JOURNAL OF MECHANICS OF CONTINUA AND MATHEMATICAL SCIENCES
www.journalimcms.org

ISSN (Online) : 2454 -7190 Vol.-18, No.-04, April (2023) pp 10-22 ISSN (Print) 0973-8975

IMAGE WATERMARKING ON DEGRADED COMPRESSED SENSING MEASUREMENTS

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https://doi.org/10.26782/jmcms.2023.04.00002

(Received: January 29, 2023; Accepted: April 05, 2023)

Abstract

This paper proposes an additive watermarking on sparse or compressible coefficients of the host image in the presence of blurring and additive noise degradation. The sparse coefficients are obtained through basis pursuit (BP). Watermark recovery is done through deblurring, and performance is studied here for Wiener and fast total variation deconvolution (FTVD) techniques; the first one needs the actual or an estimate of the noise variance, while the second one is blind. Extensive simulations are done on images for different CS measurements along with a wide range of noise variations. Simulation results show that FTVD with an optimum value for regularization parameter enables the extraction of the watermark image in visually recognizable form, while Wiener deconvolution neither restores the watermarked image nor the watermark when no knowledge of noise is used.

Keywords : Basis pursuit; CS imaging; additive watermarking; Wiener deblurring; FTVD.

I. Introduction

Imaging is the first step in all applications of image processing and analysis and plays an important role in overall performance improvement. Recently, compressive sampling/sensing (CS) based imaging has drawn the attention of the research community because sampling at the *Nyquist* rate may not always be possible (being a large number of samples) due to large storage requirements, slow sampling rate or the measurements may be extremely expensive e.g. MR imaging [IX], [I]. Furthermore, most imaging systems suffer from a common degradation such as blurring due to atmospheric turbulence, object motion and/or camera shake, lens de-focussing, etc. All these effects are also pertinent in CS imaging and become issues in image restoration problems. Sometimes, the security of the captured multimedia content becomes questionable that needs proper protection and authentication mechanism. Nowadays security is also a major issue for the storage and transmission of multimedia data in an open network environment, and sometimes when data owners outsource data storage or processing to a third-party computing service such as the cloud. To address this security issue, an auxiliary piece of data or watermark (text, image, or pattern) may be embedded in the multimedia, called digital watermarking. There are various watermark casting methods reported in the literature and among them, additive watermarking is a popular one. An optimal (adaptive) embedding strength is determined that simultaneously meets imperceptibility and robustness against various attacks. Watermarking being added suffers from the interfering effect of the host image. Faithful recovery of additive watermark information in CS imaging in the presence of degradation such as blurring and noise contamination becomes highly challenging. To the best of our knowledge, watermarking in the CS domain in the presence of blurring and additive noise has not been addressed much.

In CS-based imaging, the technique exploits the fact that images to be captured are assumed to have sparse/compressible representation on some basis. This allows their reconstruction from the small number of non-adaptive measurements taken from some basis incoherent to the sparsity basis [XIII]. The reconstruction of the signal is done, generally, following a convex optimization procedure e.g. l_p norm minimization, basis pursuit (BP), etc., and the method is known as *synthesis*-based CS problem solving [II]. Synthesis-based CS approach can be described as the problem of finding the best (minimum l_1 norm) sparse coefficients for a fixed basis, Ψ, that would synthesize the signal from a given set of measurements [XX]. This set of sparse coefficients is responsible for determining the quality of the reconstructed image, involving restoration through deblurring. Blurring in any imaging system can occur due to defocusing, relative motion between the camera and the object, camera shake, lens defocusing, etc. Image degradations of such types also make proof of content authentication difficult. Conventionally, image deblurring/deconvolution techniques are employed to restore the (estimated) original image under the assumption that noise variance is either known beforehand or is estimated [XI],[XIII],[XIX],[XIV].

The primary objective of this work is to ensure copyright protection through the faithful recovery of the watermark from the blurred watermarked image. To this aim, a grayscale watermark image on the transform domain is additively embedded on sparse or compressible coefficients of the host image in the presence of blurry and noise corruption. *Symlet*-8 filter kernel for the transformation is used. This essentially needs restoration of the watermarked image through appropriate deblurring. To achieve this a residual error minimization algorithm i.e. *Wiener* deblurring [III] and a more recent approach total variation (TV) based deconvolution technique [XXIII] with optimum regularization parameter, are studied to restore the (estimated) watermarked image. The first one needs the actual or an estimate of the noise variance and the second one is blind.

The rest of the paper is organized as follows. BP principle for the synthesis-based CS approach is discussed in Section II. Section III presents the proposed watermarking system model. Simulation results using synthetic images are presented in Section IV. Finally, the paper is concluded in Section V.

II. Basis Pursuit: A Synthesis-Based CS Approach

There are two popular CS reconstruction mechanisms one is an *analysis* based and the other one is a *synthesis*-based approach. In an *analysis*-based approach, the given sparse/compressible signal is reconstructed from low dimensional measurements, which is impractical for real-world imaging applications [XV]. On the other hand, the synthesis-based CS approach can be described as the problem of finding the best (minimum l_1 norm) sparse coefficients for a fixed basis, Ψ, that would synthesize the signal given a set of measurements obtained from the domain in which the image was captured/acquired. Basis Pursuit (BP) is used to represent the signal in over-complete dictionaries by convex optimization. It enables the decomposition that minimizes the l_1 norm of the coefficients that occurred in the representation. BP can also be applied to noisy data by solving an optimization problem, trading off a quadratic error criterion with an l_1 norm of the coefficients [XX]. Moreover, BP employs an optimization principle that can be efficiently applied as a solution to *synthesis-based* CS problem-solving.

Imaging via CS in reality works on the principle of a *synthesis-based* approach. CS works on two attributes, sparsity, i.e. concise representation of the signal in some basis 'Ψ' and the other one is the incoherence that relates to the sensing modality [II]. The working principle of CS may be considered a simple yet efficient acquisition technique. This allows the sampling of a signal in an independent measurement set at a low rate and then exploits computational power for reconstruction. Like other imaging systems, image degradation in the form of blurring is also encountered in CS-based imaging [X].

Mathematically, the synthesis-based CS reconstructs a signal $x \in R^N$ from low dimensional measurement y. It is assumed that x is sparsely represented in a certain basis Ψ, i.e. $x = \Psi\beta$ and $y = \Phi x = \Phi\Psi\beta$. Here Φ is the measurement matrix of dimension $M \times N$ matrix with $M < N$. The reconstruction follows a convex optimization procedure like BP and this can be formulated as [XX]

$$\hat{\beta} = \min_{\beta \in R^N} \|\beta\|_1; \quad subject\ to \quad y = \Phi\Psi\beta \tag{1}$$

The reconstructed sparse coefficients $\hat{\beta}$ are the particular coefficients set capable of generating the same measured data but having minimum l_1 norm.

III. Proposed Watermarking System Model

In CS-based imaging the visual quality of the reconstructed image may also degrade due to blurring [XII]. Security of the multimedia content then becomes challenging under such image degradation. Existing works on compressive image watermarking assume that only the sub-space measurements are affected by an additive noise [IV],[VII],[XVI]. The BP algorithm or its variants look for the sparsest set of coefficients that are capable of nullifying or reducing the effect of the additive noise and producing a good-quality reconstructed image [XXI]. Reconstruction quality from CS space is largely affected due to this blurring effect that degrades watermark recovery. However, it is difficult to remove the effect of blurring operation on CS measurement space [VIII]. Data embedding on blurred and noisy CS measurements need appropriate deblurring for watermark recovery.

A. Watermarking Technique

In the present work, watermarking is done in the Ψ domain in presence of blurring. The test image $I(N \times N)$ is assumed to be sparse/compressible in a known basis and is represented as following vector-matrix equation,

$$I = \Psi\beta$$

where $\beta \in R^{N2}$ is the coefficients in the sparse/compressible basis and Ψ is the basis (inverse wavelet), in which the image is assumed to be sparse/compressible. In the present work, the *Symlet-8* filtering kernel is used as the sparse/compressible basis. *Symlet* is used due to its orthogonality and its ability to represent signals (images) in sparse/compressible form [VIII]. However, any other orthogonal basis having sparsifying ability can also be used. Subspace measurements are taken using a matrix (Φ) whose columns are formed by random sampling of the independent and identically distributed (i.i.d) Gaussian distribution. Now incorporating the degradation, the low dimensional measurement vector 'y' is written as,

$$y = \Phi I = \Phi H \Psi \beta$$

Here H is the *Fourier* representation of spatially invariant defocus blur. To find out the sparsest set of coefficients, Eq. 1 can be reformulated as

$$\hat{\beta} = \min_{\beta \in R^{N^2}} \|\beta\|_1; \quad subject\ to\ \ y = \Phi H \Psi \beta \qquad (2)$$

The solution to the Eq. 2 gives the desired set of coefficients.

Fig. 1 shows the block diagram representation of the proposed watermarking technique. The grayscale watermark image, W ($p \times p$), of dimension smaller than I i.e. $p < N$, is embedded in the coefficients obtained applying Eq. 2. The transform domain coefficients of the grayscale watermark image is first obtained i.e. $\Psi^T W$. Then these coefficients are embedded in the selected sparse coefficients (basis pursuit solution) of the host image. Generally, the sparsity basis of the host image and the transform domain decomposition of the watermark is made the same (wavelet). This makes the ease of embedding process perceptually adaptive. The set of high-valued coefficients is selected for embedding, i.e.

$$\gamma_i = \max|\hat{\beta}_j|; \quad i = 1,2,...,p^2, j = 1,2,...,N^2, i \subset N j$$

The watermark strength (α) is chosen properly to meet the robustness and imperceptibility criterion of the watermark image. The watermarked coefficients are obtained in the following manner,

$$\hat{\gamma}_i = \gamma_i + \alpha\Psi^T W$$

The watermarked coefficients ($\hat{\gamma}_i$) are substituted back into their respective

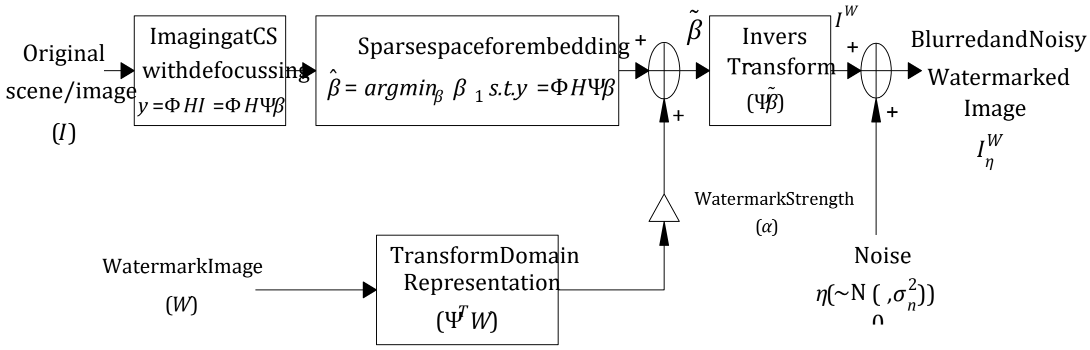


Fig. 1. Watermarking at CS defocuss space using basis pursuit

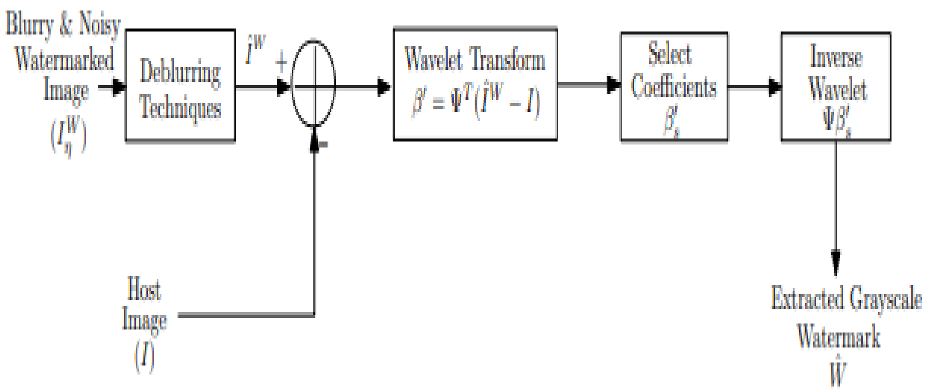


Fig. 2: Watermark extraction model

places and thus we obtain the modified sparse coefficients ($\tilde{\beta}$) containing the watermark. The blurred watermarked image I_B^W is obtained by taking the inverse transform i.e. $\Psi\tilde{\beta}$. The blurred watermarked image $I_B{}^W$, is assumed to be contaminated by an additive white *Gaussian* noise due to an error in the recording medium or transmission through a communication channel. Now watermarked images can be written as

$$I_\eta{}^W = I_B{}^W + \eta = HI^W + \eta$$

where $\eta(\sim N(0, \sigma_n{}^2))$ is the noise, I^W is the watermarked image when there was no blurring and $I_\eta{}^W$ is the blurred and noisy image.

B. Deblurring and Watermark Decoding

Fig. 2 shows the decoding of the watermark which consists of two modules, image deblurring, and watermark extraction. Image restoration or deblurring or deconvolution is well addressed in the literature [XVII]. Estimation or restoration of an image from its blurred and noisy observation is an ill-posed problem [XVII]. Deblurring techniques e.g.*Wiener* and *fast total variation deconvolution* [XXIII] are used to estimate the restored watermarked image $\hat{I}^W$. Watermark recovery is done from $\hat{I}^W$.

1) *Wiener deblurring technique:* The Wiener filter used in this technique is a linear space-invariant filter that prevents excessive noise amplification. It incorporates both the degradation function and statistical characteristics of noise into the restoration process. It can be approximated by the expression

$$\hat{I}^W = (\frac{1}{H}.\frac{|H|^2}{|H|^2 + P_\eta/P_{I^W}})I_\eta^W \tag{3}$$

where P_η and P_IW are the power spectral densities of noise and the watermarked image, respectively. However, in the majority of the cases, the power spectral densities of the image ($P_I\,W$) or the noise (P_η), or both may not be known *a priori*.

2) *Fast total variation deconvolution (FTVD):* Among the various image deconvolution/deblurring methods, the total variation (TV) regularization is used mostly for its edge preservation ability. The objective function of the TV restoration problem is given by

$$\min_{\hat{I}^W}\{\lambda \sum_{i=1}^{N^2} \| D_i \hat{I}^W \|_q + \frac{1}{2} \| H\hat{I}^W - I_\eta^W \|_2^2\} \tag{4}$$

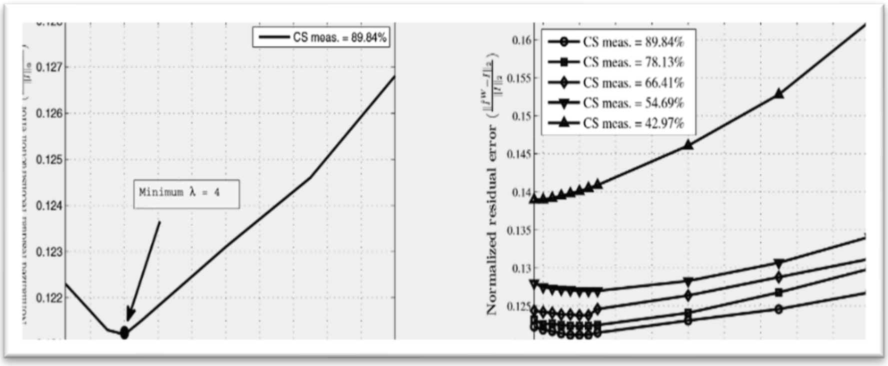
where $D_i\hat{I}^W$ denotes the discrete gradient of $\hat{I}^W$ at pixel i, and the $\sum \| D_i\hat{I}^W \|$ is the discrete TV of $\hat{I}^W$. The second term i.e. $\| H\hat{I}^W - I_\eta^W \|_2^2$ represents the data fidelity constraint. The value of q determines whether the TV is *isotropic* (q = 2) or *anisotropic* (q = 1). In this work the *anisotropic* TV deconvolution/deblurring method is utilized to obtain the estimated watermarked image.

Another critical issue in TV regularization is the selection of the regularization parameter λ, since it plays a very important role. If λ is too large, the regularized solution will be under-smoothed. On the other hand, if λ is too small, the regularized solution will not fit the observations properly. Most works [6],[18],[5] in the literature consider a fixed λ but in the present work optimal

IV. Simulation Results

The performance of the proposed method is presented in this section in terms of visual perception of both the restored watermarked image and decoded watermark. The normalized residual reconstruction error (NRRE) represented by () is calculated for a series of λ and for a different CS measurement space with fixed $\sigma_n{}^2$. The point of minimum NRRE gives the value of λ_{opt}. This is verified in Fig. 3(a) for a particular CS measurement, while

$\hat{I}^W - I \ \|_2 \ / \ \| \ I \ \|_2$



2 4 6 8 10 12 14 1618 20

Regularization parameter (λ)

(a)

24 6 8 10 12 14 1618 20

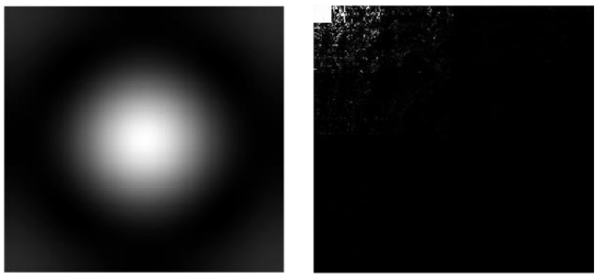
Regularisation parameter (λ)

(b)

Fig. 3. Normalized residual reconstruction error vs. regularization parameter λ (a) at 89.84% CS measurement and (b) various CS measurements at $\sigma n^2 = 10$

(a) (b)

Fig. 4. (a) Test image *Lena*. (b) Watermark image Fig. 6: Blurred and noisy watermarked image from 89.84% CSmeasurements.at $\sigma_n^2 = 10$; PSNR =25.53 dB and SSIM = 0.8173.



(a) (b)

Fig. 5. (a) Defocus blurring kernel. (b) Watermarked basis pursuit solution.

Fig. 3(b) shows the same for different CS measurements. To highlight performance results, under the influence of blurring effect and noise, the proposed study is done over a large number of 8 bits/pixel grayscale images. One such host image *Lena* with size 256×256 is shown in Fig. 4(a). The performance of the proposed algorithm for a few other images is shown at the end of this section.

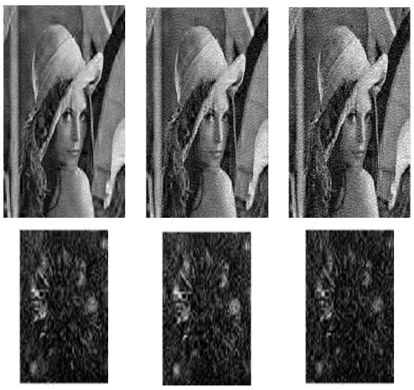
The grayscale watermark image $W\,(32 \times 32)$ shown in Fig. 4(b), is used for simulation. The blurring (Fig. 5(a)) is

incorporated in the CS measurement space and due to that the visual quality of the watermarked image degrades during capturing/acquisition phase. Then the BP algorithm finds the coefficients in the Ψ domain by solving the minimization problem (Eq. 2). Watermarking is done on the selected highest valued coefficients. Fig. 5(b), shows the watermarked coefficients. This is contrast-enhanced for better visibility of the marked coefficients. The blurred watermarked image is assumed to be contaminated by additive white *Gaussian* noise $\eta(\sim \mathrm{N}(0,10)$.

Fig. 6 shows the blurred and noisy image I_η^W with a *peak signal-to-noise ratio* (PSNR) and *structural similarity index metric* (SSIM) values of 25.53 dB, 0.8173, respectively. Now the blurred and noisy image (I_η^W) is deblurred first using *Wiener* filtering and secondly through FTVD.

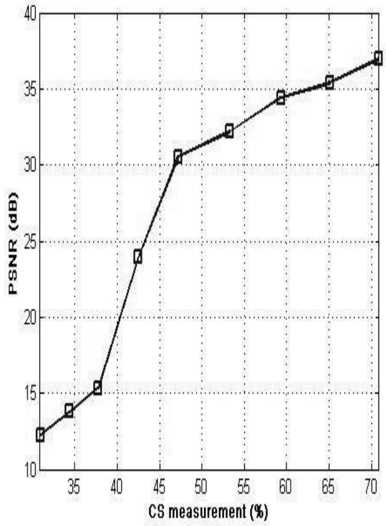
(a) (b)

Fig. 7. First row shows the restored images and the second row shows the extracted watermark images for (a) *Wiener* (b) FTVD (with $\lambda_{opt} = 4$), respectively at 89.84% CS measurements at $\sigma_n^2 = 10$.



(a) (b) (c)

Fig. 8. Top row restored image and bottom row extracted watermark by ise variance (a) 50 (b) 100 (c) 150.

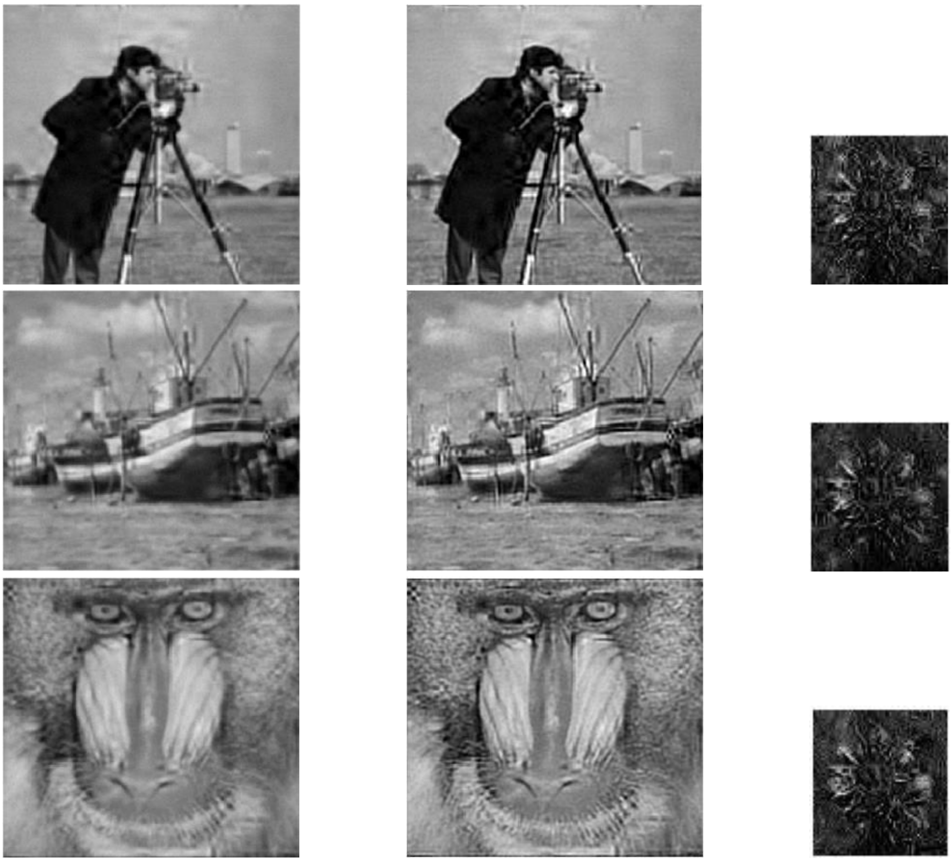


The restoration of blurred watermark images by *Wiener* filtering depends on the ratio of the spectral densities of the watermark image and noise (Eq. 3). This ratio, in most practical cases, is determined heuristically or from a large set of images. In either situation, the process of obtaining the ratio is very times consuming and purely based on guesswork [XVII]. Since the noise variance (σ_n^2) is assumed to be unknown *Wiener* filtering fails to produce the desired restored image and consequently, the extracted watermark is of very poor quality. On the other hand, FTVD (Eq. 4) with the optimum λ produces a much better-restored image and thereby the visual quality of the extracted watermark is quite recognizable, unlike the case of *Wiener* deblurring. The result of *Wiener* deblurring and the extracted watermark images from the restored image are shown in Fig. 7(a) and those for FTVD are shown in Fig. 7(b).

The proposed system is also tested under various degrees of noise contamination and the results are shown in Fig. 8. As expected with the increase in noise variance, the quality of the decoded watermark becomes gradually poor, but all the decoded watermark images are visually recognizable. The effect of the CS measurements on the quality of the reconstructed watermarked image in terms of PSNR is shown Fig. 9 and as expected PSNR values increase with the increase in CS measurements. Fig. 10 shows the importance of appropriate value for the regularization parameter (λ) on the quality of Fig. 9: PSNR values for the reconstructed watermarked image vs increasing CS measurement space at $\sigma_n^2 = 10$ restored images as well as extracted watermark images. It is also seen that the values are different for different images.

V. Conclusion

The present work proposes an additive watermarking method on CS imaging where measurements are assumed to be affected due to blurring and additive noise contamination. Watermarking on sparse or compressible coefficients obtained through BP is found to be robust and extracted watermark is visually recognizable through proper deblurring technique. Performance studies show that *Wiener* filter in absence of the knowledge of the noise variance neither restores the watermarked image nor recovers the watermark. On the other hand, FTVD with regularization parameters restores the watermarked image that leads to the watermark recovery in a visually recognizable form even for the high degree of noise contamination. Simulation results show that both restoration and watermark recovery performance depend on the optimal value of the regularization parameter which is again image dependent. The present work may be extended to find this optimal value analytically. Further, watermarking can be done on highly incomplete measurements (CS) followed by robustness performance against common and deliberate signal processing operations.



(a) (b) (c)

Fig. 10. FTVD performance with λ_{opt} =10,5,12 for cameraman, boat, and mandrill images respectively at 89.84% CS measurements and $\sigma_n{}^2$ = 10. (a) Blurred and noisy watermarked image (PSNR = 27.53, 26.84, and 24.64 dB for cameraman, boat, and mandrill respectively). (b) Deblurred images (c) Extracted watermarks.

Conflict of Interest:

There was no relevant conflict of interest regarding this paper.

References

I. E. Candès, N. Braun, and M. Wakin, "Sparse signal and image recovery from compressive samples," in *4th IEEE International Symposium on Biomedical Imaging: From Nano to Macro, ISBI*, April 2007, pp. 976–979.

II. E. Candès and M. Wakin, "An introduction to compressive sampling," *IEEE Signal Processing Magazine*, vol. 25, no. 2, pp. 21–30, 2008.

III. F. Lin and C. Jin, "An improved wiener deconvolution filter for highresolution electron microscopy images," *Micron*, vol. 50, no. 0, pp. 1 – 6, 2013.

IV. H.-C. Huang, F.-C. Chang, C.-H. Wu, and W.-H. Lai, "Watermarking for compressive sampling applications," in *Eighth International Conference on Intelligent Information Hiding and Multimedia Signal Processing (IIH-MSP),*, July 2012, pp. 223–226.

V. H. Liao, F. Li, and M. K. Ng, "Selection of regularization parameter in total variation image restoration," *Journal of the Optical Society of America A*, vol. 26, no. 11, pp. 2311–2320, Nov 2009.

VI. H. W. Engl and W. Grever, "Using the l-curve for determining optimal regularization parameters," *Numerical Mathematics*, vol. 69, no. 1, pp. 25–31, 1994.

VII. I. Orovic and S. Stankovic, "Combined compressive sampling and image watermarking," in *55th International Symposium ELMAR*, Sept 2013, pp. 41–44.

VIII. J. Ma and F.-X. Le Dimet, "Deblurring from highly incomplete measurements for remote sensing," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 47, no. 3, pp. 792–802, March 2009.

IX. J. Romberg, "Imaging via compressive sampling [introduction to compressive sampling and recovery via convex programming]," *IEEE Signal Processing Magazine*, vol. 25, no. 2, pp. 14–20, 2008.

X. J. Yang, J. Wright, T. S. Huang, and Y. Ma, "Image super-resolution via sparse representation," *IEEE Transactions on Image Processing*, vol. 19, no. 11, pp. 2861–2873, 2010.

XI. L. Ruidong, S. She, Z. Hongtai, T. Xiaomin, and Z. Lanlan, "Analysis on the affection of noise in radar super resolution though deconvolution," in *IET International Radar Conference*, April 2009, pp. 1–4.

XII. L. Spinoulas, B. Amizic, M. Vega, R. Molina, and A. Katsaggelos, "Simultaneous bayesian compressive sensing and blind deconvolution," in *Proceedings of the 20th European Signal Processing Conference (EUSIPCO)*, Aug 2012, pp. 1414–1418.

XIII. M. Duarte, M. Davenport, D. Takhar, J. Laska, T. Sun, K. Kelly, and R. Baraniuk, "Single-pixel imaging via compressive sampling," *IEEE Signal Processing Magazine*, vol. 25, no. 2, pp. 83–91, March 2008.

XIV. P. Samarasinghe, R. Kennedy, and H. Li, "On non-blind image restoration," in *3rd International Conference on Signal Processing and Communication Systems, ICSPCS*, Sept 2009, pp. 1–7.

XV. P. Sen and S. Darabi, "A novel framework for imaging using compressed sensing," in *16th IEEE International Conference on Image Processing (ICIP)*, Nov 2009, pp. 2133–2136.

XVI. Q. Wang, W. Zeng, and J. Tian, "A compressive sensing based secure watermark detection and privacy preserving storage framework," *IEEE Transactions on Image Processing*, vol. 23, no. 3, pp. 1317–1328, March 2014.

XVII. R. C. Gonzalez and R. E. Woods, *Digital Image Processing (3rd Edition)*. Upper Saddle River, NJ, USA: Prentice-Hall, Inc., 2006.

XVIII. S. Babacan, R. Molina, and A. Katsaggelos, "Variational bayesian blind deconvolution using a total variation prior," *IEEE Transactions on Image Processing*, vol. 18, no. 1, pp. 12–26, Jan 2009.

XIX. S. Cho, J. Wang, and S. Lee, "Handling outliers in non-blind image deconvolution," in *IEEE International Conference on Computer Vision (ICCV)*, Nov 2011, pp. 495–502.

XX. S. S. Chen, D. L. Donoho, Michael, and A. Saunders, "Atomic decomposition by basis pursuit," *SIAM Journal on Scientific Computing*, vol. 20, pp. 33–61, 1998.

XXI. W. Lu and N. Vaswani, "Modified basis pursuit denoising(modifiedbpdn) for noisy compressive sensing with partially known support," in *IEEE International Conference on Acoustics Speech and Signal Processing (ICASSP)*, March 2010, pp. 3926–3929.

XXII. Y. Jiang and X. Yu, "On the robustness of image watermarking via compressed sensing," in *International Conference on Information Science, Electronics and Electrical Engineering (ISEEE)*, vol. 2, April 2014, pp. 963–967.

XXIII. Y. Wang, J. Yang, W. Yin, and Y. Zhang, "A new alternating minimization algorithm for total variation image reconstruction," *SIAM Journal of Imging Science*, vol. 1, no. 3, pp. 248–272, 2008.