JOURNAL OF MECHANICS OF CONTINUA AND MATHEMATICAL SCIENCES
www.journalimcms.org

ISSN (Online) : 2454 -7190 Vol.-18, No.-12, December (2023) pp 20-31 ISSN (Print) 0973-8975

OPTIMAL POLICY OF THE INTERVAL EPQ MODEL USING *C-L* INTERVAL INEQUALITY

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https://doi.org/10.26782/jmcms.2023.12.00002

(Received: October 25, 2023; Revised: December 05, 2023; Accepted: December 09, 2023)

Abstract

The objective of this work is to study the optimal policy of the classical economic production quantity (EPQ) model under interval uncertainty using interval inequality. To serve this purpose existing arithmetic mean-geometric mean (AM-GM) inequality is extended for interval numbers using c-L interval order relation. Then, using the said AM-GM interval inequality, the optimal policy of the classical EPQ model in the interval environment is developed. Thereafter, the optimality policy of the classical EPQ model in a crisp environment is obtained as a special case of that of the interval environment. Finally, all the optimality results are illustrated with the help of some numerical examples.

Keywords: Interval order relation, Generalised AM-GM inequality, *c-L* minimizer, Interval EPQ, *c-L* optimal policy

Nomenclature

Notation	Description
$\mathbb{R}$	The set of real numbers
$\mathbb{N}$	The set of natural numbers
$[O_L, O_U]$	Interval-valued ordering cost per order
$[h_L, h_U]$	Interval-valued carrying cost per unit per unit time
$[D_L, D_U]$	Interval-valued demand rate

$\left[k_L, k_U\right]$	Interval-valued production rate
$\left[Q_L, Q_U\right]$:	Interval-valued ordering quantity
$\left[T_L, T_U\right]$:	Interval-valued cycle length
$\left[t_{1L}, t_{1U}\right]$	Interval-valued production period
$\left[t_{2L}, t_{2U}\right]$	Interval-valued no production time
$\left[TC_L, TC_U\right]$	Interval-valued total cost
$\left[AC_L\left(\cdot\right), AC_U\left(\cdot\right)\right]$	Interval-valued average cost

I. Introduction

The study of optimal policy of an inventory problem is very important for appropriate controlling of inventory. Studying of exact optimal policy for inventory problems under uncertainty is a challenging task. However, in the literature, there are available few approaches viz. stochastic, fuzzy, fuzzy-stochastic and interval approaches to get a rough estimate of the optimal policy of such problems. Among these approaches, the interval approach is simpler than other approaches and, in this approach, imprecise parameters are presented by closed intervals. In the existing literature, the optimal policies of several inventory problems [X, VII, III] with various assumptions under interval uncertainty were studied using interval differential equations and interval optimization techniques.

On the other hand, the concept of arithmetic mean-geometric mean (AM-GM) inequality plays a significant role in studying the optimal policy for inventory control problems. In this area, for the first time, Grubbstrom [V] studied the optimality of the classical EOQ model using the AM-GM inequality. Following the previous work, Grubbsrom and Erdem [II] analyzed the optimality of the EOQ model with shortages using the AM-GM inequality. Cardenas-Barron [II] used the same technique to obtain the optimal policy of the classical EPQ model. Further, Gani et al. [IV] studied the optimality of an imprecise EOQ model using fuzzy AM-GM inequality. Finally, Rahman and Khatun [XII] analyzed the optimal policy of the classical EOQ model in an interval environment using interval AM-GM inequality. However, till now, no one studied the optimal policy of the EPQ model in an interval environment using interval AM-GM inequality.

In this work, the optimal policy of the extended EPQ model in an interval environment is studied using c-L AM-GM inequality. In this extended EPQ model, all the inventory components including cycle length are considered as interval-valued. After that using interval mathematics and *c*-*L* interval inequality the optimality conditions of the interval-valued average cost of the model are obtained. Finally, all the optimal results are validated by considering a set of numerical examples.

II. Basic Concepts of Intervals

An interval number is a closed and bounded interval $A = [a_L, a_U] = \{x \in \mathbb{R}: a_L \leq x \leq a_U, a_L \& a_U \in \mathbb{R}\}$. Any real number $a \in \mathbb{R}$ can be expressed as an interval number $[a, a]$ as degenerate with zero width. Also, the interval number can be expressed in terms of center and radius form as $A = \langle a_c, a_r \rangle = \{x \in \mathbb{R}: a_c - a_r \leq x \leq a_c + a_r\}$, where $a_c = \frac{a_U + a_L}{2}$ and $a_r = \frac{a_U - a_L}{2}$.

In this section, the definition of arithmetic operations of intervals [VIII, XIII] is reported as follows:

Definition 1: Let us consider $A = [a_L, a_U], B = [b_L, b_U]$ be any two interval numbers. Then arithmetical operations such as addition, subtraction, scalar multiplication, multiplication, and division of interval numbers are given below:

(i) Addition:

$$A + B = [a_L, a_U] + [b_L, b_U] = [a_L + b_L, a_U + b_U]$$

(ii) Subtraction:

$$A - B = [a_L, a_U] - [b_L, b_U] = [a_L - b_U, a_U - b_L]$$

(iii) Scalar multiplication:

$$\lambda A = \lambda [a_L,\ a_U] = \begin{cases} [\lambda a_L,\ \lambda a_U] \text{ if } \lambda \geq 0 \\ [\lambda a_U,\ \lambda a_L] \text{ if } \lambda < 0, \end{cases} \text{ for any real number } \mathbb{R}$$

(iv) Multiplication:

$$AB = [\min\{a_L b_L, a_L b_U, a_U b_L, a_U b_U\}, \max\{a_L b_L, a_L b_U, a_U b_L, a_U b_U\}]$$

(v) Hukuhara type Division:

$$\frac{A}{B} = \left[min\left\{\frac{a_L}{b_L}, \frac{a_U}{b_U}\right\}, max\left\{\frac{a_L}{b_L}, \frac{a_U}{b_U}\right\}\right] \text{ provided } 0 \notin B$$

(vi) n-th power: $A^n = [a_L^n, a_U^n]$, for $a_L \geq 0, n \in \mathbb{N}$

(vii) n-th root: $A^{1/n} = \left[a_L^{1/n}, a_U^{1/n}\right], for\, a_L \geq 0, n \in \mathbb{N}$

III. *c-L* Interval order relation and *c-L* Arithmetic Mean (AM)-Geometric Mean (GM) inequality:

In this section, following Bhunia and Samanta's approach [I], new definitions of interval order relations have been proposed. Then using this order relation, *c-L* AM-GM inequality for interval numbers has been derived.

Definition 2: Let $A = [a_L, a_U] = \langle a_c, a_r \rangle$ and $B = [b_L, b_U] = \langle b_c, b_r \rangle$ be two intervals.

Then A is greater or equal to B which is denoted by $A \geq_L^c B$ and defined by

$$A \geq_L^c B \Leftrightarrow \begin{cases} a_c \geq b_c, if \ \ a_c \neq b_c \\ a_L \geq b_L, if \ \ a_c = b_c \end{cases}$$

and

$$A >_L^c B \Leftrightarrow A \geq_L^c B \,\&\, A \neq B.$$

Definition 3: The interval A is said to be less or equal to B which is denoted by $A \leq_L^c B$ and defined by $A \leq_L^c B \, iff \, B \geq_L^c A$.

Proposition 1: Let $\{A_i = [a_{iL}, a_{iU}] : a_{iL} \geq 0, \, i = 1, 2, ..., n\}$ be the set of n non-negative interval numbers. Then $\frac{A_1 + A_2 + ... + A_n}{n} \geq_L^c \sqrt[n]{A_1 A_2 ... A_n}$ and the equality holds iff $A_1 = A_2 = \ldots = A_n$, where the symbol $\geq_L^c$ is defined in **Definition 2**.

Proof.

Using the addition and scalar multiplication of interval numbers, $\frac{A_1 + A_2 + ... + A_n}{n}$ can be written as

$$\frac{A_1 + A_2 + ... + A_n}{n} = \left[\frac{a_{1L} + a_{2L} + ... + a_{nL}}{n}, \frac{a_{1U} + a_{2U} + ... + a_{nU}}{n} \right].$$

Again, using the multiplication and n-th root of interval numbers, one can get

$$\sqrt[n]{A_1 A_2 ... A_n} = \sqrt[n]{[a_{1L} a_{2L} ... a_{nL}, a_{1U} a_{2U} ... a_{nU}]} = \left[\sqrt[n]{a_{1L} a_{2L} ... a_{nL}}, \sqrt[n]{a_{1U} a_{2U} ... a_{nU}} \right]$$

Now, applying AM $\geq$ GM on the sets of non-negative real numbers $\{a_{1L}, a_{2L}, ..., a_{nL}\}$ & $\{a_{1U}, a_{2U}, ..., a_{nU}\}$, we get

$$\frac{a_{1L} + a_{2L} + ... + a_{nL}}{n} \geq \sqrt[n]{a_{1L} a_{2L} ... a_{nL}} \tag{1}$$

and

$$\frac{a_{1U} + a_{2U} + ... + a_{nU}}{n} \geq \sqrt[n]{a_{1U} a_{2U} ... a_{nU}} \tag{2}$$

and equality holds iff

$$a_{1L} = a_{2L} = ... = a_{nL} \;\&\; a_{1U} = a_{2U} = ... = a_{nU} \tag{3}$$

Now, (1)-(2) implies,

$$\left(\frac{a_{1L} + a_{2L} + ... + a_{nL}}{n} + \frac{a_{1U} + a_{2U} + ... + a_{nU}}{n} \right) / 2 \geq \left(\sqrt[n]{a_{1L} a_{2L} ... a_{nL}} + \sqrt[n]{a_{1U} a_{2U} ... a_{nU}} \right) / 2 \tag{4}$$

Now, two cases may arise

Case-I:

$$\left(\frac{a_{1L}+a_{2L}+\ldots+a_{nL}}{n}+\frac{a_{1U}+a_{2U}+\ldots+a_{nU}}{n}\right)/2\neq\left(\sqrt[n]{a_{1L}a_{2L}\ldots a_{nL}}+\sqrt[n]{a_{1U}a_{2U}\ldots a_{nU}}\right)/2$$

Then (4) implies that $\frac{A_1+A_2+\ldots+A_n}{n}\geq_L^c \sqrt[n]{A_1A_2\ldots A_n}$,for all $n\in\mathbb{N}$,

Case-II:

$$\left(\frac{a_{1L}+a_{2L}+\ldots+a_{nL}}{n}+\frac{a_{1U}+a_{2U}+\ldots+a_{nU}}{n}\right)/2=\left(\sqrt[n]{a_{1L}a_{2L}\ldots a_{nL}}+\sqrt[n]{a_{1U}a_{2U}\ldots a_{nU}}\right)/2,$$

then

$$\frac{a_{1L}+a_{2L}+\ldots+a_{nL}}{n}\geq\sqrt[n]{a_{1L}a_{2L}\ldots a_{nL}}$$

implies, $\frac{A_1+A_2+\ldots+A_n}{n}\geq_L^c \sqrt[n]{A_1A_2\ldots A_n}$,for all $n\in\mathbb{N}$,

Hence, in both cases,

$$\frac{A_1+A_2+\ldots+A_n}{n}\geq_L^c \sqrt[n]{A_1A_2\ldots A_n},\text{for all } n\in\mathbb{N}.$$

Now, the equality holds iff

$$\frac{a_{1L}+a_{2L}+\ldots+a_{nL}}{n}=\sqrt[n]{a_{1L}a_{2L}\ldots a_{nL}}$$

and

$$\frac{a_{1U}+a_{2U}+\ldots+a_{nU}}{n}=\sqrt[n]{a_{1U}a_{2U}\ldots a_{nU}}$$

$$\Leftrightarrow a_{1L}=a_{2L}=\ldots=a_{nL}\ \&\ a_{1U}=a_{2U}=\ldots=a_{nU}$$

$$\Leftrightarrow [a_{1L},a_{1U}]=[a_{2L},a_{2U}]=\ldots=[a_{nL},a_{nU}]$$

$$\Leftrightarrow A_1=A_2=\ldots=A_n.$$

This completes the proof.

IV. Model Formulation of Interval EPQ model

In this section, motivated by the work of Rahman and Khatun [XII], we have studied the optimal policy of the non-deterministic classical EPQ model in interval environment using *c*-*L* AM-GM inequality.

Assumptions:

The model is formulated under the following assumptions:

(i) The demand is interval-valued.

(ii) Holding/carrying cost and ordering cost/ set up cost are interval-valued.

(iii) The production rate is interval-valued.

(iii) The inventory system deals with a single item or product.

(iv) Inventory planning/time horizon is infinite.

(v) Shortages are not allowed.

(vi) Lead time is constant and Cycle length is interval valued.

(viii) Purchase price and reorder costs do not vary with the quantity ordered.

Mathematical formulation:

In this model, each interval-valued production cycle time $[T_L, T_U]$ consists of two parts $[t_{1L}, t_{1U}]$ $[t_{2L}, t_{2U}]$, where

(i) $[t_{1L}, t_{1U}]$ is the interval-valued time period during which the stock increases at a constant rate $[k_L, k_U]\Theta_{gH}[D_L, D_U]$.

(ii) $[t_{2L}, t_{2U}]$ is the period during which there is no production and the stock level decreases at the rate $[D_L, D_U]$.

Let $[S_L, S_U]$ be the stock available at the end of time $[t_{1L}, t_{1U}]$ which is consumed during the remaining period $[t_{2L}, t_{2U}]$ at the consumption rate $[D_L, D_U]$.

Now, $[k_L, k_U]\Theta_{gH}[D_L, D_U] = [\min\{k_L - D_L, k_U - D_U\}, \max\{k_L - D_L, k_U - D_U\}]$

$$= \left[k_L', k_U'\right], \text{ say.}$$

Then $$\left[k_L', k_U'\right]\cdot[t_{1L}, t_{1U}] = [S_L, S_U], \tag{5}$$

Since the total quantity produced during the production period $[t_{1L}, t_{1U}]$ is $[Q_L, Q_U]$,

$$Q_L = k_L t_{1L},\ Q_U = k_U t_{1U} \tag{6}$$

From (5) and (6), we have

$$S_L = \frac{Q_L k_L'}{k_L},\ S_U = \frac{Q_U k_U'}{k_U}$$

Again, $Q_L = D_L T_L$, $Q_U = D_U T_U$.

Therefore, the interval-valued initial order quantity is

$$[Q_L, Q_U] = [D_L T_L, D_U T_U] \quad (7)$$

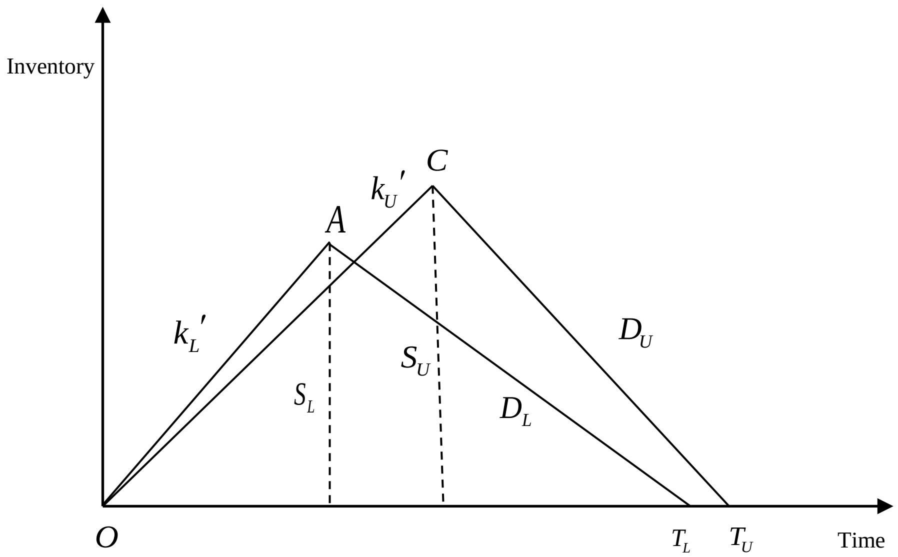

Fig.1. Status of Inventory level at any instant

Now, the related inventory costs corresponding to this model are

***Ordering cost*:** $[O_L, O_U]$

***Carrying cost*:**

Now the bounds of interval-valued inventory carrying cost for the cycle can be calculated as follows:

$$C_L = h_L \times \text{area of the triangle } OAT_L$$

$$= h_L \times \frac{1}{2} T_L S_L$$

$$= h_L \times \frac{1}{2} T_L \frac{Q_L k_L'}{k_L}$$

$$= \frac{h_L D_L k_L' T_L^2}{2k_L}$$

Similarly, $C_U = h_U \times$ area of the triangle $OCT_U = \dfrac{h_U D_U k_U{}' T_U^2}{2k_U}$

$$[C_L, C_U] = \left[\frac{h_L D_L k_L{}' T_L^2}{2k_L}, \frac{h_U D_U k_U{}' T_U^2}{2k_U}\right]$$

Hence, the total cost of this model is as follows:

$$[TC_L, TC_U] = [O_L, O_U] + [C_L, C_U] = [O_L, O_U] + \left[\frac{h_L D_L k_L{}' T_L^2}{2k_L}, \frac{h_U D_U k_U{}' T_U^2}{2k_U}\right]$$

Therefore, the average cost is given by

$$\left[AC_L(T_L, T_U), AC_U(T_L, T_U)\right] = \frac{[TC_L, TC_U]}{[T_L, T_U]} = \frac{[O_L, O_U] + \left[\frac{h_L D_L k_L{}' T_L^2}{2k_L}, \frac{h_U D_U k_U{}' T_U^2}{2k_U}\right]}{[T_L, T_U]}$$

V. Optimality Conditions

In this section, the optimality conditions of the interval-valued average cost for the present model are obtained using the proposed generalized AM-GM inequality.

The interval-valued average cost $\left[AC_L(T), AC_U(T)\right]$ can be rewritten as

$$\left[AC_L(T_L, T_U), AC_U(T_L, T_U)\right] = \frac{[O_L, O_U] + \left[\frac{h_L D_L k_L{}' T_L^2}{2k_L}, \frac{h_U D_U k_U{}' T_U^2}{2k_U}\right]}{[T_L, T_U]}$$

$$= \frac{2\frac{[O_L, O_U]}{[T_L, T_U]} + \left[\frac{h_L D_L k_L{}'}{k_L}, \frac{h_U D_U k_U{}'}{k_U}\right]\frac{[T_L^2, T_U^2]}{[T_L, T_U]}}{2}$$

Now, using generalized AM-GM inequality for interval numbers, we get

$$\left[AC_L(T), AC_U(T)\right] = \frac{\frac{2[O_L, O_U]}{[T_L, T_U]} + \left[\frac{h_L D_L k_L{}'}{k_L}, \frac{h_U D_U k_U{}'}{k_U}\right][T_L, T_U]}{2}$$

$$\geq_L^c \sqrt{\frac{2[O_L, O_U]}{[T_L, T_U]}\left[\frac{h_L D_L k_L{}'}{k_L}, \frac{h_U D_U k_U{}'}{k_U}\right][T_L, T_U]}$$

i.e.,

$$\left[AC_L(T), AC_U(T)\right] \geq_L^c \sqrt{2\left[O_L, O_U\right]\left[\frac{h_L D_L k_L'}{k_L}, \frac{h_U D_U k_U'}{k_U}\right]} \quad (8)$$

Equality holds in (8), if

$$\frac{2\left[O_L, O_U\right]}{\left[T_L, T_U\right]} = \left[\frac{h_L D_L k_L'}{k_L}, \frac{h_U D_U k_U'}{k_U}\right]\left[T_L, T_U\right]$$

$$\text{i.e., } 2\left[O_L, O_U\right] = \left[\frac{h_L D_L k_L'}{k_L}, \frac{h_U D_U k_U'}{k_U}\right]\left[T_L, T_U\right]^2$$

$$\text{i.e., } \left[T_L, T_U\right]^2 = \frac{2\left[O_L, O_U\right]}{\left[\frac{h_L D_L k_L'}{k_L}, \frac{h_U D_U k_U'}{k_U}\right]} = \left[\min\left\{\frac{2O_L k_L}{h_L D_L k_L'}, \frac{2O_U k_U}{h_U D_U k_U'}\right\}, \max\left\{\frac{2O_L k_L}{h_L D_L k_L'}, \frac{2O_U k_U}{h_U D_U k_U'}\right\}\right]$$

i.e.,

$$\left[T_L, T_U\right] = \sqrt{\left[\min\left\{\frac{2O_L k_L}{h_L D_L k_L'}, \frac{2O_U k_U}{h_U D_U k_U'}\right\}, \max\left\{\frac{2O_L k_L}{h_L D_L k_L'}, \frac{2O_U k_U}{h_U D_U k_U'}\right\}\right]}$$

Hence the optimal cycle length is

$$\left[T_L^*, T_U^*\right] = \left[\sqrt{\min\left\{\frac{2O_L k_L}{h_L D_L k_L'}, \frac{2O_U k_U}{h_U D_U k_U'}\right\}}, \sqrt{\max\left\{\frac{2O_L k_L}{h_L D_L k_L'}, \frac{2O_U k_U}{h_U D_U k_U'}\right\}}\right] \quad (9)$$

Hence the optimal interval-valued order quantity is

$$\left[Q_L^*, Q_U^*\right] = \left[D_L T_L^*, D_U T_U^*\right] \quad (10)$$

And from the equation (8), one can easily obtain the optimal interval-valued average cost by the relation (11):

$$\left[AC_L\left(T_L^*, T_U^*\right), AC_U\left(T_L^*, T_U^*\right)\right] = \left[\sqrt{\frac{2O_L h_L D_L k_L'}{k_L}}, \sqrt{\frac{2O_U h_U D_U k_U'}{k_U}}\right] \quad (11)$$

Corollary 1: If all the inventory parameters are crisp valued i.e., all input parameters are taken as degenerate intervals as follows:

$[O_L, O_U] = [O, O] \leftrightarrow O; [D_L, D_U] = [D, D] \leftrightarrow D;$

$[k_L, k_U] = [k, k] \leftrightarrow k$; $[h_L, h_U] = [h, h] \leftrightarrow h$ then $k_L' = k_U' = k - D$, and the equations (9)-(11) take the form:

$$[T_L^*, T_U^*] = \left[\sqrt{\min\left\{\frac{2Ok}{hD(k-D)}, \frac{2Ok}{hD(k-D)}\right\}}, \sqrt{\max\left\{\frac{2Ok}{hD(k-D)}, \frac{2Ok}{hD(k-D)}\right\}}\right]$$

$$= \left[\sqrt{\frac{2Ok}{hD(k-D)}}, \sqrt{\frac{2Ok}{hD(k-D)}}\right] \leftrightarrow \sqrt{\frac{2Ok}{hD(k-D)}} \tag{12}$$

$$[Q_L^*, Q_U^*] = [DT^*, DT^*] \leftrightarrow DT^* = \sqrt{\frac{2O}{h}\frac{Dk}{k-D}} \tag{13}$$

$$[AC_L(T^*), AC_U(T^*)] = \left[\sqrt{\frac{2OhD(k-D)}{k}}, \sqrt{\frac{2OhD(k-D)}{k}}\right] \leftrightarrow \sqrt{\frac{2OhD(k-D)}{k}} \tag{14}$$

Which are the optimality conditions of the classical EPQ model.

VI. Numerical Illustrations

Two numerical examples are considered to illustrate the optimality conditions of the proposed EPQ model.

Example 1: Find the optimal interval-valued cycle length, interval-valued order quantity, and corresponding average cost of the present EPQ model concerning the values of interval-valued input parameters as follows:

$[O_L, O_U] = [300, 400], [h_L, h_U] = [3, 5], [k_L, k_U] = [1500, 2000]$ and $[D_L, D_U] = [800, 1000]$.

Solution: The optimal cycle length, produced quantity, and average cost of the model are calculated by the formula (9)-(11) and its optimal values are:

$$[T_L^*, T_U^*] = [0.4, 0.536]\text{Year.}$$

$$[Q_L^*, Q_U^*] = [D_L T_L^*, D_U T_U^*] = [320, 536]\text{units}.$$

And

$$[AC_L(T^*), AC_U(T^*)] = \$[289.828, 1264.911].$$

Example 2: To validate the optimal results in an interval environment, the values of interval-valued inventory parameters are considered from the crisp environment which are given below:

$[O_L, O_U] = [350, 350], [h_L, h_U] = [4, 4], [k_L, k_U] = [1750, 1750]$ and $[D_L, D_U] = [900, 900]$.

Solution: The optimal cycle length, produced quantity, and average cost of the model are calculated by the formula (12)-(14) and its optimal values are:

$T^{*} = 0.4003$ unit.

$Q^{*} = 360.27$ units .

$AC\left(T^{*}\right) = \$1106.345.$

VII. Limitations of the work

This work studies the optimal policy of a non-deterministic classical EPQ model in an interval environment using an alternative approach which has been developed based on the AM-GM interval *c-L* inequality. Though the present article has investigated interesting works, it has limited scope to apply. This approach will face a major challenge in analyzing the optimal policy of different real-life modelling problems under realistic assumptions and unavoidable constraints (inventory model with highly non-linear/imprecise demand rate, deterioration rate, production rate, etc.).

VIII. Conclusion

In this work, the optimal policy of the classical EPQ model has been analyzed using the AM-GM interval *c-L* inequality. Here, for the first time, the cycle length including the production period has been considered as interval-valued. In this study, all the optimal results are presented in the interval form and sometimes, these derived results may be named as *c-L* optimal policy. Then, from these results, the optimality conditions of the existing classical EPQ model have been derived as a particular case. Finally, all the optimal results of the model in both interval and crisp environments are justified using a set of numerical examples.

For future research, one may try to investigate the optimality conditions of the EPQ model with shortages using this AM-GM approach. Also, the result may be generalized for the EOQ/EPQ model in the Type-2 interval [XI, IX] environment.

Conflict of Interest:

There was no relevant conflict of interest regarding this paper.

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