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BOUNDARY CONDITION ON THE CONVECTION PROCESS INVOLVING NANOFLUIDS

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Abstract

The present numerical investigation deals with the laminar natural convection flow of a nanofluid along an isothermal vertical plate. As indicated by the Boungiorno model [V], nanofluid is considered a two-part combination (base liquid in addition to nanoparticles) where the impacts of Brownian movement and thermophoresis are significant. The boundary condition on the fluid flow is new: the nanoparticle volume fraction at the plate is passively controlled by assuming that its flux there is zero. The outcome of the present study with this new boundary condition is in better agreement with the practical applications of nanofluids.

Keywords: Isothermal Vertical Plate, Natural Convection, NanoFluid, Brownian Motion, Thermophoresis.

I. Introduction

Nanofluid was introduced by Choi in 1995 [IV]. When nano-sized particles (1 – 100 nm) are strategically deployed in the base fluids (water, oil, polymer solution, biofluids, etc.), the ensuing nanofluids have been verified to achieve remarkable enhancement of thermal conductivity. This significant property makes nanofluids potentially useful in many heat transfer applications: electronics cooling, nuclear system cooling, transportation (engine cooling/vehicle thermal management), heat exchanger, fuel cells, solar water heating, biomedicine, etc. A two-phase model was proposed by Boungiorno [V], who analyzed seven mechanisms between nanoparticles and base fluid. Brownian diffusion and thermophoresis prevail over the other five mechanisms, namely, inertia, diffusiophoresis, the Magnus effect, fluid drainage, and gravity. Many researchers use this model. This model introduces a separate equation for nano-particle species diffusion.

II. Literature Review and Objective

The natural convection process plays an important role in heat and mass transfer phenomena in regular as well as nanofluids. Convective boundary-layer flow of a nanofluid along a plate was studied by Kuznetsov and Nield [III], Khan and Aziz [VI], Nield, A.V. Kuznetsov [III], etc. In those and other related papers, a constant volume fraction of nanoparticles at the surface of the plate was assumed by the authors, but no information was given as to how to achieve this. In the present numerical study, the boundary condition on nanoparticle volume fraction at the surface of the plate was reformulated such that its mass flux is zero there. The goal of this research is to build a simple, accurate numerical simulation of laminar free-convection flow and heat transfer over an isothermal vertical plate inundated in a nanofluid. The numerical computations have been carried out for different values of the thermo physical parameters relevant to the present problem, namely Prandtl number (Pr), Lewis number (Le), thermophoresis parameter (Nt), the buoyancy-ratio parameter (Nr), Brownian motion parameter (Nb). The dependence of dimensionless stream function (s), longitudinal velocity (s′), temperature (θ), and nanoparticle volume fraction (f) on these parameters are figured and delineated graphically. The reduced Nusselt number's independence on these parameters is also demonstrated. A linear regression correlation between them is also developed. The results obtained from the present numerical study are then compared with the earlier works available in the literature.

The organization of the present study is as follows: Mathematical model of the problem, its solution procedure and the development of codes in Matlab environment, results, and discussion, conclusion.

III. Materials and Methods

III.i. Mathematical Model

The natural convection flow is steady, laminar, and two-dimensional. The x-axis is aligned with the vertical plate and the direction normal to the surface to be y. Following Oberbeck-Boussinesq approximation and standard boundary layer approximations, the equations for continuity, momentum, thermal energy, and nano-particle species governing the flow [V, II] can be written as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial p}{\partial x}=\mu\frac{\partial^2 u}{\partial y^2}-\rho_f\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)+[(1-\phi_\infty)\rho_{f\infty}\beta g(T-T_\infty)-(\rho_p-\rho_{f\infty})g(\phi-\phi_\infty)] \tag{2}$$

$$\frac{\partial p}{\partial y}=0 \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\nabla^2 T+\frac{(\rho c)_p}{(\rho c)_f}\left[D_B\frac{\partial \phi}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right] \tag{4}$$

$$u\frac{\partial \phi}{\partial x}+v\frac{\partial \phi}{\partial y}=D_B\frac{\partial^2 \phi}{\partial y^2}+\left(\frac{D_T}{T_\infty}\right)\frac{\partial^2 T}{\partial y^2} \tag{5}$$

where, *u* and *v* are velocity components along *x* and *y* respectively, T_∞ and φ_∞ are temperature and nanoparticle volume fraction far away from the plate respectively, ρ_f is the density of the base fluid and ρ_p is the density of the nanoparticles, μ, *k* ,β and α are the viscosity, thermal conductivity, volumetric expansion coefficient and thermal diffusivity of the nanofluid, *g* is the acceleration due to gravity, $(\rho c)_f$ is the heat capacity of the fluid, $(\rho c)_p$ is the effective heat capacity of the nanoparticle material, D_B and D_T are the Brownian diffusion coefficient and thermophoretic diffusion coefficient, respectively.

RHS of Eq. (2) represents the stress component due to viscosity, the convective acceleration, the upward buoyancy term due to the thermal expansion of the base fluid, and the downward buoyancy term due to the variation in densities of the base fluid and the nanoparticles. The terms in the left hand of Eq. (4) are the convection terms due to temperature while the terms in the right-hand side represent the heat enthalpy, diffusion of thermal energy due to Brownian diffusion, and thermophoretic effect. A similar interpretation may be given to the terms in eq. (5).

The boundary conditions for the system of equations are:

At y = 0: u = v = 0, $T = T_w$, $D_B \frac{\partial \phi}{\partial y} + \frac{D_B}{T_\infty}\frac{\partial T}{\partial y} = 0$

For large y: u= v = 0, $T = T_\infty$, $\varphi = \varphi_\infty$ (6)

We introduce the stream function:

$$u \equiv \frac{\partial \psi}{\partial y} \qquad v \equiv -\frac{\partial \psi}{\partial x} \tag{7}$$

The continuity equation (1) is by design satisfied. Furthermore, we can dispense with p from Eqs (2) and (3) by cross-separation and acquire the subsequent equations

$$\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2} - v\frac{\partial^3 \psi}{\partial y^3} = (1-\phi_\infty)\rho f_\infty \beta g(T-T_\infty) - (\rho_p - \rho_{f\infty})g\phi \tag{8}$$

$$\frac{\partial \psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial T}{\partial y} = \alpha \nabla^2 T + \frac{(\rho c)_p}{(\rho c)_f}[D_B \frac{\partial \phi}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}(\frac{\partial T}{\partial y})^2] \tag{9}$$

$$\frac{\partial \psi}{\partial y}\frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \phi}{\partial y} = D_B \frac{\partial^2 \phi}{\partial y^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} \tag{10}$$

Where

$\alpha = \frac{k}{(\rho c)_f}$ is the thermal diffusivity of the fluid.

In order to non-dimensionalize the system of equations (8)-(10), the following similarity variables are introduced:

$$\eta = (\frac{y}{x}) Ra_x^{1/4}$$

$$s(\eta) = \frac{\psi}{\alpha Ra_x^{1/4}}$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}$$

$$f(\eta) = \frac{\varphi - \phi_\infty}{\phi_w - \phi_\infty} \tag{11}$$

Equations (8), (9), and (10) are then transformed to (with a prime denoting differentiation with respect to η)

$$s''' + \frac{1}{4\Pr}(3ss'' - 2s'^2) + \theta - Nrf = 0 \tag{12}$$

$$\theta'' + \frac{3}{4} s\theta' + Nbf'\theta' + Nt\theta'^2 = 0 \tag{13}$$

$$f'' + \frac{3}{4} Lesf' + \frac{Nt}{Nb}\theta'' = 0 \tag{14}$$

The boundary conditions are now:

$$s = 0, \quad s' = 0, \quad \theta = 1, \quad Nbf' + Nt\theta' = 0 \quad at \quad \eta = 0$$
$$s' = 0, \quad \theta = 0, \quad f = 1, \quad at \quad \eta = \infty \tag{15}$$

where Local Rayleigh number Ra_x, the buoyancy-ratio parameter (Nr), Brownian motion parameter (Nb), thermophoresis parameter (Nt), Prandtl number (Pr) and Lewis number (Le) are defined as follows:

$$Ra_x = \frac{(1 - \phi_\infty)\beta g (T_w - T_\infty) x^3}{\nu\alpha}$$

$$Nr = \frac{(\rho_p - \rho_{f\infty})(\phi_w - \phi_\infty)}{\rho_{f\infty}\beta(T_w - T_\infty)(1 - \phi_\infty)}$$

$$Nb = \frac{(\rho c)_p D_B (\phi_w - \phi_\infty)}{(\rho c)_f \alpha}$$

$$Nt = \frac{(\rho c)_p D_T (T_w - T_\infty)}{(\rho c)_f \alpha T_\infty}$$

$$\Pr = \frac{\nu}{\alpha}$$

$$Le = \frac{\alpha}{D_B} \tag{16}$$

The relevant physical quantity characterising the heat transfer rate is the Nusselt number. Local Nusselt number is defined as

$$Nu_x = \frac{q_w x}{k(T_w - T_\infty)} \tag{17}$$

where q_w is the wall heat flux. On substitution of the similarity variables (11) in (17), the reduced local Nusselt number can be written as

$$Nur = \frac{Nu}{Ra_x^{1/4}} = -\theta'(0) \tag{18}$$

III.ii. Solution Procedure

Eqs (12) (13) and (14) are coupled nonlinear higher-order ordinary differential equations. There are three unknown initial values at the wall: $s''(0)$, $\theta'(0)$ and $f(0)$.

III.ii.a. Reduction of Equations to First-order System

We define a set of new variables:

$$\begin{aligned} z_1 &= s \\ z_2 &= z_1' = s' \\ z_3 &= z_2' = z_1'' = s'' \\ z_4 &= \theta \\ z_5 &= z_4' = \theta' \\ z_6 &= f \\ z_7 &= z_6' = f' \end{aligned} \tag{19}$$

Therefore, eqs (12)-(14) can now be written as the following set:

$$\begin{aligned} z_1' &= s' = z_2 \\ z_2' &= s'' = z_1'' = z_3 \\ z_3' &= z_2'' = z_1''' = s''' = -\frac{1}{4\Pr}(3z_1z_3 - 2z_2^2) - z_4 + Nrz_6 \\ z_4' &= \theta' = z_5 \\ z_5' &= \theta'' = z_4'' = -\frac{3}{4}z_1z_5 - Nbz_5z_7 - Ntz_5^2 \\ z_6' &= f' = z_7 \\ z_7' &= f'' = z_6' = -\frac{3}{4}Lez_1z_7 + \frac{3}{4}\frac{Nt}{Nb}z_1z_5 + Ntz_5z_7 + \frac{Nt^2}{Nb}z_5^2 \end{aligned} \tag{20}$$

The third-order eq (12) is replaced by three first-order equations, whereas equations (13) and (14) are second-order each and is replaced with two first-order equations each. The corresponding transformed boundary conditions (15) are then:

$$
\begin{aligned}
&z_1(0) = s(0) = 0\\
&z_2(0) = s'(0) = 0\\
&z_2(\infty) = s'(\infty) = 0\\
&z_4(0) = \theta(0) = 1\\
&z_4(\infty) = \theta(\infty) = 0\\
&Nbz_7(0) + Ntz_5(0) = 0\\
&z_6(\infty) = f(\infty) = 0
\end{aligned} \tag{21}
$$

III.ii.b. Conversion to Initial Value Problems

Equations (20) denotes a system of first-order ODE of seven variables, so we require seven initial values in the boundary condition to solve it. Out of seven, we see three of them, namely, $s''(0)$, $\theta'(0)$ and $f(0)$ are missing. We assume them as a_1, a_2, and a_3 respectively. The arrangement of initial conditions is then, at that point:

$$
\begin{aligned}
&z_1(0) = s(0) = 0\\
&z_2(0) = s'(0) = 0\\
&z_3(0) = s''(0) = a_1\\
&z_4(0) = \theta(0) = 1\\
&z_5(0) = \theta'(0) = a_2\\
&z_6(0) = f(0) = a_3\\
&z_7(0) = f'(0) = -\frac{Nt}{Nb}\theta'(0) = -\frac{Nt}{Nb}a_2
\end{aligned} \tag{22}
$$

We will solve equations (20) with the adaptive Runge-Kutta method using the initial conditions in eq (22). The computed boundary values at $\eta = \infty$ depend on the choice of a_1, a_2 and a_3 respectively:

$$
\begin{aligned}
&z_2(\infty) = s'(\infty) = f_1(a_1)\\
&z_4(\infty) = \theta(\infty) = f_2(a_2)\\
&z_6(\infty) = f(\infty) = f_3(a_3)
\end{aligned} \tag{23}
$$

The correct choice of a_1, a_2 and a_3 yields the given boundary conditions at $\eta = \infty$; that is, it satisfies the equations

$$
\begin{aligned}
&f_1(a_1) = 0\\
&f_2(a_2) = 0\\
&f_3(a_3) = 0
\end{aligned} \tag{24}
$$

The Newton-Raphson method is issued to solve these nonlinear equations. We take 10 as infinity for integration, even if we integrate further nothing will change.

III.ii.c. Program Details

For the solution of equations (20) along with the boundary conditions (22), we develop a set of Matlab routines using Newton Raphson and adaptive Runge-Kutta methods which are shown in Table 1.

Table 1: A set of Matlab routines used sequentially to solve equations (20)

Matlab code	Brief Description
deqs.m	Describes the differential equations (20)
incond.m	Statements of initial values for integration, a_1, a_2 and a_3 are guessed values, eq (22)
runKut5.m	Integrates the initial value problem using the adaptive Runge-Kutta method
residual.m	Provides boundary residuals and approximate solutions
newtonraphson.m	Gives accurate values a_1, a_2 and a_3 using approximate solutions from residual.m
runKut5.m	Again integrates equations (20) using correct values of a_1, a_2 and a_3.

These codes give the numerical values of s, s', s'', θ, θ', f, f' in tabular format as functions of η with Pr, Le, Nr, Nb, and Nt parameters. In the simulation process, we run the codes for the following set of discrete values of the parameters, Pr, Le, Nr, Nb and Nt:

Table 2: Values of input parameters

Input parameters	values
Pr	1, 10, 100, 1000
Le	5, 10
Nr	10^{-5}, 0.1, 0.2, 0.3, 0.4, 0.5
Nb	10^{-5}, 0.1, 0.2, 0.3, 0.4, 0.5
Nt	10^{-5}, 0.1, 0.2, 0.3, 0.4, 0.5

IV. Results and Discussion

IV.i. Validation of the Numerical Procedure.

For validation of the codes developed for the solution, the numerical results for reduced Nusselt number values (eq. 18) obtained from them for regular fluid at different values of Pr are compared with the values reported in previous works [I,II] in Table 3. Table 3 shows an excellent agreement between the present computation and the earlier results on the reduced number.

Table 3: Comparison with previous works [6, 3] when Le = 10, Nr = Nb = Nt = 10^{-5}

Pr	1	10	100	1000
Nur [6]	0.401	0.465	0.490	0.499
Nur [3]	0.401	0.463	0.481	0.484
Nur [present]	0.4010	0.4633	0.4811	0.4836

IV.ii. A representative case

As mentioned in Table 2, numerical computations have been carried out for different values of the parameters involved, namely Pr, Le, Nr, Nb, and Nt that describe the flow characteristics, heat, and mass transfer, and the results are presented in graphs and tables.

We run the above-mentioned codes for the case Pr = 10, Le = 10, Nr = Nb = Nt = 0.5, and the obtained profiles of dimensionless stream function (s), longitudinal velocity (s′), temperature (θ), and nanoparticle volume fraction (f) shown in Fig. 1.

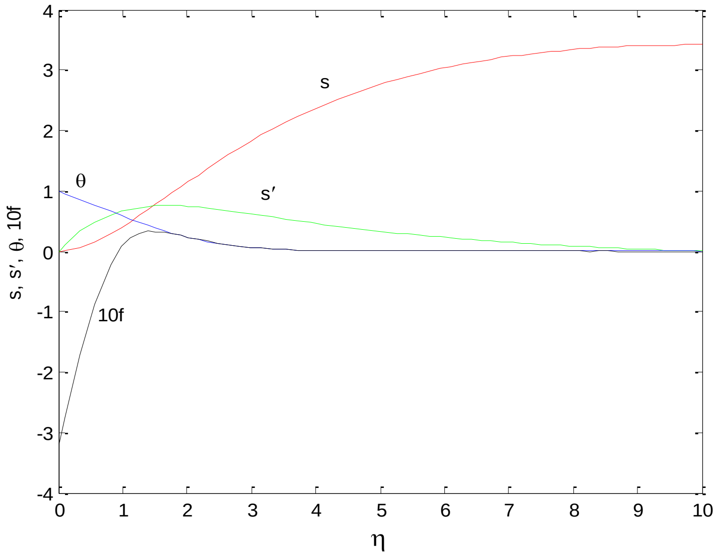

Fig. 1: Plots of dimensionless similarity functions s(η), s′(η), θ (η), 10f(η) for the case Pr = 10, Le = 10, Nr = Nb = Nt = 0.5. (The function f(η) was multiplied by a factor of 10 for better visualization).

It is clear that the profiles for the temperature function θ (η) and the stream function s(η) possess similar forms to the case of a regular fluid. It is noteworthy that the value of f, nanoparticle volume fraction at the wall is negative. This indicates that the effect of thermophoresis is such that an elevation above the ambient surface temperature leads to a reduction in the relative value of the nanoparticle fraction at the surface.

IV.iii. Correlation

We run the codes developed in the present study for 125 sets of values of Nr, Nb, and Nt in the range [0.1, 0.2, 0.3, 0.4, 0.5] with Pr = 10 and Le = 10, and the reduced Nusselt number (eq. 18) from the solutions are shown in Table 4. The linear regression, performed on the results, yielded the correlation

$$Nur_{est} = 0.465 - 0.0009Nr - 0.0029Nb - 0.0748Nt \quad (25)$$

with a maximum error of less than 0.94%. This may be compared with the correlation where the nanoparticle volume fraction at the plate is actively controlled [II], which was

$$Nur_{est} = 0.465 - 0.0055Nr - 0.256Nb - 0.160Nt \quad (26)$$

Table 4 - Values of $-\theta'(0)$ with Pr = 10 and Le = 10

$-\theta'(0)$						
Nb	Nt	Nr = 0.1	Nr = 0.2	Nr = 0.3	Nr = 0.4	Nr = 0.5
0.1	0.1	0.4560	0.4562	0.4563	0.4564	0.4565
	0.2	0.4353	0.4344	0.4336	0.4327	0.4318
	0.3	0.4164	0.4148	0.4131	0.4114	0.4097
	0.4	0.3991	0.3969	0.3946	0.3923	0.899
	0.5	0.3832	0.3805	0.3777	0.3749	0.3719
0.2	0.1	0.4705	0.4712	0.4719	0.4726	0.4733
	0.2	0.4486	0.4488	0.4489	0.4490	0.4491
	0.3	0.4288	0.4284	0.4281	0.4277	0.4274
	0.4	0.4106	0.4099	0.4092	0.4085	0.4077
	0.5	0.3939	0.3930	0.3920	0.3909	0.3899
0.3	0.1	0.4859	0.4869	0.4878	0.4888	0.4897
	0.2	0.4625	0.4630	0.4634	0.4639	0.4644
	0.3	0.4413	0.4414	0.4415	0.4416	0.4417
	0.4	0.4220	0.4218	0.4216	0.4214	0.4212
	0.5	0.4043	0.4039	0.4035	0.4030	0.4026
0.4	0.1	0.5029	0.5040	0.5051	0.5062	0.5073
	0.2	0.4775	0.4782	0.4788	0.4795	0.4802
	0.3	0.4546	0.4550	0.4553	0.4557	0.4560
	0.4	0.4340	0.4341	0.4341	0.4342	0.4343
	0.5	0.4151	0.4150	0.4149	0.4148	0.4146
0.5	0.1	0.5217	0.5230	0.5242	0.5255	0.5267
	0.2	0.4939	0.4947	0.4956	0.4964	0.4973
	0.3	0.4691	0.4697	0.4702	0.4707	0.4712
	0.4	0.4469	0.4471	0.4474	0.4477	0.4480
	0.5	0.4267	0.4268	0.4268	0.4269	0.4270

and had a maximum error of about 8%. Nur_{est} seems to be almost independent of the Brownian motion parameter Nb [eq. (25)] with the modified boundary condition on nanoparticle volume fraction at the plate, whereas this parameter has significant effects in the case of actively controlled nanoparticle volume fraction at the plate [eq (26)]. Eq (25) demonstrates the reduced Nusselt number decreases as the parameters Nr and Nt each increase, which increases the thermal boundary layer.

V. Conclusions

In the present numerical simulation, the steady, laminar, two-dimensional flow of a nanofluid along an isothermal vertical plate with realistic boundary conditions on nanoparticle volume fraction at the plate is presented. Boungiorno model of nanofluid is employed. The prevailing nonlinear partial differential equations of flow are converted into a set of nonlinear ordinary differential equations by similarity transformations. Afterward, they are abridged to a first-order system and integrated using Newton Raphson and adaptive Runge-Kutta methods. The whole numerical procedure is coded in a Matlab environment. Relevant dimensionless stream function (s), longitudinal velocity (s′), temperature (θ), and nanoparticle volume fraction (f) are registered and represented graphically for different values of five dimensionless parameters, namely, Lewis number (Le), Prandtl number (Pr), buoyancy-ratio parameter (Nr), Brownian motion Parameter (Nb), and thermophoresis parameter (Nt). The dependence of the reduced Nusselt number on these five parameters is illustrated. A linear regression correlation between them is also developed. The current review uncovers that the reduced Nusselt number (Nur) is a declining function of each of the parameters Nr and Nt (buoyancy-ratio parameter and thermophoresis parameter respectively), and nearly independent of Nb (Brownian motion parameter), whereas, in the case of actively controlled nanoparticle volume fraction at the plate, Nb has significant effects. The conclusion of the present simulation is in good concurrence with the previous reports available in the literature.

Conflicts of Interest:

The authors declared that there is no conflict of interest regarding the paper.

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