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A NEW CONCEPT TO PROVE, $\sqrt{-1} = -1$ IN BOTH GEOMETRIC AND ALGEBRAIC METHODS WITHOUT USING THE CONCEPT OF IMAGINARY NUMBERS

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Abstract :

A particular branch of mathematics is coordinate geometry where geometry is studied with the help of algebra. According to the new concept of three types of Rectangular Bhattacharyya's coordinate systems, plane coordinate geometry consists of four axes. In type – I, Rectangular Bhattacharyya's coordinate system, the four axes are all neutral straight lines having no direction; in the type – II coordinate system the four axes are all count up straight lines and, in the type – III coordinate system all the axes are countdown straight lines. The author has considered all four axes to be positive in type II and type III coordinate systems. Ultimately, the author has established relations among the three types of coordinate systems and used the extended form of Pythagoras Theorem to prove $\sqrt{-1} = -1$.

In this paper, algebra is studied with the help of geometry. The equation, $x^2 + 1 = 0$, *means* $x^2 = -1$ *and therefore, the value of* $\sqrt{-1} = -1$, *has been proved by the author with the help of geometry by using the new concept of the three types of coordinate systems without using the concept of the imaginary axis. Also, the author has given an alternative method of proof of* $\sqrt{-1} = -1$ *algebraically by using the concept of the theory of dynamics numbers.*

The square root of any negative number can be determined in a similar way. This is the basic significance of that paper. This significance can be widely used in Mathematics, Science, and Technology and also, in Artificial Intelligence (AI), and Crypto-system.

Keywords : Cartesian Coordinate System, Dynamics of Numbers, Extended form of Pythagoras Theorem, Imaginary Number, Quadratic Equation, Three Types of Rectangular Bhattacharyya's coordinate systems.

I. Introduction

The author states that Mathematics is a tool to explain the truth of nature. At present, we are using two tools : (1) Real numbers (positive and negative) and (2) Imaginary numbers. The author developed a new concept of the 'Theory of Dynamics of Numbers', a new tool to explain the truth of nature. The question may arise why this new tool is necessary? Because in the imaginary number concept, 0 + i0 is undefined and there is no order relation except an equality relation. Basically, the Theory of Dynamics of Numbers is governed by the three laws are follows :

(1) 0 (zero) is defined as the starting point of any number. There is an infinite number of directions through which the numbers can move from the starting point 0 (zero) and back to the starting point 0 (zero) with a vertically opposite direction of motion of numbers. The number which is moving away from the starting point 0 (zero) is defined as count up number and the number which is moving toward the starting point 0 (zero) is defined as a countdown number.

(2) The Count up numbers are always greater than or equal to the countdown numbers. The Count up numbers can move independently but the motion of the countdown numbers is dependent on the motion of Count up numbers. The motion of the countdown numbers exists if and only if there are motions of the Count up numbers.

(3) For every equation, the number of the Count up number is always equal to the number of the countdown number.

By using this new tool, the author has developed three types of 'Rectangular Bhattacharyya's Coordinate Systems' where there are no negative abscissas or ordinates whereas, in the Cartesian Coordinate System, there is one negative abscissa and one negative ordinate exist. Actually, we find the distance between any two points in a plane by using a coordinate system. The author has exhibited that it is possible to find the distance between any two points in a plane without taking negative abscissas or ordinates.

The concept of studying algebra with the help of geometry becomes possible depending on the following new concepts developed already by the present author, namely :

I. AN INTRODUCTION TO RECTANGULAR BHATTACHARYYA'S COORDINATES: A NEW CONCEPT . [XI]

II. AN INTRODUCTION TO THEORY OF DYNAMICS OF NUMBERS: A NEW CONCEPT. [XII]

III. A NOVEL CONCEPT IN THEORY OF QUADRATIC EQUATION. [XIII]

IV. A NOVEL METHOD TO FIND THE EQUATION OF CIRCLES. [XIV]

V. AN OPENING OF A NEW HORIZON IN THE THEORY OF QUADRATIC EQUATION : PURE AND PSEUDO QUADRATIC EQUATION – A NEW CONCEPT. [XV]

VI. A NOVEL CONCEPT FOR FINDING THE FUNDAMENTAL RELATIONS BETWEEN STREAM FUNCTION AND VELOCITY POTENTIAL IN REAL NUMBERS IN TWO DIMENSIONAL FLUID MOTIONS. [XVI]

VII. A NEW CONCEPT OF THE EXTENDED FORM OF PYTHAGORAS THEOREM [XVII]

According to the Theory of Dynamics of Numbers :

(1) 0 (zero) is the starting point of any number. 0 (zero) is a neutral number having no direction but has existence only.

(2) There are infinite numbers of directions through which the numbers can move from the starting point 0 (zero) and back to the starting point 0 (zero) with a vertically opposite direction of motion of numbers.

(3) There are three types of countable numbers : (i) Neutral numbers, (ii) Count up numbers, (iii) Countdown numbers.

(4) Symbolic representation of three types of countable numbers are as follows:

(a) Neutral numbers have no symbol over the head of the numbers.

(b) Count up numbers have a symbol ‘$\blacktriangle$’ over the head of the numbers. For example : Count up 1, count up 2, count up 3, count up 4, etc. will be represented as $\overset{\blacktriangle}{1}$, $\overset{\blacktriangle}{2}$, $\overset{\blacktriangle}{3}$, $\overset{\blacktriangle}{4}$ etc. respectively.

(c) Countdown numbers have a symbol ‘$\blacktriangledown$’ over the head of the numbers. For example : Countdown 1, countdown 2, countdown 3, countdown 4, etc. will be represented as $\overset{\blacktriangledown}{1}$, $\overset{\blacktriangledown}{2}$, $\overset{\blacktriangledown}{3}$, $\overset{\blacktriangledown}{4}$ etc. respectively.

(5) (a) The discrete numbers which have no direction of motion are called Neutral numbers. For example : Neutral 5 = 5, Neutral 6 = 6 etc.

(b) The numbers which move away from the starting point 0 (zero) are called count up numbers. For example : $\overset{\blacktriangle}{5} = +5$.

(c) The numbers which move towards the starting point 0 (zero) are called countdown numbers. For example : $\overset{\blacktriangledown}{5} = -5$.

6) For every equation the number of count up number is equal to the number of countdown number. Also, the sum of count up number and the countdown number will be equal to zero.

For example : $\overset{\uparrow}{5} + \overset{\downarrow}{5} = +5 - 5 = 0$

(7) The numerical value of neutral numbers, count up numbers and countdown numbers may be used as the coefficient of any unknown variable.

Ultimately, with the help of the new concept of three types of coordinate systems – I, II & III which are based on another new concept of the theory of dynamics of numbers, the author has proved that $\sqrt{-1} = -1$ not only geometrically but also algebraically.

The basic significance of this paper, the author studied quadratic equations with the help of the three types of Bhattacharyya's Coordinate System which is based on the theory of dynamics of numbers. Also, the author used 'Pythagoras Theorem' and 'A new concept of the extended form of Pythagoras Theorem' which may be called Bhattacharyya's Theorem to find the method of solution of a quadratic equation such as, $x^2 + 1 = 0$. Applying the new method of solution of the quadratic equation, $x^2 + 1 = 0$, the author has shown that, $\sqrt{-1} = -1$ and became successful to find square root of any negative numbers without using the concept of imaginary numbers.

II. Literature Review

Pythagoras (570 B.C. – 490 B.C.), a Greek mathematician stated that "The square on the hypotenuse of a rectangular triangle is equal to the sum of the squares on the sides containing the right angle" which is popularly known as 'Pythagoras Theorem.' The most geometrical significance of this theorem, that the sides of any right–angle triangle will exhibit this relationship amongst them, was perhaps first realized by altar–building Vedic priests. The German mathematicians A. Burk and M. Cantor discussed the question in detail and came to the conclusion that the theorem was known at the latest by 8th century B.C. i.e., the date of the oldest Sulbasutra that of Boudhyaana (Dr. T. A. Sarasvati Amma, Geometry in Ancient and Medieval India, P – 17) [XXII]. The most of the ancient people knew and used to form a right angled triangle by 3, 4, and 5 numbers. The Chinese Nine Section (1100 B.C.) mentions this triangle and Kahun Papyrus Egypt (2000 B.C.) refers to four set of numbers forming right angles (D. E. Smith, History Mathematics, Vol II, P. – 293) [XX].

The method of representing and solving algebraic and arithmetical problems geometrically is as old as geometry itself. The Sulbasutra (800 B.C. – 500 B.C., Ap. Sl.1.5) determined , $\sqrt{2}\,,\sqrt{3}$ etc. with the help of squares and rectangles (Dr. T. A. Sarasvati Amma, Geometry in Ancient and Medieval India, P – 219) [XIII]. The same method is applicable for evaluating $\sqrt{a^2 + b^2}$ and $\sqrt{a^2 - b^2}$ where a and b are any two rational numbers.

The original concept of the quadratic equation was based on the concept of rectangular area which had length and breadth. One of the unknown quantities such as area, length, or breadth can be calculated depending on the two known quantities out of three. A solution of the quadratic equation was found in the Berlin papyrus (C.A. 2160 – 1700 B.C.) for the first time (Smith, 1953, P. 443) [XXI]. In the Indus civilization (2500 B.C. – 1750 B.C.) presence of developed mathematical concepts was found in the construction of cities, construction of buildings followed by standardized measurement of bricks in the ratio of 4:2:1 (Thapar, R., 2000) [XXIV]. Although the script of Harappan has not yet been deciphered, the effect of Indian mathematicians on Babylonians cannot be ruled out.

From Babylon clay tablets (2000 – 1700 B.C.) Babylonian mathematicians constructed eight types of quadratic equations (Gandz, 1937, P. 405)[V]. Out of the eight equations, the first six equations were solved by the Diophantus method and the last two types of equations are known as Arabic type since the equations had been solved by Al – Khwarizmi (780 – 850 A.D.), a Persian Mathematician who introduced Hindu – Arabic numerals and concept of algebra into European Mathematics. A good number of Arabian mathematicians who contributed to the development of quadratic are : Al – karki, Ibn Erza, Savadorsa, Immanuel Bonfils (Gandz, 1937) [V].

The subject of algebra is originated in India. Its origin can be traced back to the Shatapatha Brahmana (2000 BCE) and Sulbasutra (800 – 500 BCE). Algebra was used to design and construct the Vedis.

Sridhara Acharya (870 – 930 AD), Indian mathematician was the first person who had given an algorithm for solving quadratic equation in Sanskrit Verse (B. B. Dutta, 1929) [III]. The mathematician of Babylon, Egypt, Greek, Arabic countries had no perception of negative numbers in quadratic equation at that time. Indian mathematician, Bhaskara – II (1114 – 1185 A.D.) introduced the negative root and had shown that the roots of quadratic equations could be both positive and negative (Katz, 1988, pp. 226 – 227) [VII] considering all previous knowledge in mathematics as far as possible by the author, the author has developed a new form of mathematical structure adopting the new concepts developed recently which are as follows :

1) Aryabhatta (476 – 550 A.D.) Indian mathematician introduced 0 (zero) in calculations not only place holder but also viewed the 0 (zero) as having a null value, called 'Sunya'. Brahmagupta was the first to show that subtracting a number from itself result is zero. Fibonacci (1170 – 1250 A.D.), an Italian mathematician help to introduce to the main stream and it later figured promptly in the work of Rene Descartes along with Sir Issac Newton and Leibniz's invention of calculus. Since then, the concept of nothing has continued to play a role in the development of everything from physics, economics, engineering, and computing.
   The author has defined 0 (zero) as the starting point of any number and a number can move in infinite directions from zero and back to zero with its vertical opposite directional motion. The author has considered dynamism in numbers. The author has discussed in detail regarding this new concept in the article, : 'An Introduction to Theory of Dynamics of Numbers: A new concept' [XII].
2) Rene Descartes (1596 – 1650 A.D.), a French mathematician provided a systematic link between Euclidean Geometry and Algebra with the help of the Cartesian Coordinate System where one abscissa is positive and the other is negative and one ordinate is positive and the other is negative. However, the author has developed a new concept in coordinate geometry system where all axes are positive and exhibited the systematic link between geometry and algebra in details in an article. 'An Introduction to Rectangular Bhattachayya's Coordinates : A New Concept' [XI].
3) The concept of imaginary numbers was first introduced by Euler (1707 – 1783 A.D.). Subsequently, Gauss. Hamilton, Cauchy, Riemann, Stokes, Navier and others used this concept because they could not find the solution of the quadratic equation, $x^2 + 1 = 0$ in real numbers. They used imaginary numbers to solve similar types of problems in mathematics and in other fields. But the author has solved any type of quadratic equation in real numbers by adopting the new concepts which are as follows : 'A Novel Concept in Theory of Quadratic Equation' [XIII] and 'An Opening of a New Horizon in the Theory of Quadratic Equation : Pure and Pseudo Quadratic Equation' [XV]. Taking the help of the concept of this article [XV] M. Janani et al published an article, 'Multivariate Crypto System Based on a Quadratic Equation to Eliminate in Outliers using Homomorphic Encryption Scheme [IX].

The Cauchy – Riemann equation $\frac{\partial \varphi}{\partial x} = \frac{\partial \Psi}{\partial y}$ and $\frac{\partial \varphi}{\partial y} = -\frac{\partial \Psi}{\partial x}$ had been solved by using imaginary numbers whereas the author has shown that the same equation can be solved in real numbers by publishing an article, ' A Novel Concept for finding the Fundamental Relations between Stream Function and Velocity Potential in Real Numbers in Two – Dimensional Fluid Motion [XVI].

The author published an article, ' A Novel Method to Find the Equation of Circle [XIV], and became successful to solve the general form of the equation of a circle, $x^2 + y^2 + 2gx + 2fy + c = 0$, where the radius of the circle, $r = \sqrt{g^2 + f^2 - c}$ if $g^2 + f^2 < c$ in real number where c is the constant term by using Bhattacharyya's Coordinate System whereas Euler solved the said problem by using the complex plane.

Ultimately, the author has proved that, $\sqrt{-1} = -1$ in the geometric method as well as in algebraic method in the present article with the help of a new concept already published, 'A New Concept of the Extended form of Pythagoras Theorem' [XVII] along with other new concepts of the author.

Formulation of the Problem and Method of Solution :

<u>Some Definitions</u>

<u>Point</u> : In geometry, a point is the location represented by a dot. A point does not have any length, width, shape, or size, it has only positional existence.

There are three types of points on a plane or space : 1) Neutral point,

2) Count up point, 3) Countdown point.

<u>Neutral Point</u> : Any discrete point on a plane is defined as a neutral point.

Note that the origin O (0,0) is always defined as a neutral point.

<u>Count up Point</u> : Any discrete point or neutral point moving away from the origin O or a fixed point is called a count up point.

<u>Countdown Point</u> : Any discrete point or neutral point moving towards the origin O or a fixed point is called countdown point.

<u>Straight Line</u> : A line that connects two points by a shortest distance is called a straight line.

There are three types of straight lines : 1) Neutral straight line, 2) Count up straight line, 3) Countdown straight line.

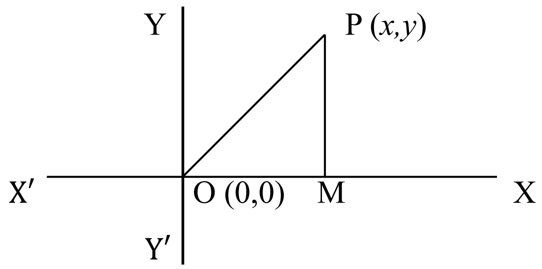

Fig. 1

Neutral straight line : If a straight line is connected by the origin O or a fixed point to another neutral point is called a neutral straight line. In Fig. 1, let us consider the origin O (0,0) and P (x, y)is any other neutral point then the line OP is called a neutral straight line.

It is symbolically represented as

Neutral OP = OP (having no symbol over the head of OP).

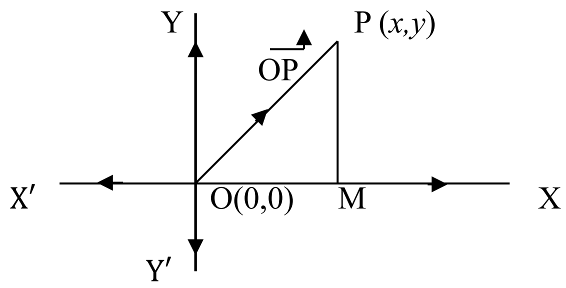

Fig. 2

Count up straight line :

If a straight line is drawn by taking points moving away from the origin or a fixed point O to another point P, then the straight line OP is called count up OP.

It is symbolically represented as OP. OP means the distance between the points O and P and the direction will be from O to P as in fig. 2.

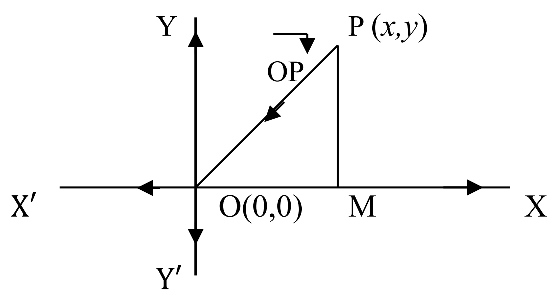

Fig. 3.

Countdown straight line :

If a straight line is drawn by taking points moving towards the origin or a fixed point O from another point P, then the straight line OP is called countdown OP.

It is symbolically represented as $\overrightarrow{OP}$. $\overrightarrow{OP}$ means the distance between the points O and P and the direction will be from the point P to the O as in fig. 3.

According to the Theory of Dynamics of Numbers another basic mathematical concept to find the root of quadratic equation which depends on the inherent nature of one unknown quality (say x) in the quadratic equation.

Identities :

let us consider three identities are as follows :

1) $x^2 = x \times x$

2) $x^2 = (+x) \times (+x)$

3) $x^2 = (-x) \times (-x)$

From identities (1), (2), and (3), the numerical values of x^2 are one and same but

(A) The inherent nature of the unknown quantity x is neutral x only of x^2 in the identity (1) but not $\pm x$ also.

(B) The inherent nature of the unknown quantity x is $+x$ only of x^2 in the identity (2) but not neutral x and $-x$ also.

(C) The inherent nature of the unknown quantity x is $-x$ only of x^2 in the identity (3) but not neutral x and $+x$ also.

Now, let us consider three quadratic equations involving x^2 where x is an unknown quantity as follows :

$$x^2 = 1 \tag{1}$$

$$x^2 = +1 \tag{2}$$

$$x^2 = -1 \tag{3}$$

According to the Theory of Dynamics of Number :

i) The inherent nature of the unknown quantity x in equation (1) is the neutral x only but not $\pm x$, also.

ii) The inherent nature of the unknown quantity x in equation (2) is the $\overrightarrow{x}$, i.e. $+x$ only but not neutral x and countdown x, also

iii) The inherent nature of the unknown quantity x in equation (3) is the $\overrightarrow{x}$, i.e. $-x$ only but not neutral x and count up x, also

Introducing these three types of inherent nature of unknown quantity (say x) for the respective identities and equations the author has solved any types of quadratic equations.

Triple roles of the point P

The author has introduced another new concept that any point P on a plane has triple roles where the point P may be neutral or count up or countdown point though the point P is one and same point on the plane.

The author has developed a new concept in the coordinate system :

Type – I : Rectangular Bhattacharyya's Coordinate System where all axes are neutral only, neither positive nor negative.

Type – II : Rectangular Bhattacharyya's Coordinate System where all axes are count up axes and positive.

Type -- III : Rectangular Bhattacharyya's Coordinate System where all axes are countdown axes and positive.

In the case of the type I coordinate system the position of point P (x, y) can be uniquely determined where the axes are neutral and the units of x and y are neutral units.

In case of finding the position of the count up point P (x, y) or countdown point P (x, y) on the plane we can uniquely determine the position of point P (x, y) with the help of neutral axes system because the measure of units of the coordinates x and y are neutral. So, it is immaterial whether the axes are count up or countdown. First of all, we shall introduce neutral axes system to find the position of the point P on the plane by the measure of neutral units x and y because we know that the point P (x, y) is one and the same for the three types – I, II, & III coordinate system.

Type – I : Rectangular Bhattacharyya's Coordinate system (in short)

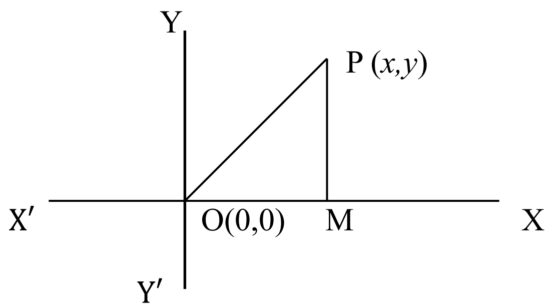

Fig. 4.

Let O (0,0) be considered as a neutral fixed point on the plane of a page. Let us draw two mutual perpendicular neutral straight lines through O. Clearly, these two neutral lines divide the plane of the page into four parts. Each of these parts is called a quadrant. The parts of are respectively called the first, second, third and fourth quadrants (Fig.4). The fixed neutral point O (0,0) where (0,0) are the neutral number zero is called the origin and the neutral straight lines are called the neutral co-ordinate axes and separately the neutral line is called the neutral X – axis, neutral X′ - axis, neutral Y – axis and neutral Y′ - axis respectively. There are four neutral axes.

Note that X – axis, Y – axis, X′ - axis and Y′ axis are all neutral axes and have no direction that means the coordinates of any point on the plane are neutral numbers. Here, X – axis and X′ - axis are called neutral abscissas; and Y – axis and Y′ - axis are called neutral ordinates, that means there are two neutral abscissas and two neutral ordinates.

In general, the statement, the neutral coordinates of the point P (x, y) means that the point P is situated at a distance x unit (neutral) from the origin O along X – axis (abscissa) and at a distance y unit (neutral) from the origin O along (or parallel) to Y – axis. Depending on the symbol '/' dash or without dash over the neutral coordinates x and y the point P may be on the first quadrant or second quadrant or third quadrant or fourth quadrant.

Conversely, if are neutral numbers the point

having neutral coordinates (x, y) lies in the first quadrant

having neutral coordinates (x', y) lies in the second quadrant

having neutral coordinates (x', y') lies in the third quadrant

having neutral coordinates (x, y') lies in the fourth quadrant

<u>For solving equation, $x^2 - 1 = 0$ by Geometric method</u>

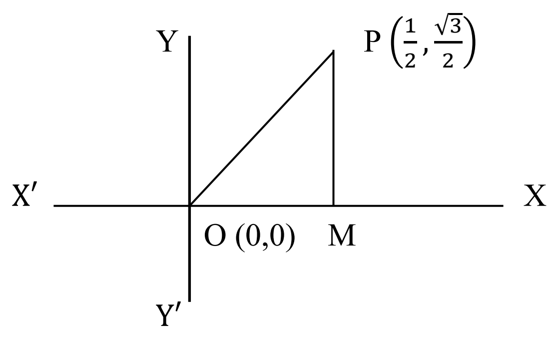

Fig. 5

From fig. 5 we have

$$OP^2 = OM^2 + MP^2 \quad (1)$$

Let, OP = x, OM = $\frac{1}{2}$ and MP = $\frac{\sqrt{3}}{2}$

$$\therefore OP = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{3}{4}} = \sqrt{1} = 1$$

Here, OP is a neutral straight line then, obviously 1 will be neutral 1,

Let us consider, OP = neutral x

Then, neutral $x = x$

$$\therefore x^2 = \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} + \frac{3}{4} = 1$$

or,

$$x^2 = 1 \quad (2)$$

$$\Rightarrow x = \sqrt{1} = 1 \quad (3)$$

From equation (2) we have

$$x^2 - 1 = 0 \quad (4)$$

and its solution

$x = 1$, where 1 is neutral 1.

Hence, the equation $x^2 - 1 = 0$ can be solved geometrically.

Note I : The question may arise what is the necessity of the neutral coordinate system? The answer is 'Yes', it is required. We know that the distance between any two points is a scalar quantity. If we need to find the coordinate of a point dividing the line segment joining two given points in a given ratio or to find the area of a triangle formed by joining three given points or any geometric shape formed by joining points we need a neutral axes system only.

For example: according to the Cartesian coordinate system, we know that the area of the triangle, Δ PQR $= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$ sq. unit. Where the coordinates of the points are P (x_1, y_1), Q (x_2, y_2) and R (x_3, y_3). The above expression for the area of the triangle, ΔPQR will be positive if the vertices P, Q, R are taken in anti – clockwise directions [Fig. 6].

i.e. Δ PQR $= +\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$ sq. unit.

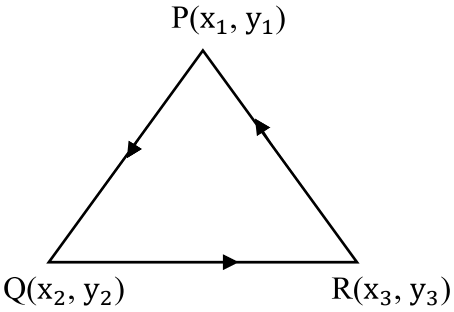

Fig. 6

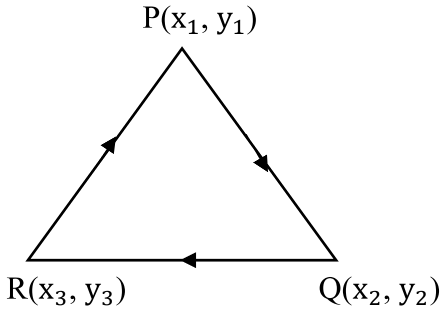

Fig. 7

On the contrary, the expression for the area of the triangle Δ PQR will be negative if the vertices P, Q, R are taken in the clockwise direction. [Fig. 7]

i.e. $\Delta\, PQR = -\frac{1}{2}\,[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$ sq. unit.

This contradiction arises because the Cartesian coordinate system are directed axes system.

<u>Note II</u> : The process of finding the position of any point P(x, y) on a plane in type II and III Rectangular Bhattacharrya's Coordinate System, first of all we have to introduce type I Rectangular Bhattacharrya's Coordinate System to find the position P(x, y). This process of finding the position of the point P(x, y) by type I will hold good in case of type II and III also.

<u>Note III</u> : <u>Alternative method to solve the equations, $x^2 – 1 = 0$ Algebraically</u>

To solve the equation

$$x^2 - 1 = 0 \qquad (1)$$

algebraically two cases may arise.

Case I : <u>When $x^2 = 1$ or x^2 = neutral 1</u>

For case I, x^2 will follow the identity

$$x^2 = x \times x \qquad (1)$$

where the inherent nature of x in x^2 will be neutral x only.

So, the root of the equation (1) will be a neutral number x only but not $\pm x$

Case II : <u>When $x^2 = 1$ or x^2 = count up $1 = \overset{\uparrow}{1} = +1$</u>

For case II, x^2 will follow the identity

$$x^2 = (+x) \times (+x) \qquad (2)$$

where the inherent nature of x in x^2 will be count up $x = + x$ only but not neutral x and $(- x)$

To solve the equation

$$x^2 + 1 = 0 \tag{2}$$

algebraically Case III may arise

Case III : <u>When $x^2 = -1$ or x^2 = Countdown 1 = 1 = -1</u>

For case III, x^2 will follow the identity

$$x^2 = (-x) \times (-x) \tag{3}$$

where the inherent nature of x in x^2 will be countdown $x = -x$ only but not neutral x and $(+ x)$.

So, the root of the equation $x^2 + 1 = 0$ will be countdown $x = - x$ only but not neutral x and $(+ x)$

For solving the equation, $x^2 - 1 = 0$ by Algebraic method (Type I)

Let,

$$x^2 - 1 = 0 \tag{1}$$

From case I : When x^2 = neutral 1, we know according to identity (1)

$$x^2 = x \times x$$

where inherent nature of x in x^2 will be neutral x only.

So, the root of the equation (1) will be neutral x only but not $\pm x$

So, from the equation (1) we have

$$x^2 - 1 = 0$$

$$\Rightarrow x^2 = 1$$

$$\Rightarrow x = \sqrt{1} = 1 = \text{neutral } 1$$

$$\therefore x = 1 \text{ (neutral)}.$$

Type – II : Rectangular Bhattacharyya/s Coordinate System

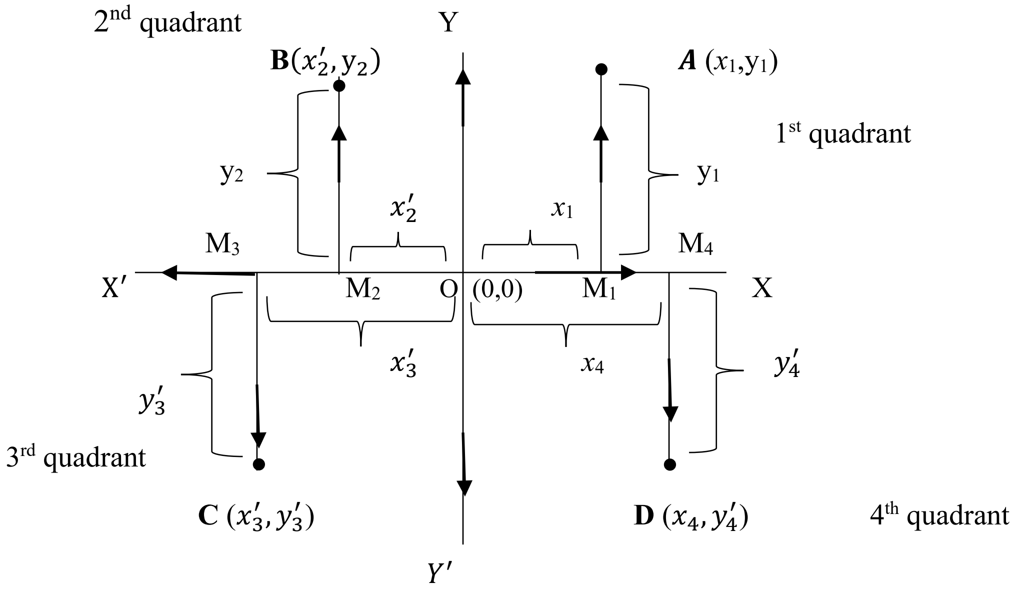

Fig 8

Let us consider O (0,0) to be a fixed point on the plane of this page which is stated as the origin in Figure 8. We draw four straight lines from the origin :

1. Towards the X direction.
2. Towards the Y direction.
3. Towards the X' direction.
4. Towards the Y' direction.

$\overrightarrow{OX}$ and $\overrightarrow{OY}$; $\overrightarrow{OX'}$ and $\overrightarrow{OY}$; $\overrightarrow{OX'}$ and $\overrightarrow{OY'}$; $\overrightarrow{OX}$ and $\overrightarrow{OY'}$, lines are mutually perpendicular to each other and $\overrightarrow{OX}$ and $\overrightarrow{OX'}$ are in the vertically opposite directions and also $\overrightarrow{OY}$ and $\overrightarrow{OY'}$ are in the vertically opposite directions.

The straight line $\overrightarrow{OX}$ is called X-axis, similarly $\overrightarrow{OY}$ is called Y-axis, $\overrightarrow{OX'}$ is called X'-axis and $\overrightarrow{OY'}$ is called Y'-axis. These four lines together divided the plane into four parts of the plane. Each of these parts is said to be a quadrant. The quadrants are named as follows: XOY is the first quadrant, X'OY is the second quadrant, X'OY' is the third quadrant and XOY' is the fourth quadrant. Note that X-axis, Y-axis, X'-axis, Y'-axis are all positive axes.

Now we can determine the position of any point on the plane uniquely concerning the co-ordinate axes drawn through O.

Let A (x_1, y_1) be any point on the first quadrant (Figure 8). From A draw $\overrightarrow{M_1 A}$ perpendicular on the X-axis. If $\overrightarrow{OM_1}$ and $\overrightarrow{M_1 A}$ measure x_1 and y_1 units respectively then the position of the point A on the plane is determined uniquely. First of all, we have to move from O through a distance x_1 unit along $\overrightarrow{OX}$ and then proceed through a distance y_1 unit in the direction parallel to $\overrightarrow{OY}$. Similarly, we can find a point B (x_2', y_2). First of all, we have to move from O through a distance x_2 unit along $\overrightarrow{OX'}$ and then proceed through a distance y_2 unit in the direction or parallel to $\overrightarrow{OY}$. Similarly, we can find the point C (x_3', y_3') and D (x_4, y_4') in the 3rd and 4th quadrant. So, it is possible to find four different points on the plane. The position of the points can be differentiated by introducing the following notations :

1) The distance measured from O along $\overrightarrow{OX}$ axis (or parallel in the direction to $\overrightarrow{OX}$ axis) putting without any notation over the unit is positive.
2) The distance measured from O along to $\overrightarrow{OY}$ axis (or parallel in the direction to $\overrightarrow{OY}$ axis) putting without any notation over the unit is positive.
3) The distance measured from O along $\overrightarrow{OX'}$ axis (or parallel in the direction to $\overrightarrow{OX'}$ axis) putting a notation dash (′) over the unit is positive.
4) The distance measured from O along $\overrightarrow{OY'}$ axis (or parallel in the direction to $\overrightarrow{OY'}$ axis) putting a notation dash (′) over the unit is positive.
5) Significance of arrows : Motions are always positive and also the units of motions are always positive. A motion which is away from the starting point is called count up motion and a motion which is towards the starting point is called countdown motion. The units of count up or countdown motion is always positive. To every motion there is a count up motion which is always greater than or equal to countdown motion.
   (i) The Symbol forward arrow (→) means direction of count up motion of a point only.
   (ii) The symbol backward arrow (←) means direction of countdown motion of a point only.

Following the above discussion, we can determine uniquely the position of a point on a plane referred to mutually perpendicular co-ordinate axes through an origin, we require without dash or with one dash (′) over positive real numbers. These two positive real numbers may be both without dash or both with one dash, also it may be one without dash and the other with a dash together are called the Rectangular Bhattacharyya's Co-ordinate. To represent a point we write two positive real numbers in braces by putting a comma between them where the first number is the distance from the origin along $\overrightarrow{OX}$ or $\overrightarrow{OX'}$ and the second number is the distance from the origin along $\overrightarrow{OY}$ or $\overrightarrow{OY'}$ respectively.

Type II -- The solution of the equation $x^2 - 1 = 0$ by Geometric method

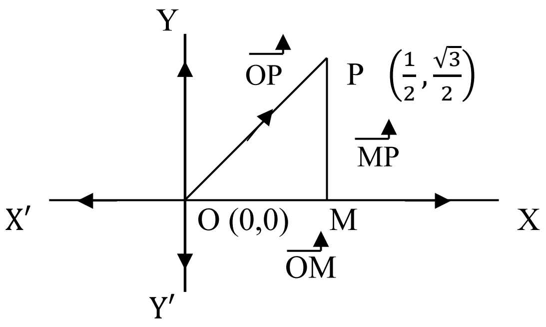

Fig. 9.

From fig. 9 we have

$$\overrightarrow{OP}^2 = \overrightarrow{OM}^2 + \overrightarrow{MP}^2 \qquad (1)$$

Let, $\overrightarrow{OP} = \overrightarrow{x} = +x$, $\overrightarrow{OM} = \overrightarrow{\left(\frac{1}{2}\right)} = +\frac{1}{2}$ and $\overrightarrow{MP} = \overrightarrow{\frac{\sqrt{3}}{2}} = +\frac{\sqrt{3}}{2}$

So, from equation (1) we have

$$\overrightarrow{x^2} = \left(+\frac{1}{2}\right)^2 + \left(+\frac{\sqrt{3}}{2}\right)^2$$

or, $\quad +x^2 = +\frac{1}{4} + \frac{3}{4}$

or, $\quad +x^2 = +1 \qquad (2)$

or, $\quad +x = \sqrt{+1} = +1 \qquad (3)$

From equation (2) we have

$$x^2 - 1 = 0 \qquad (4)$$

And its solution is $x = +1$ only.

Hence, the equation $x^2 - 1 = 0$ can be solved geometrically.

Note : According to the article [XV], the equation $x^2 - 1 = 0$ is a pure quadratic equation and it has one and only one root.

Type II : The solution of the equation $x^2 - 1 = 0$ by Algebraic method

$$x^2 - 1 = 0 \qquad (1)$$

Since, $-1 < 0$, the inherent nature of x is count up x in x^2 of equation (1) which also satisfy the identity (2)

$$x^2 = (+x) \times (+x) \qquad (2)$$

According to the Theory of Dynamics of Numbers, the equation (1) takes the form

$$\overset{\uparrow}{x^2} + \overset{\downarrow}{1} = 0 \quad (3)$$

According to the Theory of Dynamics of Numbers : number of count up number = number of countdown number.

Therefore,

$$x^2 = 1 \quad (4)$$

$$\Rightarrow x = 1 \quad (5)$$

According to the inherent nature of x, the root of the equation (1) will be count up $x = \overset{\uparrow}{x}$

Using equation (5) we have

$$\overset{\uparrow}{x} = \overset{\uparrow}{1} = +1 \quad (6)$$

Therefore, from equation (6), the root of equation (1), we have

$$x = +1$$

Note : a is the number of count up a and b is the number of countdown b then

a = b and $\overset{\uparrow}{a} + \overset{\downarrow}{b} = 0$

Type III : <u>Rectangular Bhattacharyya's Coordinate System</u> (In short)

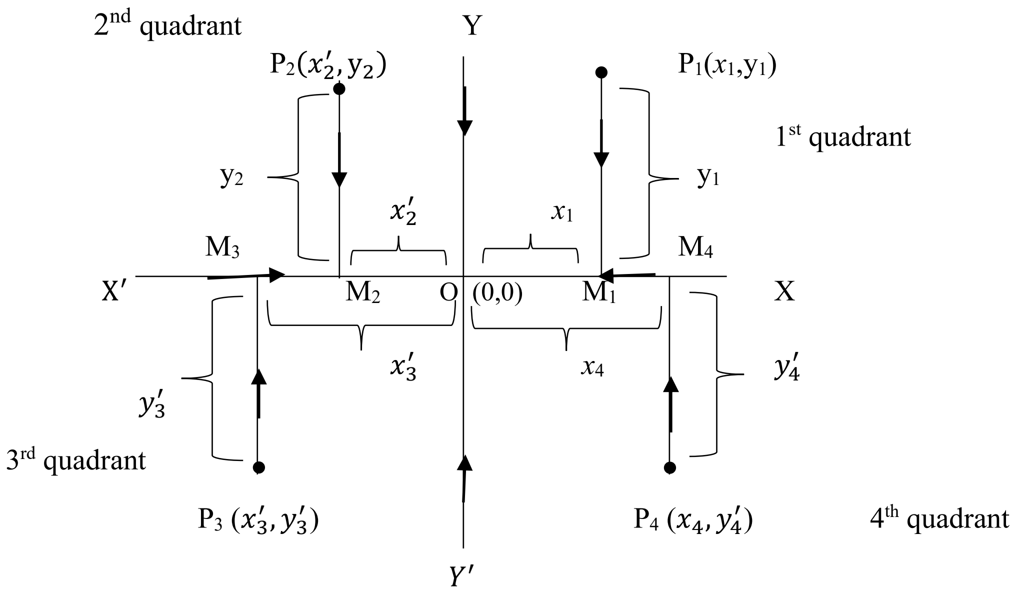

Fig 10

Let us consider O (0,0) to be a neutral point on the plane of this page which is stated as origin. We draw four straight lines towards the origin.

1. Towards the origin along $\overrightarrow{\mathrm{XO}}$ direction
2. Towards the origin along $\overrightarrow{\mathrm{YO}}$ direction
3. Towards the origin along $\overrightarrow{\mathrm{X'O}}$ direction
4. Towards the origin along $\overrightarrow{\mathrm{Y'O}}$ direction

$\overrightarrow{\mathrm{XO}}$ and $\overrightarrow{\mathrm{YO}}$; $\overrightarrow{\mathrm{X'O}}$ and $\overrightarrow{\mathrm{YO}}$; $\overrightarrow{\mathrm{X'O}}$ and $\overrightarrow{\mathrm{Y'O}}$; $\overrightarrow{\mathrm{XO}}$ and $\overrightarrow{\mathrm{Y'O}}$ lines are mutually perpendicular to each other and $\overrightarrow{\mathrm{XO}}$ and $\overrightarrow{\mathrm{X'O}}$ must meet at the origin O and also, $\overrightarrow{\mathrm{YO}}$ and $\overrightarrow{\mathrm{Y'O}}$ meet at the origin O. Though the straight lines $\overrightarrow{\mathrm{XO}}$, $\overrightarrow{\mathrm{X'O}}$, $\overrightarrow{\mathrm{YO}}$ and $\overrightarrow{\mathrm{Y'O}}$ meet at the origin 0 we shall consider the straight lines $\overrightarrow{\mathrm{XO}}$, $\overrightarrow{\mathrm{X'O}}$, $\overrightarrow{\mathrm{YO}}$ and $\overrightarrow{\mathrm{Y'O}}$ to be count up straight lines for the present type of coordinate system. The count up straight line XO is called the X – axis, similarly YO is called the Y – axis, $\mathrm{X'O}$ is called X′ - axis and $\mathrm{Y'O}$ is called Y′ - axis. These four count up lines together divided the plane into four parts of the plane. Each of these parts is said to be a quadrant. The quadrants are named as follows : XOY is the first quadrant, X′O Y is the second quadrant, X′O Y′ is the third quadrant, and X O Y′ is the forth quadrant. Note that X – axis, Y – axis, X′ - axis and Y′ - axis are all positive axes.

Now we can determine the position of any point on the plane uniquely concerning the coordinate axes meeting at the origin 0.

Let P_1 (x_1, y_1) be any point on the 1st quadrant (fig, 10). From P_1 draw $\overrightarrow{\mathrm{P_1M_1}}$ perpendicular on the X – axis. If $\overrightarrow{\mathrm{M_1O}}$ and $\overrightarrow{\mathrm{P_1M_1}}$ measure neutral x_1 unit and neutral y_1 unit respectively, then the position of the point P_1 (x_1, y_1) on the plane can be determined uniquely.

The distance between two points O to M_1 or M_1 to O is neutral *x* unit and M_1 to P_1 or P_1 to M_1 is neutral y_1 unit. So first of all, we shall introduce Type – 1 neutral axes system to find the location of the point P_1. This process of finding the location of the point P_1 by Type – I neutral axes system will hold good in the case of finding the location of the point P_1 by Type – III directed axes system also.

Prove that $\sqrt{-1} = -1$ by Geometric Method

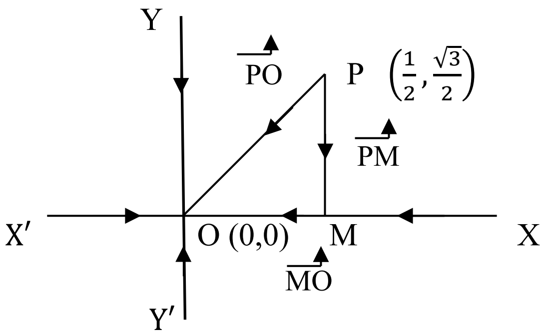

Fig. 11

From fig. (11) we have according to Pythagoras Theorem

$$\overset{\uparrow}{\mathrm{PO}} = \sqrt{\overset{\uparrow}{\mathrm{PM}}{}^2 + \overset{\uparrow}{\mathrm{MO}}{}^2} \qquad (1)$$

Let, $\overset{\uparrow}{\mathrm{MO}} = +\frac{1}{2}$ and $\overset{\uparrow}{\mathrm{PM}} = +\frac{\sqrt{3}}{2}$

$$\therefore \overset{\uparrow}{\mathrm{PO}} = \sqrt{\left(+\frac{1}{2}\right)^2 + \left(+\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{3}{4}} = \sqrt{+1} = +1$$

According to Theory of Dynamics of Numbers

$$\overset{\downarrow}{\mathrm{PO}} = -1 \qquad (2)$$

Now, we rotate $\overset{\uparrow}{\mathrm{PO}}$ by 180^0 in anticlockwise direction in the same quadrant then, according to 'A New Concept of Extended Form of Pythagoras Theorem' we shall get

$$\overset{\downarrow}{\mathrm{PO}} = \sqrt{\overset{\downarrow}{\mathrm{PM}}{}^2 + \overset{\downarrow}{\mathrm{MO}}{}^2} \qquad (3)$$

We also know that according to the Theory of Dynamics of Numbers

$\overset{\downarrow}{\mathrm{PM}}{}^2 = -\mathrm{PM}^2$ and $\overset{\downarrow}{\mathrm{MO}}{}^2 = -\mathrm{MO}^2$ where $\mathrm{PM} = +\frac{\sqrt{3}}{2}$ and $\mathrm{MO} = +\frac{1}{2}$

$$\therefore \overset{\downarrow}{\mathrm{PO}} = \sqrt{-\left(+\frac{\sqrt{3}}{2}\right)^2 - \left(+\frac{1}{2}\right)^2}$$

$$= \sqrt{-\frac{3}{4} - \frac{1}{4}}$$

$$= \sqrt{-1}$$

$$\therefore \overset{\downarrow}{\mathrm{PO}} = \sqrt{-1} \qquad (4)$$

From equation (4) and (2) we have

$$\sqrt{-1} = -1$$

Hence the proof

Note : The question may arise regarding the utility of the coordinate system where the points are moving towards the centre is positive? The answer is 'Yes'. If we take a big dram full of water whose top is completely open and if we suck the water with a small pipe having very small radius at the centre of the top level of water, then the water particles will move positively towards the centre which is known as 'Sink' in fluid dynamics. Also, in nature we can find the same thing that dust particles in space are moving positively towards the earth due to the gravitational attraction of the earth if we consider the earth as a centre point.

Type III : Alternative method using the Theory of Dynamics of Numbers

$$x^2 + 1 = 0 \tag{1}$$

$$\Rightarrow x^2 = -1$$

$$\Rightarrow x = \sqrt{-1} \tag{2}$$

Again

$$x^2 + 1 = 0 \tag{1}$$

Since $+1 > 0$, the inherent nature of x is countdown x in x^2 of equation (1) which also satisfy the identity (3)

$$x^2 = -x \times -x \tag{3}$$

According to the Theory of Dynamics of Numbers the equation (1) takes the form

$$\overset{\downarrow}{x^2} + \overset{\uparrow}{1} = 0 \tag{4}$$

According to the third law of the Theory of Dynamics of Numbers : number of count up number = number of countdown number

So that countdown number + count up number = 0

Therefore,

$$x^2 = 1 \tag{5}$$

or, $$x = 1 \tag{6}$$

According to the inherent nature of x, the root of equation (1) will be countdown $x = \overset{\downarrow}{x}$

Using equation (6) we have

$$x = 1 = -1 \tag{7}$$

We know that the inherent nature of x of the equation $x^2+1 = 0$ is the root of the equation $x^2+1 = 0$

So, $\quad x = -1 \qquad (8)$

From equation (2) and equation (8) we have,

$$\sqrt{-1} = -1$$

Hence the proof.

Conclusion

The concept of $\sqrt{-1} = -1$ is a great achievement in mathematics in this era. The square root of any negative number can be obtained by using the same method. This concept will work on failure-based upgradation in the field of mathematics, science and technology specially in computer science, artificial intelligence and crypto-system. This concept which is based on a new mathematical structure of the Theory of Dynamics of Numbers and three types of Rectangular Bhattacharyya's Coordinate Systems has wide application in any field of scientific research.

Conflict of Interest:

There is no conflict of interest regarding this paper

References:

I. Ahmad Raza, : "Research Work of Mathematics in Algebra. New Inventory Work in Quadratic Equation (P.E. Degree 2), Cubic Equation (P.E. Degree 3) & Reducible Equation General Term". *European Journal of Mathematics and Statistics*. pp. 310-315. Vol 4, Issue 1, January 2023. 10.24018/ejmath.2023.4.1.159

II. Ashwannie Harripersaud, : "The Quadratic Equation Concept". *American Journal of Mathematics and Statistics.* 2021; 11(3): 67-71. 10.5923/j.ajms.20211103.03

III. B. B. Dutta, (1929). : "The Bhakshali Mathematics". Calcutta, West Bengal: *Bulletin of the Calcutta Mathematical Society*.

IV. B. B. Datta, & A. N. Singh, (1938). : "History of Hindu Mathematics, A source book". Mumbai, Maharashtra: Asia Publishing House.

V. Gandz, S. (1937). : "The origin and development of the quadratic equations in Babylonian, Greek, and Early Arabic algebra". *History of Science Society*, 3, 405-557.

VI. H. Lee Price, Frank R. Bernhart, : "Pythagorean Triples and a New Pythagorean Theorem". arXiv:math/0701554 [math.HO]. 10.48550/arXiv.math/0701554

VII. Katz, V., J. (1998). : "A history of mathematics (2nd edition)". pp. 226-227. Harlow, England: Addison Wesley Longman Inc.

VIII. L. Nurul , H., D., (2017). : "Five New Ways to Prove a Pythagorean Theorem". *International Journal of Advanced Engineering Research and Science*. Volume 4, issue 7, pp.132-137. 10.22161/ijaers.4.7.21

IX. M. Janani et al, : "Multivariate Crypto System Based on a Quadratic Equation to Eliminate in Outliers using Homomorphic Encryption Scheme". *Homomorphic Encryption for Financial Cryptography*. pp 277–302. 01 August 2023. Springer.

X. M. Sandoval-Hernandez et al, : **"**The Quadratic Equation and its Numerical Roots". *International Journal of Engineering Research & Technology (IJERT)*. Vol. 10 Issue 06, June-2021 pp. 301-305. 10.17577/IJERTV10IS060100

XI. Prabir Chandra Bhattacharyya, : "AN INTRODUCTION TO RECTANGULAR BHATTACHARYYA'S CO-ORDINATES: A NEW CONCEPT". *J. Mech. Cont. & Math. Sci.*, Vol.-16, No.-11, November (2021). pp 76-86. 10.26782/jmcms.2021.11.00008

XII. Prabir Chandra Bhattacharyya, : "AN INTRODUCTION TO THEORY OF DYNAMICS OF NUMBERS: A NEW CONCEPT". *J. Mech. Cont. & Math. Sci.*, Vol.-17, No.-1, January (2022). pp 37-53. 10.26782/jmcms.2022.01.00003

XIII. Prabir Chandra Bhattacharyya, : 'A NOVEL CONCEPT IN THEORY OF QUADRATIC EQUATION'. *J. Mech. Cont. & Math. Sci.*, Vol.-17, No.-3, March (2022) pp 41-63. 10.26782/jmcms.2022.03.00006

XIV. Prabir Chandra Bhattacharyya. : "A NOVEL METHOD TO FIND THE EQUATION OF CIRCLES". *J. Mech. Cont. & Math. Sci.*, Vol.-17, No.-6, June (2022). pp 31-56. 10.26782/jmcms.2022.06.00004

XV. Prabir Chandra Bhattacharyya, : "AN OPENING OF A NEW HORIZON IN THE THEORY OF QUADRATIC EQUATION: PURE AND PSEUDO QUADRATIC EQUATION – A NEW CONCEPT". *J. Mech. Cont. & Math. Sci.*, Vol.-17, No.-11, November (2022). pp 1-25. 10.26782/jmcms.2022.11.00001

XVI. Prabir Chandra Bhattacharyya, : "A NOVEL CONCEPT FOR FINDING THE FUNDAMENTAL RELATIONS BETWEEN STREAM FUNCTION AND VELOCITY POTENTIAL IN REAL NUMBERS IN TWO-DIMENSIONAL FLUID MOTIONS". *J. Mech. Cont. & Math. Sci.*, Vol.-18, No.-02, February (2023) pp 1-19. 10.26782/jmcms.2023.02.00001

XVII. Prabir Chandra Bhattacharyya, : "A NEW CONCEPT OF THE EXTENDED FORM OF PYTHAGORAS THEOREM". *J. Mech. Cont. & Math. Sci.*, Vol.-18, No.-04, April (2023) pp 46-56. 10.26782/jmcms.2023.04.00004

XVIII. Salman Mahmud, : "14 New Methods to Prove the Pythagorean Theorem by using Similar Triangles". *International Journal of Scientific and Innovative Mathematical Research (IJSIMR)*. vol. 8, no. 2, pp. 22-28, 2020. 10.20431/2347-3142.0802003

XIX. S. Mahmud, (2019). : "Calculating the area of the Trapezium by Using the Length of the Non Parallel Sides: A New Formula for Calculating the area of Trapezium". *International Journal of Scientific and Innovative Mathematical Research*. volume 7, issue 4, pp. 25-27. 10.20431/2347-3142.0704004

XX. Smith, D. (1953). : "History of mathematics", Vol. 2. Pp. 293. New York: Dover

XXI. Smith, D. (1953). : "History of mathematics", Vol. 2. pp. 443. New York: Dover.

XXII. T. A. Sarasvati Amma, : "Geometry in Ancient and Medieval India". Pp. – 17. Motilal Banarasidass Publishers Pvt. Ltd. Delhi.

XXIII. T. A. Sarasvati Amma, : "Geometry in Ancient and Medieval India". pp. – 219. Motilal Banarasidass Publishers Pvt. Ltd. Delhi.

XXIV. Thapar, R., (2000). : "Cultural pasts: Essays in early Indian History", New Delhi: Oxford University Press.