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NUMERICAL INVESTIGATION OF THE GROWTH-DIFFUSION MODEL

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Abstract

In this article, a numerical solution to the growth-diffusion problem is investigated by obtaining the results of computational experiments for the non-homogeneous growth-diffusion problem and finding its approximate solution by using the modified finite difference method. In this article, a numerical study is carried out by the modified finite difference method. The numerical scheme used a second-order central difference in space with a first-order in time.

Keywords: growth-diffusion problem; modified finite difference method; central difference; non-classical variational.

I. Introduction

The paper will implement a numerical study of the mathematical model of growth diffusion:

$$\frac{\partial}{\partial t} u(x, t) = D\frac{\partial^2}{\partial x^2} u(x, t) + k\, u(\theta - u) / \theta \tag{1}$$

with the initial condition

$$u(x, 0) = f(x), \quad a < x < b \tag{2}$$

and the boundary conditions:

$$\text{and, } t > 0 \quad \left.\begin{aligned} u(a, t) &= h(t) \\ u(b, t) &= g(t) \end{aligned}\right\} \tag{3}$$

where D and k are constants, and θ represents a saturation population per unit area length, and a, b are real numbers.

Equation (1) describes the combination of population (u) growth and diffusion. The generalization of equation (1) is

$$u_t = D\nabla^2 u + k\, u\, (\theta - u) / \theta \qquad (4)$$

A nonlinear eigenvalue problem is solved analytically to obtain the shock-like traveling waves of Fisher's nonlinear diffusion equation. The analytic solution is asymptotically accurate in the limit of infinitely large characteristic speeds [I]. An equation (1) which processes quadratic nonlinearity has been discussed [II].

Several people have investigated analytically and numerically diffusion equations, and there have been a lot of studies and research [IX,V,VIII,VII,IV,III]. A similar result and observation was obtained by Mohammad Izadi [VI].

This article aims to solve problems (1)-(3) approximately by using the Numerical scheme used is a second-order central difference in space with a first order in time.

II. Exact Solvability

The mathematical model relates to mathematical models of Sobolev type

$$L\dot{u} = Mu, \qquad (5)$$

where the operator $L \in L(U;F)$ (i.e. linear and bounded), the operator $M \in Cl(U;F)$ (i.e. linear, closed, and densely defined), and F - Banach space.

Let U –Banach space and $L(U)$ - space of bounded linear operators. A mapping $\mathrm{U} \in C(U; L(U))$ is called a semigroup of operators, if for all $s,t \in R_+$

$$\mathrm{U}^s \mathrm{U}^t = \mathrm{U}^{s+t}. \qquad (6)$$

Usually, the semigroup of operators is identified with its graph $\{U^t:\ t \in R_+\}$. semigroup of operators is called holomorphic, if it is continuous analytically with preserving of property (0.1), In some sectors of the complex plane that contain half-axis R_+. A holomorphic semigroup is called degenerate if its identity $P = s - \lim\limits_{t \to 0+} U^t$ is a projector U .

Let U and F are quasi-Banach spaces of sequences; the operators M and L defined as in the above. Then the operator M is strongly $(L,0)$-sectorial. Consider a weakened (in the sense of S.G. Crane) Showalter-Sidorov problem.

$$\lim_{t\to 0+} P(u(t)-u_0)=0 \tag{7}$$

for a nonhomogeneous linear evolution equation of the Sobolev type

$$L\dot{u}=Mu+f, \tag{8}$$

where a vector function $f:[0,\tau]\to U$, $f=f^0+f^1$, $f^1=Qf$, $f^0=f-f^1$, will defined below, $\tau\in R_+$.

Theorem 4.1. For any vector function $f=f(t)$ such that $f^0\in C^1((0,\tau);F^0)$, $f^1\in C((0,\tau);F^1)$ and for any vector $u_0\in U$ there exists a unique solution $u\in C^1((0,\tau);U)$ for the equation (5) with the condition (4), which has the form

$$u(t)=-M_0^{-1}f^0(t)+U^t u_0+\int_0^{\tau} U^{t-s}L_1^{-1}f^1(s)\,ds. \tag{9}$$

III. Numerical Scheme

On the rectangle [0, T] × [0, 1], we introduce a uniform $j-layer$ with step h in the variable x and with step s in the variable t a grid, we defined a uniform $j-layer$ with a step h in the variable x and with a step in s in the variable t a grid

$wi,j=\{(xi,tj): xi=ih, tj=js, i=0,1,\ldots,n, j=0,1,\ldots,m\}$.We introduce the grid function $uij=u(xi,tj)$ on the grid wi,j. We write the following approximations for the derivatives as follows

$$u_t(x_i,t_j)=\frac{u_{i,j+1}-u_{i,j}}{s}+O(s) \tag{10}$$

$$u_{xx}(x_i,t_j)=\frac{u_{i+1,j}-2u_{i,j}-u_{i-1,j}}{h^2}+O(h^2) \tag{11}$$

With the condition

$$s_j\le\frac{h^2}{2\max\limits_{0\le i\le n}\{(u_{i,j})\}} \tag{12}$$

Therefore, equation (1) takes the form

$$\frac{u_{i,j+1}-u_{i,j}}{s}+O(s)=\frac{u_{i+1,j}-2u_{i,j}-u_{i-1,j}}{h^2}+O(h^2)+\frac{\mathrm{k}\theta u_{i,j}-\mathrm{k}\mathrm{u}_{\mathrm{i,j}}{}^2}{\theta} \tag{13}$$

The initial condition (2) becomes as follows

$$u_{i,0}=u_0(x_i) \tag{14}$$

and the boundary conditions (3)

$$u_{0,j}=u_{1,j}=0 \tag{15}$$

where h(t)=g(t)=0, a=0 and b=1

IV. Numerical Experiments

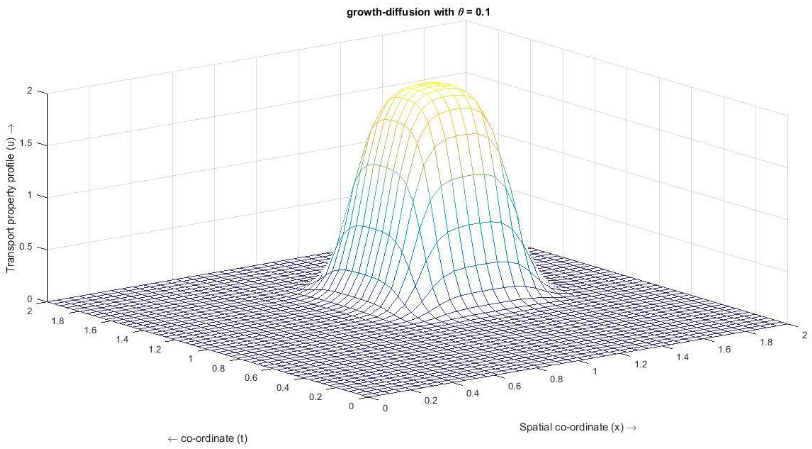
A few numerical computations experiments were implemented on the problems by using the modified finite difference method.

Example 1.

Problem (1)-(3) with the following data:

Initial perturbationf(x) $= 1 - x$, n=50, m= 60 and h= 0.01, $\theta = 0.01$

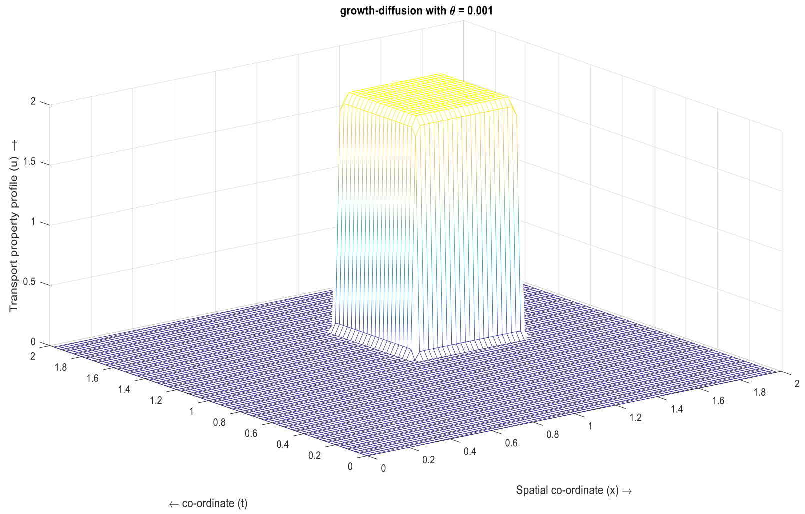
and the boundary conditions h(t)=g(t)=0. Construct an approximate solution as shown in Fig. 1. We noted that the initial state of the phenomenon changes only its form but not its support. The supremum of diffusion decreases as the growth increases. The solution already exists on all-time layers in the segment [0, 3].



Example 2.

Problem (1)-(3) with the following data:

Initial perturbation f(x) $= 1 - x$, n=80, m= 70 and h= 0.001, $\theta = 0.001$
and the boundary conditions h(t)=g(t)=0. Construct an approximate solution as shown in Fig. 2. We noted that the initial phenomenon changes only its form but not its support. The supremum of diffusion decreases as the growth increases. The solution already exists on all time layers in the segment [0, 4].



V. Conclusions:

The numerical results can be improved and obtain a better degree of accuracy when a large number of time and space steps are selected and also small and different numbers for saturation population per unit area length θ. It should be noted that the proposed procedure for solving a growth-diffusion problem is easily and effectively to be implemented. As one can see from the numerical results the parameter θ plays an important role in determining the approximate solution of the growth-diffusion problem, where the solution is dependent on θ.

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Conflict of Interest:

The author declares that there was no conflict of interest regarding this paper.

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