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A NOVEL CONCEPT OF THE THEORY OF DYNAMICS OF NUMBERS AND ITS APPLICATION IN THE QUADRATIC EQUATION

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Abstract

Considering the basic role of numbers in Mathematics, Science, and Technology the author developed a new structure of numbers named as 'Theory of Dynamics of Numbers.' According to the Theory of Dynamics of Numbers, the author defined 0 (zero) is the starting point of any number and also defined 0 (zero) as a neutral number. The numbers can move in infinite directions from the starting point 0 (zero) and back to 0 (zero). The author has defined the three types of numbers: 1) Neutral Numbers, 2) Count Up Numbers, and 3) Count Down Numbers. These three types of numbers cover the entire numbers in the number system where there is no necessity for the concept of imaginary numbers. Introducing this new concept the author solved the quadratic equation in one unknown (say x) in the form $ax^2 + bx + c = 0$, even if the numerical value of the discriminant $b^2 – 4ac < 0$ in real numbers without using the concept of imaginary numbers. Already the author solved the quadratic equation $x^2 + 1 = 0$ and proved that $\sqrt{-1} = -1$ by using the Theory of Dynamics of Numbers. The Theory of Dynamics of Numbers is a more powerful tool than that of the real and imaginary number system to explain the truth of nature.

Keywords: Cartesian Coordinate System, Imaginary Numbers, Quadratic Equation, Rectangular Bhattacharyya's Coordinate System, Theory of Numbers, Theory of Dynamics of Numbers.

I. Introduction.

A number is an arithmetical value that is used to represent the quantity of any object. A number is a mathematical concept expressed in words, symbols, or figures to represent a particular quantity of objects used in counting and calculations. We can't find any negative object in the universe. Therefore, negative numbers cannot exist independently. So, – 1, – 2, – 3, – 4, etc can not exist independently according to the present author.

Mathematics is a tool to explain the truth of nature. In mathematics, we do nothing but addition only. Four operations in mathematics: 'Addition', 'Multiplication', 'Subtraction', and 'Division' can be represented by addition only. For example:

1) $2 + 2 + 2 + 2 + 2 = 10$
2) $2 \times 5 = 2 + 2 + 2 + 2 + 2 = 10$
3) $10 - 2 = 8$, we subtract 2 from 10, which means that we add 2 with vertically opposite direction of motion of number 10.
4) $10 \div 2 = 5$, we subtract 2 by 5 times from 10 but we know that subtraction is nothing but addition.

Based on the conventional method of operations on numbers, especially regarding subtraction there exists a direction of motion of numbers in a vertically opposite direction of the independent number. The author concluded that the numbers are in dynamic nature which is true.

Considering the works of the conventional mathematician of the world the author developed a new mathematical structure in the number system in the present article which is known as the 'Theory of Dynamics of Numbers'.

In the Theory of Dynamics of Numbers, the author is the first person who define 0 (zero) is the starting point of any number and also, defined 0 (zero) as a neutral number. There are three types of numbers: 1) Neutral Numbers, 2) Count Up Numbers, 3) Count Down Numbers. Details of this structure of numbers have been given in the 'Formulation and its Solution' part of the article.

With the help of this new mathematical structure of numbers in mathematics, the author solved the problem of finding the distance between any two points in a plane by Rectangular Bhattacharyya's Coordinate System where all four axes are positive instead of the Cartesian Coordinate System. [XV]

The question may arise what is the necessity of this new concept of the new structure of the number system in mathematics. 'Yes', there is a necessity for this new concept. At present, we are using conventionally two tools in calculations to explain the truth of nature which are not sufficient :

1) Real Numbers (Positive and Negative) system. In this number system, the negative number can exist independently which is absurd according to the author. The positive numbers can exist independently but the negative numbers can exist only depending on the positive numbers.

2) Imaginary Numbers : In the concept of imaginary numbers 0 + i0 is undefined and there is no order relation except an equality relation.

The concept of imaginary numbers have been developed since the mathematician of the world could not find the solution of a quadratic equation, $x^2 + 1 = 0$ in real numbers.

But with the new concept of the Theory of Dynamics of Numbers, the author solved $x^2 + 1 = 0$ in real number [XVI] and recently proved that $\sqrt{-1} = -1$ by publishing an article : "A New Concept to Prove, $\sqrt{-1} = -1$ in both Geometric and Algebraic Method without using the Concept of Imaginary Numbers" [XXII].

Using this new concept it is possible to solve the problem of quadratic equation in one unknown number (say x) of the quadratic equation in the form of $ax^2 + bx + c = 0$ whether the numerical value of the discriminant is $b^2 - 4ac \geqslant 0$ or $b^2 - 4ac < 0$, in real number only without using the concept of imaginary numbers. [XVII, XIX]

II. Literature Review

It is highly plausible that more than 40,000 years in the past, the progression of the number system occurred, shifting from the use of figures and tally marks to a set of glyphs that were proficient in representing any conceivable number. Most probably unambiguous notations for numbers emerged in Mesopotamia about 5000 – 6000 years ago.

The number system provides a unique representation of every number and represents the arithmetic and algebraic structure of figures. The numbers are natural numbers, rational numbers, irrational numbers, imaginary numbers, etc. 0 (zero) is also a number that represents a null value. The numbers are used to perform arithmetical calculations such as addition, subtraction, multiplication, and division.

Recently the author developed a new number system such as Neutral numbers, Count up numbers, and Countdown numbers which are the outcome of the 'Theory of Dynamics of Numbers'. According to the Theory of Dynamics of Numbers 0 (zero) is a neutral number and also, 0 (zero) is defined as the starting point of any number.

Aryabhata – I (476 CE – 550 CE) was the first of the major mathematicians and astronomers from the classical age of Indian mathematics. Aryabhata – I is credited for using 0 (zero) in the decimal system and introducing zero in mathematics. Brahmagupta (598 CE – 668 CE) is the first person who invented zero and used 0 (zero) in mathematical operations like addition and subtraction [III, IV]. Sridhara Acharya (870 CE – 930 CE), a Bengali mathematician of India was the first person who had given an algorithm for solving quadratic equations. In his book 'Trisatika' which contains three hundred (300) Sanskrit slokas where he discussed counting of numbers, natural numbers, zero, measure, multiplication, fraction, division, squares, cubes, rule of three, and mensuration which is the main part of geometry concerned with ascertaining size, length, areas, and volume [VI].

Imaginary number, $\sqrt{-1}$ = i, is an ambiguous notation for numbers that emerged in mathematics since conventional mathematicians could not find the root of the quadratic equation $x^2 + 1 = 0$ in numerical value. At present the author solved the quadratic equation, $x^2 + 1 = 0$, and found the root of the quadratic equation in real numerical value based on the new concept of the Theory of Dynamics of Numbers. Also, the author developed this new concept in different fields of mathematics with the following articles:

I. AN INTRODUCTION TO RECTANGULAR BHATTACHARYYA'S CO-ORDINATES: A NEW CONCEPT. [XV]

II. AN INTRODUCTION TO THEORY OF DYNAMICS OF NUMBERS: A NEW CONCEPT. [XVI]

III. A NOVEL CONCEPT IN THEORY OF QUADRATIC EQUATION. [XVII]

IV. A NOVEL METHOD TO FIND THE EQUATION OF CIRCLES. [XVIII]

V. AN OPENING OF A NEW HORIZON IN THE THEORY OF QUADRATIC EQUATION: PURE AND PSEUDO QUADRATIC EQUATION – A NEW CONCEPT. [XIX]

VI. A NOVEL CONCEPT FOR FINDING THE FUNDAMENTAL RELATIONS BETWEEN STREAM FUNCTION AND VELOCITY POTENTIAL IN REAL NUMBERS IN TWO-DIMENSIONAL FLUID MOTIONS. [XX]

VII. A NEW CONCEPT OF THE EXTENDED FORM OF PYTHAGORAS THEOREM. [XXI]

VIII. A NEW CONCEPT TO PROVE, $\sqrt{-1} = -1$ IN BOTH GEOMETRIC AND ALGEBRAIC METHODS WITHOUT USING THE CONCEPT OF IMAGINARY NUMBERS. [XII]

These new concepts will generate some new structures in the number system. It may be the opening of a new system in the field of mathematics for present and future mathematicians.

III. Formulation of the Problem and its Solution

According to the Theory of Dynamics of Numbers :

1) O (zero) is the starting point of any number. 0 (zero) is a neutral number having no direction but has existence only having a null value.

2) there is an infinite number of directions through which the number can move from the starting point 0 (zero) and back to the starting point 0 (zero) with a vertically opposite direction of motion of numbers.

3) There are three types of numbers : (a) Neutral Number, (b) Count up Number, (c) Count down Number.

4) Neutral Numbers : The discrete numbers which have no direction of motion are called neutral numbers. For example, Neutral 1 = 1, Neutral 2 = 2, Neutral 3 = 3, Neutral 4 = 4, Neutral 5 = 5, etc. The neutral numbers have no symbol over the head of the numbers.

5) Count up Numbers : The numbers which move away from the starting point 0 (zero) are called 'Count up Numbers'. The Count up numbers have a symbol '$\overset{\uparrow}{\_\_}$' over the head of the numbers. For example, count up 2, count up 3, count up 4, count up 5, etc. will be represented as $\overset{\uparrow}{2}$, $\overset{\uparrow}{3}$, $\overset{\uparrow}{4}$, $\overset{\uparrow}{5}$, etc. respectively. The numerical value of count up number is defined as Count up number when the neutral number is prefixed with the addition operation (+).

Therefore $\overset{\uparrow}{2} = +2$, $\overset{\uparrow}{3} = +3$, $\overset{\uparrow}{4} = +4$, $\overset{\uparrow}{5} = +5$ etc.

6) Countdown Numbers: The numbers which move towards the starting point 0 (zero) are called 'Countdown Numbers'. The countdown numbers have a symbol '$\overset{\downarrow}{\_\_}$' over the head of the numbers. For example, Countdown 2, Countdown 3, Countdown 4, Countdown 5, etc. will be represented as $\overset{\downarrow}{2}$, $\overset{\downarrow}{3}$, $\overset{\downarrow}{4}$, $\overset{\downarrow}{5}$ etc. respectively. The numerical value of the Countdown number is defined as the Countdown number when the neutral number is prefixed with the subtraction operation (–). Therefore, $\overset{\downarrow}{2} = -2$, $\overset{\downarrow}{3} = -3$, $\overset{\downarrow}{4} = -4$, $\overset{\downarrow}{5} = -5$ etc. respectively.

7) Neutral numbers may be used as the coefficient of any unknown countable object (quantity). For example: $5x$, $+7x$, $-6x$, $3x^2$, $+4x^2$, $-5x^2$ where 5, 7, 6, 3, 4, 5 are the coefficients of x, x, $-x$, x^2, x^2, $-x^2$ respectively.

We know that multiplication can be represented by the addition of numbers where the number of Count up numbers or numbers of Countdown numbers are always Neutral numbers.

<u>Case</u> – I (In case of Neutral number)

Neutral 3 = 3

(Neutral 3)^2 = $(3)^2 = 3 + 3 + 3 = 9$

(Neutral 3)^3 = $(3^2) \times 3 = 9 + 9 + 9 = 27$

(Neutral 3)^m = 3^m, where m is any integer or fraction number.

Similarly, (Neutral x)^m = x^m, where m is any integer or fraction number.

<u>Case – II</u> (In case of Count up number)

$\overset{\uparrow}{3} = +3$, where 3 is a neutral number

$(\overset{\uparrow}{3})^2 = (\overset{\uparrow}{3}) \times (\overset{\uparrow}{3}) = \overset{\uparrow}{3} + \overset{\uparrow}{3} + \overset{\uparrow}{3} = +3+3+3 = +9 = \overset{\uparrow}{9} = \overset{\uparrow}{3^2}$

$(\overset{\uparrow}{3})^3 = (\overset{\uparrow}{3})^2 \times \overset{\uparrow}{3} = \overset{\uparrow}{9} \times \overset{\uparrow}{3} = \overset{\uparrow}{9} + \overset{\uparrow}{9} + \overset{\uparrow}{9} = +9+9+9 = +27 = \overset{\uparrow}{27} = \overset{\uparrow}{3^3}$

$(\overset{\uparrow}{3})^m = \overset{\uparrow}{3^m}$, where m is any integer or fraction number.

Similarly, $(\overset{\uparrow}{x})^m = \overset{\uparrow}{x^m}$, where m is any integer or fraction number.

Therefore, Count up Number X Count up Number = Count up Number.

<u>Case – III</u> (In case of Countdown number)

$\overset{\downarrow}{3} = -3$ where 3 is a neutral number

$(\overset{\downarrow}{3})^2 = (\overset{\downarrow}{3}) \times (\overset{\downarrow}{3}) = \overset{\downarrow}{3} + \overset{\downarrow}{3} + \overset{\downarrow}{3} = -3 - 3 - 3 = -9 = \overset{\downarrow}{9} = \overset{\downarrow}{3^2}$

$(\overset{\downarrow}{3})^3 = (\overset{\downarrow}{3})^2 \times \overset{\downarrow}{3} = \overset{\downarrow}{9} \times \overset{\downarrow}{3} = \overset{\downarrow}{9} + \overset{\downarrow}{9} + \overset{\downarrow}{9} = -9 - 9 - 9 = -27 = \overset{\downarrow}{27} = \overset{\downarrow}{3^3}$

$(\overset{\downarrow}{3})^m = \overset{\downarrow}{3^m}$, where m is any integer or fraction number.

Similarly, $(\overset{\downarrow}{x})^m = \overset{\downarrow}{x^m}$, where m is any integer or fraction number.

Therefore, Countdown number X Countdown number = Countdown number.

<u>Note</u> :

(a) Count up Number X Count up Number = Count up Number.
(b) Countdown Number X Countdown Number = Countdown Number.
(c) Count up of Count up Number = Count up Number.
(d) Count up of Countdown Number = Countdown Number.
(e) Countdown of Count up Number = Countdown Number.
(f) Countdown of Countdown Number = Count up Number.
(g) Countdown Number X Countdown Number ≠ Countdown of Countdown Number

IV. Fundamental three laws of the Theory of Dynamics of Numbers

(1) 0 (zero) is defined as the starting point of any number. There is an infinite number of directions through which the numbers can move from the starting point 0 (zero) and back to the starting point 0 (zero) with a vertically opposite direction of motion of numbers. The number that is moving away from the starting point 0 (zero) is defined as a count-up number and the number which is moving toward the starting point 0 (zero) is defined as a Countdown number.

(2) The Count up numbers are always greater than or equal to the Countdown numbers. The Count up numbers can move independently but the motion of the Countdown numbers is dependent on the motion of Count up numbers. The motion of the Countdown numbers exists if and only if there are motions of the Count up numbers.

(3) For every equation, the number of the Count up number is always equal to the number of the Countdown number.

Problem – I :

Prove that $\sqrt{-2} = -\sqrt{2}$

Solution :

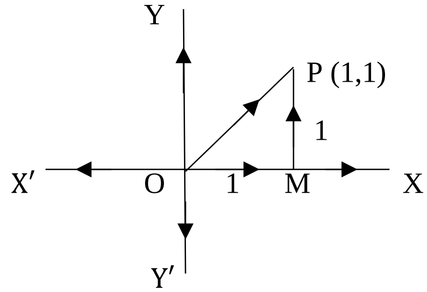

Fig.1 (a).

Let OMP be a right-angled triangle where

$\overrightarrow{OM} = \overrightarrow{I}$ and $\overrightarrow{MP} = \overrightarrow{I}$

According to Pythagoras Theorem from Fig. 1 (a)

$$(\overrightarrow{OP})^2 = (\overrightarrow{OM})^2 + (\overrightarrow{MP})^2 \quad (1)$$

or, $$(\overrightarrow{OP})^2 = (\overrightarrow{1})^2 + (\overrightarrow{1})^2 = +(1)^2 + (1)^2 = +2$$

or, $$\overrightarrow{OP} = \sqrt{+2} = +\sqrt{2} \quad (2)$$

So, $$\overrightarrow{OP} = -\sqrt{2} \quad (3)$$

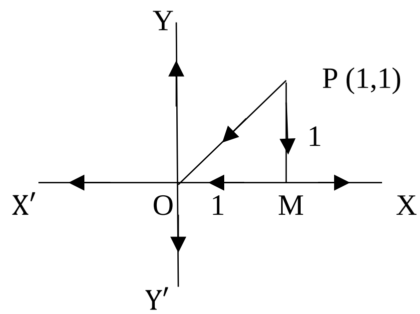

Fig.1 (b).

Again from Fig. 1(b). we have

$$(\overset{\blacktriangle}{\overline{PO}})^2 = (\overset{\blacktriangle}{\overline{PM}})^2 + (\overset{\blacktriangle}{\overline{MO}})^2 \qquad (4)$$

and

$$\overset{\blacktriangle}{\overline{PO}} = \overset{\blacktriangledown}{\overline{OP}},\ \overset{\blacktriangle}{\overline{PM}} = \overset{\blacktriangledown}{\overline{MP}} \text{ and } \overset{\blacktriangle}{\overline{MO}} = \overset{\blacktriangledown}{\overline{OM}}$$

So, $(\overset{\blacktriangledown}{\overline{OP}})^2 = (\overset{\blacktriangledown}{\overline{MP}})^2 + (\overset{\blacktriangledown}{\overline{OM}})^2$

or, $(\overset{\blacktriangledown}{\overline{OP}})^2 = \overset{\blacktriangledown}{\overline{MP^2}} + \overset{\blacktriangledown}{\overline{OM^2}}$ [since, $(\overset{\blacktriangledown}{\overline{x}})^2 = \overset{\blacktriangledown}{\overline{x^2}}$]

or, $(\overset{\blacktriangledown}{\overline{OP}})^2 = \overset{\blacktriangledown}{\overline{(1)^2}} + \overset{\blacktriangledown}{\overline{(1)^2}} = -1 - 1 = -2$ (5)

or, $\overset{\blacktriangledown}{\overline{OP}} = \sqrt{-2}$ (6)

From the equation (6) and (3) we have

$$\sqrt{-2} = -\sqrt{2}$$

<u>Problem</u> – II

Solve : $x^2 + 2 = 0$ by geometric and algebraic method

<u>Solution by Geometric Method</u>

$x^2 + 2 = 0$ (1)

or, $x^2 = -2$

or, $x = \sqrt{-2}$

Therefore $x = -\sqrt{2}$ [Using Problem – 1, the geometric method, we get $\sqrt{-2} = -\sqrt{2}$]

<u>Solution by Algebraic Method</u>

$x^2 + 2 = 0$ (1)

Since, 2 > 0, the inherent nature of x in x^2 will be Countdown $x = \overset{\blacktriangledown}{\overline{x}}$

So, according to the Theory of Dynamics of Numbers, the equation (1) takes the form

$(\overset{\blacktriangledown}{\overline{x}})^2 + \overset{\blacktriangle}{\overline{2}} = 0$ (2)

or, $\overset{\blacktriangledown}{\overline{x^2}} + \overset{\blacktriangle}{\overline{2}} = 0$ [Since, $(\overset{\blacktriangledown}{\overline{x}})^2 = \overset{\blacktriangledown}{\overline{x^2}}$] (3)

According to the 3rd law of the Theory of Dynamics of Numbers, we have

$x^2 = 2$

or, $x = \sqrt{2}$

In the equation (1), x represents Countdown $x = \overset{\downarrow}{x} = -x$

$\therefore x = -\sqrt{2}$

<u>Problem – 3</u>

Prove that $\sqrt{-25} = -5$

Solution

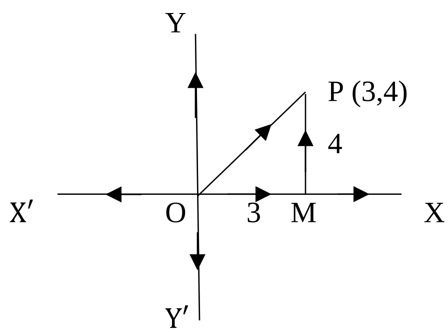

Fig. 2(a)

Let OMP is a right-angled triangle where

$\overset{\uparrow}{\text{OM}} = \overset{\uparrow}{3}$ and $\overset{\uparrow}{\text{MP}} = \overset{\uparrow}{4}$

According to the Pythagoras Theorem

$$(\overset{\uparrow}{\text{OP}})^2 = (\overset{\uparrow}{\text{OM}})^2 + (\overset{\uparrow}{\text{MP}})^2 \quad (1)$$

$$(\overset{\uparrow}{\text{OP}})^2 = (+3)^2 + (+4)^2 = +9 + 16 = +25$$

$$\therefore \overset{\uparrow}{\text{OP}} = \sqrt{+25} = +5 \quad (2)$$

So, $\overset{\downarrow}{\text{OP}} = -5$ (3)

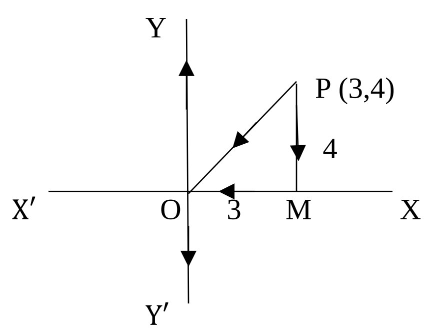

Fig. 2 (b).

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From Fig. 2(b). we have

$$\overset{\blacktriangle}{\overline{PO}} = \overset{\blacktriangledown}{\overline{OP}},\ \overset{\blacktriangle}{\overline{MO}} = \overset{\blacktriangledown}{\overline{OM}} \text{ and } \overset{\blacktriangle}{\overline{PM}} = \overset{\blacktriangledown}{\overline{MP}} \qquad (4)$$

Now,

$$\left(\overset{\blacktriangle}{\overline{PO}}\right)^2 = \left(\overset{\blacktriangle}{\overline{PM}}\right)^2 + \left(\overset{\blacktriangle}{\overline{MO}}\right)^2$$

$$= \left(\overset{\blacktriangledown}{\overline{PM}}\right)^2 + \left(\overset{\blacktriangledown}{\overline{MO}}\right)^2$$

$$= \overset{\blacktriangledown}{\overline{MP}}{}^2 + \overset{\blacktriangledown}{\overline{OM}}{}^2 \ \left[\because \left(\overset{\blacktriangledown}{\overline{x}}\right)^2 = \overset{\blacktriangledown}{\overline{x}}{}^2\right]$$

$$= \overset{\blacktriangledown}{\overline{4}}{}^2 + \overset{\blacktriangledown}{\overline{3}}{}^2$$

$$= -16 - 9$$

$$= -25 \qquad (5)$$

or, $\left(\overset{\blacktriangledown}{\overline{OP}}\right)^2 = -25 \ \left[\because \overset{\blacktriangle}{\overline{PO}} = \overset{\blacktriangledown}{\overline{OP}}\right]$

or, $\overset{\blacktriangledown}{\overline{OP}} = \sqrt{-25}$ (6)

From the equation (6) and (3) we get

$$\sqrt{-25} = -5$$

Hence the proof.

Problem – 4

Solve : $x^2 + 25 = 0$ by the geometric and algebraic methods.

Solution by Geometric Method

$$x^2 + 25 = 0 \qquad (1)$$

or, $x^2 = -25$

or, $x = \sqrt{-25}$

Using Problem 3, the geometric method, we get $\sqrt{-25} = -5$

Therefore, $x = -5$

Solution by Algebraic Method

$$x^2 + 25 = 0 \qquad (1)$$

Since, 25 > 0, the inherent nature of x in x^2 is Countdown $x = \overset{\blacktriangledown}{\overline{x}}$

So, according to the Theory of Dynamics of Numbers, the equation (1) takes the form

$$(\overset{\downarrow}{\overline{x}})^2 + \overset{\uparrow}{\overline{25}} = 0 \qquad (2)$$

or, $\overset{\downarrow}{\overline{x}}{}^2 + \overset{\uparrow}{\overline{25}} = 0 \quad [\because (\overset{\downarrow}{\overline{x}})^2 = \overset{\downarrow}{\overline{x^2}}\,]$

According to the 3 rd law of the Theory of Dynamics of Numbers, we have

$$x^2 = 25 \qquad (4)$$

or, $x = \sqrt{25}$

or, $x = 5 \qquad (5)$

We know that according to the inherent nature of the root of the equation (1) is $\overset{\downarrow}{\overline{x}}$

From the equation (5) we have

$$x = \overset{\downarrow}{\overline{5}} = -5$$

So, $x = -5$

Significance : According to the Theory of Dynamics of Numbers, we can find the square root of any negative number in the geometric and algebraic methods.

Working Rule : The square root of any negative number is equal to square root of that neutral numbe with prefixed subtraction operator (–) i.e. $\sqrt{-7} = -\sqrt{7}$, where 7 is the neutral number of -7.

Definition of Quadratic Equation (According to the conventional method)

An equation in one unknown quantity (say x) in the form of $ax^2 + bx + c = 0$ is called a quadratic equation or an equation of the second degree. Here a, b, c are constants and $a \neq 0$ while b and c may be zero; a is called the coefficient of x^2, b, the coefficient of x and c, the constant term.

Root of the Quadratic Equation (According to the conventional Method)

The value of x which satisfy the equation are called the roots of the quadratic equation

According to the Theory of Dynamics of Numbers the three identities are as follows :

i) $x^2 = (\text{neutral } x)^2$

ii) $x^2 = (\overset{\uparrow}{\overline{x}})^2$

iii) $x^2 = (\overset{\downarrow}{\overline{x}})^2$

<u>Definition of the Quadratic Equation</u> (According to the Theory of Dynamics of Numbers)

The numerical value of the inherent nature of x which satisfies the quadratic equation is called the root of the quadratic equation. The inherent nature of x can be determined uniquely from the nature of the constant term of the quadratic equation $ax^2 + bx + c = 0$ provided the character of the structure of the second – degree expression, $ax^2 + bx + c$ of the quadratic equation $ax^2 + bx + c = 0$ must be in the second degree.

In case of identity (1), the solution of the identity (1) will be neutral $x = x$.

The inherent nature of the unknown quantity of x in x^2 will be neutral $x = x$.

In case of identity (2), the solution of identity (2) will be count up $x = \overset{\uparrow}{x}$.

The inherent nature of the unknown quantity of x in x^2 will be $x = \overset{\uparrow}{x} = +x$.

In case of identity (3), the solution of the identity (3) will be countdown $x = \overset{\downarrow}{x}$.

The inherent nature of the unknown quantity of x in x^2 will be countdown $x = \overset{\downarrow}{x} = -x$

According to the Theory of Dynamics of Numbers three cases may arise to find the inherent nature of root x of the quadratic equation $ax^2 + bx + c = 0$.

<u>Case – I</u>

If $c < 0$, the inherent nature of the root x will be count up $x = \overset{\uparrow}{x} = +x$

<u>Case – II</u>

If $c > 0$, the inherent nature of the root x will be countdown $x = \overset{\downarrow}{x} = -x$

<u>Case – III</u>

If $c = 0$, the inherent nature of the root x will be 0 (zero).

We can find two forms of the quadratic equations : From – I and II

<u>Form – I</u> :

$$ax^2 + bx - c = 0$$

Solve :

$$ax^2 + bx - c = 0$$

Solution :

$$ax^2 + bx - c = 0 \qquad (1)$$

Since, $-c < 0$, the inherent nature of x of the quadratic equation is Count up x or $\overset{\uparrow}{x}$. According to the Theory of Dynamics of Numbers, the quadratic equation (1) takes the form.

$$a\left(\overset{\uparrow}{x}\right)^2 + b\left(\overset{\uparrow}{x}\right) + \overset{\downarrow}{c} = 0$$

or, $\overset{\uparrow}{a x^2} + b\left(\overset{\uparrow}{x}\right) + \overset{\downarrow}{c} = 0$ [since, $\left(\overset{\uparrow}{x}\right)^2 = \overset{\uparrow}{x^2}$]

or, $$\overset{\uparrow}{ax^2 + bx} + \overset{\downarrow}{c} = 0 \qquad (2)$$

According to law III of the Theory of Dynamics of Numbers, we have

$$ax^2 + bx = c \qquad (3)$$

$$\Rightarrow x^2 + \frac{b}{a}x = \frac{c}{a}$$

$$\Rightarrow x^2 + 2\frac{b}{2a}x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 + \frac{c}{a}$$

$$\Rightarrow \left(x + \frac{b}{2a}\right)^2 = \frac{b^2+4ac}{4a^2}$$

$$\Rightarrow x + \frac{b}{2a} = \sqrt{\frac{b^2+4ac}{4a^2}}$$

$$\Rightarrow x = -\frac{b}{2a} + \frac{\sqrt{b^2+4ac}}{2a}$$

$$\Rightarrow x = \frac{-b+\sqrt{b^2+4ac}}{2a}\text{, a neutral number} \qquad (4)$$

From the quadratic equation (2) we know that the inherent nature of an unknown quantity

$$x = \frac{-b+\sqrt{b^2+4ac}}{2a} \text{ will be Count up } x$$

So, the value of x will be $\overset{\uparrow}{x} = +x$

$$\therefore \overset{\uparrow}{x} = \overset{\uparrow}{\frac{-b+\sqrt{b^2+4ac}}{2a}} = +\frac{-b+\sqrt{b^2+4ac}}{2a}$$

$$\therefore x = +\frac{-b+\sqrt{b^2+4ac}}{2a}$$

<u>Form – II</u> :

$$ax^2 + bx + c = 0 \ [\ a > 0\ ]$$

Solve

$$ax^2 + bx + c = 0$$

Solution

$$ax^2 + bx + c = 0 \qquad (1)$$

Since c > 0, the inherent nature of x in the quadratic equation (1) will be Countdown $x = \overset{\searrow}{x}$. According to the Theory of Dynamics of Numbers, the quadratic equation (1) takes the form.

$$a\,(\overset{\searrow}{x})^2 + b\,(\overset{\searrow}{x}) + \overset{\nearrow}{c} = 0$$

or, $$a\,\overset{\searrow}{x^2} + b\,(\overset{\searrow}{x}) + \overset{\nearrow}{c} = 0 \quad [\text{since, } (\overset{\searrow}{x})^2 = \overset{\searrow}{x^2}\,]$$

$$\overset{\searrow}{ax^2 + bx} + \overset{\nearrow}{c} = 0 \qquad (2)$$

According to the law – III of the Theory of Dynamics of Numbers we have,

$$ax^2 + bx = c \qquad (3)$$

$$\Rightarrow x^2 + \frac{b}{a}x = \frac{c}{a}$$

$$\Rightarrow x^2 + 2.\frac{b}{2a}x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 + \frac{c}{a}$$

$$\Rightarrow \left(x + \frac{b}{2a}\right)^2 = \frac{b^2 + 4ac}{4a^2}$$

$$\Rightarrow x + \frac{b}{2a} = \sqrt{\frac{b^2+4ac}{4a^2}}$$

$$\Rightarrow x = -\frac{b}{2a} + \frac{\sqrt{b^2+4ac}}{2a}$$

$$\Rightarrow x = \frac{-b + \sqrt{b^2+4ac}}{2a}, \text{ a neutral number} \qquad (4)$$

From the quadratic equation (2) we know that the inherent nature of unknown quantity

$$x = \frac{-b + \sqrt{b^2+4ac}}{2a}$$

will be countdown $x = \overset{\searrow}{x}$

Therefore, the numerical value of $\overset{\searrow}{x} = -x$

So, $x = -\frac{-b + \sqrt{b^2+4ac}}{2a}$

<u>Problem – 5</u>

Solve :

$x^2 + 2x - 2 = 0$

Solution :

$$x^2 + 2x - 2 = 0 \qquad (1)$$

Since, $-2 < 0$ the inherent nature of one unknown quantity x will be count up $x = \overset{\uparrow}{x}$

Now, the discriminant of the quadratic equation (1) compared with the general form of the quadratic equation $ax^2 + bx + c = 0$ will be $b^2 - 4ac = (2)^2 - 4.1.(-2) = 4 + 8 = 12$. Therefore, $\sqrt{b^2 - 4ac} = \sqrt{12}$, is a quadratic surd. According to the Theory of Dynamics of Numbers the equation (1) is a pure quadratic equation and according to the inherent nature of x the equation (1) will take the form.

$$(\overset{\uparrow}{x})^2 + 2(\overset{\uparrow}{x}) + \overset{\downarrow}{2} = 0 \qquad (2)$$

or, $\overset{\uparrow}{x^2} + 2(\overset{\uparrow}{x}) + \overset{\downarrow}{2} = 0$ {since, $(\overset{\uparrow}{x})^2 = \overset{\uparrow}{x^2}$}

or, $\overset{\uparrow}{x^2 + 2x} + \overset{\downarrow}{2} = 0 \qquad (3)$

According to the law – III of the Theory of Dynamics of Numbers

$x^2 + 2x = 2$

or, $x^2 + 2x + 1 = 2 + 1$

or, $(x + 1)^2 = 3$

or, $x + 1 = \sqrt{3}$

or, $x = -1 + \sqrt{3} \qquad (4)$

Since the inherent nature of x is Count up $x = \overset{\uparrow}{x}$ and the numerical value of $\overset{\uparrow}{x} = +x$, we have

$\overset{\uparrow}{x} = \overset{\uparrow}{(-1 + \sqrt{3})} = +(-1 + \sqrt{3}) = -1 + \sqrt{3}$

$\therefore\ x = -1 + \sqrt{3}$

or, $x = -1 + 1.732$

or, $x = +0.732$ (approx.)

is the root of the pure quadratic equation (1)

Now, let us verify whether $\overset{\uparrow}{x}$ satisfies the pure quadratic equation $x^2 + 2x - 2 = 0$ or not.

Let us put the value of $\overset{\uparrow}{x}$ instead of x in the expression $x^2 + 2x - 2$ of the quadratic equation $x^2 + 2x - 2 = 0$, we get

$(\vec{x})^2 + \overrightarrow{2x} - 2$

$= \{+ (- 1 + \sqrt{3}) \}^2 + 2\{+ (- 1 + \sqrt{3})\} - 2$

$= + \{ 1 - 2\sqrt{3} + 3\} \ - 2 + 2\sqrt{3} \ - 2$

$= + 4 - 2\sqrt{3} \ - 4 + 2\sqrt{3}$

$= 0$

So, the numerical value of $\vec{x} = - 1 + \sqrt{3}$ satisfies the pure quadratic equation (1)

Hence, $x = - 1 + \sqrt{3}$

or, $x = - 1 + 1.732 = +0.732$ (approx)

is the root of the quadratic equation (1).

Now, let us solve the equation (1) by conventional method.

$$x^2 + 2x - 2 = 0 \qquad (1)$$

So, $x = \frac{- 2 \pm \sqrt{2^2 - 4.1.-2}}{2.1} = \frac{- 2 \pm \sqrt{12}}{2} = \frac{- 2 \pm 2\sqrt{3}}{2} = -1 \pm \sqrt{3}$.

<u>Problem – 2</u>

Find the length and breadth of a square land whose area is 81 square ft.

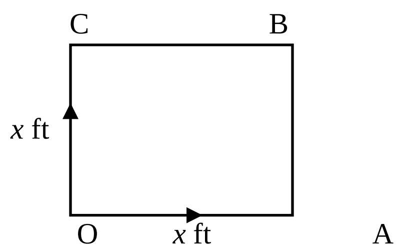

Fig. 3

Solution :

Let us construct a square land with a length and breadth of x ft. each and the area of the square land is 81 sq. ft.

Let us consider the square land whose length OA = x ft and breadth OC = x ft.

Therefore, the area of the square land is x^2 Sq. ft.

According to the problem

$$x^2 = 81 \qquad (1)$$

So, $x = \pm \sqrt{81} = \pm 9$ (2)

Therefore, $x = +9$ ft. and $x = -9$ ft.

$\therefore$ The length OA = + 9 ft. and breadth OC = + 9 ft.

and also

The length OA = – 9 ft. and breadth OC = – 9 ft.

Now, the question arise whether we can construct a square land with length = – 9 ft. and breadth = – 9 ft.

The answer will be "absurd' though according to conventional method $x = -9$ ft. is true.

Why it has happened ?

Because in conventional method the inherent nature of the root of the equation (1) has not been considered.

Now, let us solve the equation $x^2 = 81$ according to the Theory of Dynamics of Numbers.

$$x^2 = 81 \qquad (1)$$

or, $$x^2 - 81 = 0 \qquad (2)$$

Since, $-81 < 0$, the inherent nature of x is Count up $x = \overset{\uparrow}{x} = +x$ only.

$\therefore$ The equation (2) takes the form : $(\overset{\uparrow}{x})^2 + \overset{\downarrow}{81} = 0$

$$\overset{\uparrow}{x^2} + \overset{\downarrow}{81} = 0 \text{ [Since, } (\overset{\uparrow}{x})^2 = \overset{\uparrow}{x^2} \text{]} \qquad (3)$$

According to the Law – III of the Theory of Dynamics of Numbers

$x^2 = 81$, when x^2 is a neutral number.

Or, $x = \sqrt{81} = 9$

Here, the inherent nature of x is $\overset{\uparrow}{x} = +x$ only.

$\therefore$ $x = +9$ ft. only, is the root of the equation (1)

$\therefore$ Length and breadth of the square land = + 9 ft. only.

<u>Observation</u> :

(A) In the conventional method the inherent nature of one unknown quantity (say x) of the quadratic equation has not been taken into account though the numerical value of the roots of the equation $x = -1 + \sqrt{3}$ and $x = -1 - \sqrt{3}$ satisfy the quadratic equation (1). On the contrary, we can find in problem – 2 where two roots of the quadratic equations do not justify according to the conventional definition of the root of the quadratic equations.

(B) The square root of the discriminant of the quadratic equation $x^2 + 2x - 2 = 0$ is $\sqrt{12}$ which is a quadratic surd. According to the Theory of Dynamics of Numbers, equation (1) represents a pure quadratic equation. Hence the quadratic equation (1) must have one root only, i.e., $x = -1 + \sqrt{3}$ which satisfies not only the equation (1) but also the inherent nature of the root of the equation (1).

Problem – 3

Solve :

$$x^2 + 2x + 2 = 0$$

Solution :

$$x^2 + 2x + 2 = 0 \qquad (1)$$

Since 2 > 0, the inherent nature of one unknown quantity x of the quadratic equation (1) will be Countdown $x = \overset{\downarrow}{x}$.

Here, the discriminant of the quadratic equation (1) compared with the general form of quadratic equation $ax^2 + bx + c = 0$ will be $b^2 - 4ac = (2)^2 - 4.1.2 = 4 - 8 = -4 < 0$. According to the novel concept in the theory of quadratic equations, the equation (1) is a pure quadratic equation. According to the Theory of Dynamics of Numbers, the equation (1) takes the form

$$x^2 + \overset{\downarrow}{2x} + \overset{\uparrow}{2} = 0 \qquad (2)$$

According to the law – III of the Theory of Dynamics of Numbers

$$x^2 + 2x = 2 \qquad (3)$$

$$\Rightarrow x^2 + 2x + 1 = 1 + 2$$

$$\Rightarrow (x + 1)^2 = 3$$

$$\Rightarrow x + 1 = \sqrt{3}$$

$$\Rightarrow x = -1 + \sqrt{3} \qquad (4)$$

Since the inherent nature of x is Countdown $x = \overset{\downarrow}{x}$ and the numerical value of $\overset{\downarrow}{x} = -x$, we have

$$\overset{\downarrow}{x} = \overset{\downarrow}{(-1 + \sqrt{3})} = -(-1 + \sqrt{3}) = -(-1 + 1.732) = -0.732 \text{ (approx.)}$$

Therefore, $x = -0.732$ (approx.) (5)

or, $\overset{\downarrow}{x} = 0.732$ (6)

is the root of the pure quadratic equation (1)

Let us verify whether x satisfies the equation (1) or not

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Now, let us put the value of x from the equation (6) in the expression :

$$x^2 + 2x + 2 \qquad (7)$$

We get,

$$(0.732)^2 + 2.(0.732) + 2$$

$$= (0.732)^2 + 2.(0.732) + 2$$

$$= 0.535824 + 2.(0.732) + 2$$

$$= -0.535824 + 2.(-0.732) + 2$$

$$= -0.535824 - 1.464 + 2$$

$$= -1.999824 \text{ (approx.)} + 2$$

$$\cong 2 - 2 = 0$$

Again, let us solve the equation (1) by conventional method using imaginary number.

$$x^2 + 2x + 2 = 0$$

Then,

$$x = \frac{-2 \pm \sqrt{2^2 - 4.1.2}}{2}$$

or, $$x = \frac{-2 \pm \sqrt{-4}}{2}$$

or, $$x = \frac{-2 \pm \mathrm{i}\,2}{2}$$

or, $$x = -1 \pm \mathrm{i}$$

<u>Observations</u> :

I. We can find the numerical value of the root of the quadratic equation by using the Theory of Dynamics of Numbers in the quadratic equations.

II. We cannot find the numerical value of the root of the quadratic equation by using imaginary numbers since the numerical value of $\sqrt{-1} = \mathrm{i}$ is unknown.

V. Conclusion

The Theory of Dynamics of Numbers is one of the greatest achievements in mathematics in this era. Imaginary number, $\sqrt{-1} = \mathrm{i}$, is an ambiguous notation for numbers that emerged in mathematics because the conventional mathematicians failed to find the root of the quadratic equation, $x^2 + 1 = 0$ in its numerical value. But the Theory of

Dynamics of the Number system provides a unique representation of the square root of every number such as neutral numbers, Countup numbers (positive numbers), Countdown numbers (negative numbers) in numerical values, and represents the arithmetic and algebraic structure of the figure. By using the new concept we can find the root or roots of the quadratic equation $ax^2 + bx + c = 0$ even if the discriminant $b^2 - 4ac < 0$ in numerical values without using the concept of imaginary numbers.

Based on the Theory of Dynamics of Numbers the author developed a new structure of the plane coordinate geometry system where all four axes are positive which is known as the 'Rectangular Bhattacharyyas Coordinate System' instead of the 'Cartesin Coordinate System. The Theory of Dynamics of Numbers has wide application in mathematics, science, and technology especially in computer science, artificial intelligence, and cryptosystem. The Theory of Dynamics of Numbers opened a new door for researchers to develop new concepts in mathematics to perform in various fields of scientific innovations.

Conflict of Interest:

The author declares that there was no conflict of interest regarding this paper.

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