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# A NEW CONCEPT OF THE EXTENDED FORM OF PYTHAGORAS THEOREM

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**Abstract**

*According to Pythagoras Theorem : In a right-angled triangle* $x^2 + y^2 = z^2$ *, where, base = x, altitude = y, and hypotenuse = z. In the present paper, the author states that* $x^2 + y^2 = -z^2$ *is the extended form of the Pythagoras Theorem.*

**Keywords :** Countup and countdown straight line, circle, Dynamics of Numbers, Pythagoras Theorem

## I. Introduction

What is mathematics? According to the author, mathematics is a tool to explain the truth of nature. At present, we are using two tools, 1) real numbers (positive or negative) and 2) complex numbers.

The author has developed a new tool to explain the truth of nature with the help of the Theory of Dynamics of Numbers. Why this tool is necessary? In the case of complex numbers not only 0 + i0 is undefined but also there is no order relation in a complex number system except for equality relation. For example, we can not find whether 2 + i3 is greater or less than 3 + i2. These are the drawback in the case of a complex number. That is why the new tool: the Theory of Dynamics of Numbers is necessary.

What is the Theory of Dynamics of Numbers? Basically, it is governed by three laws.

**(1)** 0 (zero) is defined as starting point of any number. There is an infinite number of directions through which the numbers can move from the starting point 0 (zero) and back to the starting point 0 (zero) with a vertically opposite direction of motion of numbers. The number which is moving away from the starting point 0 (zero) is defined as countup number and the number which is moving toward the starting point 0 (zero) is defined as countdown number.

**(2)** The Countup numbers are always greater than or equal to the countdown numbers. The Countup numbers can move independently but the motion of the countdown numbers is dependent on the motion of Countup numbers. The motion of the countdown numbers exists if and only if there are motions of the Countup numbers.

**(3)** For every equation, the Countup numbers are always equal to the countdown numbers.

With the help of this tool, the author has developed Rectangular Bhattacharyya's Coordinate System [XIII] where there is no negative axis or abscissa whereas in Cartesian System there is a negative axis or abscissa.

In a coordinate system, we find actually the distance between any two points. The author has shown that it is possible to find the distance between any two points in a plane without taking the negative axis or abscissa.

With this new concept, the author has established the relation between stream function and velocity potential $\frac{\partial \varphi}{\partial x} = \frac{\partial \Psi}{\partial y}$ and $\frac{\partial \varphi}{\partial y} = -\frac{\partial \Psi}{\partial x}$ in real numbers without using imaginary numbers concept which has been done by Cauchy – Riemann using imaginary number concept [XVIII]. The concept of complex numbers was first introduced by Euler and subsequently, Hamilton, Gauss, Cauchy, Riemann, Navier, Stokes, and others used this concept because they could not find the solution of quadratic equation, $x^2 + 1 = 0$ in real numbers, so they used complex numbers to solve any problem in fluid dynamics and in other fields. But the author has shown that $x^2 + 1 = 0$ can be solved in real numbers with the help of the Theory of Dynamics of Numbers by publishing a paper [XV].

The author has developed a new concept of the extended form of Pythagoras Theorem using that tool.

However, The method of solution of the extended form of Pythagoras Theorem has not been investigated by any author previously with a similar approach.

## II. Formulation of the problem and its Solution :

To understand the extended form of Pythagoras Theorem the author used some new concepts and definitions. The new concepts are used namely

1. Rectangular Bhattacharyya's Coordinates [XIII]
2. Theory of Dynamics of Numbers [XIV]
3. New Concept of Quadratic Equation [XV, XVII ]
4. New Concept of Circle. [XVI]

## III. Some Definitions

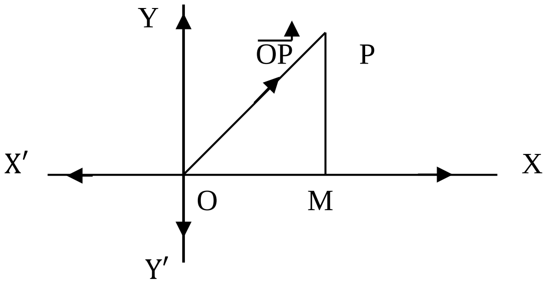


**Fig. 1.**

### (A) Count up straight line :

If a straight line is drawn by taking points moving away from the origin or a fixed point O to another point P, then the straight line OP is called countup OP.

It is symbolically represented as $\overset{\uparrow}{\overline{\mathrm{OP}}}$. $\overset{\uparrow}{\overline{\mathrm{OP}}}$ means the distance between the points O and P and the direction will be from O to P as in fig.. 1.

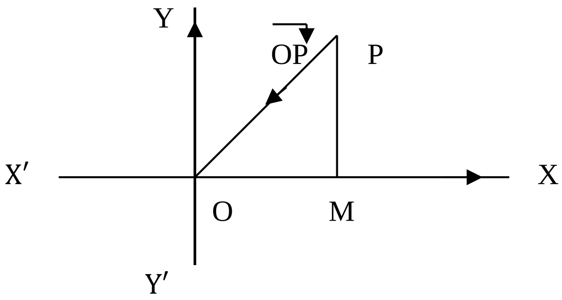


**Fig. 2.**

### (B) **Countdown straight line :**

If a straight line is drawn by taking points moving towards the origin or a fixed point O from another point P, then the straight line OP is called countdown OP.

It is symbolically represented as $\overset{\downarrow}{\overline{\mathrm{OP}}}$. $\overset{\downarrow}{\overline{\mathrm{OP}}}$ means the distance between the points O and P and the direction will be from the point P to the O as in fig. 2.

**(C) Angle between countup and Countdown straight lines :**

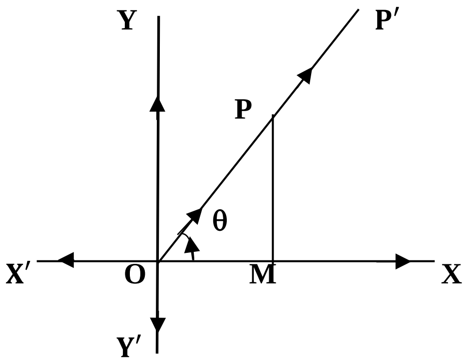


**Fig. 3.**

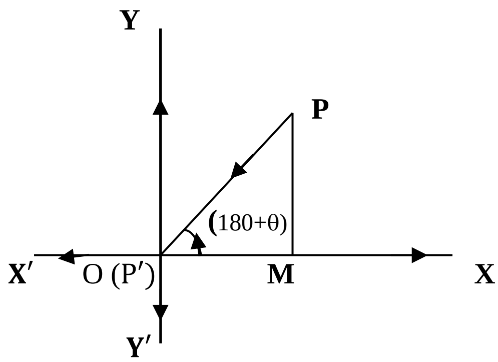


**Fig. 4.**

In fig. 3, a line $\overrightarrow{OPP'}$ is drawn in the first quadrant such that $\overrightarrow{OP} = \overrightarrow{PP'}$, i.e. P is the midpoint of $\overrightarrow{OPP'}$ and $\measuredangle P'OX = \theta$. Now consider point P as the centre and then rotate the line $\overrightarrow{PP'}$ $180^0$ in anti-clockwise direction with respect to $\overrightarrow{OPP'}$. So, it can be found that the point P′ will coinside with the origin O. It means that $\overset{\blacktriangledown}{\overline{OP}}$ can be obtained after rotating $180^0$ with $\overset{\blacktriangle}{\overline{OP}}$. So the angle between $\overset{\blacktriangledown}{\overline{OP}}$ and $\overset{\blacktriangle}{\overline{OP}}$ is $180^0$ but both $\overset{\blacktriangledown}{\overline{OP}}$ and $\overset{\blacktriangle}{\overline{OP}}$ will lie in the first quadrant of the plane and $\overset{\blacktriangledown}{\overline{OP}}$ will make an angle $180^0 + \theta$ with the X – axis as shown in fig. 4 and symbolically denoted as $\measuredangle POX = \theta$ and $\measuredangle POX = 180^0 + \theta$.

**(D) Double Role of the Point P**

For example : I am a father to my son as well as I am a son to my father but I am one and the same person in my family. So, I have a double role in my family.

Similarly, From Fig. 3. and Fig. 4. we find that point P has a double role with respect to the same frame of reference. In Fig. 3. Point P is a countup point whereas in Fig. 4. P is the countdown point though they are one and the same point in the same frame of reference. The role of point P will depend on the inherent nature of motion whether point P is considered as moving away from the origin or moving towards the origin.

**Symbols :** ‘ $\overset{\blacktriangle}{\overline{\quad}}$ ’ means a countup straight line that is a bar with an upward arrow.
‘ $\overset{\blacktriangledown}{\overline{\quad}}$ ’ means a countdown straight line that is bar with down ward arrow.

Note that $\overset{\blacktriangledown}{\overline{OP}}$ is vertically opposite to $\overset{\blacktriangle}{\overline{OP}}$ having equal distance and $\overset{\blacktriangledown}{\overline{OP}} = -\overset{\blacktriangle}{\overline{OP}}$.

Also note that $\angle$ POX = θ means the angle between $\overset{\uparrow}{\overline{OP}}$ and $\overset{\uparrow}{\overline{OX}}$

and $\angle$ POX = $180^0 + \theta$ means angle between $\overset{\downarrow}{\overline{OP}}$ and $\overset{\uparrow}{\overline{OX}}$

**<u>Note</u> :** (1) Significance of the forward arrow (→) and backward arrow (←) over the head of a line means the direction of the line only.

(2) If there is no symbol or a bar only over the head of a line such as OP or $\overline{OP}$ means a line with the distance between two points O and P only.

**IV. Formulation of the Extended form of Pythagoras Theorem and its solution :**

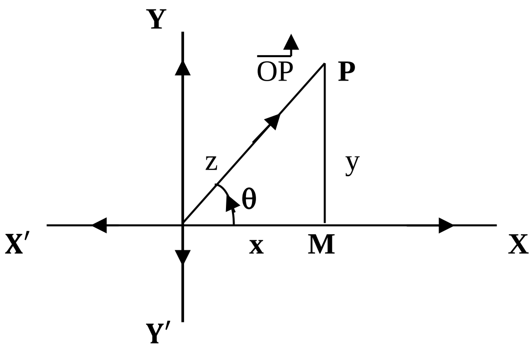


**Fig. 5.**

According to the Pythagoras Theorem

$$x^2 + y^2 = z^2 \quad (1)$$

Where, base = OM = x, altitude = MP = y and hypotenuse = OP = z.

Suppose, $\angle$ POX = θ

Therefore, x = z cosθ and y = z sinθ.

According to the Theory of Dynamics of Numbers the Pythagoras Theorem takes the form

$$\overset{\uparrow}{\overline{x^2}} + \overset{\uparrow}{\overline{y^2}} = \overset{\uparrow}{\overline{z^2}}$$

and

$$\overset{\uparrow}{\overline{x}} = \overset{\uparrow}{\overline{z}} \cos\theta \text{ and } \overset{\uparrow}{\overline{y}} = \overset{\uparrow}{\overline{z}} \sin\theta$$

Now, suppose

$$\overset{\uparrow}{\overline{z^2}} = \overset{\uparrow}{\overline{r^2}} \quad (2)$$

So,

$$\overset{\uparrow}{\overline{x^2}} + \overset{\uparrow}{\overline{y^2}} = \overset{\uparrow}{\overline{r^2}} \quad (3)$$

and we have,

$$\overset{\uparrow}{x} = \overset{\uparrow}{r}\cos\theta \quad \text{and} \quad \overset{\uparrow}{y} = \overset{\uparrow}{r}\sin\theta \tag{4}$$

Conventionally we know that

$$x^2 + y^2 = r^2$$

Represents the equation of a circle. But according to the theory of dynamics of numbers, the equation (4) takes the form

$$\overset{\uparrow}{x^2} + \overset{\uparrow}{y^2} = \overset{\uparrow}{r^2} \tag{5}$$

which represents a count up circle.

Now, suppose

$$z^2 = -r^2,\ r > 0 \tag{6}$$

We can write with the equation (6) as

$$z^2 + r^2 = 0,\ r > 0 \tag{7}$$

According to theory of dynamics of numbers we have

$$\overset{\downarrow}{z^2} + \overset{\uparrow}{r^2} = 0 \tag{8}$$

According to 3rd law of theory of dynamics of numbers we have

$$z^2 = r^2$$

Or, $\quad z = r$

So, the solution of equation (7) will be

$$z = \overset{\downarrow}{r} = -r \tag{9}$$

therefore,

$$x = -r\cos\theta = r\cos(180^0 + \theta) \tag{10}$$

and

$$y = -r\sin\theta = r\sin(180^0 + \theta) \tag{11}$$

The equations (10) and (11) prove that $\overset{\downarrow}{OP}$ and $\overset{\uparrow}{OP}$ exist in the first quadrant and $\overset{\downarrow}{OP}$ is obtained by rotation of $180^0$ with $\overset{\uparrow}{OP}$. The line $\overset{\downarrow}{OP}$ makes $(180^0 + \theta)$ angle with X – axis.

According to the author the equation $x^2 + y^2 = -r^2$ represents a countdown circle whereas $x^2 + y^2 = r^2$ represents, a count up circle.

**Problem – 1**

**Find the coordinates of the centre and its location on the real plane and the length of the radius of the circle.**

**Solution :**

$$5x^2 + 5y^2 - 8x + 6y + 15 = 0 \qquad (1)$$

or,

$$x^2 + y^2 - \frac{8}{5}x + \frac{6}{5}y + 3 = 0$$

Here,

$$g = -\frac{4}{5}, f = \frac{3}{5} \text{ and } c = 3$$

So,

$$g^2 + f^2 = \left(-\frac{4}{5}\right)^2 + \left(\frac{3}{5}\right)^2 = \frac{16}{25} + \frac{9}{25} = 1 \text{ and } c = 3$$

Here,

$$g^2 + f^2 < c$$

Since, $g^2 + f^2 < c$, the inherent nature of x and y are count down x and countdown y in equation (1).

According to the Theory of Dynamics of Numbers, equation (1) takes the form

$$x^2 + y^2 - \frac{8}{5}x + \frac{6}{5}y + 3 = 0 \qquad (2)$$

or,

$$\left(x - \frac{4}{5}\right)^2 + \left(y + \frac{3}{5}\right)^2 + 1 + 3 = 0$$

$$\left(x - \frac{4}{5}\right)^2 + \left(y + \frac{3}{5}\right)^2 = 4 = -4 \qquad (3)$$

Suppose,

$$r^2 = -4 \qquad (4)$$

or,

$$r^2 + 4 = 0 \qquad (5)$$

According to the Theory of Dynamics of Numbers, equation (5) takes the form

$$r^2 + 4 = 0 \qquad (6)$$

According to the third law of the Theory of Dynamics of Numbers

$$r^2 = 4$$

or, $r = 2$

So, the solution of equation (4) will be

$$r = 2 = -2 \qquad (7)$$

According to Rectangular Bhattacharyya's Coordinate System, the equation (3) takes the form

$$\left(x - \frac{4}{5}\right)^2 + \left(y + \frac{3}{5}\right)^2 = r^2 = -r^2 \tag{8}$$

which is extended form of Pythagoras Theorem where $r = -2$.

Equation (8) represents a countdown circle. Here, the point (x, y) lie on the first quadrant of the real rectangular plane and the coordinates of the centre, $C\left(\frac{4}{5}, \frac{3'}{5}\right)$ lies on the forth quadrant of the real rectangular plane and its radius, $r = -2$, according to Rectangular Bhattacharyya's Coordinate System.

Now, let us solve the said problem by convensional method

$$5x^2 + 5y^2 - 8x + 6y + 15 = 0 \tag{1}$$

$$x^2 + y^2 - \frac{8}{5}x + \frac{6}{5}y + 3 = 0$$

$$\left(x - \frac{4}{5}\right)^2 + \left(y + \frac{3}{5}\right)^2 = -4$$

$$\left(x - \frac{4}{5}\right)^2 + \left(y + \frac{3}{5}\right)^2 = r^2 \tag{2}$$

Where, $r^2 = -4$ or $r = 2i$, $i = \sqrt{-1}$

It is clear from equation (2) that the coordinates of the centre of the circle are $\left(\frac{4}{5}, \frac{-3}{5}\right)$ and its radius, r = 2i and the point P (x, y) lie in the first quadrant of the real rectangular plane and the centre $C\left(\frac{4}{5}, \frac{-3}{5}\right)$ lie in the forth quadrant of the real rectangular plane.

**Observation :**

**I.** We know that the distance between any two points having two real coordinates on a real rectangular plane cannot be imaginary quantity according to Rectangular Bhattacharyya's Coordinate System where as we find that the radius, r = 2i by convensional method.

**II.** In case of the same problem if we solve it according to Rectangular Bhattacharyya's Coordinate System which is based on the Theory of Dynamics of Numbers that the numerical value of the radius of the countdown circle, r = – 2 but we can not find the numerical value of radius, r = 2i

## V. Conclusion

The author states that any problem related to equation of a circle $x^2 + y^2 = -r^2$ Which is primarily based on extended form of Pythagoras Theorem can be solved without using the concept of complex numbers.

**Conflict of Interest:**

The author declared that there was no relevant conflict of interest regarding this paper.

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