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MATHEMATICAL ANALYSIS AND STUDY OF THE NUMEROUS TRAVELING WAVE BEHAVIOR FOR DIFFERENT WAVE VELOCITIES OF THE SOLITON SOLUTIONS FOR THE NONLINEAR LANDAU-GINSBERG-HIGGS MODEL IN NONLINEAR MEDIA

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Abstract

In this study, the nonlinear Landau-Ginsberg-Higgs (LGH) model is proposed and examined. The stated model is applied to analyze superconductivity and drift cyclotron waves in radially inhomogeneous plasma for coherent ion-cyclotron waves. This is undeniably a robust mathematical model in real-world applications. The generalized exponential rational function method (GERFM) is utilized to extract the suitable, useful, and further general solitary wave solutions of the LGH model via the traveling wave transformation. Furthermore, we investigate the effects of wave velocity in a particular time limit through a graphical representation of the examined solutions of the model to understand the dynamic behavior of the system. The attained results confirm the effectiveness and reliability of the considered scheme.

Keywords: The nonlinear Landau-Ginsberg-Higgs (LGH) model; the generalized exponential rational function method (GERFM); the traveling wave transformation; the soliton solutions.

I. Introduction

Most of the physical phenomena around us can be modeled by the nonlinear partial differential equations (NLPDEs). With the development of human knowledge, the applications of nonlinear partial differential equations increased rapidly. The NLPDEs play an elementary role in quantum mechanics, fluid mechanics and mechanics of water waves, plasma physic, material energy, biological science, geo-optical filaments, optical fibers, and more. In this way, the exact solutions of NPDEs gain much interest from scholars.

Accordingly, numerous efficient, powerful, and rising techniques have been established and applied to explore exact and explicit solutions of NLPDEs by means of Maple and Mathematica software. The existing techniques are: the generalized exponential rational function method(GERFM) [XIII, IX-X], the extended tanh-function method [XII], the test function method [XXXIII], the Fourier transform method [XXXV], the exp-function approach [XVIII], the auxiliary equation method [XIV, XIX-XXI], the generalized (G'/G)-expansion method [XXII, XXIII, XXXII], the logarithmic transformation method [XXXVI], the unified method [XXIV], the tanh-coth method [XXXIV], the modified extended tanh expansion scheme [XXIX], the modified simple equation approach [XXX], the newly extended direct algebraic technique [XXXVII], the trial equation approach [XXXIX], the improved Adomian decomposition method [I], the sine-gordon expansion method [XV], the $\exp_a$ function method and hyperbolic function method, IBSEF method [II], the modified $exp^{(-\Omega(\xi))}$-expansion function approach [XVI], the finite difference approach [XL], the Laplace-Adomian decomposition method [XXXI], the Ricatti and generalized Bernoulli sub-ODE method and generalized Tanh method [III], the ansatz functions method [XXV], the Adomian-based Method [IV] and more.

In the literature, many scholars have investigated the LGH model through several approaches for an instant, the technique of improved Bernoulli sub-equation function (IBSEF) [XXVI], the multi-symplectic Runge-Kutta technique [XXXVIII], the first integral technique [V], the new modified simple technique [VI], the sine-cosine and extended tanh function methods [VII], the two variables $(G'/G\,,1/G)$ expansion technique [VIII], etc. From the statistical assessment we have found, the LGH model has not yet been investigated through the generalized exponential rational function method (GERFM) [XIII, IX-X].

The objective of our study is to construct adequate, more general, and wide-ranging soliton solutions of the LGH model with the aid of the GERFM which are not established in the previous literature. We also show that, the effect and variation of the solutions for the different velocities of the traveling wave in a specific time limit by the graphical depiction of the attained solutions using the Mathematica software.
The residue of the paper is organized as: the methodology of the stated scheme is outlined in the second section. The mathematical analysis of traveling wave solutions and discussions of the results graphically are presented in the third section. Finally, the article ends with some concluding remarks.

II. The algorithm of the method

In this part, we briefly discuss the generalized exponential rational function method (GERFM) to study the useful, typical, and further generic closed-form wave solutions of the model. The generalized exponential rational function method (GERFM) which is first proposed by Ghanbari and Mustafa [XI]. This is a newly proposed efficient technique to handle nonlinear partial differential equations. These successful experiences influenced us to utilize the method of solving nonlinear models.

Consider a general NLDE of the form:

$$H\left(u, u_t, u_x, u_y, u_{tt}, u_{xx}, u_{yy}, \dots\right) = 0, \tag{1}$$

where $u = u(t, x, y)$ is a function of wave variable, H is a polynomial in $u(t, x, y)$ and its partial derivatives which contains of the highest order derivatives and the highest order nonlinear terms, the subscripts denote partial derivatives. To find the solution of (1) by using the GERFM, it performs the subsequent steps:

Stage 1: Let us assess the traveling wave variable

$$u(x, y, t) = u(\xi), \text{ where } \xi = x - \omega t, \tag{2}$$

where ω is the velocity of the traveling wave. The above transformation allows us to change Eq. (1) into the ordinary differential equation (ODE) as:

$$\mathcal{F}(u, u', u'', \dots) = 0, \tag{3}$$

where $\mathcal{F}$ is a polynomial in $u(\xi)$ and its derivatives, wherein $u'(\xi) = \frac{du}{d\xi}$.

Stage 2: Our most important assumption in this method is the structure of the wave solution of Eq. (3) is considered as the form:

$$u(\xi) = \mathcal{B}_0 + \sum_{i=1}^{N} \mathcal{B}_i f^i(\xi) + \sum_{i=1}^{N} \frac{\mathcal{A}_i}{f^i(\xi)}, \tag{4}$$

where

$$f(\xi) = \frac{c_1 e^{(d_1\xi)} + c_2 e^{(d_2\xi)}}{c_3 e^{(d_3\xi)} + c_4 e^{(d_4\xi)}}, \tag{5}$$

the values of the constants c_n, d_n $(1 \leq n \leq 4)$, $\mathcal{B}_0, \mathcal{B}_i$ *and* $\mathcal{A}_i (1 \leq i \leq \text{N})$ are determined. The form of the solution (4) always persuades Eq. (3). By applying the homogenous balance principle the value of N can be determined.

Stage 3: Substituting Eq. (4) into Eq. (3) with Eq. (5), yields a polynomial equation $S(P_1, P_2, P_3, P_4) = 0$ in terms of $P_j = e^{(d_j\xi)}$ for $j = 1, \dots, 4$. by equating all the coefficients of this polynomial, a system of nonlinear equations in terms of c_n, d_n $(1 \leq n \leq 4)$, and $\mathcal{B}_0, \mathcal{B}_i$, $\mathcal{A}_i (1 \leq i \leq N)$ is generated.

Stage 4: By solving the above system of equations using any computer package like Maple or Mathematica. The values of c_n, d_n $(1 \leq n \leq 4)$, $\mathcal{B}_0, \mathcal{B}_i$ *and* $\mathcal{A}_i (1 \leq i \leq \text{N})$ are obtained. Inserting these values in Eq. (4) introduces the soliton solutions of NLEEs.

III. Mathematical analysis of traveling wave solutions of the LGH model and results discussion

The Landau-Ginzburg-Higgs (LGH) equation was first constructed by Lev Devidovich Landau and Vitally Lazarevich Ginzburg having a very wide range of applications in radially inhomogeneous plasma having an invariable phase relation of ion-cyclotron waves. It demonstrated superconductivity and unidirectional wave propagation in nonlinear media [XXXVIII].

In this section, we study the nonlinear Landau-Ginsberg-Higgs (LGH) model to attain some different suitable, fresh, and further general closed-form traveling wave solutions by executing the new rising method likely, the generalized exponential rational function method (GERFM). Moreover, for more clarification, we illustrate the three-dimensional and combined two-dimensional structures to visualize the inner mechanism and physical explanation of the results obtained through GERFM.

Mathematical analysis of the traveling wave solutions of the model:

Our earlier stated nonlinear Landau-Ginsberg-Higgs (LGH) model is the form [XVII, XXVII, XXVIII]:

$$u_{tt} - u_{xx} - m^2 u + n^2 u^3 = 0, \tag{6}$$

where $u(x,t)$ express the electrostatic potential of the ion-cyclotron wave and x, t indicate the non-linearized spatial and temporal coordinates respectively and m and n are real parameters.

To examine the exact wave solutions of the model (6), we consider the following defined transformation,

$$u(x,t) = u(\xi), \text{ where } \ \xi = x - \omega t. \tag{7}$$

Utilizing the wave transformation (7) the model (6) is reduced to an ODE as:

$$(\omega^2 - 1)u'' - m^2 u + n^2 u^3 = 0, \tag{8}$$

where m and n are real parameters and ω is the wave velocity.

Now, balancing the two highest-order linear and nonlinear terms occurring in (8) yields $N = 1$. Then the solution of Eq. (8) in the form:

$$u(\xi) = a_0 + a_1 f(\xi) + \frac{b_1}{f(\xi)} \tag{9}$$

By pursuing the required steps of this method, the traveling wave solutions of the model are arranged below:

Family 1: suppose $c = [-1, -1, 1, -1]$ and $d = [1, -1, 1, -1]$

$$f(\xi) = -\frac{\cosh(\xi)}{sinh(\xi)}. \tag{10}$$

Segment 1.1: For $\omega = \pm\frac{\sqrt{(4+m^2)}}{2}, a_0 = 0, a_1 = \pm i\frac{m}{n\sqrt{2}}, b_1 = \mp i\frac{m}{n\sqrt{2}}$, the solution can be extracted as the form:

$$u(x,t) = \pm i\frac{m}{n\sqrt{2}}\left(tanh\left(x \mp \frac{\sqrt{(4+m^2)}}{2}t\right) - coth\left(x \mp \frac{\sqrt{(4+m^2)}}{2}t\right)\right). \tag{11}$$

Family 2: suppose $c = [1, -3, -1, 1]$ and $d = [1, -1, 1, -1]$

$$f(\xi) = \frac{cosh(\xi) - 2sinh(\xi)}{sinh(\xi)}. \tag{12}$$

Segment 2.1: When $\omega = \pm\frac{\sqrt{(4-2m^2)}}{2}, a_0 = \frac{2m}{n}, a_1 = \frac{m}{n}, b_1 = 0$, we achieved the solution:

$$u(x,t) = \frac{m}{n}\left(2 + \frac{\left(cosh\left(x \pm \frac{\sqrt{(4-2m^2)}}{2}t\right) - 2sinh\left(x \pm \frac{\sqrt{(4-2m^2)}}{2}t\right)\right)}{sinh\left(x \pm \frac{\sqrt{(4-2m^2)}}{2}t\right)}\right). \tag{13}$$

Family 3: suppose $c = [-2,0,-1,1]\ and\ d = [1,-1,1,-1]$

$$f(\xi) = \frac{cosh(\xi)+sinh(\xi)}{sinh(\xi)}. \tag{14}$$

Segment 3.1: When $\omega = \pm\frac{\sqrt{(4-2m^2)}}{2}, a_0 = \frac{m}{n}, a_1 = -\frac{m}{n}, b_1 = 0$, we reach the soliton solution:

$$u(x,t) = \frac{m}{n}\left(1 - \frac{cosh\left(x\pm\frac{\sqrt{(4-2m^2)}}{2}t\right)+sinh\left(x\pm\frac{\sqrt{(4-2m^2)}}{2}t\right)}{sinh\left(x\pm\frac{\sqrt{(4-2m^2)}}{2}t\right)}\right). \tag{15}$$

Family 4: suppose $c = [2-i,-2-i,-1,1] and\ d = [i,-i,i,-i]$

$$f(\xi) = \frac{-2sin(\xi)+cos(\xi)}{sin(\xi)}. \tag{16}$$

Segment 4.1: For $\omega = \pm\frac{\sqrt{(4+2m^2)}}{2}, a_0 = \pm i\frac{2m}{n}, a_1 = \pm i\frac{m}{n}, b_1 = 0$, the solution can be extracted as the form:

$$u(x,t) = \pm i\frac{2m}{n}\left(1 - \frac{sin\left(x\pm\frac{\sqrt{(4+2m^2)}}{2}t\right)+cos\left(x\pm\frac{\sqrt{(4+2m^2)}}{2}t\right)}{sin\left(x\pm\frac{\sqrt{(4+2m^2)}}{2}t\right)}\right). \tag{17}$$

Family 5: suppose $c = [-1-i,1-i,-1,1]\ and\ d = [i,-i,i,-i]$

$$f(\xi) = \frac{sin(\xi)+cos(\xi)}{sin(\xi)}. \tag{18}$$

Segment 5.1: When $\omega = \pm\frac{\sqrt{(4+2m^2)}}{2}, a_0 = \mp i\frac{m}{n}, a_1 = 0, b_1 = \pm i\frac{2m}{n}$, we get the solution:

$$u(x,t) = \mp i\frac{m}{n}\left(1 - \frac{2sin\left(x\pm\frac{\sqrt{(4+2m^2)}}{2}t\right)}{cos\left(x\pm\frac{\sqrt{(4+2m^2)}}{2}t\right)+sin\left(x\pm\frac{\sqrt{(4+2m^2)}}{2}t\right)}\right). \tag{19}$$

Family 6: suppose $c = [-2-i,2-i,-1,1]\ and\ d = [1,-1,1,-1]$

$$f(\xi) = \frac{cos(\xi)+2sin(\xi)}{sin(\xi)}. \tag{20}$$

Segment 6.1: When $\omega = \pm\frac{\sqrt{(4+2m^2)}}{2}, a_0 = \mp i\frac{2m}{n}, a_1 = 0, b_1 = \pm i\frac{5m}{n}$, we reach the soliton solution:

$$u(x,t) = \mp i\frac{m}{n}\left(2 - \frac{5sin\left(x\pm\frac{\sqrt{(4+2m^2)}}{2}t\right)}{cos\left(x\pm\frac{\sqrt{(4+2m^2)}}{2}t\right)+2sin\left(x\pm\frac{\sqrt{(4+2m^2)}}{2}t\right)}\right). \tag{21}$$

Family 7: suppose $c = [1-i, -1-i, -1,1]$ and $d = [i, -i, i, -i]$

$$f(\xi) = \frac{cos(\xi)-sin(\xi)}{sin(\xi)}. \tag{22}$$

Segment 7.1: For $\omega = \pm\frac{\sqrt{(4+2m^2)}}{2}, a_0 = \pm i\frac{m}{n}, a_1 = \pm i\frac{m}{n}, b_1 = 0$, we find the soliton solution:

$$u(x,t) = \pm i\frac{m}{n}\left(1 + \frac{cos\left(x\pm\frac{\sqrt{(4+2m^2)}}{2}t\right)-sin\left(x\pm\frac{\sqrt{(4+2m^2)}}{2}t\right)}{sin\left(x\pm\frac{\sqrt{(4+2m^2)}}{2}t\right)}\right). \tag{23}$$

Family 8: suppose $c = [-3,-2,1,1]$ and $d = [0,1,0,1]$

$$f(\xi) = \frac{-3-2e^{\xi}}{1+e^{\xi}}. \tag{24}$$

Segment 8.1: When $\omega = \pm\sqrt{(1-2m^2)}, a_0 = -\frac{5m}{n}, a_1 = 0, b_1 = -\frac{12m}{n}$**,** our achieved solution:

$$u(x,t) = -\frac{m}{n}\left(5 - 12\frac{3+2e^{\left(x\pm\sqrt{(1-2m^2)}t\right)}}{1+e^{\left(x\pm\sqrt{(1-2m^2)}t\right)}}\right). \tag{25}$$

Family 9: suppose $c = [-1,-2,1,1]$ and $d = [1,0,1,0]$

$$f(\xi) = \frac{-e^{\xi}-2}{e^{\xi}+1}. \tag{26}$$

Segment 9.1: For $\omega = \pm\sqrt{(1-2m^2)}, a_0 = -\frac{3m}{n}, a_1 = -\frac{2m}{n}, b_1 = 0$, we get the solution:

$$u(x,t) = -\frac{m}{n}\left(3 - 2\left(\frac{e^{\left(x\pm\sqrt{(1-2m^2)}t\right)}+2}{e^{\left(x\pm\sqrt{(1-2m^2)}t\right)}+1}\right)\right). \tag{27}$$

Family 10: suppose $c = [2,1,1,1]$ and $d = [1,0,1,0]$

$$f(\xi) = \frac{2e^{\xi}+1}{e^{\xi}+1}. \tag{28}$$

Segment 10.1: When $\omega = \pm\sqrt{(1-2m^2)}, a_0 = -\frac{3m}{n}, a_1 = 0, b_1 = \frac{4m}{n}$, the solution can be extracted as the form:

$$u(x,t) = -\frac{m}{n}\left(3 - 4\left(\frac{e^{\left(x\pm\sqrt{(1-2m^2)}t\right)}+1}{2e^{\left(x\pm\sqrt{(1-2m^2)}t\right)}+1}\right)\right). \tag{29}$$

Family 11: suppose $c = [3,2,1,1]$ and $d = [1,0,1,0]$

$$f(\xi) = \frac{3e^{\xi}+2}{e^{\xi}+1}. \tag{30}$$

Segment 11.1: For $\omega = \pm\sqrt{(1-2m^2)}, a_0 = \frac{5m}{n}, a_1 = -\frac{2m}{n}, b_1 = 0$, we reach the soliton solution:

$$u(x,t) = \frac{m}{n}\left(5 - 2\left(\frac{3e^{\left(x\pm\sqrt{(1-2m^2)}t\right)}+2}{e^{\left(x\pm\sqrt{(1-2m^2)}t\right)}+1}\right)\right). \tag{31}$$

Family 12: suppose $c = [-1,0,1,1]$ and $d = [0,1,0,1]$

$$f(\xi) = -\frac{1}{e^{\xi}+1}. \tag{32}$$

Segment 12.1: When $\omega = \pm\sqrt{(1-2m^2)}, a_0 = -\frac{m}{n}, a_1 = -\frac{2m}{n}, b_1 = 0$, the solution turns into the form:

$$u(x,t) = -\frac{m}{n}\left(1 - 2\left(\frac{1}{e^{\left(x\pm\sqrt{(1-2m^2)}t\right)}+1}\right)\right). \tag{33}$$

The above-established soliton solutions of the LGH model are further general, broad-spectral, useful, and new-fangled and have not been determined in the previous literature.

The physical significance of the results

This section presents the possible physical significance of attained solutions for the earlier-mentioned model. We represent 3D surface plots together with combined 2D graphs of attained solutions for analyzing the effects of different wave velocities on traveling waves within suitable time intervals. Indeed, the plots represent different shapes such as singular soliton, bell shape soliton, kink waves, periodic soliton, multiple singular periodic solitons, and more. We find abundant distinct closed-form soliton solutions such as exponential, hyperbolic, complex hyperbolic, and trigonometric solitons. For simplicity, we depict the obtained solutions of the stated model such as (11), (13), (25), (29), (33), and the figures of residual solutions are omitted here. The internal structure of the associated physical phenomenon is analyzed with the aid of the Wolfram Mathematica software is presented below:

The profiles of the solution (11) for different wave velocities ω are:

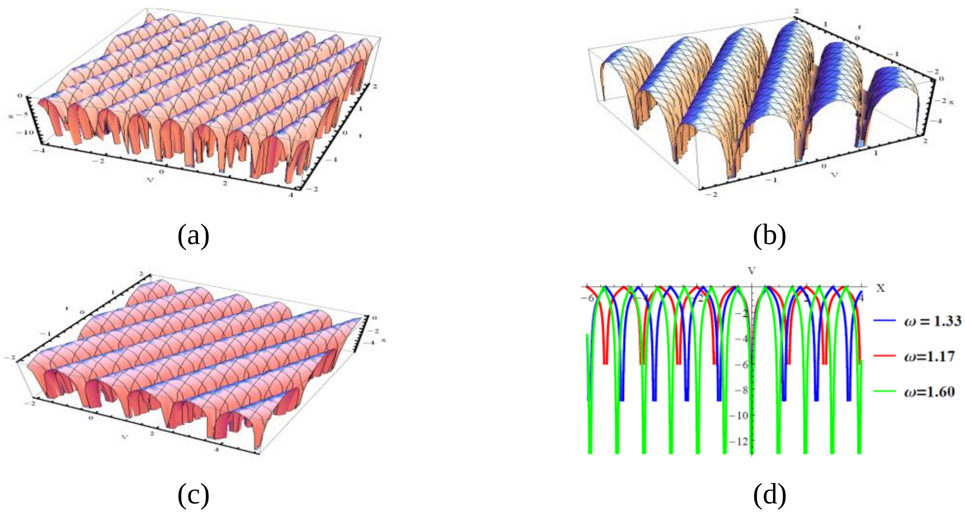

Fig. 1. Dynamic behavior of the solution (11) where a, b, and c indicate 3D plot and d shows 2D combined line graph.

The solution function (11) represents multiple singular periodic solitons for the wave velocity $\omega = 1.60, 1.33,$ and 1.17 respectively (see Figs. 1(a-c)). This shows that with the decrease in wave velocity, the amplitude of a wave is decreased. All the depicted 3D figures vary in shape with the wave velocity ω varies. And Fig. 1-d represents 2D line chart of the solution concerning the wave velocity ω by unchanging the other involved parameters of the considered model. The soliton solution demonstrates its different dynamical behavior.

The profiles of the solution (13) for different wave velocities ω are:

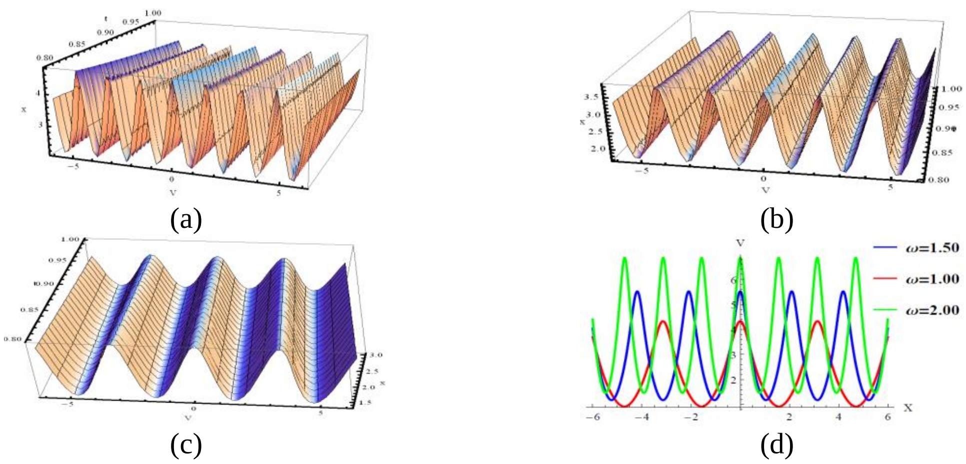

Fig. 2. Dynamic activities of the solution (13) where a, b, and c indicate 3D plot and d shows 2D combined line graph.

The solution function (13) represents the multiple periodic soliton for the wave velocity $\omega = 2.00, 1.50$ and 1.00 respectively (see Figs. 2(a-c)). This shows that with the decrease of wave velocity, the wave turns into a smooth periodic shape. These sketched 3D figures vary in shape with the wave velocity ω varies. And Fig. 2-d represents 2D line chart of the solution concerning the wave velocity ω by unchanging other involved parameters of the stated model. The soliton solution exhibits its different dynamical characteristics.

The profiles of the solution (25) for different wave velocities ω are:

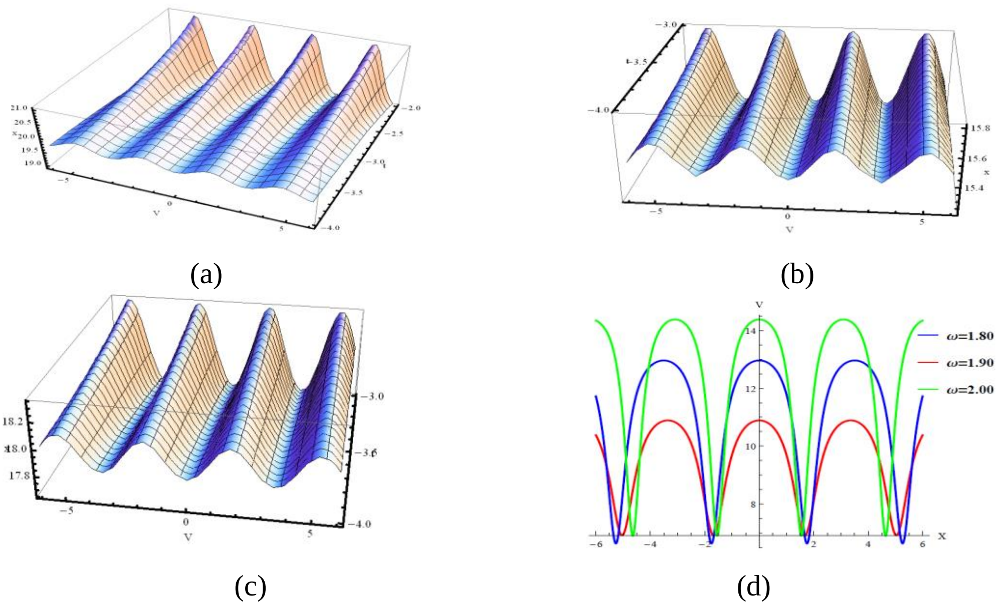

Fig. 3. Dynamical characteristics of the solution (25) where a, b, and c indicate 3D plot and d shows 2D combined line chart.

Again, the solution function (25) represents the multiple periodic solitons for the wave velocity $\omega = 2.00, 1.90,$ and 1.80 respectively (see Figs. 3(a-c)). This depiction illustrates that with the decrease in wave velocity, the amplitude of a wave is decreased. All the sketched 3D figures vary in shape with the wave velocity ω varies. And Fig. 3-d represents 2D line chart of the solution concerning the wave velocity ω by unchanging other involved parameters of mentioned model. This solution demonstrates its diverse dynamical behavior.

The profiles of the solution (29) for different wave velocities ω are:

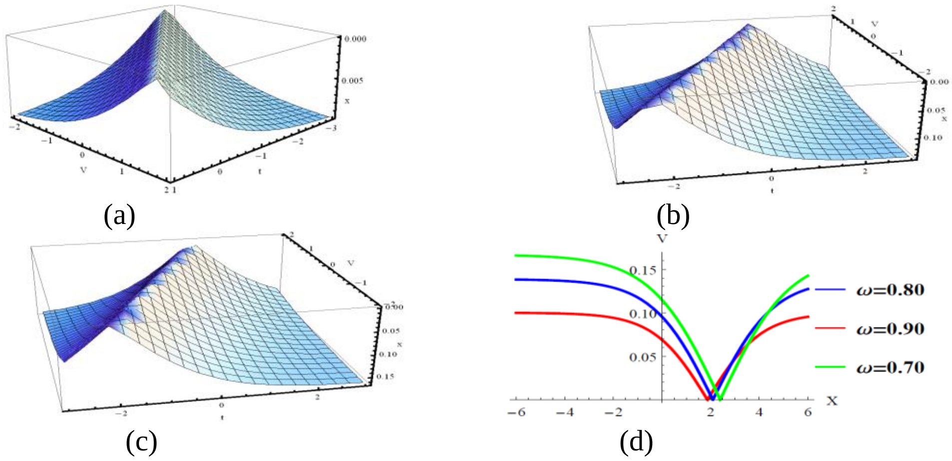

Fig. 4. Dynamic behavior of the solution (29) where a, b, and c indicate 3D plot and d shows 2D combined line chart.

The above graphical illustration shows that the solution function (29) represents bell shape soliton for the wave velocity $\omega = 0.90, 0.80,$ and 0.70 respectively (see Figs. 4(a-c)). These sketching 3D figures vary in shape with the wave velocity ω varies. And Fig. 4-d represents 2D line chart of the solution concerning the wave velocity ω by unchanging other involved parameters of the model. The soliton solution demonstrates its different dynamical characteristics.

The profiles of solution (33) for different wave velocities ω are:

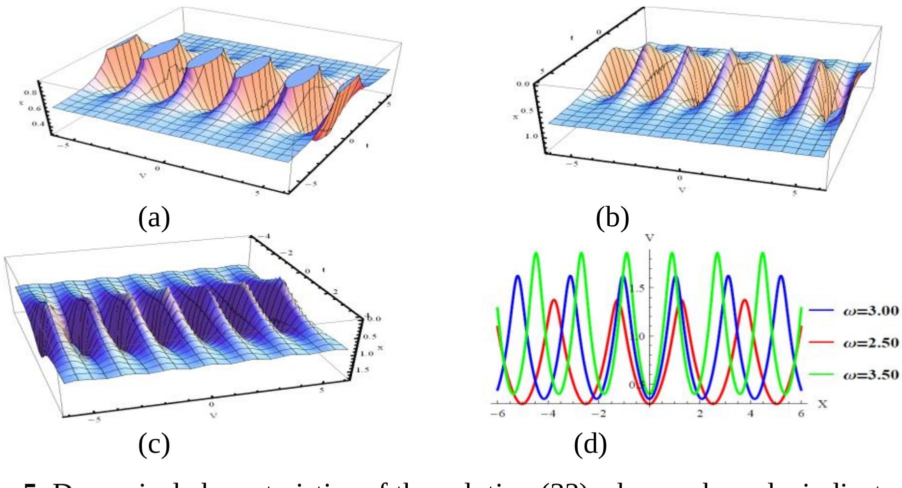

Fig. 5. Dynamical characteristics of the solution (33) where a, b, and c indicate 3D plot and d shows 2D combined line graph.

This graphical depiction shows that the solution function (33) represents singular periodic soliton for the wave velocity $\omega = 2.50, 3.00,$ and 3.50 respectively (see

Figs. 5(a-c)). All the plotted 3D figures vary in shape with the wave velocity ω varies. And Fig. 5-d represents 2D line graph of the solution concerning the wave velocity ω by unchanging other involved parameters of mentioned model. The soliton solution demonstrates its diverse dynamical behavior.

The obtained solutions of the mentioned model are straightforward, useful, efficient, and further general and have not been reported in prior works. This might be supportive to analyze the internal mechanisms of the physical phenomenon and disclose a variety of novel properties of complex incidents connected with the stated model. All the plotted 3D figures of the solutions in this study carry diverse natures of well-known shapes of soliton.

IV. Conclusion

This study has demonstrated the nonlinear Landau-Ginsberg-Higgs (LGH) model with numerous wide-ranging, adequate typical, advanced, exclusive, and further general solitary wave solutions such as the multiple periodic waves, the multiple singular periodic waves, the kink wave, and the bell shape soliton. The generalized exponential rational function method (GERFM) has been successfully implemented in the mentioned model and constructs different novel closed-form traveling wave solutions. The results of the article might be helfful to investigate superconductivity and drift cyclotron waves in radially inhomogeneous plasma for coherent ion-cyclotron waves. Furthermore, the numerical simulations of the attained solutions give an explanation for the physical significance of the stated model to study the dynamic behavior of the systems. Finally, it can be summarized that GERFM is effective, practical, and powerful and offers many precise solutions for nonlinear models that arise in mathematical physics, engineering, solid-state physics, nonlinear science, shallow-water wave mechanics, and other related areas.

Conflict of Interest:

There was no relevant conflict of interest regarding this paper.

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