subdivided into small capillaries, known as sinusoids which lead to lobules. Lobules are the functional units of the liver. Each of the lobules is made up of nutrients from the gastrointestinal tract, spleen and pancreas fibroelastic connective tissue layer which ultimately forms the structure of a liver. Veins, arteries, ducts and nerves through the hepatic portal constitute a capsule, known as Glisson's capsule. The function of the liver is to detoxification of various metabolites and toxic matter, regulate glycogen storage, decompose red blood cells, and hormones production and produce of biochemical

needed for digestion. The liver breaks down ammonia into areas which are excreted in the urine. It is the only human internal organ that can regenerate lost tissue naturally [I].

The hepatic artery is generated from the celiac trunk and it branches into the left hepatic and right hepatic arteries close to the porta hepatis. This artery may also be subdivided into a common hepatic artery from the celiac trunk to the origin of the gastroduodenal artery and the other is the proper hepatic artery to the bifurcation point [III]. The common hepatic artery is a blood vessel which supplies oxygenated blood to the liver, pylorus of the stomach, duodenum and pancreas. On the other hand, portal venous blood provides 50% of the oxygen requirements of the liver [V]. As the hematocrit increases, blood in the hepatic arteries remote from the heart shows a non-Newtonian character of power-law type and the flow is, in general, two-phase. The pressure in the artery is about 100 mm 0f Hg with a pressure-dependent autoregulation of blood flow.

It is to be noted that hepatic can be classified into two types-Hepatitis-A and Hepatitis-B. Of these Hepatitis-B infection is most dangerous. People with this virus are at increased risk of cirrhosis and liver cancer.

The present study deals with a model of a two-phase slip flow of blood in the hepatic artery remote from the heart and proximate to the liver for hepatic disease. Blood is supposed to be non-Newtonian of the power-law type. Solutions of the constitutive equations are obtained in analytical as well as in numerical forms. In this connection, it may be mentioned that a similar type of problem has also been considered by Upadhyay et al [IV], but the solutions obtained are incorrect in most cases as a result of which we aim to rectify and generalize these by taking into account slip flow of blood. However, we have taken the clinical data collected by them for numerical discussions. It is observed that hematocrit and slip velocity on the arterial surface play crucial roles in blood pressure drop in the case of the patients of hepatic disease Hepatitis-B.

II.   The Problem and Constitutive Equations.

Blood is a mixed fluid of two phases. The first phase is plasma while the other phase is blood cells enclosed with a semi-permeable membrane whose density is greater than that of plasma. Here we consider two-phase blood flow in a hepatic artery with slip velocity on its surface. Since hematocrit plays a vital role on blood flow, so the assumption of two-phase flow is more realistic than that of single phase. Moreover, as hematocrit increases, blood in arteries remote from the liver shows non-Newtonian character of power law type and the effective viscosity of blood remote from the liver depends upon the strain rate. In such a case if the strain rate lies between 5 to 200 per second, the power law is given by

$$T' = \eta_m e^n, \quad \text{where } 0.68 \le n \le 0.80,$$

T' being the stress and η_m is the viscosity of the two-phase mixture.

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In the present problem, it is supposed that the blood flow through the hepatic circular cylindrical artery of radius R is axially symmetric, i.e. the velocity component $v_\theta = 0$ and the other velocity components v_r, v_z and the pressure p are independent of θ. In addition, the motion is assumed to be steady so that all functions are independent of time t and the blood flows along the axial direction which we take to be the z-axis.

Keeping in view of the above facts, the governing equations in cylindrical coordinates read as follows:

Equation of continuity-

$$\frac{\partial v_z}{\partial z} = 0 \tag{1}$$

Equations of motion

$$0 = -\frac{\partial p}{\partial r}, \tag{2}$$

$$0 = -\frac{\partial p}{\partial z} + \frac{\eta_m}{r} \frac{\partial}{\partial r} \left\{ r \left(\frac{\partial v_z}{\partial r} \right)^n \right\}, \tag{3}$$

where η_m, the viscosity of mixture of blood is given by $\eta_m = X\eta_c + (1 - X)\eta_p$, η_c and η_p are the viscosities of blood cells and plasma respectively, $X = H/100$ is the volume ratio of blood cells and H is the hematocrit.

Integrating equation (1), we have

$v_z = v(r)$, a function of r only.

Also integration of (2) leads to

$p = p(z)$, a function of z only.

The pressure gradient $-\frac{dp}{dz} = P$ of blood flow in the artery remote from the liver can be taken as constant and hence the equation (3) gives

$$\frac{d}{dr} \left\{ r \left(\frac{dv}{dr} \right)^n \right\} = -\frac{Pr}{\eta_m} \ . \tag{4}$$

III. Solutions

The equation (4), on integration concerning to r, leads to

$$r \left(\frac{dv}{dr} \right)^n = -\frac{Pr^2}{2\eta_m} + A,$$

A being integration constant. Since the velocity of blood in the cylindrical artery is maximum and constant on its axis, so $= V_0$, a constant on r = 0 and so A = 0 . Thus

$$\left(\frac{dv}{dr}\right)^n = -\frac{Pr}{2\eta_m} \Rightarrow \frac{dv}{dr} = -\left(\frac{Pr}{2\eta_m}\right)^{1/n}.$$

Integrating again with respect to r and then using the slip condition $v = v_s$ on the inner wall r=R of the artery, we have

$$v = v_s + \frac{n}{n+1}\left(\frac{P}{2\eta_m}\right)^{1/n}\left(R^{\frac{n+1}{n}} - r^{\frac{n+1}{n}}\right). \tag{5}$$

This determines the velocity of blood within the artery remote from the liver.

IV. Numerical Results and Discussions

The total flux of blood through the transverse section of the arteries is

$$Q = \int_0^R v.\, 2\pi r\, dr$$

$$\text{i.e.}\, Q = \pi\left[v_s R^2 + \left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}}\frac{n}{3n+1}R^{\frac{3n+1}{n}}\right]. \tag{6}$$

To have a glimpse of the solutions obtained above, we follow the data and records given by Upadhyay et al [IV].

Average Systolic Pressure = 14443.0 pa

Average Diastolic Pressure = 9554.6 pa

H= Average Hematocrit = 24.7

P_i = Pressure on Artery = Average Systolic Pressure = 14443.0 pa

P_f = Pressure on Arterioles = $\frac{S+D}{2} = \frac{14443.0+9554.6}{2} = 11998.8$ pa

η_m = Viscosity of Mixture = 0.0035 p.s.

η_p = Viscosity of Plasma = 0.0015 p.s.

Length of common hepatic Arteries = 0.0347 m.

Noting that $\eta_m = \eta_c X + \eta_P(1 - X)$ with X $= \frac{H}{100}$, it follows that

η_c= Viscosity of cells = 0.009597 p.s.

We note that

$$\eta_m = 0.009597\frac{H}{100} + 0.0015\left(1 - \frac{H}{100}\right)$$

i.e. $\eta_m = (8.097 \times 10^{-5})H + 0.0015.$

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Now from equation (6), flow flux is given by

$$Q = \pi \left[v_s R^2 + \left(\frac{P_i - P_f}{2\eta_m(Z_f - Z_i)} \right)^{\frac{1}{n}} \frac{n}{(3n+1)} R^{\frac{3n+1}{n}} \right]. \tag{7}$$

On solving equation (7), we get n = 0.712721, whence Q = 0.01666, π = 3.14, P_i = 14443, P_f = 11998.8, η_m= 0.0035, $Z_f - Z_i$ = 0.0347, R = 0.0025, v_s = 5.

Thus from equation (7) it follows that

$$Q = \pi \left[v_s R^2 + \left(\frac{\Delta P}{2\eta_m \Delta Z} \right)^{\frac{1}{n}} \frac{n}{(3n+1)} R^{\frac{3n+1}{n}} \right]$$

$$\Rightarrow \Delta P = 2\eta_m \Delta Z \left\{ \frac{(3n+1)}{n\pi} \frac{1}{R^{\frac{(3n+1)}{n}}} (Q - \pi v_s R^2) \right\}^n.$$

Substituting the values of the data given above we have

$$\Delta P = 71.7324 \text{ H} + 852.593 \tag{8}$$

where $\eta_m = (8.097 \times 10^{-5})H + 0.0015$

Putting values of H in the equation (8), we get the following table of blood pressure drop as earlier:

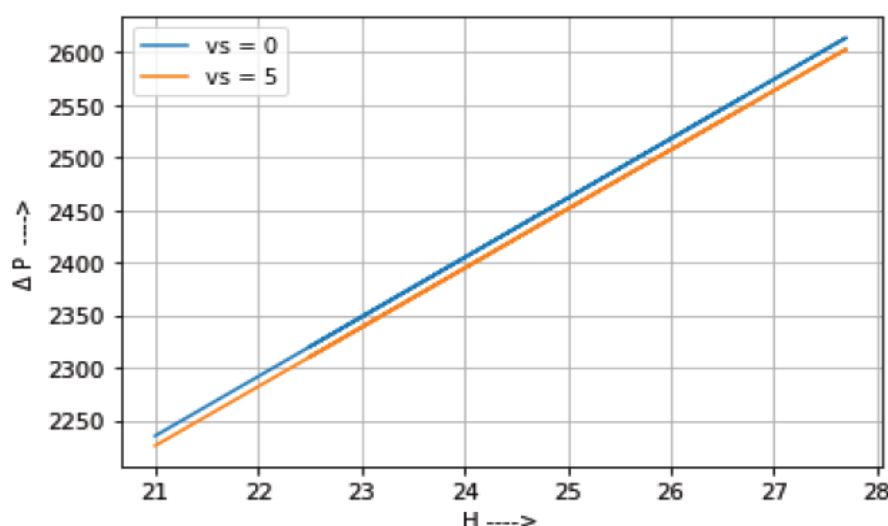| Hematocrit	26.7	22.5	24.3	26.1	27.6	21.0
Blood Pressure Drop (ΔP) ($v_s = 0$)	2557.37	2319.87	2421.65	2523.44	2613.91	2235.05
Blood Pressure Drop (ΔP) ($v_s = 5$)	2546.62	2310.12	2411.47	2512.83	2602.93	2225.65

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Fig.1. Pressure drop fluctuation

In above table and graph (Fig-1) show that pressure drop is fluctuating in nature. It is some times decreasing and sometimes increasing in the sense of a straight line. On the other hand, slip velocity decreases blood pressure drop. It is therefore evident that slip velocity plays an effective role in blood propagation in the hepatic artery. If blood pressure is increased, then for Hepatitis-B patients, operation of the liver is not suggested by physician; it is possible if the pressure drop is in a decreasing sense because hematocrit is directly proportional to blood pressure drop.

Conflict of Interest:

The author declares that there was no conflict of interest regarding this paper.

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