JOURNAL OF MECHANICS OF CONTINUA AND MATHEMATICAL SCIENCES
www.journalimcms.org

ISSN (Online) : 2454 -7190 Vol.-18, No.-6, June (2023) pp 1-13 ISSN (Print) 0973-8975

MIXED CONVECTION ANALYSIS OF HYBRID NANOFLUID IN A LID-DRIVEN CAVITY WITH A HOT BLOCK INSIDE

K. Bouaraour[1], D. Lalmi[2]

[1] Laboratory of Materials, Energetic Systems Technology and Environment. Université of Ghardaia. Ghardaia. Algeria

[2] Laboratory of Materials, Energetic Systems Technology and Environment. Université of Ghardaia. Ghardaia. Algeria

Email: [1]bouaraourk@yahoo.fr, [2]eldjemoui@gmail.com

Corresponding author: **Kamel Bouaraour**

https://doi.org/10.26782/jmcms.2023.06.00001

(Received: March 22, 2023; Accepted: June 02, 2023)

Abstract

The purpose of this study is the investigation of heat transfer and fluid flow around a heated solid block inside a lid-driven cavity filled with hybrid TiO2-Cu/water nanofluid. The considered geometry is a two-dimensional cavity with an aspect ratio of 5. The upper wall translates with uniform velocity Ulid. The solid block attached to the bottom wall of the cavity is maintained at a high temperature compared to the temperature of the upper and lower walls, whereas the other walls are kept insulated. The hybrid nanofluid flow is assumed to be Newtonian, laminar, and incompressible. The effect of the Richardson number is considered by fixing the Reynolds number to 100, and by varying the Grashof number from 102 to 104. Volume fractions for both nanoparticles are varied from 0% to 8%. Results are shown in terms of streamlines, isotherms, and profiles of the average Nusselt number. Numerical results show that clockwise and counterclockwise cells are generated within the rectangular enclosure due to the combined effects of natural and forced convection. Furthermore, increasing the Richardson number from Ri = 0.01 to Ri = 1, which results from an increase in the buoyancy effect, leads to an increase in the Nusselt number of about 4.5%. Moreover, for each Richardson number, an increase of 8% in nanoparticles volume fraction leads to an enhancement of the heat transfer rate by about 9.8%.

Keywords: Nanoparticles, Richardson number, rectangular cavity, Nusselt number.

Nomenclature

ℓ	length of the hot block, m
h	height of the hot block, m
g	is the acceleration due to gravity, m/s^2
H	Height of the cavity, m
L	Length of the cavity, m
Nu	Nusselt number, dimensionless
Re	Reynolds number, dimensionless
Pr	Prandtl number, dimensionless
Gr	Grashof number, dimensionless
T	temperature, dimensionless
P	Pressure, dimensionless
u	velocity component in x direction, m/s
v	velocity component in y direction, m/s
x	coordinate in the horizontal direction, m
y	coordinate in the vertical direction, m
X	coordinate in the horizontal direction, dimensionless
Y	coordinate in the vertical direction, dimensionless

Greek Symbols

θ	Temperature, K
α	thermal diffusivity, m^2/s
μ	dynamic viscosity, $N.s/m^2$
ψ	stream function, m^2/s
ρ	density, kg/m^3
φ	nanoparticle volume fraction
ρ_f	is the density of the base fluid, kg/m^3
ρ_{hnf}	is the density of the hybrid nanofluid, kg/m^3
β	volumetric expansion coefficient of the nanofluid, 1/K
$(\rho c)_f$	the heat capacity of the fluid
$(\rho c)_{hnf}$	the effective heat capacity of the hybrid nanofluid

I. Introduction

Over the past three decades, much research has been aimed at developing efficient cooling of electric, electronic, and nuclear devices and controlling the fluid flow and heat exchange of solar thermal operations and thermal storage. Early studies demonstrated analytically and experimentally the potential of nanofluids for cooling high-power density devices [XXI, XIX, X].

Twenty years ago, researchers began to explore convection inside lid-driven cavities. This type of convection involves both natural and forced convection currents in the same environment and is a useful tool in many different fields of engineering, particularly in heat transfer applications. These studies typically focused on mixed convection and utilized various numerical methods, such as the finite difference method [XII], the finite element method [XVIII], and other methods [V, XV] to simulate the fluid flow and heat transfer in the cavity.

In recent years, researchers have continued to investigate the effects of using a nanofluid in a lid-driven cavity with a focus on more complex phenomena such as MHD [III], Joule heating [XXII], and the hybrid nanofluid approach [VI]. The studies found that the use of a nanofluid in the cavity can enhance heat transfer and fluid flow in the system. These studies have also made use of more advanced numerical methods such as the Lattice Boltzmann Method LBM, Artificial Neural Network ANN [XIV], and High-Order Compact Scheme [XVII].

The LBM is used for the first time by Karimipour et al. [IX] to investigate the nanofluid mixed convection in an inclined lid-driven cavity. This method is developed to predict the fluid flow and heat transfer of air through the inclined lid-driven 2-D cavity. The Boltzmann collision term and hydrodynamic boundary conditions should be modified to include both buoyancy forces and inclination angle effects together with the cavity lid motion. Results show that Nusselt numbers increase for high values of the inclination angle and nanoparticles volume fraction at free convection domination. The same method is used by Goodarzi et al. [VII] to simulate the heat transfer induced by a heated obstacle within a lid-driven cavity. In addition to the ability of the LBM model to simulate the supposed domain, results revealed that the effects of inclination angle are more important at higher values of Richardson numbers.

The studies have also explored the influence of different parameters such as temperature distribution, curvature of the cavity [I, XI], and magnetic field on the fluid flow and heat transfer in the cavity. The heat transfer and laminar flow of Water/Al_2O_3 nanofluid in a T-shaped enclosure with lid-driven under the influence of applying a magnetic field have been numerically investigated [XX]. The numerical solving domain has been simulated two-dimensionally, and the Richardson number takes the following values: 0.1, 1, and 10. nanoparticles volume fraction varied between 0 and 6%. The effect of applying a homogeneous magnetic field on the natural and forced heat transfer parameters of nanofluid has been analyzed for various Hartman numbers. The results of this research revealed that applying a magnetic field has a significant effect on the temperature domain and fluid flow and considerably reduces the circulation mechanisms of fluid. By increasing Richardson and Hartmann numbers, the transfer momentum of cap in the bottom layers of fluid penetrates lesser and the amount of heat transfer reduces. The enhancement of volume fraction of nanoparticles and the reduction of Richardson and Hartmann numbers, significantly enhance the heat transfer of enclosure with cold fluid. Similar results are obtained by Bakar et al [II]. Overall, the study of lid-driven cavities with a heated block inside and subjected to nanofluid flow has evolved over the years, with a focus on the use of more advanced numerical methods and the investigation of more complex phenomena.

The present research, reports the results of mixed convection in a lid-driven cavity filled with hybrid nanofluid with a hot obstacle inside. The cavity aspect ratio is 5, and the hybrid nanofluid is formed by Copper and TiO_2 nanoparticles in water. The effect of various Grashof numbers and volume fractions of nanoparticles has been studied.

II. Geometrical Configuration

The mixed convection of hybrid nanofluid of titania (TiO_2) and Copper (Cu) with water as base fluid is numerically investigated in a lid-driven cavity. A solid rectangular block is attached to the bottom wall of the cavity and maintained at high temperature than the upper wall, which translates with uniform velocity U_{lid}.

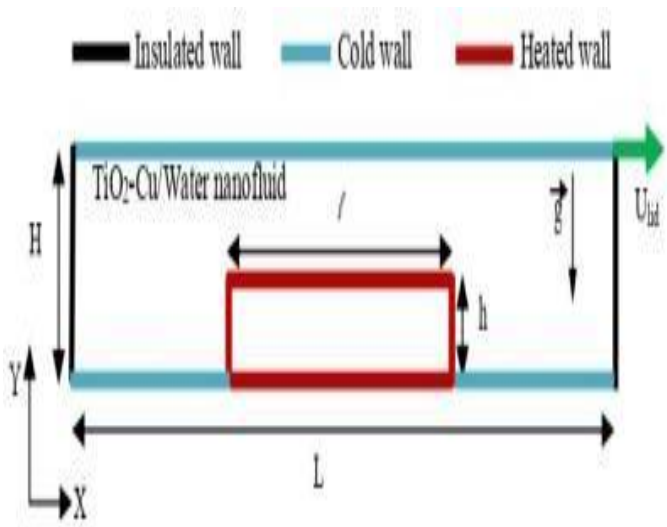

Fig. 1. Configuration of the lid-driven cavity with a hot block inside.

The side walls of the cavity are kept insulated. The aspect ratio of the cavity is fixed to L/H=5, where L is the length and H is the height of the cavity. The length and height of the solid rectangular block are respectively: h=H/2 and ℓ=L/2. A schematic view of the studied cavity is sketched in Fig 1. The thermophysical properties of water and nanoparticles are listed in Table 1.

Table 1: Thermo-physical properties of water and nanoparticles at T=300 K ([VIII])

	H_2O	Cu	TiO_2
C_p (J.Kg^{-1}.K^{-1})	4179	385	686.2
ρ (Kg/m^3)	997.1	8933	4250
k (W.m^{-1}.K^{-1})	0.613	401	8.95
β (K^{-1})	21 x10^{-5}	1.67x10^{-5}	0.9 x10^{-5}
μ (Kg.m^{-1}.s^{-1})	0.855x10^{-3}	-	-

III. Governing Equations and Boundary Conditions

The governing equations for the laminar, Newtonian hybrid nanofluid flow inside the lid-driven cavity are mathematically described by the Navier-Stokes equations, and by energy equation (1-4). It is assumed that the flow is single phase and the water is in thermal balance with nanoparticles. In addition, the flow is assumed to be incompressible and the Boussinesq approximation is considered.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\rho_{hnf}(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y})=-\frac{\partial p}{\partial x}+\mu_{hnf}\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right] \tag{2}$$

$$\rho_{hnf}(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y})=-\frac{\partial p}{\partial y}+\mu_{hnf}\left[\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right]+(\rho\beta)_{hnf}g(\theta-\theta_c) \tag{3}$$

$$u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\alpha_{hnf}\left[\frac{\partial^2 \theta}{\partial x^2}+\frac{\partial^2 \theta}{\partial y^2}\right] \tag{4}$$

The following equations are used to calculate the density, the effective thermal conductivity, and the specific heat capacity of hybrid nanofluid respectively:

$$\rho_{hnf}=(1-\varphi)\rho_f+\varphi_1\rho_1+\varphi_2\rho_2 \tag{5}$$

$$\rho_{hnf}=(1-\varphi)\rho_f+\varphi_1\rho_1+\varphi_2\rho_2 \tag{6}$$

$$(\rho C_p)_{hnf}=(1-\varphi)(\rho C_p)_f+\varphi_1(\rho C_p)_1+\varphi_2(\rho C_p)_2 \tag{7}$$

$$(\rho\beta)_{hnf}=(1-\varphi)(\rho\beta)_f+\varphi_1(\rho\beta)_1+\varphi_2(\rho\beta)_2 \tag{8}$$

For estimating dynamic viscosity, the model of Brinkman [IV] is used:

$$\mu_{hnf}=\mu_f(1-\varphi)^{-2.5} \tag{9}$$

The thermal conductivity of the hybrid nanofluid k_{hnf} was given by the Maxwell-Garnet model [XIII]**:**

$$k_{hnf} = k_f \frac{\left(\frac{\varphi_1 k_1+\varphi_2 k_2}{\varphi}\right)+2k_f+2(\varphi_1 k_1+\varphi_2 k_2)-2\varphi k_f}{\left(\frac{\varphi_1 k_1+\varphi_2 k_2}{\varphi}\right)+2k_f-(\varphi_1 k_1+\varphi_2 k_2)+\varphi k_f} \tag{10}$$

In the previous equations: $\varphi=\varphi_1+\varphi_2$

φ_1 and φ_2 denote respectively the volume fraction of the Cu nanoparticles and TiO_2 nanoparticles. The hybrid nanofluid consisting an equal solid volume fraction of TiO_2 and Cu nanoparticles.

The boundary conditions for the considered problem can be expressed as:

- A slip velocity is considered for the upper wall: $u=U_{lid}$
- No slip boundary conditions are considered over all the other walls.
- The temperature of the upper wall is $\theta=\theta_c$ and the temperature of the heated block is $\theta=\theta_h$.
- The side walls of the cavity are kept insulated.

The dimensionless variables for the considered problem are:

$X=\frac{x}{H},\ Y=\frac{y}{H}, P=\frac{p}{\rho_{nf}U_{lid}^2}\ , U=\frac{u}{U_{lid}},\ V=\frac{v}{U_{lid}}, T=\frac{\theta-\theta_c}{\theta_h-\theta_c}$.

By using the dimensionless parameters, the governing equations are written in the following form:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{11}$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\frac{1}{Re}\frac{\mu_{hnf}}{\rho_{hnf}}\frac{\rho_f}{\mu_f}\left[\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right] \tag{12}$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\frac{1}{Re}\frac{\mu_{hnf}}{\rho_{hnf}}\frac{\rho_f}{\mu_f}\left[\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right]+\frac{Gr}{Re^2}\frac{(\rho\beta)_{hnf}}{\rho_{hnf}\beta_f}T \tag{13}$$

$$U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial Y}=\frac{1}{Re.Pr}\frac{\alpha_{hnf}}{\alpha_f}\left[\frac{\partial^2 T}{\partial X^2}+\frac{\partial^2 T}{\partial Y^2}\right] \tag{14}$$

IV. Numerical Methods

The finite volumes method is used for the discretization of the governing equations. The solver is the commercial CFD code FLUENT 16.0. For coupling of mass and momentum equations, the SIMPLE algorithm is used [XVI]. The convergence criterion was taken 10^{-6} for the normalized residual of each equation. We have used relaxation factors of 0.7 for velocities and energy equation and 0.3 for the pressure.

Code validation

To validate the mathematical model and the numerical method, it has been tested against the numerical results of Karimipour et al. (2014) for the laminar heat transfer within a lid-driven cavity. The cavity with aspect ratio L/H= 5 is filled with Cu/water nanofluid which is assumed to be laminar and incompressible. The upper wall of the cavity translates with uniform velocity U_{lid}, and maintained at high temperature T_h than the temperature of the lower wall T_c. The other walls of the cavity are kept insulated.

For a fixed Reynolds number (Re=100), the Richardson numbers investigated are 1 and 10, with different volume fractions (φ =0, 0.02, and 0.04). Good agreement was found between our numerical results of the mean Nusselt number and the numerical results of Karimipour et al. (2014) as shown in Fig. 2.

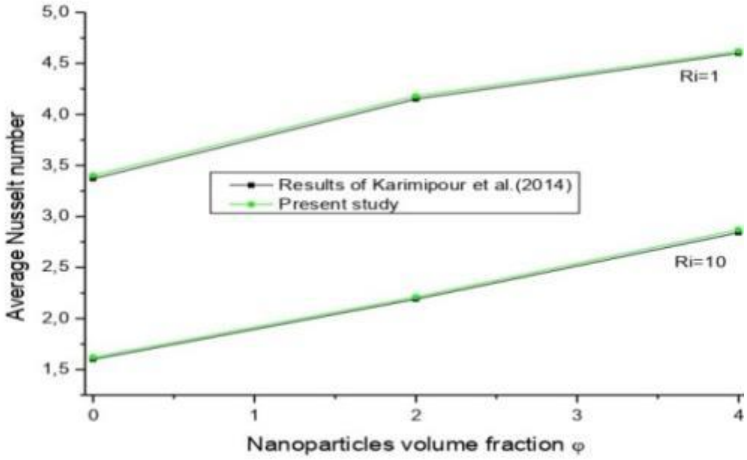

Fig. 2. The validation of the mean Nusselt numbers for Re=100.

Grid Independence

To select the adequate number of nodes, a grid-independent treatment is used where three meshes of different grid sizes are used. Reynolds number based on the inlet velocity and the microchannel height was taken Re = 100. For φ=0%, the average deviation was less than 0.40% between the two first meshes, and less than 0.70% between the two last meshes for the mean Nusselt number of the heated surface as indicated in Table 2. This is why the last mesh was adopted.

Table 2: Effect of the mesh size on the mean Nusselt number.

Mesh size	Mean Nusselt number	Deviation %
500x100	7.15	1.24
550x110	7.24	0.68
600x120	7.29	-

V. Results and Discussions

The numerical results of the current simulation are presented for the laminar mixed convection of hybrid TiO_2-Cu/Water nanofluid within a lid-driven cavity is carried out. For a fixed Reynolds number (Re=100), the considered Richardson numbers in the present work are varied from: Ri=0.01 to Ri=1. This change in Richardson number makes the Grashof number varied between 10^2 and 10^4. The nanoparticles effect on the heat transfer rate is considered, by varying the nanoparticle's fraction from 0% to 8%. The local Nusselt number of each horizontal part of the lower wall can be expressed as:

$$\mathrm{Nu(x)} = -\frac{k_{hnf}}{k_f}\left(\frac{\partial\theta}{\partial y}\right) \quad (15)$$

The local Nusselt number of each vertical part of the lower wall can be expressed as:

$$\mathrm{Nu(y)} = -\frac{k_{hnf}}{k_f}\left(\frac{\partial\theta}{\partial x}\right) \quad (16)$$

The average Nusselt number of the heated surface is defined by:

$$Nu_{ave} = Nu_m|_x + Nu_m|_y \quad (17)$$

Where $Nu_m|_x$ and $Nu_m|_y$ denote the integration of the formula (15) and (16) over the length of each horizontal and vertical part of the hot block respectively.

Effect of Richardson Number

Fig. 3 indicates the evolution of the average Nusselt number with nanoparticles volume fraction. For all considered Richardson numbers, the Nusselt number increases almost linearly with nanoparticles volume fraction. The increase of Copper and TiO_2 hybrid nanoparticles in the receiving fluid improves heat transfer influenced by the enhancement of the overall heat conductivity from the hybrid nanofluids. Also, in higher Richardson values, changes in heat transfer and Nusselt number are more important.

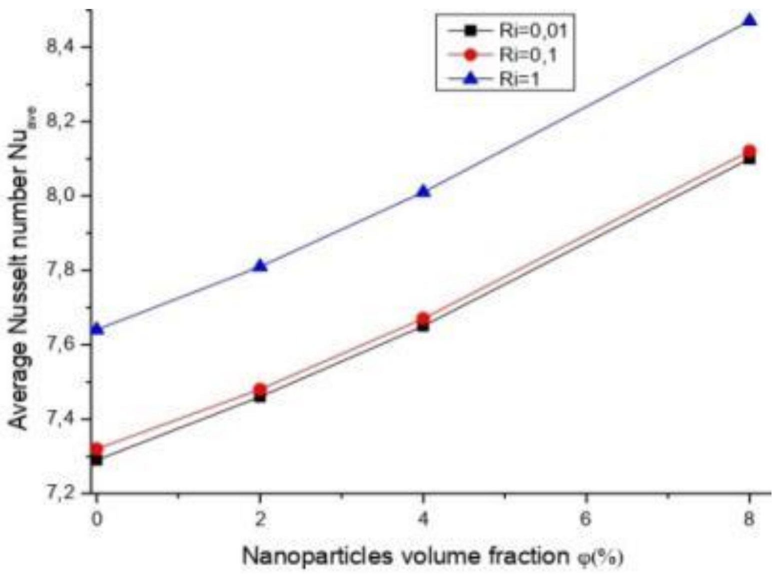

Fig. 3. Average Nusselt number for different volume fractions

For a fixed nanoparticle volume fraction (φ = 0.08), fig. 4 shows the streamlines and the isotherms for different Richardson numbers. In this state, the flow pattern displays two clockwise cells which have almost occupied the entire space of the cavity. The first one is centered in the upper left corner of the cavity and elongated to the right side under the influence of shear force caused by the lid wall velocity. The second cell is located on the right side of the heated obstacle. With an increasing buoyancy effect, a small counterclockwise cell is created in the left lower corner of the cavity.

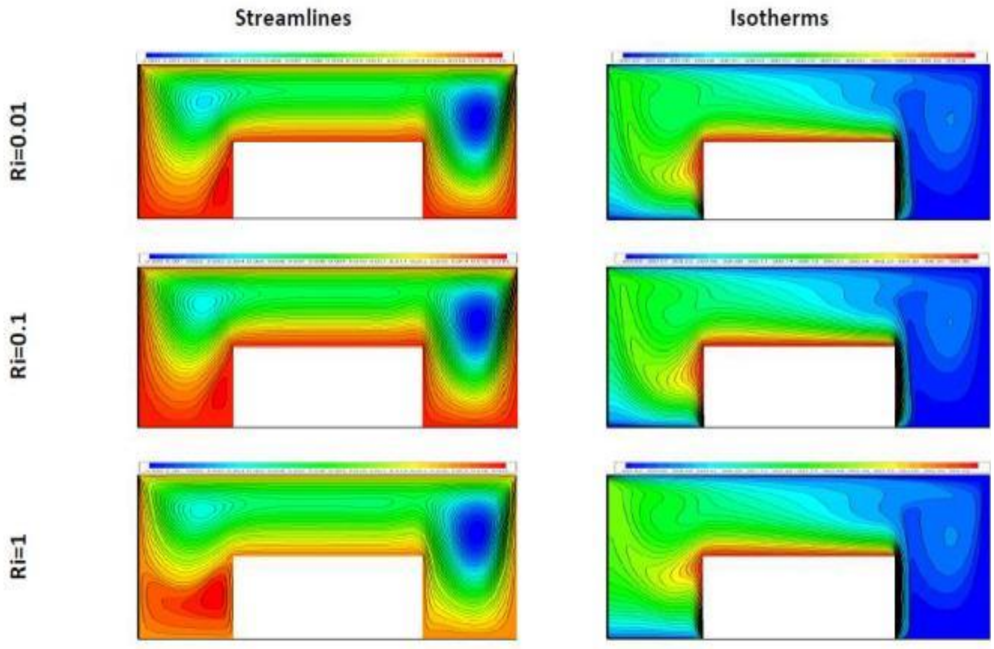

Fig. 4. Streamlines and isotherms for φ = 0.08

This cell got bigger when increasing the Richardson number. In addition, temperature lines are highly intense in the areas close to the hot block, reflecting the important temperature gradient. Isotherm lines separate slightly from each other by moving away from the hot obstacle, and the temperature gradient decreases.

Effect of nanoparticles fraction

Streamlines and isotherms for Ri=1 are given in Fig. 5. Two large cells dominate the whole domain of the cavity with no significant changes seen with volume fraction variation, and a small cell is created on the left side of the heated block can be seen. Moreover, isotherms are dense and compact around the heated block due to the resulting temperature gradient.

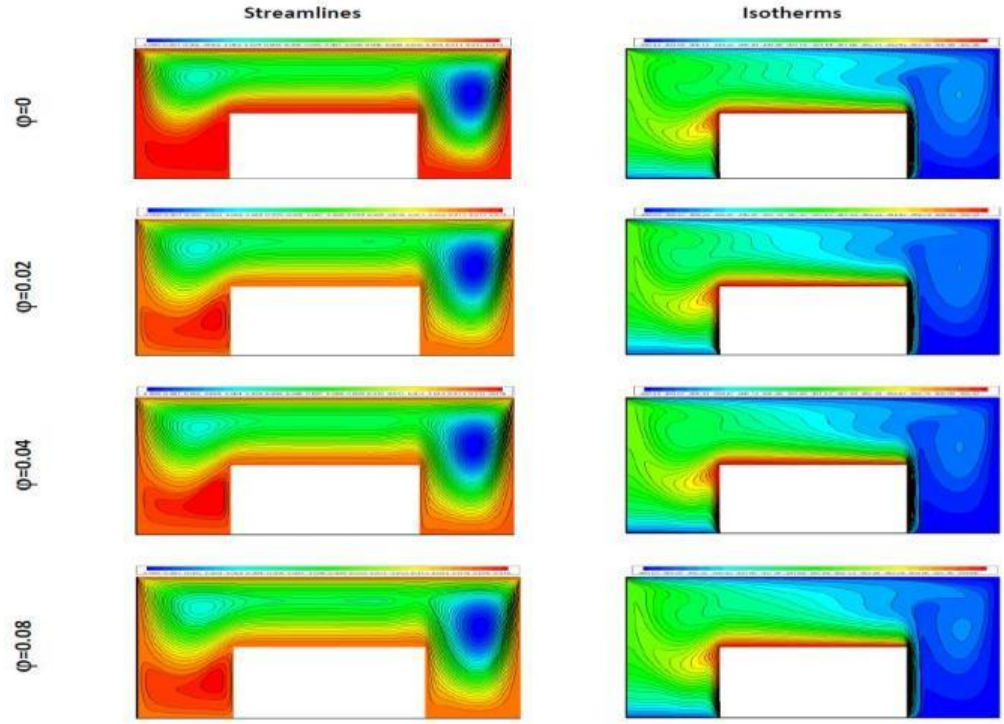

Fig. 5. Streamlines and isotherms for **Ri=1**

Fig. 6 displays the variation of the average Nusselt number with the Richardson number for different nanoparticle fractions. For a given value of volume fraction, it can be seen that the application of hybrid nanofluids enhanced heat transfer in the cavity compared to the Cu-based nanofluid and TiO_2-based nanofluid. Note that the heat rate is a monotonically increasing function of Ri number due to the increased buoyancy effect.

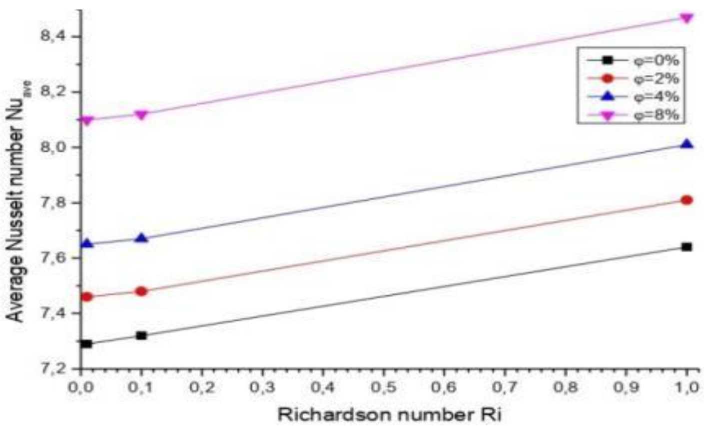

Fig. 6. Average Nusselt number for different Richardson numbers

VI. Conclusions

In the present research, the mixed convection of hybrid nanofluid flow inside a rectangular lid-driven cavity is numerically investigated. A hot block is attached to the lower wall of the cavity, whereas the side walls are insulated. The hybrid nanofluid is composed of Copper and TiO_2 nanoparticles in thermal balance with water as the base fluid. The flow is considered a single phase and governed by the Naviers-Stokes equations and by the energy equation in the two-dimensional case. The considered Reynolds number in the present simulations is fixed to Re=100 and the Grashof number was varied from $Gr=10^2$ to $Gr=10^4$. The nanoparticles effect is considered by varying the nanoparticles volume fraction from 0% to 8%. Numerical results revealed that due to the combined effect of natural and forced convection, clockwise and counterclockwise cells are created inside the rectangular enclosure. In addition, increasing the Richardson number from Ri=0.01 to Ri=1 causes an increase in the buoyancy effect, which leads to an enhancement of the Nusselt number by about 4.5% for all volume fractions. Furthermore, the heat transfer rate is enhanced with the augmentation of nanoparticles volume fraction by about 9.8% for each Richardson number.

Conflict of Interest:

There was no relevant conflict of interest regarding this paper.

References

I. Aljabair, S., Ekaid, A. L., Hasan ibrahim, S. and Alesbe, I : MIXED CONVECTION IN SINUSOIDAL LID- DRIVEN CAVITY WITH NON-UNIFORM TEMPERATURE DISTRIBUTION ON THE WALL UTILIZING NANOFLUID. *Heliyon 7, e06907, 2021*.

II. Bakar, N. A., Karimipour, A. and Roslan, R. : EFFECT OF MAGNETIC FIELD ON MIXED CONVECTION HEAT TRANSFER IN A LID-DRIVEN SQUARE CAVITY. *Journal of Thermodynamics, Article ID 3487182, 2016*. 10.1155/2016/3487182.

III. Bakar, N. A., Roslan, R., Karimipour, A. and Hashim, I. : MIXED CONVECTION IN LID-DRIVEN CAVITY WITH INCLINED MAGNETIC FIELD. *Sains Malaysiana, 48(2), pp 451–471, 2019*. 10.17576/jsm-2019-4802-24.

IV. Brinkman, H. C. : THE VISCOSITY OF CONCENTRATED SUSPENSIONS AND SOLUTIONS. *Journal of Chemical Physics, 3, pp 571–581, 1952*. 10.1063/1.1700493.

V. Dumon, A., Allery, C. and Ammar, A. : SIMULATION OF HEAT AND MASS TRANSPORT IN A SQUARE LID-DRIVEN CAVITY WITH PROPER GENERALIZED DECOMPOSITION (PGD). *Numerical Heat Transfer, Part B:* Fundamentals *An International Journal of Computation and Methodology, 63(1), pp 18-43, 2013*. 10.1080/10407790.2012.724991.

VI. Geridonmez, B. P. and Oztop, H. F. : ENTROPY GENERATION DUE TO MAGNETO-CONVECTION OF A HYBRID NANOFLUID IN THE PRESENCE OF A WAVY CONDUCTING WALL. *Mathematics, 10(24), 4663, 2022.* 10.3390/math10244663.

VII. Goodarzi, M., D'orazio, A., Keshavarzi, A., Mousavi, S. and Karimipour, A. : DEVELOP THE NANO-SCALE METHOD OF LATTICE BOLTZMANN TO PREDICT THE FLUID FLOW AND HEAT TRANSFER OF AIR IN THE INCLINED LID DRIVEN CAVITY WITH A LARGE HEAT SOURCE INSIDE, TWO CASE STUDIES: PURE NATURAL CONVECTION & MIXED CONVECTION. *Physica A, 509, pp 210–233, 2018*. 10.1016/j.physa.2018.06.013.

VIII. Incropera, F. P. and De witt, D. P. (2002), Introduction to Heat Transfer, *Wiley 4th edition, New York.*

IX. Karimipour, A., Hemmat esfe, M., Reza Safaei, M., Toghrai, D., Jafari, S. and Kazi, S. N. : MIXED CONVECTION OF COPPER–WATER NANOFLUID IN A SHALLOW INCLINED LID DRIVEN CAVITY USING THE LATTICE BOLTZMANN METHOD. *Physica A, 402, pp 150–168, 2014*. 10.1016/j.physa.2014.01.057.

X. Khanafer, K. and Vafai, K. : A CRITICAL SYNTHESIS OF THERMOPHYSICAL CHARACTERISTICS OF NANOFLUIDS, *Int. J. Heat Mass Trans., 54, pp 4410-4442, 2012*. 10.1016/j.ijheatmasstransfer.2011.04.048.

XI. Korei, Z. and Benissaad, S. : ENTROPY GENERATION OF A HYBRID NANOFLUID ON MHD MIXED CONVECTION IS A LID-DRIVEN CAVITY WITH PARTIAL HEATING HAVING TWO ROUNDED CORNERS. *E3S Web of Conferences 321, 02004, 2021*. 10.1051/e3sconf/202132102004

XII. Kosti, S. and Rathore, V. S. : NUMERICAL STUDY OF LID DRIVEN CAVITY AT DIFFERENT REYNOLDS NUMBER. *Trends in Mechanical Engineering and Technology, 5(3), pp 1-5, 2015.*

XIII. Maxwell, J. C. (1954), : A Treatise on Electricity and Magnetism, *Unabridged, Dover*.

XIV. Nahak, P., Triveni, M. K. and Panua, R. : NUMERICAL INVESTIGATION OF MIXED CONVECTION IN A LID-DRIVEN TRIANGULAR CAVITY WITH A CIRCULAR CYLINDER USING ANN MODELING. *International Journal of Heat and Technology, 35(4), pp 903-918, 2017*. 10.18280/ijht.350427.

XV. Pashaie, P., Jafari, M., Baseri, H. and Farhadi, M. : NUSSELT NUMBER ESTIMATION ALONG A WAVY WALL IN AN INCLINED LID-DRIVEN CAVITY USING ADAPTIVE NEURO-FUZZY INFERENCE SYSTEM (ANFIS). *IJE TRANSACTIONS A*: *Basics, 26(4), pp 383-392, 2013*.

XVI. Patankar, S. V. (1980), : Numerical Heat Transfer and Fluid Flow, *Mac Graw Hill, New York.*

XVII. Rashidi, M. M., Sadri, M. and Sheremet, M. A. : NUMERICAL SIMULATION OF HYBRID NANOFLUID MIXED CONVECTION IN A LID-DRIVEN SQUARE CAVITY WITH MAGNETIC FIELD USING HIGH-ORDER COMPACT SCHEME. *Nanomaterials, 11(9), 2250, 2021*.

XVIII. Saha, L. K., Somadder, M. C. and Salah uddin, K. M. : MIXED CONVECTION HEAT TRANSFER IN A LID DRIVEN CAVITY WITH WAVY BOTTOM SURFACE. *American Journal of Applied Mathematics, 1(5), pp 92-101, 2013*.

XIX. Saidur, R., Leong, K. Y. and Mohammad, H. A. : A REVIEW ON APPLICATIONS AND CHALLENGES OF NANOFLUIDS. *Renew. Sustain. Energy. Rev.,15, pp 1646-1668, 2011*.

XX. Sarlak, R., Yousefzadeh, S., Akbari, O. A., Toghraie, D., Sarlak, S. and Assadi, F. : THE INVESTIGATION OF SIMULTANEOUS HEAT TRANSFER OF WATER/AL_2O_3 NANOFLUID IN A CLOSE ENCLOSURE BY APPLYING HOMOGENEOUS MAGNETIC FIELD. *International Journal of Mechanical Sciences, 133, pp 674-688, 2017*. 10.1016/j.ijmecsci.2017.09.035.

XXI. Tuckerman, D. B. and. Pease, R. F. W. : HIGH PERFORMANCE HEAT SINKING FOR VLSI. *IEEE Electron Dev. Letters., 2, pp 126- 129, 1981*.

XXII. Zahan, I., Nasrin, R. and Alim, M. A. : MIXED CONVECTIVE HYBRID NANOFLUID FLOW IN LID-DRIVEN UNDULATED CAVITY: EFFECT OF MHD AND JOULE HEATING. *Journal of Naval Architecture and Marine Engineering, 16, pp 109-126, 2019*. 10.3329/jname.v16i2.40585.